ROBUST PRIMARY USER IDENTIFICATION USING COMPRESSIVE SAMPLING FOR COGNITIVE RADIOS

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ABSTRACT

In cognitive radio (CR), the problem of limited spectral resources is solved by enabling unlicensed systems to opportunistically utilize the unused licensed bands. Compressive Sensing (CS) has been successfully applied to alleviate the sampling bottleneck in wideband spectrum sensing leveraging the sparseness of the signal spectrum in open-access networks. This has inspired the design of a number of techniques that identify spectrum holes from sub-Nyquist samples. However, the existence of interference emanating from low-regulated transmissions, which cannot be taken into account in the CS model because of their non-regulated nature, greatly degrades the identification of licensed activity. Capitalizing on the sparsity described by licensed users, this paper introduces a feature-based technique for primary user's spectrum identification with interference immunity which works with a reduced amount of data. The proposed method detects which channels are occupied by primary users' and also identify the primary users transmission powers without ever reconstructing the signals involved. Simulation results show the effectiveness of the proposed technique for interference suppression and primary user detection.

1. INTRODUCTION

Cognitive Radio (CR) [1] resolves the problem of limited spectral resources by enabling unlicensed systems to opportunistically utilize the unused licensed bands. The task of accurately detecting the presence of licensed user is encompassed in spectrum sensing. Among the challenges related to spectrum sensing implementation, the most critical is the need to process very wide bandwidth, which involves sampling many points on the radio spectrum [2].

Compressive Sensing (CS) [3] has emerged as a promising signal processing technique to simultaneous sensing and compressing sparse signals thus allowing sampling rates significantly lower than Nyquist rate. CS has been successfully applied to alleviate the sampling bottleneck in wideband spectrum sensing leveraging the sparseness of the signal spectrum in open-access networks [4]. This has inspired the design of a number of techniques that identify spectrum holes from sub-Nyquist samples [5–8]. These approaches assume that the received signal can be modeled as a superposition of a small number of sinusoids contaminated with noise (either bounded noise or Gaussian with known power). However, the blind nature of the Fourier basis impedes discrimination between sources of received energy. In practical settings, primary signals must be detected even with the presence of low-regulated transmissions from secondary systems. The existence of interference emanating from low-regulated transmissions, which cannot be taken into account in the CS model because of their non-regulated nature, greatly degrade the identification of licensed activity [9]. There are few approaches which successfully mitigate the interference contribution assuming some prior information about the interference [10, 11]. To the best of our knowledge, interference mitigation in conjunction with CS assuming no prior information about the interference has never been considered in the literature.

This paper presents a feature-based technique for primary user's spectrum identification with interference immunity which works with a reduced amount of data. The spectrum characteristics of the primary signals, which can be obtained by identifying its transmission technologies, are used as features. The basic strategy is to compare the a priori known spectral shape of the primary user with the power spectral density (PSD) of the received signal. To save us from computing the PSD of the received signal, the comparison is made in terms of autocorrelation by means of a correlation matching. Thus, the occupied channels are directly detected from the sample autocorrelation matrix avoiding the complete signal reconstruction. We propose the use of an overcomplete dictionary that contains tuned prototype spectral shapes of the primary user in order to achieve sparse representation of primary user spectral support. In essence, the use of an overcomplete dictionary avoids the traditional channel-by-channel scanning where the multiple patterns are matched to the received data independently, thus allowing the different matched-filters to be applied simultaneously to the compressive data. Extraction of the primary user frequency locations needs to be performed based on sparse signal recovery algorithms. The spirit of the novel interference rejection mechanism lies in preserving the positive semidefinite character of the difference between the reference and the sample autocorrelation matrices, which is achieved by introducing weights to the l_1-norm and suppling a new stopping criteria for conventional CS-based iterative reconstruction techniques.

This work extends previous authors' publications [12,13], where the sparsity condition was not taken into account in the reconstruction process.

2. PROBLEM STATEMENT

Our goal is to decide whether a given frequency band is occupied by a primary user or not based on sub-Nyquist samples of the received signal. Let us denote $x(t)$ the wideband signal represent-

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ing the superposition of different primary systems in a CR network. Some of the channels left idle by primary users may be occupied by low-regulated transmissions generated by the unlicensed radios. The non-regulated activity of the spectrum will be denoted as $i(t)$ and henceforth is considered interference. Let $y(t) = x(t) + i(t) + \eta(t)$ be the continuous-time received signal, where $\eta(t)$ is additive white Gaussian noise (AWGN), $\mathcal{N}(0, \sigma_\eta^2)$. Signal $x(t)$ is considered to be a bandlimited sparse multiband signal, i.e., its spectral support is small relative to the overall signal bandwidth. Assuming that $y(t)$ is bandlimited to $F = [0, f_{max}]$, its Nyquist sampling period is given by $T = 1/f_{max}$. The problem of determining weather a given frequency is occupied or not by a licensed radio can be modeled into a binary hypothesis test,

$$y(t) = \begin{cases} i(t) + w(t) & (H_0) \\ x(t) + i(t) + w(t) & (H_1) \end{cases} \tag{1}$$

where (H_0) stands for the absence of primary signal and (H_1) stands for the presence of a primary signal. The presence of unknown interference adds additional challenges to the conventional spectrum sensing problem.

3. COMPRESSIVE SAMPLING

We consider the sub-Nyquist periodic nonuniform sampling strategy known as multi-coset (MC) sampling [14]. Given the received signal $y(t)$, the MC sampler samples are obtained at the time instants,

$$t_i(n) = (nL + c_i)T \tag{2}$$

where $L > 0$ is a suitable integer, $i = 1, 2, \ldots, \kappa$ and $n \in \mathbb{Z}$. The set $\{c_i\}$ contains κ distinct integers chosen from $\{0, 1, \ldots, L-1\}$. The MC sampling process can be viewed as a classical Nyquist sampling followed by a block that discards all but κ samples in every block of L samples periodically. Thus, a sequence or coset of equally-spaced samples is obtained for each c_i. The period of each one of these sequences is equal to LT. Therefore, one possible implementation consists of κ parallel ADCs, each working uniformly with period LT.

The complete observation consists of a data record of N_f blocks of κ non-uniform samples noted as $\mathbf{y}_f$. In order to relate the acquired samples $\mathbf{y}_f$ with the original Nyquist-sampled signal, let us consider $\mathbf{z}_f$ as the f-th block of L uniform Nyquist samples of $y(t)$,

$$\mathbf{z}_f = \begin{bmatrix} y(t_1^f) & \ldots & y(t_L^f) \end{bmatrix}^T \tag{3}$$

where $t_n^f = (fL + n)T$. Thus, the relation between the Nyquist samples and the sub-Nyquist samples is given by,

$$\mathbf{y}_f = \mathbf{\Phi} \mathbf{z}_f \tag{4}$$

where $\mathbf{\Phi}$ is a matrix that randomly selects κ samples of $\mathbf{z}_f$ ($\kappa < L$). This matrix $\mathbf{\Phi}$ is known as sampling matrix and is given by randomly selecting κ rows of the identity matrix $\mathbf{I}_L$.

4. SPECTRAL MATCHING DETECTION

In this paper, the primary user detection problem is approached from a feature-based perspective. We assume that the spectral shape of the transmitted primary signal is known a priori at the receiver of a CR. The spectral shape of linearly modulated signals depends mainly on the transmission rate and the modulation pulse, which can be analytically extracted from physical layer standardization of primary services. To detect the presence of licensed activity in the spectrum, we propose to compare the a priori known spectral shape of the primary user with the power spectral density of the received signal by shifting the reference spectrum over subsequent channel positions. To save us from computing the power spectral density of the received signal, the comparison is made in terms of autocorrelation by means of a correlation matching. Note that if there is interference from another secondary user, the feature detector would be able to distinguish the primary signal from the interfering system. Following the notation of (4), the sample autocorrelation matrix $\hat{\mathbf{R}}_y \in \mathbb{C}^{\kappa \times \kappa}$ can be obtained as,

$$\hat{\mathbf{R}}_y = \frac{1}{N_f} \sum_{f=1}^{N_f} \mathbf{y}_f \mathbf{y}_f^H \tag{5}$$

In order to obtain the frequency location of each primary user, the baseband reference autocorrelation $\mathbf{R}_b$ (extracted from spectral features of the primary user) is modulated by a rank-one matrix formed by the steering frequency vector at the sensed frequency ω as follows,

$$\mathbf{R}_c(\omega) = \left[\mathbf{R}_b \odot \mathbf{e}(\omega) \mathbf{e}^H(\omega) \right] \tag{6}$$

where $\odot$ denotes the elementwise product of two matrices and $\mathbf{e}(\omega) = \begin{bmatrix} 1 & e^{j\omega} & \ldots & e^{j(L-1)\omega} \end{bmatrix}^T$ is the steering vector.

According to this notation, the corresponding model for the sample autocorrelation matrix defined in (5) is given by,

$$\mathbf{R}_y = \sum_{k=1}^{K} \gamma(\omega_k) \mathbf{\Phi} \mathbf{R}_c(\omega_k) \mathbf{\Phi}^H + \mathbf{R}_{\boldsymbol{\epsilon}} \tag{7}$$

where $\mathbf{R}_{\boldsymbol{\epsilon}}$ represents the contribution of the sub-Nyquist-sampled interference and noise autocorrelation matrices and $\gamma(\omega_k)$ is the power level at frequency ω_k, which denotes the tentative frequency of the k-th active primary user.

The model in (7) can be conveniently rewritten into a sparse notation as follows,

$$\mathbf{r}_y = \mathbf{kron}(\mathbf{\Phi}, \mathbf{\Phi}) \mathbf{B} \mathbf{S} \mathbf{p} + \mathbf{r}_\epsilon = \mathbf{A}\mathbf{p} + \mathbf{r}_\epsilon \tag{8}$$

where $\mathbf{r}_y$ is a $\kappa^2 \times 1$ vector formed by the concatenation of the columns of $\mathbf{R}_y$. From now on, to clarify notation, the concatenation of columns will be denoted with the operator $\mathbf{vec}(\cdot)$. Therefore, $\mathbf{r}_y = \mathbf{vec}(\mathbf{R}_y)$. $\mathbf{B}$ contains the spectral information of the primary signals and is defined as diag$(\mathbf{r}_b)$ where $\mathbf{r}_b = \mathbf{vec}(\mathbf{R}_b)$. Matrix $\mathbf{S}$ defines the frequency scanning grid,

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}(\omega_0) & \mathbf{s}(\omega_1) & \cdots & \mathbf{s}(\omega_{M-1}) \end{bmatrix} \tag{9}$$

where $\mathbf{s}(\omega_m) = \mathbf{vec}(\mathbf{e}(\omega_m)\mathbf{e}^H(\omega_m))$. ω_m is defined as $\omega_m = \omega_0 + m\Delta_\omega$, $m = 0, \ldots, M-1$, where ω_0 and Δ_ω denote the lowest frequency in the bandwidth of interest, and the frequency resolution, respectively. The variable $\mathbf{r}_\epsilon$ encompasses interference and noise contribution.

Vector $\mathbf{p} = \begin{bmatrix} p(\omega_0) & p(\omega_1) & \cdots & p(\omega_{M-1}) \end{bmatrix}^T$ can be viewed as the output of an indicator function, whose elements different from zero correspond to the frequencies where the reference is present. Moreover, the values different from zero correspond to the power of each primary user that is present. This is,

$$p(\omega_m) = \begin{cases} \gamma(\omega_k) & \text{if } \omega_m = \omega_k \quad (H_1) \\ 0 & \text{otherwise} \quad (H_0) \end{cases} \tag{10}$$

Since $\kappa < L$, there are infinitely many solutions to the following problem,

$$\min_{\mathbf{p} \geq 0} \|\hat{\mathbf{r}}_y - \mathbf{A}\mathbf{p}\|_{l_2} \tag{11}$$

Among all the solutions of (11), we are interested in the solution that meets the following requirements: (1) $\mathbf{p}$ must be sparse, and (2) the solution must not include interference. The interference immunity is not achieved only with the spectral shape dictionary $\mathbf{A}$ because, although the spectral shapes of the license-holder users are assumed to be different from that of the opportunistic users, they might not be orthogonal.

4.1. Enforcing positive semidefinite residual correlation

Correlation matrices are hermitian positive-semidefinite matrices by definition. The set of autocorrelation matrices is a convex cone [15]. Thus, the difference between $\hat{\mathbf{R}}_y$ and the compressed frequency-shifted reference matrix $p(\omega_m)\mathbf{\Phi}\mathbf{R}_c(\omega_m)\mathbf{\Phi}^H$ must lie in the surface of the cone too. In other words, the residual matrix must be positive semi-definite. The problem can be formulated as,

$$\begin{aligned} \max_{p(\omega_m)\geq 0} \quad & p(\omega_m) \\ \text{s.t.} \quad & \hat{\mathbf{R}}_y - p(\omega_m)\mathbf{\Phi}\mathbf{R}_c(\omega_m)\mathbf{\Phi}^H \succeq 0 \end{aligned} \qquad (12)$$

and the solution is the maximum eigenvalue of $(\hat{\mathbf{R}}_y^{-1}(\mathbf{\Phi}\mathbf{R}_c(\omega_m)\mathbf{\Phi}^H))$, that is,

$$p(\omega_m) = \lambda_{max}^{-1}(\omega_m) \qquad m = 0, \dots, M-1 \qquad (13)$$

Note that the values $\lambda_{max}^{-1}(\omega_m)$ are coarse estimates of the primary user power. The meaning of (13) is that if the values of $p(\omega_m)$ are chosen lower than $\lambda_{max}^{-1}(\omega_m)$, the residual matrix will be a positive semidefinite matrix. Next, we show how to impose the positive semidefinite restriction together with the sparsity constraint.

The sparsity restriction is usually imposed by adding a l_1-norm constraint to the optimization problem [16]. Thus, a common restriction to impose sparsity in $\mathbf{p}$ is $\|\mathbf{p}\|_{l_1} \leq \beta$, where $\|\mathbf{p}\|_{l_1} = \sum_{m=0}^{M-1} |p(\omega_m)|$. The theoretical value of β is the summation of the primary users' power, i.e. $\beta = \sum_{k=1}^{K} \gamma(\omega_k)$. However, neither the number of primary users K nor the transmitted power $\gamma(\omega_k)$ are known a priori. In fact, they are unknowns to be determined by the spectrum sensing mechanisms.

The l_1-norm tends to penalized large coefficients to the detriment of smaller coefficients [17]. Weighted l_1-norm have been proposed to democratelly penalize nonzero entries. Let us consider the following weighted l_1-norm,

$$\sum_{m=0}^{M-1} w_m \cdot p(\omega_m) \leq \alpha \qquad (14)$$

where $w_0, \dots, w_{M-1}$ are positive weights. Note that the values of $p(\omega_m)$ must be greater than or equal to zero and, thus, the absolute value is removed for simplicity. The value of α depends on the chosen weights. One way to rectify the dependence on magnitude of the l_1-norm is to enforce each product $w_m \cdot p(\omega_m)$ be equal to 1. Thus, the weights must be estimates of the inverse power corresponding to the primary users present in the band under scrutiny. Ideally,

$$w(\omega_m) = \begin{cases} \frac{1}{\gamma(\omega_k)} & \text{if } \omega_m = \omega_k \quad (H_1) \\ 0 & \text{otherwise} \quad (H_0) \end{cases} \qquad (15)$$

Initial estimates of the powers can be obtained from the upperbound defined in (13),

$$w_m = \lambda_{max}(\omega_m) \qquad m = 0, \dots, M-1 \qquad (16)$$

With the weights defined in (16), the value of α in (14) is approximately equal to the number of primary users present in the band under scrutiny, K, which determines the sparsity level of vector $\mathbf{p}$. However, as mentioned earlier on this section, the value of K is unknown. The level of sparsity is typically unknown in CS problems even though it plays a fundamental role in solving the sparse vector recovery problem. In this paper, sparse reconstruction is performed using the iterative algorithm known as Weighted Orthogonal Matching Pursuit (WOMP) [18] which provides fast reconstruction with low computational complexity and is appropriate to the problem at hand. Since conventional CS assumes sparse signals corrupted by noise, robust stopping criteria for iterative reconstruction algorithms are usually based on information about the noise magnitude. Our model considers noise and interference (both unknown) and, as such, impedes the application of conventional stopping criteria.

The weights defined in (16) impose the licensed-holder users to be selected by WOMP before the interference. Interestingly, only when a licensed-holder user is selected, the weighted l_1-norm increases in one with respect to the previous iteration (see (14) and (16)). Consequently, after K iterations, the weighted l_1-norm is expected to be equal to K and the residual is expected to contain noise plus interference. We propose to stop the WOMP when the difference between the present weighted l_1-norm and the weighted l_1-norm of the previous iteration does not fall into the following interval,

$$1 - \xi_{down} \leq \Delta_{l_1}^{(t)} \leq 1 + \xi_{up} \qquad (17)$$

where $\Delta_{l_1}^{(t)} = \sum_{m=0}^{M-1} w_m \cdot p^{(t)}(\omega_m) - \sum_{m=0}^{M-1} w_m \cdot p^{(t-1)}(\omega_m)$, with $\mathbf{p}^{(t)}$ being the estimated sparse vector at iteration t. The values of $\xi_{down} \in [0, 1)$ and $\xi_{up} \in [0, \infty)$ determine the detector performance.

5. SIMULATION RESULTS

The performance of the proposed scheme is evaluated in this section. The spectral band under scrutiny has bandwidth equal to $f_{max} = 20$ MHz. The size of the observation $\mathbf{z}_m$ is $L = 33$ samples. The sampling rates of $\mathbf{y}_m$ and $\mathbf{z}_m$ are related through the compression rate $\rho = \frac{\kappa}{L}$. To strictly focus on the performance behavior due to compression and remove the effect of insufficient data records, the size of the compressed observations is forced to be the same for any compression rate. Therefore, we set $M = 2L\delta\rho^{-1}$ where δ is a constant (in the following results $\delta = 10$). Thus, for a high compression rate, the estimator takes samples for a larger period of time. The simulation parameters are summarized in Table 1. To test the ability of the proposed sparse spectral matching technique to properly label licensed users, we first consider a scenario with two primary user in the presence of noise and interference. The interference is included as a 10 dB carrier at frequency 7.5 MHz. The primary user is assumed to be a Binary Phase Shift Keying (BPSK) signal with a rectangular pulse shape and 8 samples per symbol. The SNR of the desired users are 10 dB and 7 dB, respectively, and their carrier frequencies are 2.5 MHz and 12.5 MHz, respectively. The spectral occupancy for this particular example is 0.5 (the primary users are

$\boldsymbol{\rho}$	**1**	**0.76**	**0.52**	**0.24**
$\boldsymbol{\kappa}$	33	25	17	8
M	660	871	1281	2723
Acquisition Time (ms)	1.1	1.4	2.1	4.5

Table 1. Simulation Parameters

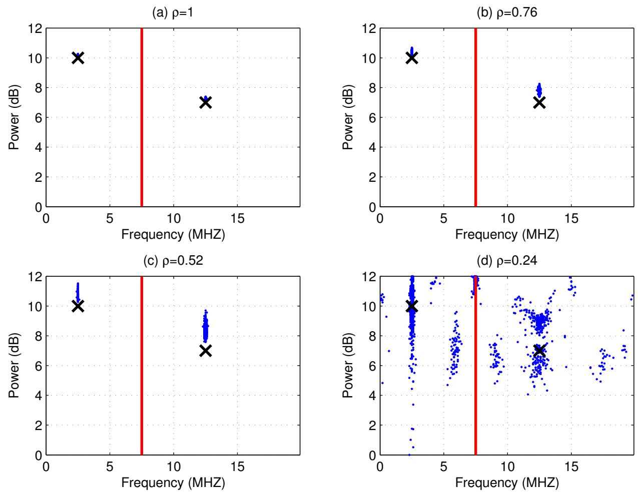

Fig. 1. Primary user detection with sparse spectral matching.

using half of the available spectrum). The values of ξ are chosen $\xi_{up} = 2$ and $\xi_{down} = 0.5$.

Fig. 1 shows the result of the proposed detector for 1000 Monte Carlo runs for different compression rates. Blue points indicate the output of the detector, black crosses represent the true primary users location and the red solid line indicate the interference location. From Fig. 1 we observe that, in agreement with the Landau's lower bound [19], the proposed technique works well until the compression exceeds the limit given by the spectral occupancy. When the compression rate surpass $\rho = 0.5$, the acquisition procedure loses part of the information, which translates into a degradation of the detector performance. The results obtained in Fig. 1 can be compared with the performance of the conventional periodogram spectral estimation. To this end, Fig. 2(a) shows the periodogram for the scenario under consideration for different compression rates. Besides presenting low resolution (becoming worse as the compression rate decreases), note also that they are not robust to the strong interference. In contrast, the proposed method provides a clear frequency and power estimate and makes the interference disappear because of their feature-based nature.

The interference rejection characteristic is linked to the reliability of the weights, which must provide a coarse estimate of the inverse transmitted power of the primary users. Fig. 2(b) shows the inverse of the weights for the scenario under consideration for different compression rates. As expected, the maximums of the inverse weights are located at the frequencies where a primary user is present and its value coincides with the primary users' SNR. However, the dynamic range of the weights significantly decrease when the compression rate surpass the limit of $\rho = 0.5$. High detection sensibility is a fundamental aspect in spectrum sensing for CR. To evaluate the probability of detection (P_d) versus SNR we have run 5000 simulations, each in the presence of the primary user (H1), and 5000 records of the same length with noise (H0). Fig. 3 shows the results for a primary user (BPSK) located at 10 MHz, free from interference, for a fixed probability of false alarm $P_{fa} = 10^{-3}$. It is clear that, even for low compression rates, the proposed sparsity-based primary user detection approach is able to reliably detect very low primary transmission. For comparison, we plot in the same figure the results for the energy detection without noise uncertainty and the performance of the coarse estimate given by the inverse of the

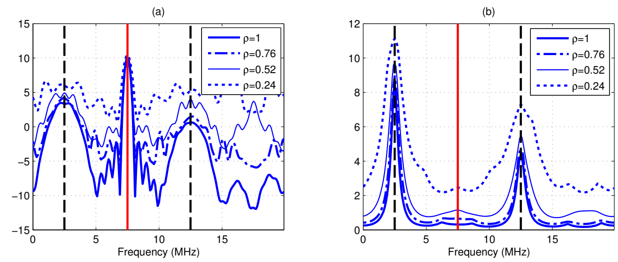

Fig. 2. (a) Periodogram and (b) Inverse of weights

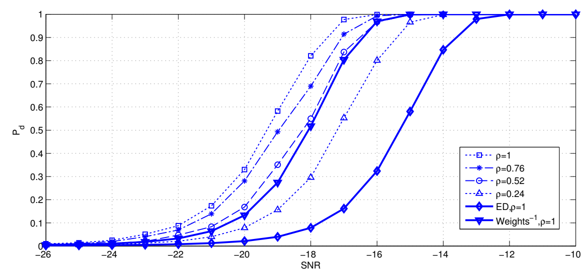

Fig. 3. Probability of detection versus SNR.

weights for the same scenario and for $\rho = 1$. As shown in the figure, the energy detection and the estimates given by the weights are much worse than that of the proposed technique.

6. CONCLUSION

This paper introduces a feature-based technique for primary user's spectrum identification with interference immunity which considers CS. Results based on computer simulations were presented, which showed the effectiveness in primary user detection and proved the interference rejection capabilities of the proposed method.

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