3D Crack Detection Using an XFEM Variant and Global Optimizaion Algorithms

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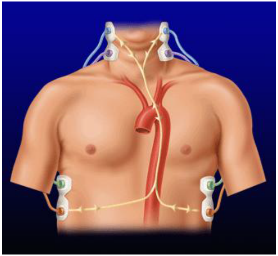
Outline

Background

Nondestructive evaluation (NDE)

Methods used to examine an object, material or system without impairing its future usefulness



- SHM - Damage Detection: Monitor changes in the dynamic properties of a structure

- Available Techniques: impedance tomography, radiography, ultrasounds, acoustic emission

Inverse problem

→ Detection of cracks in existing structures

→ Measurements are available

→ A computational model is employed

→ The difference between the two is minimized

→ Information regarding the cracks is obtained

Inverse problem

Mathematical formulation:

$$\textit{Find } \beta_i \textit{ such that}$$
$$\mathcal{F}\left(r\left(\beta_i\right)\right) \rightarrow \min$$

where

- β_i Parameters describing the crack geometry
- $r\left(\cdot\right)$ Norm of the difference between measurements and computed values
- $\mathcal{F}$ Some function of the residual

The CMA-ES algorithm is employed to solve the problem.

Inverse problem

Solution process:

→ Generation of initial population (β_i) with CMA-ES

→ Fitness function ($\mathcal{F}(r(\beta_i))$) evaluation using XFEM and measurements

→ Population is updated with CMA-ES

→ The procedure is repeated until convergence

Inverse problem

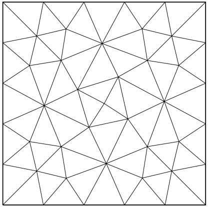
During the optimization proccess:

- A large number of crack geometries is tested
- The computational model is solved several times
- An efficient and robust method is required

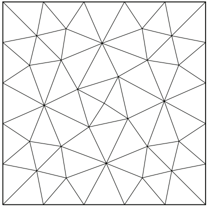
FEM vs XFEM for fracture:

- No crack

FEM
⇒



XFEM
⇒

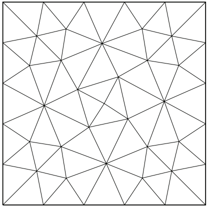


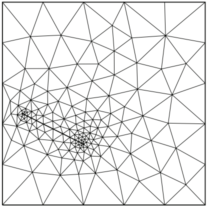
XFEM

FEM vs XFEM for fracture:

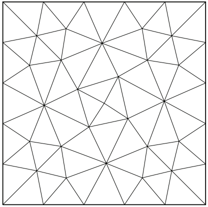
- No crack
- Crack 1

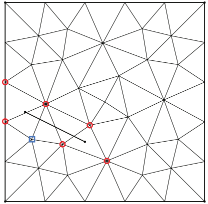
FEM ⇒





XFEM ⇒



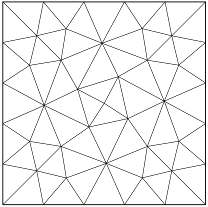


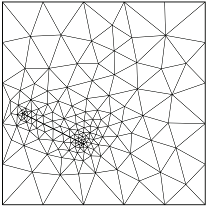
XFEM

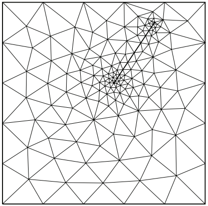
FEM vs XFEM for fracture:

- No crack
- Crack 1
- Crack 2

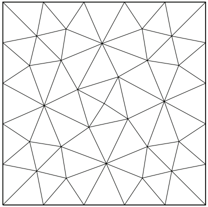
FEM ⇒

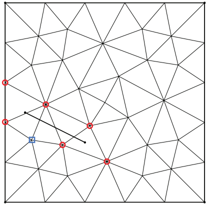


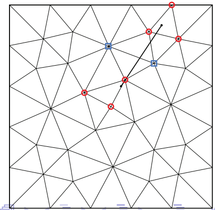




XFEM ⇒







XFEM approximation

XFEM approximation:

$$\mathbf{u}(\mathbf{x}) = \underbrace{\sum_{\forall I} N_I(\mathbf{x})\,\mathbf{u}_I}_{\text{FE approximation}} + \underbrace{\sum_{\forall I} N_I^*(\mathbf{x})\,\Psi(\mathbf{x})\,\mathbf{b}_I}_{\text{enriched part}}$$

where:

$N_I(\mathbf{x})$ are the FE shape functions

$\mathbf{u}_I$ are the nodal displacements

$N_I^*(\mathbf{x})$ are functions forming a PU

$\Psi(\mathbf{x})$ are the enrichment functions

$\mathbf{b}_I$ are the enriched dofs

Enrichment functions

Jump enrichment functions:

$$H(\phi) = \begin{cases} 1 & \text{for } \phi > 0 \\ -1 & \text{for } \phi < 0 \end{cases}$$

Tip enrichment functions:

$$F_j\left(r,\theta\right) = \left[\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right]$$

XFEM

Some drawbacks of XFEM:

- The use of tip enrichment in a fixed area around the crack front (geometrical enrichment) is required for optimal convergence
- The use of geometrical enrichment causes conditioning problems
- Blending problems the enriched and the standard part of the approximation

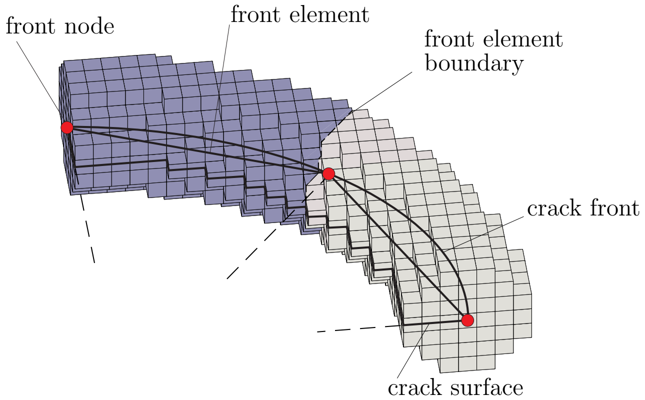
Global enrichment XFEM

An XFEM variant is employed which:

- Enables the application of geometrical enrichment to 3D
- Employs weight function blending
- Employs enrichment function shifting

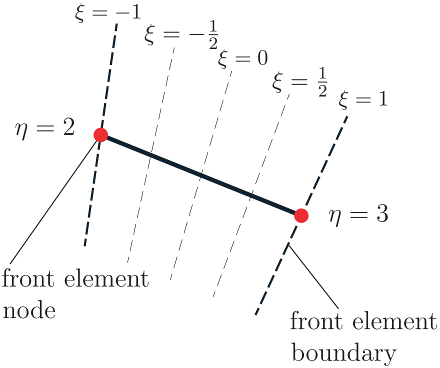
Global enrichment XFEM

Special front elements are introduced:



Global enrichment XFEM

Front element shape functions:



$$\mathbf{N}^{g}(\xi) = \begin{bmatrix} \frac{1-\xi}{2} & \frac{1+\xi}{2} \end{bmatrix}$$

Global enrichment XFEM

Displacement approximation:

$$\mathbf{u}(\mathbf{x}) = \sum_{I \in \mathcal{N}} N_I(\mathbf{x}) \mathbf{u}_I + \bar{\varphi}(\mathbf{x}) \sum_{J \in \mathcal{N}^j} N_J(\mathbf{x}) (H(\mathbf{x}) - H_J) \mathbf{b}_J +$$

$$+ \varphi(\mathbf{x}) \left(\sum_{K \in \mathcal{N}^s} N_K^g(\mathbf{x}) \sum_j F_j(\mathbf{x}) - \sum_{T \in \mathcal{N}^t} N_T(\mathbf{x}) \sum_{K \in \mathcal{N}^s} N_K^g(\mathbf{x}_T) \sum_j F_j(\mathbf{x}_T) \right) \mathbf{c}_{Kj}$$

where:

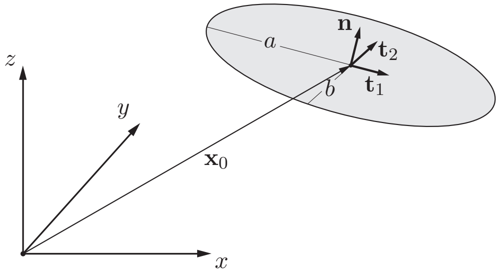
$\bar{\varphi}, \varphi$ are weight functions

N_K^g are front element shape functions

H_J, F_j are nodal values of the enrichment functions

Problem parametrization

Elliptical cracks are considered:



Parameters:

- Coordinates of center point $\mathbf{x}_0$ ($\{x_0, y_0, z_0\}$)
- Rotation about the three axes θ_x, θ_y and θ_z
- Lengths a and b

Problem parametrization

Scaling of parameters:

$$p_i = \frac{p_{i_1} + p_{i_2}}{2} + \frac{p_{i_2} - p_{i_1}}{2} \sin\left(\frac{\beta_i}{10} \cdot \frac{\pi}{2}\right)$$

where:

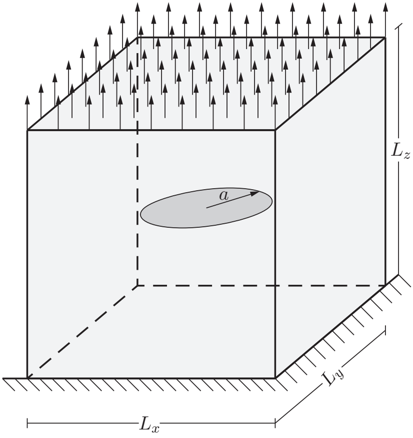
β_i are design variables

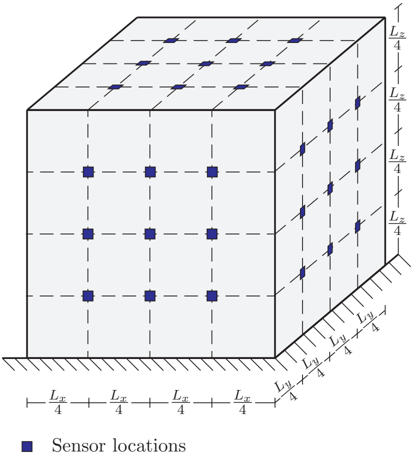
p_i are geometrical parameters of the crack

p_{i_1}, p_{i_2} are lower and upper values for the parameters

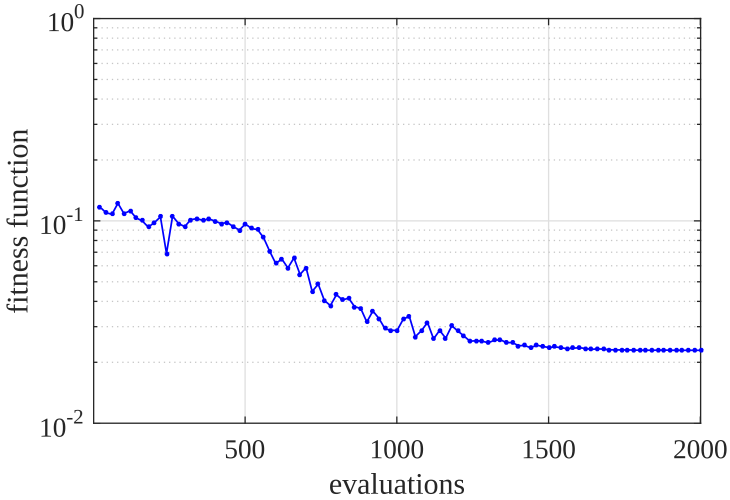
Penny crack in a cube

Geometry and sensors:



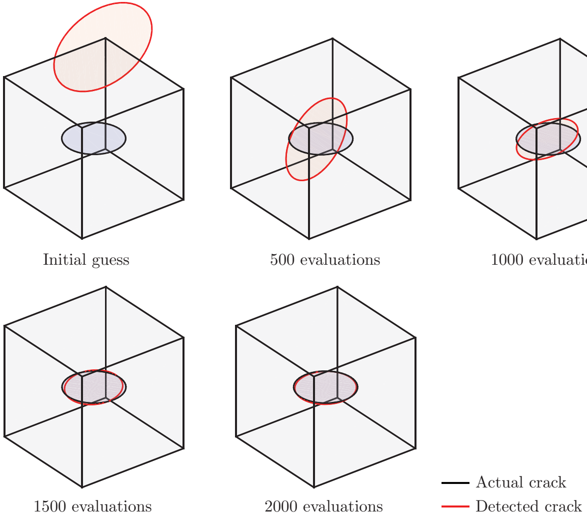


Optimization problem convergence:



Penny crack in a cube

Best solution after different numbers of iterations



Conclusions

→ A 3D crack detection scheme was presented

→ Promising results were obtained

→ Extension to practical problems would increase computational cost

→ Computational cost of forward problem solutions should be reduced