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Airport Congestion and Inefficiency in Slot Allocation*

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Abstract

This paper analyzes optimal slot allocation in the presence of airport congestion. We model peak and offpeak slots as vertically differentiated products, and congestion limits the number of peak slots that the airport can allocate. Inefficiency emerges when the airport does not exploit all its slots. We show that for a private airport, inefficiency may arise if the airport is not too congested and the per-passenger fee is small enough, while with a public airport it does not emerge. Furthermore the airport, irrespective of its ownership, tends to give different slots to flights with same destination if the underlying market is a duopoly, and a single slot if the underlying market is served by a monopoly.

JEL classification: R41; H23; H21.

Keywords: Slot allocation; Airport congestion; Vertical differentiation.

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1 Introduction

Over the past decades, airline traffic growth has outpaced capacities at many of the world's major airports.[1] As a result, airport congestion has become a major issue faced by many airports worldwide. Airport congestion is likely to get worse in the coming decades, being generated by an expanding demand due to increase of income, and the growth of some developing countries.[2]

In the analysis of airport congestion, the economic literature focused mainly on "congestion pricing" (for which carriers pay a toll according to their contribution to congestion) as a regulatory tool to deal with congestion.[3] However, despite its theoretical feasibility, congestion pricing has not been practised in the real world. By contrast, slot allocation is the usual approach to management of congestion at airports. According to IATA World Scheduling Guidelines, a slot is "the permission given by a managing body for a planned operation to use airport infrastructure that is necessary to arrive or depart at a airport on a specific date and time".[4] Under a slot system, the airport authority determines the total number of slots to make available, and slots are distributed among the airlines according to some allocation rule.[5] Given the prevalence of slot systems, a theoretical analysis investigating the interaction between slot allocation and congestion seems highly policy relevant.

In this paper we analyse endogenous slot allocation in the presence of airport congestion, and we investigate the conditions under which the allocation choice is inefficient. We refer to a "slot" as the permission granted to a certain airline to use airport infrastructure for a planned operation at a specific time window

[1] For instance, over half of Europe's 50 largest airports have already reached or are close to their saturation points in terms of declared ground capacity (Madas and Zografos, 2008).

[2] The European Commision estimated that half of the world's new traffic will come from Asia Pacific region in the next 20 years. They expect that air traffic in Europe will roughly double by 2030, and that 19 key airports will be at saturation. See MEMO/11/857.

[3] For an early contribution on congestion pricing see Levine (1969). Recent representative studies include Brueckner (2002, 2005). Under congestion pricing, carriers could place as many flights as they wish provided they pay the toll, thus the overall level of congestion is determined by airline decisions.

[4] See Worldwide slot guidelines.

[5] For example, FAA (Federal Aviation Administration) capped peak hour flight movements at New York La Guardia, J.F.Kennedy, and Neward airports. As for Chicago's O'Hare airport, FAA persuaded two major airlines United and American Airlines to reduce peak flight activities while prohibiting smaller airlines from increasing flights to fill the gap.

of the day.[6] We examine a setting where an airport wants to maximize the number of passengers, and sorts slots according to different departure flights, while airlines compete in the flights market. As in Brueckner (2002), we model peak and off-peak slots as products of different qualities in a model of vertical differentiation.[7] Peak slots are congested, mirroring the situation of capacity shortages at peak hours faced by many airports. We analyze both a private and a public airport being restricted to levy a uniform per-passenger fee for flight activities, this being pre-determined by administrative bodies.[8] We consider separately the case where two flights towards a same destination (denoted as "pairwise flights" in the paper) are served by two airlines, and where they are served by a monopoly. In this complete information setting, airlines know the total provision of slots and the fact that each participating airline receives a single slot. Finally, we define as "allocative inefficiency" the situation in which not all the slots available are exploited.

Because peak slots are preferred by passengers to off-peak slots, the airport's slot assignment creates an exogenous quality differential between the carriers when the assigned slots are for different periods. The carriers compete conditional on this quality differential, and the resulting prices and passenger volumes therefore depend on the slot assignments. Depending on parameter values, the total passenger volume (and hence fee revenue for the airport) could be higher when the airport withholds a peak slot that it could allocate to the carries, leaving the slot unused and the airport's peak capacity thus not fully exploited. This outcome is inefficient from society's point, but it is a possible feature of equilibrium in the model.

[6] Though in practice congestion is not exclusively confined to runway congestion, and might embody other capacity dimensions such as environmental concerns, we nevertheless focus on runway congestion.

[7] Our approach differs from Brueckner (2002) as follows. In Brueckner (2002)'s framework, a monopoly airport chooses the critical points on the continuum that respectively define whether to fly or not and whether to fly in peak slots or off-peak slots. Focusing on finding the optimal congestion pricing, he implicity assumes that airport capacity is sufficient to meet peak hour demands. Unlike Brueckner (2002), our interest stems from the scarcity of peak hour slots. Thus we focus on the allocation instead of using the pricing tool.

[8] The analysis of a private airport also seems relevant. Although airports have long been owned by governments, there has been a significant worldwide trend towards government facilities privatization beginning from the middle of the 1980s. Following the United Kingdom, many major airports in Europe, Australia and Asia have followed suit and have undergone privatization or are in the process of being privatized. In principle, privatization is characterized by the transfer of ownership structure from state-owned to private enterprises.

When the airport is private and each destination is served by a duopoly airline market, the results depend on whether the number of slots is lower than the number of destinations (the airport is "busy") or not (the airport is "not too busy"). A busy airport uses all the available peak slots to implement "peak/off-peak" configuration. The results are driven by differentiation which, on the one hand, increases the number of passengers, but on the other hand softens competition. The first effect more than outweighs the second effect, thus the airport prefers to adopt differentiation rather than to allocate both peak slots for the same destination.

A not-too-busy airport chooses its allocative strategy according to the amount of per-passenger fee. In particular, it implements a mix of "peak/off-peak" and "peak/peak" market configuration if the per-passenger fee is high, and "peak/off-peak" configuration in each market if the per-passenger fee is low. In the latter case, given that the number of destinations is lower than the number of slots, allocative inefficiency emerges. These results can be explained as follows. Low per-passenger fees imply cheap flight tickets, thus the airport can allocate slots in the peak/off-peak configuration without losing passengers, even with less competition. Allocative inefficiency is due to the fact that a not-too-busy airport does not need the extra slots to reach the optimal slot allocation. With high per-passenger fees, the airport allocates some peak/peak market configuration (undifferentiated slots) in order to induce more competition and to keep the price of flight tickets sufficiently low.

The emergence of allocative inefficiency in our results corresponds to the common practice in airport management of declaring a number of slots being lower than an airport's full capacity (Mac Donald, 2007, and De Wit and Burghouwt, 2008). Indeed, as De Wit and Burghouwt (2008) point out, "an efficient use of the slots at least requires a neutral and transparent determination of the declared capacity".

If each destination is served by a monopolist and one peak slot is assigned to them, the monopoly airline would choose to operate in the peak slot only, thus leaving unused the off-peak slot. Indeed, given the same (marginal) per-passenger fee for operating at a peak or off-peak hours, the airline prefers to put all seats in the peak flight. Moreover, if two peak slots are assigned to the monopoly airline, and assuming a preference for operating one flight at peak slot rather than two (for operating costs not modelled here), then the monopoly airline would also leave unused the extra peak slot. In turn, the airport would assign a single peak slot to each destination market, as long as peak slots are available. Naturally, in this case allocative inefficiency does not occur. Finally, the results are similar when the airport is public, with the exception than inefficiency does not emerge.

To illustrate the empirical relevance of this mechanism, we have briefly investigated the slot allocation in the city-pair markets of the 5 most busiest US airports and of a random sample of 10 mid-range US airports (see Appendix A). Table 1 shows the numbers of origin-destination routes operated by monopoly, duopoly and oligopoly, and the pattern of slot occupancy. While a large bunch of city-pair service is supplied by monopoly airlines, there exists a significant portion of flight activity served by competing airlines.

	# Origin-destinations served by			# Slots occupation(%)	
	Mon. (%)	Duo.	Olig.	Peak	offpeak
Top 5	525 (62%)	161(19%)	157(19%)	29%	71%
Mid sized 10	200 (73%)	53 (20%)	19 (7%)	28%	72%

Table 1. Pattern of market structure and slot occupation.

Our theoretical analysis claims that slot allocation can be used by congested airports as a discrimination tool in markets where several firms compete. To highlight this, we focus on duopoly city-pair markets and construct an index I that measures whether the competing airlines are put in the same slot. Typically, along the discussion in Section 3, two competing airlines, each operating one flight to the same destination, are allocated in the same time slot if $I = 0$ and are separated in a peak and an off-peak slot if $I = 1$. A higher aggregated index I measures stronger use of a slot discrimination. Excluding inter-hub traffic and high frequency destinations, we then find evidence consistent with our model prediction that slot discrimination is stronger in the largest airports, see Table 2

	Average Index I	
	Largest 5 airports	Mid 10 airports
5-11pm	0.38	0.23
11-16pm	0.32	0.2
16-23pm	0.44	0.44

Table 2. Average index I.

So far, slot allocation has drawn relatively little interest in the economic literature, with few but noteworthy contributions. Barbot (2004) models slots for airline activities as products of either high or low quality, and carriers choose the number of flights they operate. She shows that slot allocation improves efficiency according to the criteria for assessment, and welfare in fact decreases after re-allocation. Unlike the present paper, in Barbot (2004)'s model carriers could operate as many flights as they want. Our paper limits the number of peak slots, in order to address congestion. Verhoef (2008) and Brueckner (2009) compare the pricing and slot policy regimes. They show that the first best congestion pricing and slot trading/auctioning generate the same amount of passenger volume and total surplus. They investigate a single congested period. Their contributions do not distinguish between peak and off-peak hours, and allow the airport to allocate slots without charges. Although this seems a plausible description of some public airports, non-profit behavior does not seem likely for a private airport. Departing from Verhoef (2008) and Brueckner (2009), we assume that each airline operates a single flight. Our approach models certain time intervals that are most desired by all passengers as the peak period. In particular, the total number of slots that an airport could grant in the peak period does not meet the demand of passengers.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the baseline results, in which the airport is assumed to be private and an airline duopoly serves each destination. Section 2 and 5 show the changes in the results when either a monopoly airline serves for each destination or the airport is publicly owned, respectively. In section 6, we show the changes in the results when density is heterogenous among different flights. Section 7 concludes.

2 The model

In the baseline model, we consider a private airport that links N destinations $d \in D$.[9] Each destination is served by two flights f and $f' \in \mathcal{F}$ operated by independent airlines.[10] There are therefore $2N$ flights and $2N$ airlines ($\#(D) = N$ and $\#(\mathcal{F})$ =$2N$). Formally, we define the mapping I from flights f to destinations d such that $I(f) = d$. The inverse mapping from flights to destinations is defined as $A(d) = \{f : I(f) = d\}$. We assume that airports at destinations are uncongested, so that the allocation decisions do not affect the flight scheduling of destination airports. Furthermore, we focus on the case of single trip departing flights. Return trip flights can be dealt with by either an identical analysis with two runways, or simply by adding a scale factor if there is a single runway. Destinations are independent in the sense that they are neither substitutes nor complements; therefore the demand for one destination is irrelevant to demands for other destinations. We assume that the quality differential is characterized only by the departing time. Although the quality of an airline depends on many factors, this approach allows us to concentrate on the congestion issue. There are two travel periods, denoted as peak and off-peak. A peak period represents the time window that consists of the most desirable travel times in a day, for instance early morning and late afternoon. The peak period may contain a collection of disjoint time intervals like 7:00-9:00 and 17:00-19:00. The off-peak period, by contrast, contains all other time intervals that do not belong to the peak period. In order to address the problem of peak slots congestion, the off-peak period is assumed to be uncongested, i.e., airport capacity can serve all flights in off-peak time intervals. Conversely, the peak period is congested in the sense that airport capacity cannot serve all flights within peak period. This assumption captures traffic patterns at many airports nowadays. All potential passengers agree that peak hours (denoted as subscripts h for higher quality) are more preferable than off-peak hours (denoted as subscripts l for lower quality) at an equal price. At peak hours the demand to use airport runways is much higher than off-peak hours, so that the perceived "qualities" of slots, s_l and s_h, satisfy exogenously $s_h > s_l > 0$. Finally, a slot allocation is defined as

[9] Section 5 analyses the case with a public airport.

[10] Section 4 extends the analysis to the case where a private airport interacts with airline monopolies.

the mapping g from airline f to a slot type i, $g : \mathcal{F} \rightarrow \{l, h\}$, so that $g(f) = i$. For instance, $g(f) = h$ reads as airline f is allocated a peak time slot.

We assume that in each destination market the airlines engage in (seat) quantity competition.[11] We denote $p^f_{ii'}$ as the price charged for flight f flying to destination $d = I(f)$ that takes off at slot i while its competitor on the same destination takes off at slot i', $i, i' = \{l, h\}$. Similarly, $q^f_{ii'}$ denotes the number of passengers served by this flight. Following the general framework of vertical differentiation (Gabszewicz and Thisse, 1979), in each destination market the demand is generated from a unit mass of passengers indexed by a type parameter v. Passengers differ in tastes, the taste parameter is described by $v \in [0,\, 1]$, v being uniformly distributed with unit density. We assume each passenger flies at most once, and, if a passenger refrains from flying, her reservation utility is zero. Formally, a potential passenger in the destination market d with flights f and f', $d = I(f) = I(f')$, has the following preferences:

$$U^d = \begin{cases} vs_i - p^f_{ii'} & \text{if she takes flight } f \text{ in slot } s_i \text{ at price } p^f_{ii'} \\ vs_{i'} - p^{f'}_{i'i} & \text{if she takes flight } f' \text{ in slot } s_{i'} \text{ at price } p^{f'}_{i'i} \\ 0 & \text{if she does not fly.} \end{cases}$$

Without loss of generality, suppose $g(f') = h$ and $g(f) = l$. Pairwise flights obtain slots of different qualities. Under our assumption, flight f' obtains a peak slot, while flight f obtains an off-peak slot, so that $i' = h$ and $i = l$. It follows that the passenger with a taste parameter v flies with f' when $vs_h - p^{f'}_{hl} > vs_l - p^f_{lh} > 0$ and flies on flight f when $vs_l - p^f_{lh} > vs_h - p^{f'}_{hl} > 0$. The passenger indifferent between f' and f has taste

$$v_{hl} = \frac{p^{f'}_{hl} - p^f_{lh}}{s_h - s_l}. \tag{1}$$

Likewise, a passenger is indifferent between not flying and flying with airline f when $vs_l - p^f_{lh} = 0$, so that

$$v_{lh} = \frac{p^f_{lh}}{s_l}. \tag{2}$$

[11] The assumption of quantity competition is common in the airline economics literature. See Brueckner (2002), Pels and Verhoef (2004). Brander and Zhang (1990) find empirical evidence that the rivalry between duopoly airlines is consistent with Cournot behavior.

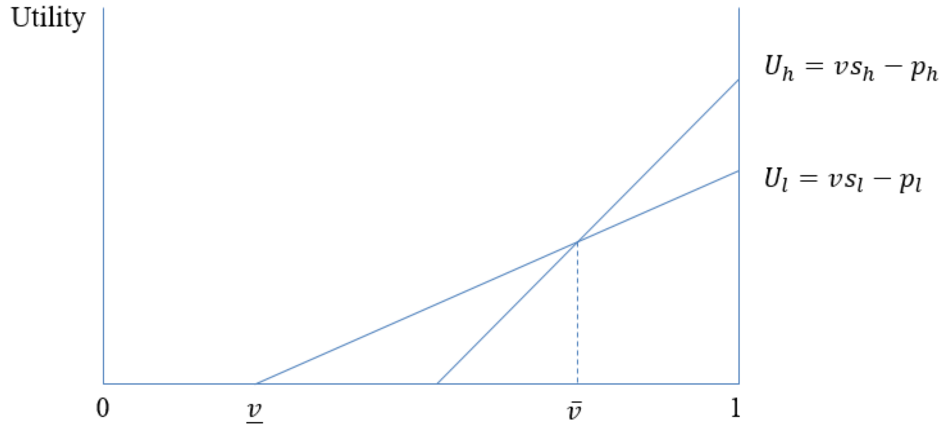

Figure 1: Valuation of slot quality

Hence with differentiated flights, the demand for flight f' is $1 - v_{hl}$, while the demand for flight f is $v_{hl} - v_{lh}$ (see Figure 1). Moreover, there are $\underline{v}$ passengers that do not fly.[12]

For $g(f') = g(f) = i \in \{h, l\}$, i.e., pairwise flights obtain slots of same quality, then a passenger is indifferent between flights. In this case, the pairwise flights are homogeneous and hence evenly share the destination market. There are two possible configurations: both flights obtain either peak or off-peak slots. In the peak/peak configuration, a passenger v is willing to fly when

$$v \geq v_{hh} \equiv \frac{p_{hh}^{f}}{s_h} \left(= \frac{p_{hh}^{f'}}{s_h} \right), \tag{3}$$

while in the off-peak/off-peak configuration, she is willing to fly when

$$v \geq v_{ll} \equiv \frac{p_{ll}^{f}}{s_l} \left(= \frac{p_{ll}^{f'}}{s_l} \right). \tag{4}$$

The demand for each flight is thus $\frac{1}{2}\left(1 - \frac{p_i}{s_i}\right)$. Hence the three configura-

[12]Motta (1993) shows that Cournot competition can be studied only with partial market coverage, since the demand function can not be inverted with full market coverage.

tions are characterized by the following demand functions, respectively:

$$(i) \begin{cases} q_{lh}^{f}(p_{lh}^{f}, p_{hl}^{f'}) & = \dfrac{p_{hl}^{f'} - p_{lh}^{f}}{s_h - s_l} - \dfrac{p_{lh}^{f}}{s_l} \\ q_{hl}^{f'}(p_{lh}^{f}, p_{hl}^{f'}) & = 1 - \dfrac{p_{hl}^{f'} - p_{lh}^{f}}{s_h - s_l}, \end{cases} \tag{5}$$

$$(ii)\ q_{ll}^{f}(p_{ll}) + q_{ll}^{f'}(p_{ll}) = 1 - \frac{p_{ll}}{s_l}, \tag{6}$$

$$(iii)\ q_{hh}^{f}(p_{hh}) + q_{hh}^{f'}(p_{hh}) = 1 - \frac{p_{hh}}{s_h}. \tag{7}$$

Solving (5)-(7) for prices, the inverse demand functions corresponding to the three possible destination market structures are given respectively as follows:

$$(i) \begin{cases} p_{lh}^{f} & = s_l \left(1 - q_{lh}^{f} - q_{hl}^{f'}\right) \\ p_{hl}^{f'} & = s_h \left(1 - \frac{s_l}{s_h} q_{lh}^{f} - q_{hl}^{f'}\right), \end{cases} \tag{8}$$

$$(ii)\ p_{ll} = s_l \left(1 - q_{ll}^{f} - q_{ll}^{f'}\right), \tag{9}$$

$$(iii)\ p_{hh} = s_h \left(1 - q_{hh}^{f} - q_{hh}^{f'}\right). \tag{10}$$

Airlines choose the number of seats in order to maximize profits. Airline costs include airport per-passenger charge ϕ, while marginal operating costs are normalized to zero.[13] We do not consider the entry of airlines in the airport and assume that fixed costs are sunk. Thus the profit of a flight f competing in the destination market d with another flight f', $d = I(f) = I(f')$, is given by:

$$\begin{aligned} \pi_{lh}^{f} &= \left(p_{lh}^{f}(q_{lh}^{f}, q_{hl}^{f'}) - \phi\right) q_{lh}^{f} \text{ if } g(f) = l \text{ and } g(f') = h \\ \pi_{hl}^{f'} &= \left(p_{hl}^{f'}(q_{lh}^{f}, q_{hl}^{f'}) - \phi\right) q_{hl}^{f'} \text{ if } g(f) = h \text{ and } g(f') = l, \end{aligned} \tag{11}$$

[13] In Section 3.2 we also consider positive operating costs. These have been assumed in some contributions, such as Pels and Verhoef (2004), Brueckner and Van Dender (2008), and Basso (2008).

for an peak/off-peak slot configuration and

$$\begin{aligned}\pi_{ii}^{f} &= \left(p_{ii}^{f}(q_{ii}^{f}, q_{ii}^{f'}) - \phi\right) q_{ii}^{f} \text{ if } g(f) = g(f') = i \in \{l, h\} \\ \pi_{ii}^{f'} &= \left(p_{ii}^{f'}(q_{ii}^{f}, q_{ii}^{f'}) - \phi\right) q_{ii}^{f'} \text{ if } g(f) = g(f') = i \in \{l, h\}\end{aligned} \tag{12}$$

for the peak/peak and off-peak/off-peak slot configuration.

The airport earns the charge ϕ for each passenger. It chooses the slot allocation mapping $g(\cdot)$ that maximizes its profits. We get the following program:

$$\max_{g(\cdot)} \Pi = \Sigma_{d=1}^{N} \phi \left(q_{g(f),g(f')}^{f} + q_{g(f'),g(f)}^{f'} \right) \text{ where } I(f) = I(f') = d, \tag{13}$$

subject to the peak slot capacity constraint

$$\#\{f : g(f) = h\} \leq M, \tag{14}$$

where M is the total number of peak slots. To avoid a cumbersome discussion of ties, we assume that M is even. Constraint (14) implies that the overall allocated peak slots cannot exceed the total number of available peak slots. Peak capacity cannot accommodate all flights ($M < 2N$), while off-peak capacity can accommodate all flights (there is no constraint for the off-peak slots).

We then define allocative inefficiency as follows.

Definition 1 *Allocative inefficiency emerges when at least one peak slot is not used.*

This definition seems natural. In the presence of airport congestion, leaving some peak slots unused represents a degree of inefficiency.

Figure 2 shows the timing of the game. In the first stage the airport allocates peak and off-peak slots for a given fee ϕ. In the second stage airlines choose number of seats to supply $q_{ii'}^{f}$ based on the slot allocation. In the third stage passengers in each destination decide whether to fly with a peak period airline, an off-peak period airline, or not to fly at all. The equilibrium concept is the subgame perfect equilibrium.

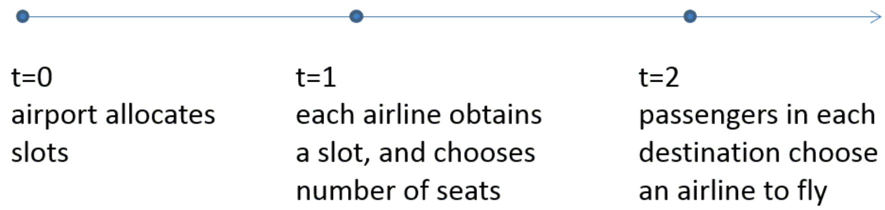

Figure 2: Timing

3 Results

In this section we show the baseline results of the model. As mentioned earlier, we first focus on a private airport that links destinations operated by duopoly airlines with symmetric demand and cost structures. We first discuss the competition between the airlines and then the optimal slot allocation the airport.

3.1 Duopoly airlines

In the second stage, airlines set their optimal supply of seats. We analyze each possible configuration (peak/off-peak; peak/peak; off-peak/off-peak) separately. Consider first a destination market d where pairwise flights $(f, f') = A(d)$ obtain different slots. Then according to (11), airlines' profits are expressed by:

$$\pi_{lh}^{f} = \left[s_l(1 - q_{lh}^{f} - q_{hl}^{f'}) - \phi \right] q_{lh} \tag{15}$$

$$\pi_{hl}^{f'} = \left(s_h - s_l q_{lh}^{f} - s_h q_{hl}^{f'} - \phi \right) q_{hl}. \tag{16}$$

Airlines choose the number of seats to maximise profits, for any given ϕ. The first-order conditions are:

$$\frac{\partial \pi_{lh}^{f}}{\partial q_{lh}^{f}} = -\phi + (1 - q_{lh}^{f} - q_{hl}^{f'})s_l - q_{lh}^{f} s_l = 0, \tag{17}$$

$$\frac{\partial \pi_{hl}^{f'}}{\partial q_{hl}^{f'}} = -\phi + s_h - 2q_{hl}^{f'} s_h - q_{lh}^{f} s_l = 0. \tag{18}$$

Solving (17) and (18) simultaneously with respect to q_{lh}^f and $q_{hl}^{f'}$ yields:

$$q_{lh}^f = q_{lh} \equiv \frac{s_h s_l - \phi\left(2s_h - s_l\right)}{(4s_h - s_l)s_l}, \tag{19}$$

$$q_{hl}^{f'} = q_{hl} \equiv \frac{2s_h - \phi - s_l}{4s_h - s_l}. \tag{20}$$

To ensure interior solutions, we assume the condition $0 < \phi < \phi_1 \equiv \dfrac{s_h s_l}{2s_h - s_l}$. Note that $q_{lh} < q_{hl}$ for all $0 < \phi < \phi_1$. If a destination market obtains different slots, then the airline with the peak slot serve more passengers in equilibrium than its off-peak competitor. Since quantities are symmetric across destinations, prices and profits are also symmetric and we can dispense the variables with the superscripts f and f' in the sequel without loss of clarity. Plugging (19) and (20) into (8) yields

$$p_{lh} = \frac{s_h(2\phi + s_l)}{4s_h - s_l}, \tag{21}$$

$$p_{hl} = \frac{2s_h^2 + \left(3s_h - s_l\right)\phi - s_h s_l}{4s_h - s_l}, \tag{22}$$

both of which are positive, and where

$$p_{hl} - p_{lh} = \frac{\left(s_h - s_l\right)\left(s_h + 2\phi\right)}{4s_h - s_l} > 0.$$

Naturally, prices are functions of ϕ, with $\frac{\partial p_{lh}}{\partial \phi}, \frac{\partial p_{hl}}{\partial \phi} > 0$.

Since $q_{lh} < q_{hl}$ and $p_{lh} < p_{hl}$, the airline flying during the peak slot earns higher profit than its off-peak slot counterpart.

Consider next the optimal number of seats provided in the same destination d market where both airlines $(f, f') = A(d)$ obtain the same slots. The airlines face the demand

$$q_{ii}^f + q_{ii}^{f'} = 1 - \frac{p_i}{s_i}. \tag{23}$$

Plugging (23) and (25) into airline profits (12) yields:

$$\begin{cases} \pi_{ii}^f = \left[(1 - q_{ii}^f - q_{ii}^{f'})s_i - \phi\right] q_{ii}^f \\ \pi_{ii}^{f'} = \left[(1 - q_{ii}^f - q_{ii}^{f'})s_i - \phi\right] q_{ii}^{f'}. \end{cases} \tag{24}$$

The first order conditions of π_{ii}^f and $\pi_{ii}^{f'}$ with respect to q_{ii}^f and $q_{ii}^{f'}$, respectively, are:

$$\begin{cases} -\phi - q_{ii}^f s_i + s_i(1 - q_{ii}^f - q_{ii}^{f'}) = 0 \\ -\phi - q_{ii}^{f'} s_i + s_i(1 - q_{ii}^f - q_{ii}^{f'}) = 0. \end{cases}$$

By solving the above two equations for q_{ii}^f and $q_{ii}^{f'}$ we obtain the optimal number of seats served by two airlines, which are identical due to symmetry:

$$q_{ii}^f = q_{ii}^{f'} = q_{ii} \equiv \frac{s_i - \phi}{3s_i}. \tag{25}$$

To ensure interior solutions, we make the assumptions $0 < \phi < s_i$ and $s_i > \phi_1$, $i \in \{h, l\}$. Again quantities are symmetric across destinations so that prices and profits are symmetric and can be dispensed with the superscript f and f'. Hence $0 < \phi < \phi_1$ is a sufficient condition for q_{lh}, q_{hl}, q_{ii} to be positive. For all $0 < \phi < \phi_1$, we have $q_{hl} > q_{hh} > q_{ll} > q_{lh}$. Plugging (25) into (24) and (10) yields:

$$p_{ll} = \frac{s_l + 2\phi}{3}, \; p_{hh} = \frac{s_h + 2\phi}{3}. \tag{26}$$

Again, prices are functions of ϕ, with $\frac{\partial p_{ii}}{\partial \phi} > 0$.

3.2 Airport

In the first stage, the airport maximizes its profit by allocating peak slots subject to congestion. In this set-up, destination markets can have only three types of slot allocations: n_1 destination markets have peak/off-peak allocations, n_2 get peak/peak allocations and n_3 receive off-peak/off-peak allocations. The airport allocation problem (13) simplifies to the following linear program:

$$\max_{n_1, n_2, n_3} \left[n_1 \left(q_{lh} + q_{hl} \right) + 2n_2 q_{hh} + 2n_3 q_{ll} \right] \phi \tag{27}$$

subject to

$$n_1 + n_2 + n_3 = N \tag{28}$$

$$n_1 + 2n_2 \leq M \tag{29}$$

$$0 \leq n_1, n_2, n_3 \leq N. \tag{30}$$

where N denotes the number of destinations, $2N$ the number of flights, and M the number of available peak slots. The first constraint checks the count of destination markets while the second one expresses the airport peak slot capacity. The optimal slot allocation depends on how the number of passengers in each type of slot allocation ($q_{lh}+q_{hl}$, $2q_{hh}$ and $2q_{ll}$) compare with each other.

According to (19) and (20), the number of passengers in a destination market is given by

$$q_{lh} + q_{hl} = \frac{s_h(3s_l - 2\phi) - s_l^2}{s_l(4s_h - s_l)}, \tag{31}$$

whereas with configuration peak/peak, or off-peak/off-peak, the number of passenger in a destination market is

$$2q_{ii} = \frac{2(s_i - \phi)}{3s_i}, \ i \in \{h, l\}. \tag{32}$$

To get intuition, consider that the capacity constraint (29) is not binding. Reallocation of flights must satisfy only the constraint (28). The airport can reorganize the slot in three ways. First it can add a peak/off-peak configuration at the expense of an off-peak/off-peak configuration (i.e. $\Delta n_1 = 1$, $\Delta n_3 = -1$). Since $s_h > s_l > 0$, the additional number of passengers is given by

$$q_{lh} + q_{hl} - 2q_{ll} = \frac{(s_h - s_l)(2\phi + s_l)}{3s_l(4s_h - s_l)} > 0,$$

Second, it can add a peak/peak configuration at the expense of the same off-peak/off-peak configuration (i.e. $\Delta n_2 = 1$, $\Delta n_3 = -1$) and gain

$$2q_{hh} - 2q_{ll} = \frac{2\phi(s_h - s_l)}{3s_h s_l} > 0$$

passenger. As a result, the off-peak/off-peak configuration should never be chosen by the airport. Finally, the airport may substitute a peak/off-peak configuration for an peak/peak configuration (i.e. $\Delta n_1 = 1$, $\Delta n_2 = -1$) and gain $(q_{lh} + q_{hl}) - 2q_{hh}$ passengers. Using (31) and (32), this gain is shown to be positive if and only if

$$\phi \leq \phi_2 \equiv \frac{s_l s_h}{6s_h - 2s_l}$$

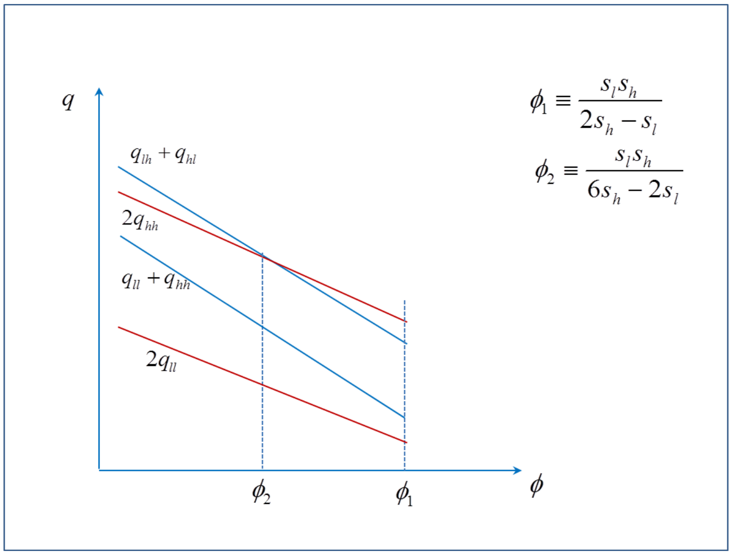

Figure 3: Passenger seats to a destination in each slot configuration

where

$$\phi_1 - \phi_2 = \frac{s_l (2s_h - s_l)}{2(3s_h - s_l)} > 0.$$

At low airport fees ϕ, the number of passengers is larger when the flights to the same destination are differentiated in their departure times. The opposite holds for high fees. There are two forces in this setting. On the one hand, competition is softer under peak/off-peak configuration as each airline targets either the high or low valuation passengers. In equilibrium they offer seats to "low cost" passengers. On the other hand, airlines intensively compete for the same high valuation passengers under a peak/peak configuration. While they attract more consumers with high valuation they do not reach the "low cost" ones. As a consequence, when the airport fee ϕ is small enough ($\phi < \phi_2$), flight fares remain sufficiently low to attract many low valuation passengers. The first effect outweighs the second effect, so that $q_{lh} + q_{hl}$ is greater than $2q_{hh}$. The airport then has an incentive to separate departing schedules. By contrast, if the airport fee ϕ is large ($\phi > \phi_2$), the flight fares are too high to attract many low valuation passengers. The airport favors high valuation passengers and offers the most valued departure slot to all of them.

Figure 3 plots number of passengers $q_{lh} + q_{hl}$, $q_{hh} + q_{hh}$ and $q_{ll} + q_{ll}$ as the fee $\phi \in (0, \phi_1)$ varies. The off-peak/offpeak configuration is always dominated. When ϕ is small ($\phi < \phi_2$) the airport gets a larger number of passenger in a peak/off-peak configuration $q_{lh} + q_{hl}$ than with a peak/peak one $q_{hh} + q_{hh}$.

Consider now that the airport hits its capacity constraint (29). Feasible slot changes must then satisfy $\Delta n_1 = -(\Delta n_2 + \Delta n_3)$ and $\Delta n_2 = \Delta n_3$.[14] For the sake of clarity, consider a slot re-allocation such that $\Delta n_1 = 2$, $\Delta n_2 = -1$ and $\Delta n_3 = -1$, which involves two destination markets and four slots, including two peak and two off-peak slots. The airport uses the slots of two peak/peak and off-peak/off-peak destinations and re-allocate only one peak slot to each destination. Doing this, it increases the number of passengers by $2(q_{lh} + q_{hl})$ and decreases it by $2q_{hh}$ and $2q_{ll}$. The comparison yields:

$$(q_{lh} + q_{hl}) - (q_{hh} + q_{ll}) = \frac{(s_h - s_l)\left[s_h s_l - (2s_h - s_l)\phi\right]}{3s_h s_l (4s_h - s_l)} > 0 \quad \text{for} \quad \phi < \phi_1.$$

As a result, when the airport reaches its capacity, it always benefits from allocating peak/off-peak slot configurations. This effect can be visualized in Figure 3 comparing $q_{lh} + q_{hl}$ and $q_{hh} + q_{ll}$.

The formal solution of the program (27) is derived in Appendix B as it follows:

(i) $n_1 = \min\{M, N\}, n_2 = 0, n_3 = N - n_1$ if $\phi < \phi_2$;

(ii) $n_1 = M$, $n_2 = 0$ and $n_3 = N - M$ if $\phi_2 < \phi < \phi_1$ and $N \geq M$;

(iii) $n_1 = N - n_2, n_2 = M - N, n_3 = 0$ if $\phi_2 < \phi < \phi_1$ and $M > N$.

When the airport is below capacity ($M > N$) and imposes a low fee ($\phi < \phi_2$), it grants a single peak slot for all destinations ($n_1 = N$ in (i)). This reflects the above incentive to set peak/off-peak slot configurations. However, in this case, some peak slots are *inefficiently* discarded whereas they have a value to all passengers. As mentioned above, by differentiating the departure time the airport increases the number of "low cost" passengers. Thus the airport does not internalize the passengers' value. By contrast, for higher fees, it grants a first peak slot to all destinations and a second one to $M - N$ destinations ($n_1 = N + (N - M)$ and $n_2 = M - N$ in (iii)). In this case flight fares are too

[14] The constraints yield $n_1 = 2N - M - 2n_3$ and $n_2 = M - N + n_3$.

high to attract the "low cost" passengers and the airport prefers offering the best departure times to the maximum number of passengers.

When the airport is at capacity ($N \geq M$) and imposes a low fee ($\phi < \phi_2$), it gives a single peak slot for the maximum number of destinations and allocates the rest of destinations into the off-peak slot pool ($n_1 = M$, $n_3 = N - M$ in (iii)). The airport makes exactly the same decision for higher fees ($n_1 = M$, $n_3 = N - M$ in (ii)).

The foregoing discussion can be summarized in the following proposition.

Proposition 1 *Suppose all destination markets are served by duopoly airlines and the airport is private. For $M \leq N$, the airport uses all available peak slots and implements the "peak/off-peak" configuration in each destination market. For $M > N$ and*

- $\phi \in (0, \phi_2]$*, the airport does not use all available peak slots (inefficiency), and the configuration is "peak/off-peak" in each destination market;*
- $\phi \in [\phi_2, \phi_1)$*, the airport uses all available peak slots, and implements $(M - N)$ "peak/peak" and $(2N - M)$ "peak/off-peak" configuration.*

Figure 4 describes the equilibria in the space (ϕ, M). When $\phi \in [\phi_2, \phi_1)$, the airport favors peak/peak allocations and as a consequence no peak slots would be optimally left unused. For per-passenger fees smaller than ϕ_2, the result depends on the relationship between peak slots M and number of destinations N. In a very busy airport where available peak slots are scarce relative to total demand ($M \leq N$), it is optimal to allocate all available peak slots. When peak slots are not scarce relative to the number of destination markets ($M > N$), a private airport would leave a number of peak slots unused when the predetermined fee is small, thereby resulting in allocative inefficiency. In reality, such behavior may be expressed by misreporting true airport handling capacity. This is in line with De Wit and Burghouwt (2008), who find that efficient slot use can be affected by capping available slots through capacity declaration.

3.3 Airline operating costs

For the sake of completeness, we end the section by discussing the case where airlines have non-zero operating costs $c > 0$. The analysis can be developed

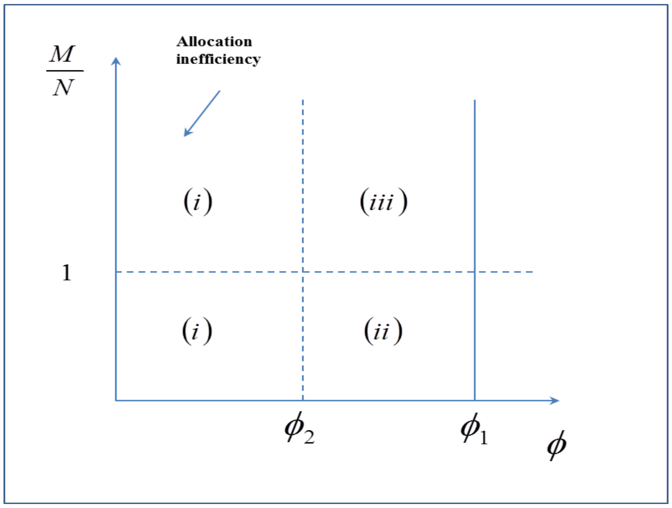

Figure 4: Equilibria: duopoly airlines and private airport

in a similar vein as before, where the airline marginal cost is now $c+\phi$ rather than ϕ only. Naturally, in both configurations the volume of passengers is larger without operating cost. The conclusion drawn from the comparison of peak-off peak configuration also applies here. It follows that, with positive operating costs airport profit is also smaller in each configuration. The condition required to guarantee positive passenger volumes in equilibrium is

$$0 < \phi < \phi_1' \equiv \frac{s_h s_l}{2s_h - s_l} - c,$$

while the threshold determining the preference between peak/peak and peak/off-peak is

$$\phi_2' \equiv \frac{s_h s_l}{6s_h - 2s_l} - c.$$

Therefore, the above proposition now reads with ϕ_1' and ϕ_2' substituting for ϕ_1 and ϕ_2. If the cost c is small enough so that $\phi_2' > 0$, the proposition presents the same configurations and the same issue of allocative inefficiency. The configuration peak/off-peak induces more passenger volume than configuration peak/peak so that the airport does not distribute all available peak slots and inefficiency arises. However, if c is large enough so that ϕ_2' becomes negative, all available peak slots are distributed and allocative inefficiency never arises.

4 Airline monopolies

Having examined competition between duopolists in each destination market, we shall now investigate the case in which each destination market is served by a single airline that acts as a monopolist operating two flights. As with the baseline model, we analyze the second stage in each possible configuration separately, while the analysis of the third stage remains unchanged.

Suppose that an airline is offered both a peak and an off-peak slot. Its profit is given by $\pi = \pi_{lh}^{f} + \pi_{hl}^{f'}$ where π_{lh}^{f} and $\pi_{hl}^{f'}$ are defined in (15) and (16). The airline solves the profit maximization problem with constraints on the positivity of outputs:

$$\max_{q_{lh}^{f},q_{hl}^{f'}} \pi = \left[s_l(1 - q_{lh}^{f} - q_{hl}^{f'}) - \phi \right] q_{lh}^{f} + (s_h - s_l q_{lh}^{f} - s_h q_{hl}^{f'} - \phi) q_{hl}^{f'} \tag{33}$$
$$\text{s.t. } q_{lh}^{f} \geq 0, q_{hl}^{f'} \geq 0.$$

The maximum profit (see Appendix C) is obtained for

$$q_{lh}^{m} = 0, \tag{34}$$
$$q_{hl}^{m} = \frac{s_h - \phi}{2 s_h}, \tag{35}$$

where the superscript m stands for "monopoly". The monopolistic airline allocates all seats in the peak flight. Indeed, it does not have any advantage to decrease consumers' value by offering a flight off peak. This result is due to the fact that the cost of boarding a passenger (in terms of per-passenger fee) is linear and equivalent between a peak and an off-peak flight. Given the same (marginal) per-passenger fee, the airline prefers to put all seats in the peak flight. In the real world, this implies that the monopoly airline assigns a "big" airplane flying on that destination in peak time rather than put two "small" airplanes flying one in the peak and the other in the off-peak slot. Plugging $q_{hl}^{m} = \frac{s_h - \phi}{2 s_h}$ into p_{hl}^{m} yields

$$p_{hl}^{m} = \frac{s_h + \phi}{2}. \tag{36}$$

Consider next the case where the monopoly airline is offered two slots at the same time for the same destination.[15] A small (unmodeled here) fixed cost per aircraft movement will entice the airline to operate only one aircraft on one slot. We denote the slot type (h or l) the monopolist obtains by i. The monopolist's profit is given by:

$$\pi_i = [(1 - q_i)s_i - \phi]\, q_i, \text{ with } i \in \{h, l\}\,.$$

Taking the first-order condition for number of seats we get the equilibrium number of seats the monopoly would provide:

$$q_i^m = q_{hl}^m \equiv \frac{s_i - \phi}{2s_i}, \tag{37}$$

which is the same result as (35) for $i = h$. The aircraft capacity is larger at peak time: $q_h^m > q_l^m$. To ensure interior solutions, suppose condition $0 < \phi < s_l$ is satisfied. Plugging (37) into (24) and (10) yields:

$$p_i^m = \frac{s_i + \phi}{2}. \tag{38}$$

which same price as (36) for $i = h$. It is easy to check that peak flights are more expensive and transport more passengers.

The airport allocation problem simplifies to allocating peak and off-peak slots to the monopoly airlines:

$$\max_{m_1, m_2} [m_1 q_h^m + m_2 q_l^m]\, \phi$$

subject to

$$\begin{aligned} & m_1 + m_2 = N \\ & m_1 \leq M \\ & 0 \leq m_1, m_2 \leq N. \end{aligned}$$

where m_1 and m_2 are the number of flights in the peak and off-peak slots. Since $q_h^m > q_l^m$, the solution is (i) $m_1 = N$ if $N < M$; (ii) $m_1 = M$ otherwise. The

[15] This configuration is mainly made for the sake of comparison. It is certainly the case in configurations where there are two (morning and evening) peak slots per day. The case where the airline merges the two flights is left for future research.

airport fills the peak slots until capacity is reached.

Proposition 2 *Suppose all destination markets are served by monopoly airlines. Then, airlines operate one flight peer destination and the private airport uses all available peak slots.*

Proposition 2 shows that, if the destination market is served by a monopoly airline, the optimal slot allocation is to assign one peak slot to each destination market, while no off-peak flights operate. The intuition lies in the fact that the monopoly airline has no incentive in exploiting an off-peak slot, given that same marginal cost as operating during peak hours.

5 Public airport

In this section, we investigate the case of a pubic, welfare-maximizing airport. This allows us to obtain some insights on how the airport's ownership influences slot allocation. In this regard, social welfare W is represented by the sum of airport' profits Π, passenger surplus CS and airlines' profits:

$$W = \Pi + CS + n_1 \left(\pi_{hl} + \pi_{lh}\right) + 2\left(n_2 \pi_{hh} + n_3 \pi_{ll}\right).$$

Since airport and airlines operating costs are normalized to zero, airport profits come from total per-passenger fees, whereas airline profits are the ticket income less total per-passenger fees paid to the airport. In turn, passenger surplus is represented by the total gross utility generated from flying minus all ticket payments. Since monetary transfers between airlines and airport cancel out, and so do transfers between passengers and airlines, then social welfare equals the sum of passengers' gross utility in all $2N$ destination markets. Thus W can be rewritten as:

$$W = n_1 \left(\int_{v_{lh}}^{v_{hl}} v s_l dv + \int_{v_{hl}}^{1} v s_h dv \right) + n_2 \int_{v_{hh}}^{1} v s_h dv + n_3 \int_{v_{ll}}^{1} v s_l dv, \tag{39}$$

where v_{lh}, v_{hl}, v_{hh} and v_{ll} are given by (1), (2), (3) and (4). Note that the surplus in the peak/peak (resp. off-peak/off-peak) configurations includes the value of all consumers from 1 to v_{hh} (resp. from 1 to v_{ll}) as all passengers take the same time slot.

In destination markets with the peak/offpeak configuration, the expression $\int_{v_{lh}}^{v_{hl}} vs_l dv$ represents the gross passenger surplus from taking the off-peak period flight, while $\int_{v_{hl}}^{1} vs_h dv$ is the gross passenger surplus from taking the peak period flight in the same destination market. In destination markets with the peak/peak configuration, the surplus is $\int_{v_{hh}}^{1} vs_h dv$,which is the second term on the right hand side of (39). The last term of (39) thus represents the gross passenger surplus in destination markets with off-peak/off-peak configuration.

The analysis of the second and third stage remains the same as in the baseline model. For notational simplicity we define $W_{lh} \equiv \int_{v_{lh}}^{v_{hl}} vs_l dv$, $W_{hl} = \int_{v_{hl}}^{1} vs_h dv$, $W_{hh} \equiv \int_{v_{hh}}^{1} vs_h dv$ and $W_{ll} \equiv \int_{v_{ll}}^{1} vs_l dv$,so that (39) becomes

$$W = n_1 \left(W_{lh} + W_{hl}\right) + n_2 W_{hh} + n_3 W_{ll}.$$

We will consider first the case with duopoly airlines and then the case with monopoly airlines.

5.1 Airline duopolies

Putting (21), (22) and (26) together with (1), (2), (3) and (4) yields:

$$v_{lh} = \frac{s_h \left(s_l + 2\phi\right)}{s_l \left(4s_h - s_l\right)}, v_{hl} = \frac{2s_h + \phi}{4s_h - s_l}, v_{hh} = \frac{s_h - 2\phi}{3s_h}, v_{ll} = \frac{s_l - 2\phi}{3s_l}. \quad (40)$$

Substituting (40) into (39) and solving the integrals yields:

$$W_{hl} + W_{lh} = \frac{4\phi s_h s_l (s_l - 2s_h) + s_h s_l \left(12s_h^2 - 5s_h s_l + s_l^2\right) - \phi^2 \left(4s_h^2 + s_h s_l - s_l^2\right)}{2s_l(4s_h - s_l)^2},$$

and

$$W_{ii} = \frac{2(\phi + s_i)(2s_i - \phi)}{9s_i}, \ i \in \{h, l\}.$$

Assuming $\phi < \phi^P \equiv \frac{s_l}{2}$ ensures that B_{ii} is positive. By comparing the number of seats obtained in each configuration, we get (see Appendix C) $W_{hh} > W_{hl} + W_{lh} > W_{ll} > 0$ and $\frac{(B_{hh}+B_{ll})}{2} < B_{hl+lh}$. We thus arrive at a situation similar to the private airport case with $\phi \in [\phi_2, \phi_1)$. In particular:

- $n_1 = M$, $n_2 = 0$ and $n_3 = N - M$ if $N \geq M$ (Case (ii)).

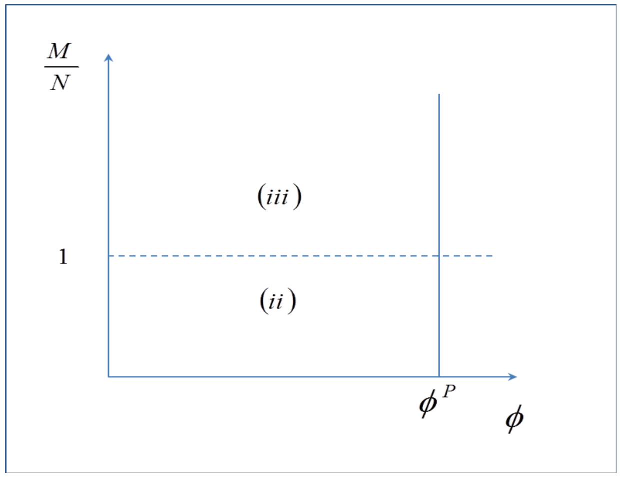

Figure 5: Equilibria: duopoly airlines and public airport

- $n_1 = N - n_2, n_2 = M - N, n_3 = 0$ if $M > N$ (Case (iii)).

The following proposition summarizes the features of the equilibrium.

Proposition 3 *Suppose all destination markets are served by duopoly airlines, and the airport is public. For $M \leq N$, the airport uses all available peak slots and favors "peak/off-peak" configuration. For $M > N$ the airport implements a mix of $(M - N)$ "peak/peak" and $(2N - M)$ "peak/off-peak" configurations.*

Figure 5 describes the equilibria in the case of public airport with duopoly destination markets. The results are qualitatively similar to the case with private airport for $\phi_2 < \phi < \phi_1$. However, now the public airport would use all available peak slots in any case, hence, inefficiency does not emerge when the airport is public. This can be explained as follows. The consumers lose when they are presented departure time away from their preferences. The public airport internalizes this loss and avoids empty peak slots. The private airport rather implement empty peak slots because it can attract more (low valuation) passengers, even though those passengers would prefer travelling at peak time.

5.2 Airline monopolies

We now turn to the interplay between a public airport and airline monopolies. Compared to the private case, the market stage in each destination market does not change: given the opportunity of operating in peak slots, each monopoly airline uses one peak slot only. Thus W can be rewritten as:

$$W^m = m_1 W_h^m + m_2 W_l^m, \tag{41}$$

where $W_h^m = \int_{v_h}^1 v s_h dv$, $W_l^m = \int_{v_l}^1 v s_l dv$, m_1 and m_2 are the number of flights in the peak and off-peak slots, and v_h and v_l are the same as (3) and (4), respectively, but when only one flight operates. Putting $p_h^m = \frac{s_h+\phi}{2}$ and $p_l^m = \frac{s_l+\phi}{2}$ from (38 into (3) and (4), respectively, yields

$$v_h = \frac{s_h + \phi}{2s_h}, v_l = \frac{s_l + \phi}{2s_l}. \tag{42}$$

Substituting (42) into (41) and solving the integrals yields:

$$W_i^m = \frac{(s_i - \phi)(3s_i + \phi)}{8s_i},\ i \in \{h, l\},$$

where

$$W_h^m - W_l^m = \frac{(s_h - s_l)\left(3s_h s_l + \phi^2\right)}{8s_h s_l} > 0.$$

Since $W_h^m > W_l^m$, the solution is (i) $m_1 = N$ if $N < M$; (ii) $m_1 = M$ otherwise. The airport fills the peak slots until capacity is reached, as in the private case (with same intuition).

Proposition 4 *Suppose all destination markets are served by monopoly airlines. Then, airlines operate one flight peer destination and the public airport uses all available peak slots.*

6 Heterogeneous Density

In this section, we assume heterogeneous density across destination markets. The point is to confirm that peak/off-peak slot configurations and allocative inefficiencies also occur when destination markets differ in sizes. For simplicity we study an economy with only two destination markets, $d \in D = \{1, 2\}$, that exhibit different densities: the small destination market 1 has a lower density δ_1 , the large destination market 2 has a higher density $\delta_2 > \delta_1$. We discuss three examples: the airport allocates either one, two or three peak slots in the two markets.

To begin with, we first look at the duopoly case where peak slots are highly scarce, where one peak slot is available compared to four slot demands, $M = 1$, $N = 2$. Let us label the large market flights by "11J, "12J $\in A(1)$ and the small market ones by "21J, "22J $\in A(2)$. There are two ways to allocate the peak slot: (1) one of the two airlines in the small destination market, and (2) one of the two airlines in the large destination market.

The demand functions for each airline can be in two possible configurations. First if the smaller destination market $d = 1$ has the peak slot we have

$$q_{lh}^{11} = \delta_1 \left(\frac{p_{hl}^{12} - p_{lh}^{11}}{s_h - s_l} - \frac{p_{lh}^{11}}{s_l} \right), \; q_{hl}^{12} = \delta_1 \left(1 - \frac{p_{hl}^{12} - p_{lh}^{11}}{s_h - s_l} \right),$$

and for destination market 2:

$$q_{ll}^{21} + q_{ll}^{22} = \delta_2 \left(1 - \frac{p_{ll}^{21}}{s_l} \right),$$

(where $p_{ll}^{21} = p_{ll}^{22}$). Second, if larger destination market $d = 2$ has the peak slot we get the same quantities where δ_1 substitutes for δ_2 and $(q_{lh}^{11}, q_{hl}^{12}, q_{ll}^{21}, q_{ll}^{22})$ is replaced by $(q_{lh}^{21}, q_{hl}^{22}, q_{ll}^{11}, q_{ll}^{12})$. In the similar manner with (19), (20) and (25), we could derive the optimal passenger volumes served by each airline and consequently each destination market in equilibrium. A comparison of the equilibrium passenger volumes in the two configurations yields:

$$\begin{aligned} & q_{lh}^{11} + q_{hl}^{12} + q_{ll}^{21} + q_{ll}^{22} - \left(q_{ll}^{11} + q_{ll}^{12} + q_{lh}^{21} + q_{hl}^{22} \right) \\ = & \frac{(\delta_1 - \delta_2)\left[s_l \left(9 + s_h + 2 s_l \right) + 2\phi \left(s_h - s_l \right) \right]}{3 s_l (4 s_h - s_l)} < 0. \end{aligned}$$

implying configuration 2 yields a higher number of passenger than configuration 1. The ranking in this simple framework suggests

Proposition 5 *Consider an economy with two duopolies with different density levels, a private airport and a single peak slot. Then the airport allocates the* *peak slot to one of the airlines in the large destination market, and inefficiency would not arise.*

We investigate next the case where peak slots have a moderate scarcity at the airport. In this setting there are two peak slots available for two destination markets, $M = N = 2$. It is straightforward to see that allocation "two peak slots to destination market 2" dominates allocation "two peak slots to destination market 1". Indeed, the large market has a bigger multiplier for density $\delta_2 > \delta_1$. This, together to the fact that off-peak/off-peak is strictly dominated by peak/peak and peak/off-peak, implies that there are two possible allocations: (1) two peak slots to the large destination market, and (2) one peak slot to each destination market. The demand functions for each airline in these two configurations are:

1. **Configuration 1.** Destination market 1 :

$$q_{ll}^{11} + q_{ll}^{12} = \delta_1 \left(1 - \frac{p_{ll}^{11}}{s_l}\right);$$

   destination market 2:

$$q_{hh}^{21} + q_{hh}^{22} = \delta_2 \left(1 - \frac{p_{hh}^{21}}{s_h}\right).$$

2. **Configuration 2.** Destination market 1:

$$q_{lh}^{11} = \delta_1 \left(\frac{p_{hl}^{12} - p_{lh}^{11}}{s_h - s_l} - \frac{p_{lh}^{11}}{s_l}\right), \; q_{hl}^{12} = \delta_1 \left(1 - \frac{p_{hl}^{12} - p_{lh}^{11}}{s_h - s_l}\right);$$

   destination market 2:

$$q_{lh}^{21} = \delta_2 \left(\frac{p_{hl}^{22} - p_{lh}^{21}}{s_h - s_l} - \frac{p_{lh}^{21}}{s_l}\right), \; q_{hl}^{22} = \delta_2 \left(1 - \frac{p_{hl}^{22} - p_{lh}^{21}}{s_h - s_l}\right).$$

Comparing the two configurations we obtain

$$q_{ll}^{11} + q_{ll}^{12} + q_{hh}^{21} + q_{hh}^{22} - \left(q_{lh}^{11} + q_{hl}^{12} + q_{lh}^{21} + q_{hl}^{22}\right) > 0$$

for

$$\phi > \phi_3 \equiv \frac{s_h s_l (\delta_1 + \delta_2)(s_h + 2s_l + 9)}{2(s_h - s_l)[\delta_2 (3s_h - s_l) - \delta_1 s_h]}.$$

This result can be summarized as follows.

Proposition 6 *Consider an economy with two duopolies with different density levels, a private airport and two peak slots. Then, for either $\phi_3 > \phi_1 > \phi$ or $\phi_1 > \phi_3 > \phi$, the airport allocates one peak slot to each market; for $\phi_1 > \phi > \phi_3$, the airport allocates two peak slots to the large market, and inefficiency would not arise.*

When the per-passenger fee is sufficiently small, the allocation is fair and does not favor any market so that each destination is equally served. For high per-passenger fees, the denser market obtains all available slots. As a consequence, passengers in the small market have no chance to fly at peak hours, while passengers in the big market cannot fly at off-peak hours.

Finally, we examine the example where peak slots are relatively abundant. In particular, there are three peak slots to be allocated to two markets, $M = 3$, $N = 2$. Given that configuration off-peak/off-peak is strictly dominated, we can set aside the situation where the airport leaves one slot unused and gives two slots to the big market. Indeed, the airport could be better off by giving the unused one to the small market. It follows that there are three plausible configurations: (1) two peak slots to market 2- one peak slot to market 1, (2) two peak slots to market 1- one peak slot to market 2, and (3) one peak slot to each market.

1. **Configuration 1.** Market 1 :

$$q_{lh}^{11} = \delta_1 \left(\frac{p_{hl}^{12} - p_{lh}^{11}}{s_h - s_l} - \frac{p_{lh}^{11}}{s_l} \right), \; q_{hl}^{12} = \delta_1 \left(1 - \frac{p_{hl}^{12} - p_{lh}^{11}}{s_h - s_l} \right);$$

   market 2:

$$q_{hh}^{21} + q_{hh}^{22} = \delta_2 \left(1 - \frac{p_{hh}^{21}}{s_h} \right).$$

2. **Configuration 2.** Market 1:

$$q_{hh}^{11} + q_{hh}^{12} = \delta_1 \left(1 - \frac{p_{hh}^{11}}{s_h} \right);$$

market 2:

$$q_{lh}^{21} = \delta_2 \left(\frac{p_{hl}^{22} - p_{lh}^{21}}{s_h - s_l} - \frac{p_{lh}^{21}}{s_l} \right),\ q_{hl}^{22} = \delta_2 \left(1 - \frac{p_{hl}^{22} - p_{lh}^{21}}{s_h - s_l} \right);$$

3. **Configuration 3.** market 1:

$$q_{lh}^{11} = \delta_1 \left(\frac{p_{hl}^{12} - p_{lh}^{11}}{s_h - s_l} - \frac{p_{lh}^{11}}{s_l} \right),\ q_{hl}^{12} = \delta_1 \left(1 - \frac{p_{hl}^{12} - p_{lh}^{11}}{s_h - s_l} \right);$$

market 2:

$$q_{lh}^{21} = \delta_2 \left(\frac{p_{hl}^{22} - p_{lh}^{21}}{s_h - s_l} - \frac{p_{lh}^{21}}{s_l} \right),\ q_{hl}^{22} = \delta_2 \left(1 - \frac{p_{hl}^{22} - p_{lh}^{21}}{s_h - s_l} \right).$$

Comparing the three configurations we obtain

$$\text{configuration } 1 > 2 > 3 \text{ when } \phi > \phi_4;$$
$$\text{configuration } 1 < 2 < 3 \text{ when } \phi < \phi_4,$$

with

$$\phi_4 \equiv \frac{s_h s_l (s_h + 2s_l + 9)}{2 (s_h - s_l)(3s_h - s_l)}.$$

Therefore

Proposition 7 *Suppose an economy with two duopolies with different density levels, a private airport and three peak slots. Then, for* $\min(\phi_1, \phi_4) > \phi$*, the airport allocates one peak slot to each market, and leaves one peak slot unused (inefficiency); for* $\phi_1 > \phi > \phi_4$*, the airport allocates two peak slots to the large market and one peak slot to the small market.*

Proposition 7 implies that when markets have different consumer densities, allocative inefficiency would arise if the per-passenger fee is sufficiently small. On the other hand, if the per-passenger fee lies in a certain range, the allocation outcome is efficient, with the denser market obtaining both peak slots and the smaller market obtaining one peak slot. Such allocation favors the denser market, which is a result of airport's profit maximizing behavior.

7 Concluding remarks

We have explored the optimal slot allocation in the presence of airport congestion in a model where peak and off-peak slots are modelled as products of different qualities in a vertically differentiated setting. Allocative inefficiency emerges when the airport does not exploit all its slots. In particular in a private airport, allocative inefficiency may emerge if the airport is not too congested and the per-passenger fee is small enough. In a public airport, allocative inefficiency does not emerge. Furthermore we have found that the airport, regardless of its ownership, tends to give different slots to flights with same destination if the underlying destination market is a duopoly, and one single slot if the underlying market is served by a monopoly.

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Appendix A

For our empirical illustration, we examine the airline market structure and slot occupation. Towards this aims, we gather data from 15 US airport's websites on three consecutive weekdays May 18,19 and 20, 2015 for flights' departure information. Weekdays are chosen to exclude irregular influx for air travel happens on weekends. We adopt the definition for peak load according to website of O'Hare International Airport (ORD)[16] and apply to all 15 airports. Our dataset contains airport level observations on air traffic: departure airport, service airline, destination airport, departure time. Among the 15 primary airports in our dataset,[17] 5 airports are the most busiest airports in US by total passenger traffic, according to ACI (Airports Council International North America) ranking in calender year 2013. The other 10 mid-sized airports are taken arbitrarily from the range 30nd-60nd on the same rank, scattered to 9 federal states. We apply the below rules to filter improper observations: (1) delete all cargo, private jet charter, aircraft Rental Service, etc.; (2) for code sharing airlines, keep the operating airline and delete all other (code sharing) partner airlines. The total number of observations used for our analysis is 5990 departure activities, of which monopoly is the primary feature of airline market. Table 5 shows per airport market structure. Table 1, reported in the Introduction, lists the numbers of origin-destination routes operated by monopoly, duopoly and oligopoly, and the pattern of slot occupancy.

[16] ORD defines 8-9am, 15-16pm, 17-18pm, and 19-22pm as peak hours.

[17] FAA defines a primary airport as having more than 10,000 passenger boardings each year.

Airport	#Mon.	#Duo.	#Olig.	Rank(2013)
Atlanta International Airport (ATL)	148	33	25	1
Los Angeles International Airport (LAX)	46	31	32	2
O'Hare International Airport (ORD)	101	45	51	3
Dallas/Fort Worth International Airport (DFW)	144	20	18	4
Denver International Airport (DEN)	86	32	31	5
Kansas City International Airport (MCI)	29	11	3	35
Oakland International Airport (OAK)	24	7	0	36
John Wayne Airport (SNA)	17	3	1	38
Luis Muñoz Marín International Airport (SJU)	21	6	6	43
San Antonio International Airport (SAT)	24	5	2	45
Indianapolis International Airport (IND)	28	5	2	48
Kahului Airport (OGG)	10	4	5	51
Buffalo Niagara International Airport (BUF)	15	4	0	55
Jacksonville International Airport (JAX)	18	5	0	56
Eppley Airfield (OMA)	14	3	0	60

Table 5. Per airport market structure

A further deletion was made to exclude all flights between the 5 largest airlines, as they are mostly inter-hub connection flights; and high frequency flights with frequency above and equal to 7 flights towards one destination airport. We then construct an index to measure allocative discrimination:

$$I = \frac{\left|x_h^a - x_h^b\right| + \left|x_l^a - x_l^b\right|}{2} \in \{0, 0.5, 1\},$$

where the subscripts h and l denote peak and offpeak time slots. x_h^a is a dummy that takes value 1 if airline a obtains a peak slot, 0 if not. The same logic applies to n_h^b, x_l^a and x_l^b. Hence when $I = 1$,allocative discrimination is largest, while, when $I = 0$, allocative discrimination doesn't exist. A higher average value of this index indicates a higher magnitude of slot discrimination. We then find evidence consistent with our model prediction that allocative discrimination over 10 smaller airports is smaller than that of the larger 5 airports, see Table 2 in the Introduction.

Appendix B

Linear programming

In this section we describe the general problem of the airport without getting into the specific cases considered in the paper. The result of each case will depend on the relationship between M and N and the level of per-passenger fee ϕ. This will be examined and linked to the general solution in the next section. We define as Q_1, $2Q_2$ and $2Q_3$ the equilibrium number of passengers in a peak/off-peak, peak/peak and off-peak/off-peak configuration, respectively, for a general problem. The airport has the following linear programming problem to solve

$$\max_{n_1,n_2,n_2} \Pi = n_1Q_1 + 2n_2Q_2 + 2n_3Q_3,$$
$$\text{s.t.}$$
$$n_1 + n_2 + n_3 = N,$$
$$n_1 + 2n_2 \leq M,$$
$$0 \leq n_1, n_2, n_3 \leq N.$$

Using $n_3 = N - n_1 - n_2$ we can re-write

$$\mathcal{P} \equiv \max_{n_1,n_2,n_2} \Pi = n_1 (Q_1 - 2Q_3) + n_2(2Q_2 - 2Q_3) + 2NQ_3 \ ,$$
$$\text{s.t.}$$
$$n_1 + 2n_2 \leq M \ ,$$
$$0 \leq n_1 + n_2 \leq N \ .$$

We get the following solution:

1. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) > 0$, and $Q_1 > 2Q_2$ then $n_1 = \min\{M, N\}$, $n_2 = 0$, $n_3 = N - n_1$;

2. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) > 0$, and $Q_2 + Q_3 > Q_1$ then $n_2 = \min\{M/2, N\}$, $n_1 = 0$, $n_3 = N - n_2$;

3. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) > 0$, $2Q_2 > Q_1 > Q_2 + Q_3$ and $N \geq M$, then $n_1 = M$, $n_2 = 0$ and $n_3 = N - M$;

4. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) > 0$, $2Q_2 > Q_1 > Q_2 + Q_3$ and $M > N > M/2$, then $n_1 = 2N - M$, $n_2 = M - N$ and $n_3 = 0$;

5. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) > 0$, $2Q_2 > Q_1 > Q_2 + Q_3$ and $M/2 > N$, then $n_1 = 0$, $n_2 = M/2$ and $n_3 = N - M/2$.

6. If $(Q_1 - 2Q_3) < 0$ and $(2Q_2 - 2Q_3) > 0$, then $n_2 = \min\{M/2, N\}$, $n_1 = 0$, $n_3 = N - n_2$.

7. If $(Q_1 - 2Q_3) < 0$ and $(2Q_2 - 2Q_3) < 0$, then $n_1 = n_2 = 0$ and $n_3 = N$.

8. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) < 0$, then $n_1 = \min\{M, N\}$, $n_2 = 0$, $n_3 = N - n_1$.

Note that Solution 5. is not applicable because we assumed $M < 2N$.

Applications

The relevant results of the linear programming depend on the structure of the economy (public/private airport, and duopoly markets), the relationship between peak slots M and destinations N, and the level of per passenger fees ϕ. What follows helps to understand which solution applies to each case considered in the paper.

Duopolies and private airport

For the duopoly case, let $Q_1 = q_{lh}^f + q_{hl}^{f'}$, $Q_2 = q_{hh}^f$ and $Q_3 = q_{ll}^f$. We know that $Q_1 > 2Q_2 > 2Q_3$ for $\phi < \phi_2$ and $2Q_2 > Q_1 > 2Q_3$ for $\phi_2 < \phi < \phi_1$. Also, $Q_1 > Q_2 + Q_3$. If $\phi < \phi_2$ we get $Q_1 > 2Q_2$, so that checking Section 7 result 1. applies: $n_1 = \min\{M, N\}$, $n_2 = 0$, $n_3 = N - n_1$;

If $\phi_2 < \phi < \phi_1$, we get:

i. if $N > M$, then $n_1 = M$, $n_2 = 0$ and $n_3 = N - M$;

ii. if $M > N > M/2$, then $n_1 = 2N - M$, $n_2 = M - N$ and $n_3 = 0$;

Checking the corresponding results in Section 7, this yields the solution:

- $n_1 = \min\{M, N\}, n_2 = 0, n_3 = N - n_1$ if $\phi < \phi_2$ or if $\phi_2 < \phi < \phi_1$ and $N > M$;

- $n_1 = 2N - M, n_2 = M - N, n_3 = 0$ if $\phi_2 < \phi < \phi_1$ and $M > N$.

Duopolies and public airport

For the duopoly case, let $Q_1 = B_{hllh}$, $Q_2 = B_{hh}/2$ and $Q_3 = B_{ll}/2$. We know $2Q_2 > Q_1 > 2Q_3$ if $\phi < \phi_1^P$ and $2Q_2 > 2Q_3 > Q_1$ if $\phi > \phi_1^P$. Also, $Q_2 + Q_3 > Q_1$ for $\phi > \phi_4^P$.

Checking the corresponding results in Section 7, this yields the following solution: for $\phi > \phi_1^P$, we have (result 6.) $n_2 = \min\{M/2, N\}$, $n_1 = 0$, $n_3 = N - n_2$. For $\phi_1^P > \phi > \phi_4^P$, we have (result 2.) $n_2 = \min\{M/2, N\}$, $n_1 = 0$, $n_3 = N - n_2$. For $\phi < \phi_4^P$, then the relationship between destinations and peak slots matters. For $M > N$, the solution (result 4.) is $n_1 = 2N - M$, $n_2 = M - N$ and $n_3 = 0$. For $N > M$, result 3. occurs, according to which $n_1 = M$, $n_2 = 0$ and $n_3 = N - M$.

Appendix C

Monopoly airlines

The Lagrangian of problem (33) and its derivatives write as

$$\begin{aligned}
L &= \left[s_l(q_{lh}^f - q_{lh}^f q_{lh}^f - q_{hl}^{f'} q_{lh}^f) - \phi q_{lh}^f \right] \\
&\quad + (s_h q_{hl}^{f'} - s_l q_{lh}^f q_{hl}^{f'} - s_h q_{hl}^{f'} q_{hl}^{f'} - \phi q_{hl}^{f'}) \\
&\quad + \lambda_{lh}^f q_{lh}^f + \lambda_{hl}^{f'} q_{hl}^{f'} \\
\frac{\partial L}{\partial q_{lh}^f} &= s_l(1 - 2q_{lh}^f - 2q_{hl}^{f'}) - \phi + \lambda_{lh}^f \\
\frac{\partial L}{\partial q_{hl}^{f'}} &= s_h - 2s_l q_{lh}^f - 2s_h q_{hl}^{f'} - \phi + \lambda_{hl}^{f'}
\end{aligned}$$

where $\lambda_{lh} \geq 0$ and $\lambda_{hl} \geq 0$ are the Khun-Tucker multiplicators. The Hessian matrix is

$$H = \begin{bmatrix} -2s_l & -2s_l \\ -2s_l & -2s_h \end{bmatrix},$$

with determinant $4s_l\,(s_h - s_l) > 0$. Therefore H is definite positive and we have unique maximum.

The unique root of $\dfrac{\partial L}{\partial q_{lh}^{f}} = \dfrac{\partial L}{\partial q_{hl}^{f'}} = 0$ is given by

$$q_{lh}^{f} = -\frac{1}{2s_l\left(s_h - s_l\right)}\left(\phi s_h - \phi s_l - s_h\lambda_{hl}^{f} + s_l\lambda_{hl}^{f'}\right),$$
$$q_{hl}^{f'} = \frac{1}{2\left(s_h - s_l\right)}\left(s_h - s_l + \lambda_{hl}^{f'} - \lambda_{hl}^{f}\right)$$

The maximum solution $\lambda_{lh}^{f} \geq 0$, $q_{lh}^{f} \geq 0$, $\lambda_{lh}q_{lh}^{f} = 0$, and $\lambda_{hl}^{f'} \geq 0$, $q_{hl}^{f'} \geq 0$, $\lambda_{hl}^{f'}q_{hl}^{f'} = 0$. Suppose $\lambda_{lh}^{f} = 0$ and $\lambda_{hl}^{f'} = 0$ while $q_{lh}^{f} \geq 0$ and $q_{hl}^{f'} \geq 0$. Then, we get $q_{lh}^{f} = -\frac{1}{2}\frac{\phi}{s_l} < 0$ and $q_{hl}^{f'} = \frac{1}{2}$, which is impossible for $\phi > 0$. Hence the two flights f and f' are not operated together. Suppose $\lambda_{lh}^{f} > 0$ and $\lambda_{hl}^{f'} = 0$ while $q_{lh}^{f} = 0$ and $q_{hl}^{f'} \geq 0$. Then, we get $q_{hl}^{f'} = \frac{1}{2s_h}\left(s_h - \phi\right)$ and $\lambda_{hl}^{f} = \phi\frac{s_h - s_l}{s_h} > 0$,which is possible for $s_h > \phi$. Suppose $\lambda_{lh}^{f} = 0$ and $\lambda_{hl}^{f'} > 0$ while $q_{lh}^{f} \geq 0$ and $q_{hl}^{f'} = 0$. Then, we get $q_{lh}^{f} = \frac{1}{2s_l}\left(s_l - \phi\right)$ and $\lambda_{hl}^{f'} = -\left(s_h - s_l\right) < 0$, which is impossible. Suppose $\lambda_{lh}^{f} > 0$ and $\lambda_{hl}^{f'} > 0$ while $q_{lh}^{f} = 0$ and $q_{hl}^{f'} = 0$. Then, we get $\lambda_{lh}^{f} = -s_l + \phi$ and $\lambda_{hl}^{f'} = -s_h + \phi$, which is impossible for $\phi < s_l$.

Public Airport

Duopoly

By comparing the number of seats obtained in each configuration, we get:

$$B_{hh} - B_{ll} = \frac{2\left(s_h - s_l\right)\left(\phi^2 + 2s_hs_l\right)}{9s_hs_l} > 0,$$
$$B_{hh} - B_{hl+lh} =$$
$$\frac{\left(s_h - s_l\right)\left(s_h^2s_l\left(20s_h + s_l\right) + 4s_hs_l\phi\left(2s_h + s_l\right) + \phi^2\left(36s_h^2 - 19s_hs_l + 4s_l^2\right)\right)}{18s_hs_l(s_l - 4s_h)^2} > 0$$
$$B_{hl+lh} - B_{ll} =$$
$$\frac{\left(s_h - s_l\right)\left(108s_h^2s_l + s_l\left(8s_l^2 - 4s_l\phi - 13\phi^2\right) + s_h\left(28\phi^2 - 65s_l^2 - 8s_l\phi\right)\right)}{18s_l(s_l - 4s_h)^2} > 0,$$

Hence we obtain $B_{hh} > B_{hl+lh} > B_{ll}$ for all $0 < \phi < \frac{s_l}{2}$.

Next, we evaluate the differences in the allocation of two peak slots:

$$B_{hh} + B_{ll} - 2B_{hl+lh} =$$

$$\frac{(s_h - s_l)\left(-s_l s_h\left(44s_h^2 - 33s_h s_l + 4s_l^2\right) + 4s_l s_h \phi\left(2s_h + s_l\right) + \phi^2\left(4s_h^2 - 3s_l s_h + 2s_l^2\right)\right)}{9s_h s_l(s_l - 4s_h)^2} < 0$$

for all $0 < \phi < \frac{s_l}{2}$. Hence, $B_{hh} + B_{ll} < 2B_{hl+lh}$.