# Parafermion bound states and the fractional Josephson effect in Rashba spin-orbit coupled nanowires

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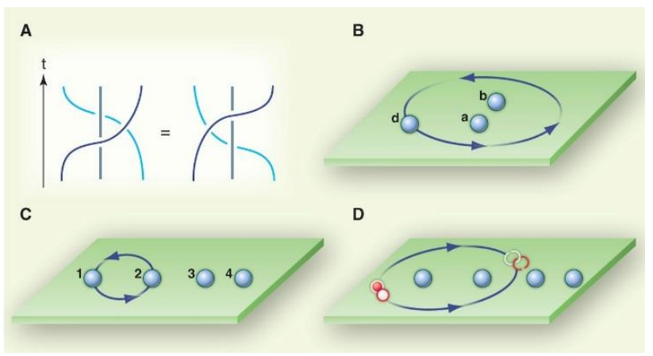
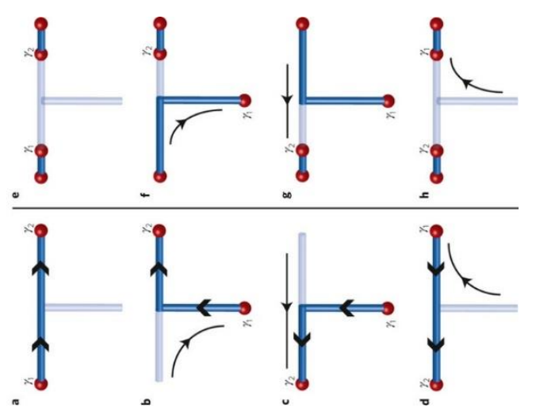
## Motivation - Theory

Majorana bound states

- Way to realise topological quantum computation $\{\gamma_i, \gamma_j\} = \delta_{ij}$
  - Braiding exploits non-Abelian statistics – 90° qubit rotation



A. Stern & N.H. Lindner, Science **339,** 1179-1184 (2013)



J. Alicea *et al*, Nature Physics **7**, 412–417 (2011)

- Majorana particles have simplest non-Abelian statistics $Z_2$ - can we do better (or at least more exotic)? $\psi_i\psi_j + e^{i\delta}\psi_j\psi_i = \delta_{ij}$
  - Yes: Parafermions have $Z_{2n}$ statistics – more qubit rotations.
  - They arise from fractionalized excitations and clock models.

$\frac{1}{2}$  = 

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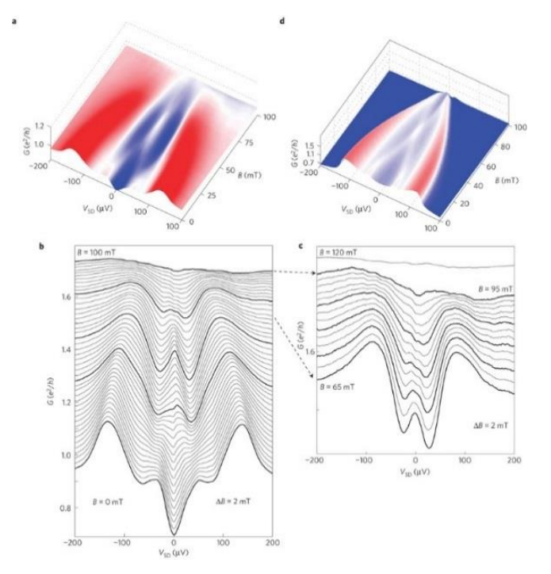
**Fundamental question – can we escape Kitaev & Fidkowski[1]?**
**"The only possible states in 1d with arbitrary interactions and no special symmetry are the trivial phase, and the topological SC phase (Majorana)" [Spoiler: yes, degeneracy not exclusively topological]**

[1] L.Fidkowski & A. Kitaev PRB 83 075103 (2011)

## Motivation - Experiment

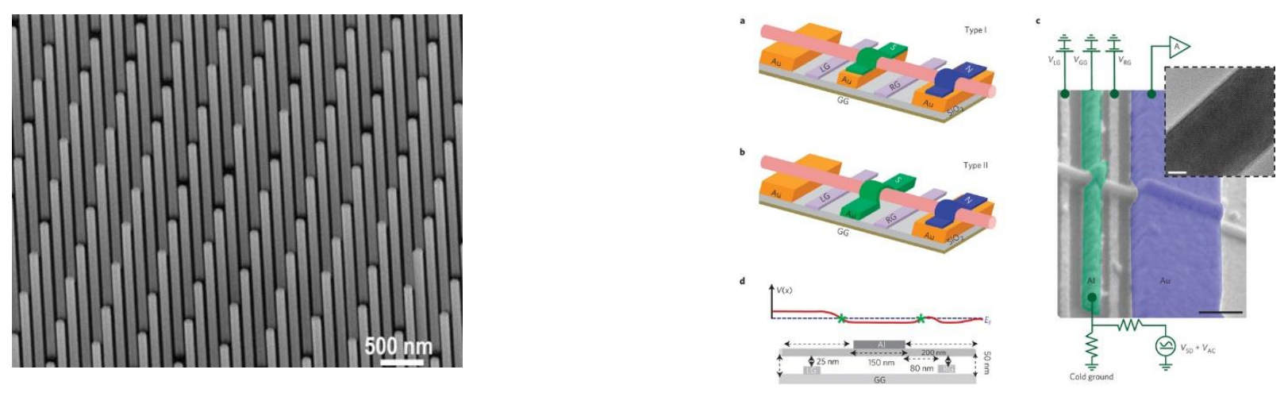
- Majorana hints in experimental systems (but no smoking gun, yet!)
  - Zero bias peaks in 1D wires with strong SOC
    [Mourik et al, Science **336**, 1003 (2012)]
  - Zero bias peaks in shiba chains
    [Najd-Perge et al Science **346**, 602 (2014)]



A. Das et al Nature Physics **8**, 887–895 (2012)

- Parafermionic excitations much harder – early proposals required fractionalized states, either:
  - a fractional QH state coupled to a SC or
  - A fractional TI coupled to a SC

- Newer theoretical proposals use electron-electron interactions to spontaneously break TRS [Zhang *et al* PRL **113** 036401, Orth *et al* PRB **91** 081406] . Quantum wires with strong interactions & strong SOC already exist (e.g. GaAs Das *et al* **Nature Phys** 2479/InSb)

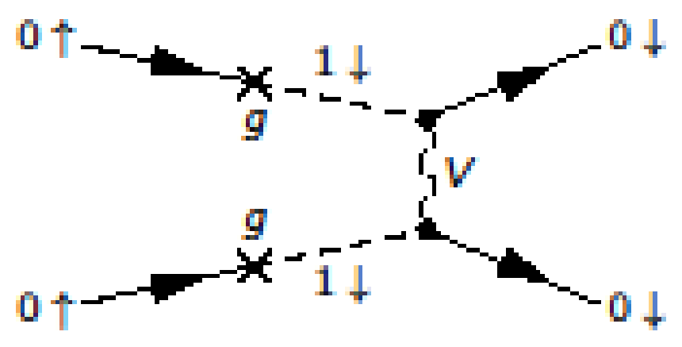


A. Das et al Nature Physics **8**, 887–895 (2012)

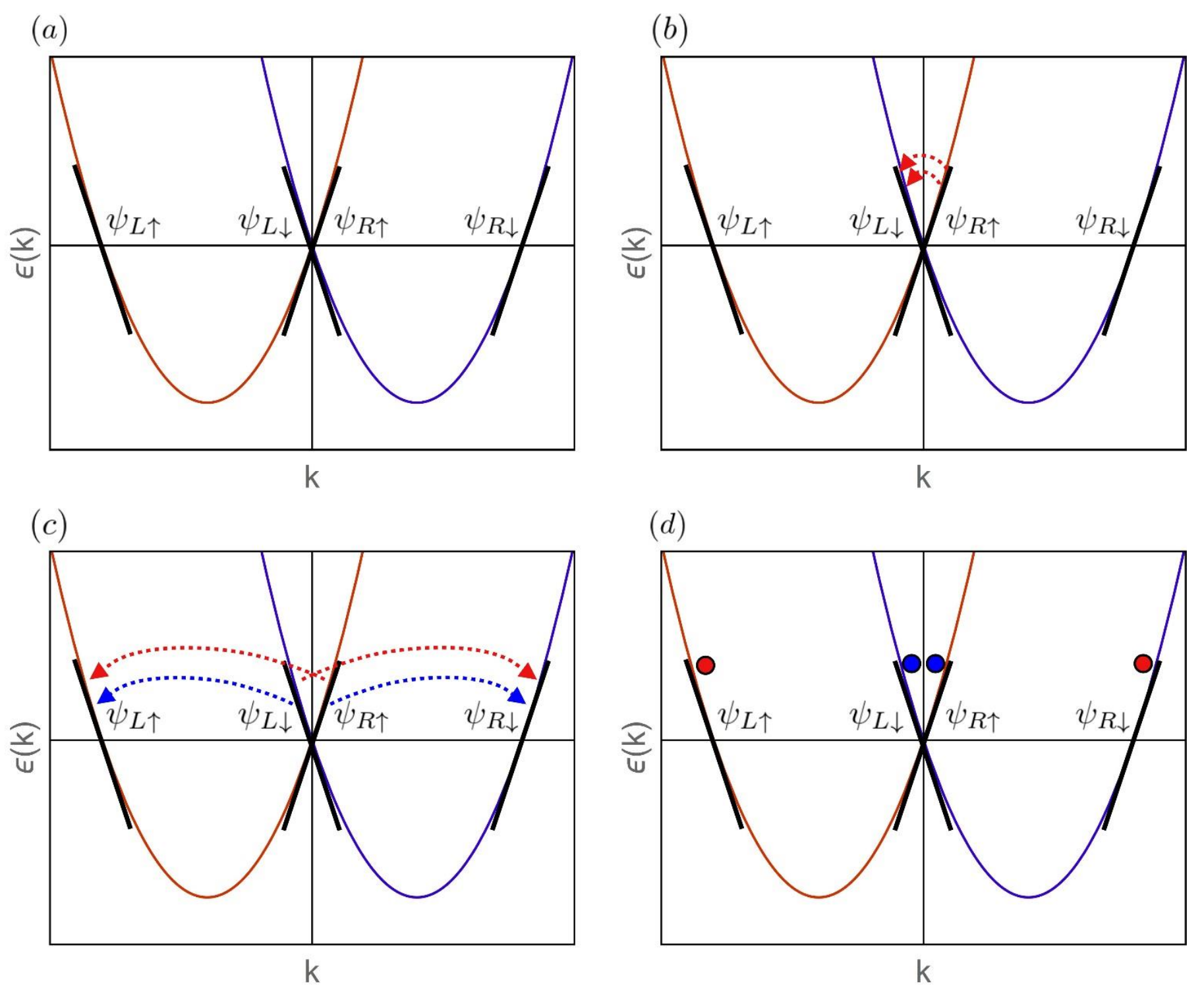
**By including physical/experimental constraints, can we provide a simpler route to non-Abelian symmetry-protected parafermions?**

## Model

- Model a finite-width wire in 2d including Rashba SOC and transversal confinement by a harmonic potential.
- Include a density-density Hubbard interaction from the screened Coulomb repulsion -> strong electron-electron interactions
- Virtual transitions to higher sub-bands of the confinement potential allow spin non-conserving *umklapp* processes.



- Integrate out transitions to give effective model for lowest sub-band



- Four low energy modes with chemical potential at Dirac point (a)
- Allowed interactions
  - (b) spin umklapp scattering
  - (c) "spin density wave" inter-band int.
  - (d) proximity-induced superconductivity
- No B-field; no external TRS breaking

C.J. Pedder, T.L. Schmidt *et al.*, *in press* (2015), S. Gangadharaiah *et al.*, PRB **78** (5), 054436 (2008)

## Renormalization Group

Bosonized model (Abelian).

Kinetic Hamiltonian
$$H_k = \sum_{a=\sigma,\rho} \frac{v_a}{2\pi} \int dx \left[ \frac{(\partial_x \phi_a)^2}{K_a} + K_a (\partial_x \theta_a)^2 \right]$$

Spin density wave
$$H_{SDW} = \frac{g_s}{(2\pi a)^2} \int dx \cos[2\sqrt{2}\theta_\sigma]$$

Umklapp
$$H_U = \frac{g_u}{(2\pi a)^2} \int dx \cos[2\sqrt{2}(\phi_\rho - \phi_\sigma)]$$

Superconductivity
$$H_{SC} = \frac{g_{sc}}{(2\pi a)^2} \int dx \cos[\sqrt{2}(\theta_\rho + \theta_\sigma)] + \{\theta_\sigma \to -\theta_\sigma\}$$

Coupled first-order RG equations

- Umklapp relevant if $K_\sigma + K_\rho < 1$ (i.e. strong interactions)
- SDW term irrelevant for $K_\sigma < 1$
- Proximity-induced superconductivity weakly irrelevant $K_\rho \sim K_\sigma \sim 1/2$

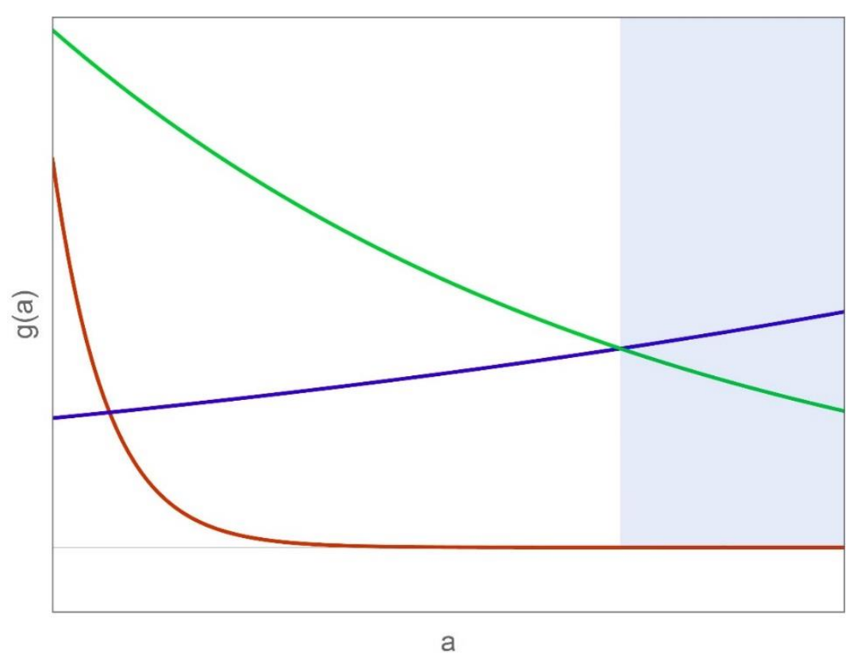
$$\frac{dg_s}{d\ell} = \left(2 - \frac{2}{K_\sigma}\right) g_s$$
$$\frac{dg_u}{d\ell} = 2\left(1 - K_\sigma - K_\rho\right) g_u$$
$$\frac{dg_{sc}}{d\ell} = \left(2 - \frac{1}{2K_\sigma} - \frac{1}{2K_\rho}\right) g_{sc}$$

Umklapp opens gap at k=0, superconductivity opens gaps at $k_F$ – localized edge states.

## Proposed Realization & Detection

Numerical RG Flow

- For weak umklapp, stronger superconductivity, and strong e-e interactions, flow to regime where we find Z4 parafermions



Typical flow: for umklapp (blue), sc (green) and sdw (red) couplings. Ks=0.551, Kr=0.45. Blue shaded region is where Z4 parafermions exist.

Unfolding transformation

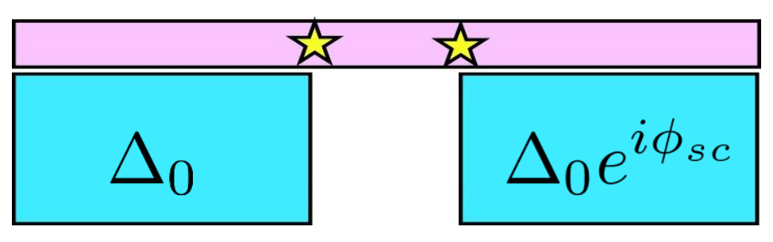
- To see localization of Majorana modes, map complete system with open bcs on line of length L onto a circle with periodic bcs of circumference 2L.
  - Modes gapped by the spin-umklapp live on [0,L]
  - Modes gapped by superconductivity live on [L,2L]
  - Majoranas live on boundaries at 0,L.

Refermionization

- Cosine term $\cos[4\phi_+]$ can be rewritten as a two-fermion term for new, free quasiparticles $\tilde{\psi}^\dagger_\pm = e^{\pm 2i\phi_+ - i\theta_+/2}$ so that $cos[4\phi_+] \sim \tilde{\psi}^\dagger_+\tilde{\psi}_-$
- Commutator with number operator is $[N, \tilde{\psi}^\dagger_\pm] = \frac{1}{2}\tilde{\psi}^\dagger_\pm$ - charge fractionalization!

Fractional Josephson effect

- Usual Josephson effect hinges on tunneling charge 2e Cooper pairs through insulating link.
- In our arrangement, our quasiparticles (kinks) have charge e/2; need to tunnel FOUR of them satisfy bcs on the link.
- Corresponds to an 8π periodicity of the Josephson current.



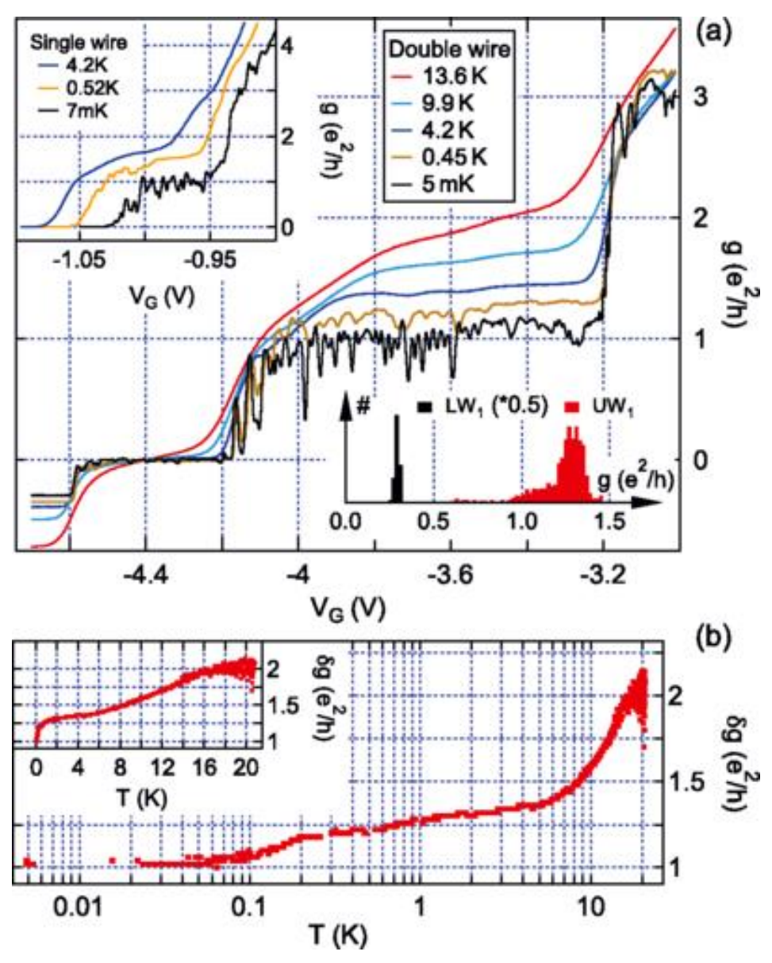
N.B. Breaking of TR symmetry gives us a pair of Majorana modes with a 4π periodic Josephson effect – unbroken TR symmetry essential to fourfold degeneracy.

## Future Directions

Nanowire realization?

- Opening of the k=0 spin-umklapp gap is pretty generic to quasi-1D systems with Rashba SOC
- Nanowires with strong intrinsic SOC already exist. Rashba SOC arises from intrinsic SOC combined with asymmetric environment (wire on substrate, electric field gating etc) – can be experimentally controlled.
- InAs nanowires with strong electron-electron interactions (K~0.4) also already exist [Hevroni *et al* Arxiv:1504.03463].

- A related phenomenon (reduction of conductance from $2e^2/h$ to $e^2/h$) may already have been seen in GaAs nanowires [Scheller *et al* PRL **112** 066801 (2014)] – could be due to gapping of k=0 modes by umklapp scattering.
  - Proximity coupling these wires to an s-wave superconductor could show 8π-periodic Josephson effect.



Scheller *et al* PRL **112** 066801 (2014)

Cold Atomic gasses?

- Can generate Rashba SOC by laser manipulation.
- In 1D, interactions can be controlled by the confinement induced resonance – analog of the Feshbach resonance.
- Require repulsive interactions at k=0, but attractive at $k=k_F$ to get umklapp and pairing
  This is difficult!