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Preference-based argumentation: Arguments supporting multiple values ☆

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Abstract

In preference-based argumentation theory, an argument may be preferred to another one when, for example, it is more specific, its beliefs have a higher probability or certainty, or it promotes a higher value. In this paper we generalize Bench-Capon's value-based argumentation theory such that arguments can promote multiple values, and preferences among values or arguments can be specified in various ways. We assume in addition that there is default knowledge about the preferences over the arguments, and we use an algorithm to derive the most likely preference order. In particular, we show how to use non-monotonic preference reasoning to compute preferences among arguments, and subsequently the acceptable arguments, from preferences among values. We show also how the preference ordering can be used to optimize the algorithm to construct the grounded extension by proceeding from most to least preferred arguments.
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1. Introduction

Dung's theory of abstract argumentation [14] is based only on a set and a binary relation defined over the set. Due to this abstract representation, it can and has been used in several ways, for example as a general framework for non-monotonic reasoning, as a framework for argumentation, and as a component in agent communication, dialogue, or decision making. Dung called the elements of his set *arguments* and elements of his relation represent that an argument *attacks* another argument. In this paper we follow the common convention in preference-based argumentation (see below) and say that in the abstract theory an argument *defeats* another argument.

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Simari and Loui [21] introduce preference relations over arguments, and various proposals have been made how to compute these preferences. Prakken and Sartor [20], among others [22], consider arguments composed of defeasible rules, and use the argument *structure* to compute preference relations among arguments, since being more specific about the evidence makes an argument stronger. Alternatively, several authors [9,2,1] build arguments from beliefs pervaded with explicit priorities, such as certainty levels. The arguments using more certain beliefs are stronger than arguments using less certain beliefs. Other authors argued that arguments can also be related to values they promote [19,12]. In particular Bench-Capon [5] does not consider the structure of arguments, but derives a preference ordering over arguments from a preference ordering over the values they promote. Since in his theory arguments promote only a single value, an argument is preferred to another one if and only if the value promoted by the former argument is preferred to the value promoted by the latter one.

In this paper we introduce a generalization of Bench-Capon's value-based argumentation theory [5] in which arguments can promote multiple values and preferences among values or arguments can be represented in various ways. Moreover we assume in addition that we have default knowledge about the preferences over the arguments, and use algorithms to derive the most likely preference order.

In particular, we represent preferences among values by distinguishing various kinds of preferences in the preference specification, and to reason about ordered values we use insights from the non-monotonic logic of preferences. When value v_1 is promoted by the arguments $A_1, \ldots, A_n$, and value v_2 is promoted by arguments $B_1, \ldots, B_m$, then "value v_1 is preferred to value v_2" means that the set of arguments $A_1, \ldots, A_n$ is preferred to the set of arguments $B_1, \ldots, B_m$. In other words, the problem of reducing ordered values to a preference relation among arguments comes down to reducing a preference relation over *sets* of arguments to a preference relation over *single* arguments. We use both minimal and maximal specificity principles to define the preference relation. To calculate the acceptable arguments we combine algorithms for reasoning about preferences with algorithms developed in argumentation theory.

The layout of this paper is as follows. In Section 2 we summarize the work on abstract argumentation, preference and value as far as relevant for this paper, and in Section 3 we present our extensions to Bench-Capon's value-based argumentation framework. In Section 4 we discuss non-monotonic reasoning with the preferences, we introduce algorithms for directly computing the set of acceptable arguments using grounded semantics, and we show how the algorithm can be optimized when proceeding from most to least preferred arguments. We present also an algorithm for ordering the arguments following a pessimistic way of reasoning. In Section 5 we discuss the case of inconsistent preferences over values in our approach. Finally we discuss related work and conclude.

2. Argument, preference and value

In this section we summarize the work on abstract argumentation, preference and value as far as relevant for this paper. We start with some common definitions on preference relations. Let $\succeq$ (respectively $\succ$) be a binary relation on a finite set $\mathscr{X} = \{x, y, z, \ldots\}$ such that $x \succeq y$ (respectively $x \succ y$) means that x is at least as preferred as (respectively strictly preferred to) y. $x \simeq y$ means that both $x \succeq y$ and $y \succeq x$ hold, i.e. x and y are equally preferred. Lastly $x \sim y$ means that neither $x \succeq y$ nor $y \succeq x$ holds, i.e. x and y are incomparable. $\succeq$ is a pre-order on $\mathscr{X}$ iff $\succeq$ is reflexive ($x \succeq x$) and transitive (if $x \succeq y$ and $y \succeq z$ then $x \succeq z$). A pre-order $\succeq$ on $\mathscr{X}$ is said to be total if and only if all pairs are comparable i.e. $\forall x, y \in \mathscr{X}$, we have $x \succeq y$ or $y \succeq x$. A strict order $\succ$ may be defined from a pre-order $\succeq$ as $x \succ y$ if $x \succeq y$ holds but $y \succeq x$ does not.

The set of the best (respectively worst) elements of A with respect to $\succeq$, denoted $\max(A, \succeq)$ (respectively $\min(A, \succeq)$), is defined by:

$$\max(\mathscr{X}, \succeq) = \{x | x \in \mathscr{X}, \nexists y \in \mathscr{X}, y \succ x\}$$
$$(\text{respectively } \min(\mathscr{X}, \succeq) = \{x | x \in \mathscr{X}, \nexists y \in \mathscr{X}, x \succ y\})$$

where $\succ$ is the strict order associated to $\succeq$.

Definition 1 illustrates how a *total* pre-order on $\mathcal{X}$ can also be represented in an equivalent way by a well ordered partition of $\mathcal{X}$. This equivalent representation as an ordered partition makes some definitions easier to read.

Definition 1 (*Ordered partition*). A sequence of subsets of $\mathcal{X}$ of the form $(E_0, \ldots, E_n)$ is an ordered partition of $\mathcal{X}$ if and only if each subset is non-empty, $E_i \neq \emptyset$ for $i = 0 \ldots n$, the union of the subsets is the set $\mathcal{X}$, $E_0 \cup \cdots \cup E_n = \mathcal{X}$, and the subsets are disjoint, $E_i \cap E_j = \emptyset$ for $i \neq j$.

An ordered partition $(E_0, \ldots, E_n)$ of $\mathcal{X}$ is associated with total pre-order $\succeq$ on $\mathcal{X}$ if and only if

$$\forall x, x' \in \mathcal{X} \text{ with } x \in E_i \text{ and } x' \in E_j \text{ we have } i \leqslant j \text{ if and only if } x \succeq x'$$

2.1. Dung's abstract argumentation framework

Argumentation is a reasoning model based on constructing arguments, determining potential conflicts between arguments and determining acceptable arguments.

Dung's framework [14] is based on a binary *defeat* relation among arguments (called the *attack* relation by Dung). In Dung's framework, an argument is an abstract entity whose role is determined only by its relation to other arguments. Its structure and its origin are not known.

We restrict ourselves to *finite* argumentation frameworks, i.e., in which the set of arguments $\mathcal{A}$ is *finite*.

Definition 2 (*Argumentation framework*). An argumentation framework is a tuple $\langle \mathcal{A}, \mathcal{D} \rangle$ where $\mathcal{A}$ is a finite set of arguments and $\mathcal{D}$ is a binary defeat relation defined on $\mathcal{A} \times \mathcal{A}$.

The various semantics of an argumentation framework are all based on the notion of defence. A set of arguments $\mathcal{S}$ defends an argument A when for each defeater B of A, there is an argument in $\mathcal{S}$ that defeats B.

Definition 3 (*Defence*). Let $\langle \mathcal{A}, \mathcal{D} \rangle$ be an argumentation framework. Let $\mathcal{S} \subseteq \mathcal{A}$. $\mathcal{S}$ defends A if $\forall B \in \mathcal{A}$ such that $B \mathcal{D} A$, $\exists C \in \mathcal{S}$ such that $C \mathcal{D} B$.

A semantics of an argumentation theory consists of a conflict-free set of arguments, i.e., a set of arguments that does not contain an argument defeating another argument in the set.

Definition 4 (*Conflict-free*). Let $\langle \mathcal{A}, \mathcal{D} \rangle$ be an argumentation framework. The set $\mathcal{S} \subseteq \mathcal{A}$ is conflict-free if and only if there are no $A, B \in \mathcal{S}$ such that $A \mathcal{D} B$.

The following definition summarizes the most widely used acceptability semantics of arguments given in the literature.

Definition 5 (*Acceptability semantics*). Let $\mathcal{F} = \langle \mathcal{A}, \mathcal{D} \rangle$ be an argumentation framework. Let $\mathcal{S} \subseteq \mathcal{A}$.

- $\mathcal{S}$ is an *admissible* extension if and only if it is conflict-free and defends all its elements.
- $\mathcal{S}$ is a *complete extension* if and only if it is conflict-free and it contains precisely all the elements it defends, $\mathcal{S} = \{A \mid \mathcal{S} \text{ defends } A\}$.
- $\mathcal{S}$ is a *grounded extension* of $\mathcal{F}$ if and only if $\mathcal{S}$ is the smallest (for set inclusion) complete extension of $\mathcal{F}$.
- $\mathcal{S}$ is a *preferred extension* of $\mathcal{F}$ if and only if $\mathcal{S}$ is maximal (for set inclusion) among admissible extensions of $\mathcal{F}$.
- $\mathcal{S}$ is a *stable extension* of $\mathcal{F}$ if and only if $\mathcal{S}$ is conflict-free and defeats all arguments of $\mathcal{A} \setminus \mathcal{S}$.

Which semantics is most appropriate in which circumstances depends on the application domain of the argumentation theory. The grounded semantics is the most basic one, in the sense that its conclusions are not controversial, each argumentation framework has a grounded extension (it may be the empty set), and this extension is unique. Grounded extensions therefore play an important role in the remainder of this paper. Preferred semantics is more credulous than the grounded semantics. There always exists at least one preferred extension but it does not have to be unique. Stable semantics have an intuitive appeal, but its drawbacks are that extensions do not have to be unique and do not have to exist. Stable semantics are used, for example, in answer set programming.

The output of $\langle \mathscr{A}, \mathscr{D} \rangle$ is derived from the set of selected acceptable arguments with respect to some acceptability semantics.

2.2. Preference-based argumentation framework

Preference-based argumentation theory can either be seen as an extension or as an instantiation of Dung's argumentation theory. In the former case there is besides the binary defeat relation also a preference ordering on the arguments, and the defeat and preference relation together are used to define new notions of acceptable arguments. In the latter case there is another binary relation – typically called the attack relation in preference-based argumentation – and a preference ordering from which Dung's defeat relation is derived. For example, Amgoud and Cayrol [1] define a preference relation on the set of arguments on the basis of the evaluation of arguments, and say that an argument A defeats an argument B if and only if A attacks B and B is not preferred to A with respect to the preference relation. In this paper we follow the latter approach.

Definition 6 (*Preference-based argumentation framework*). A preference-based argumentation framework is a triplet $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ where $\mathscr{A}$ is a set of arguments, $\mathscr{R}$ is a binary attack relation defined on $\mathscr{A} \times \mathscr{A}$ and $\succeq$ is a (total or partial) pre-order (preference relation) defined on $\mathscr{A} \times \mathscr{A}$.

A preference-based argumentation framework can represent an argumentation framework.

Definition 7. A preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ represents $\langle \mathscr{A}, \mathscr{D} \rangle$ if and only if

$$\forall A, B \in \mathscr{A}, \text{ we have } A\mathscr{D}B \text{ if and only if } A\mathscr{R}B \text{ and it is not the case that } B \succ A$$

There are always preference-based argumentation frameworks representing an argumentation framework and vice versa.

Lemma 1. *For each preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ there is an argumentation framework $\langle \mathscr{A}, \mathscr{D} \rangle$ it represents, and for each argumentation framework $\langle \mathscr{A}, \mathscr{D} \rangle$ there is a preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ that represents it.*

All proofs are given in the Appendix.

On the one hand each preference-based argumentation framework represents only one argumentation framework, but on the other hand each argumentation framework can be represented by various preference-based argumentation frameworks.

Lemma 2. *If preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ represents argumentation framework $\langle \mathscr{A}, \mathscr{D}_1 \rangle$ and argumentation framework $\langle \mathscr{A}, \mathscr{D}_2 \rangle$, then we have $\mathscr{D}_1 = \mathscr{D}_2$.*

Note however that if both $\langle \mathscr{A}, \mathscr{R}_1, \succeq_1 \rangle$ and $\langle \mathscr{A}, \mathscr{R}_2, \succeq_2 \rangle$ represent $\langle \mathscr{A}, \mathscr{D} \rangle$, then we do not necessarily have that $\mathscr{R}_1 = \mathscr{R}_2$ and $\succeq_1 = \succeq_2$. Let $\langle \mathscr{A}, \mathscr{D} \rangle$ be an argumentation framework such that $\mathscr{A} = \{A, B\}$ and $A\mathscr{D}B$. Let $\langle \mathscr{A}, \mathscr{R}_1, \succeq_1 \rangle$ and $\langle \mathscr{A}, \mathscr{R}_2, \succeq_2 \rangle$ with $A\mathscr{R}_1B$, $A \simeq_1 B$, $A\mathscr{R}_2B$, $B\mathscr{R}_2A$ and $A \succ_2 B$. Both $\langle \mathscr{A}, \mathscr{R}_1, \succeq_1 \rangle$ and $\langle \mathscr{A}, \mathscr{R}_2, \succeq_2 \rangle$ represent $\langle \mathscr{A}, \mathscr{D} \rangle$.

Summarizing, each preference-based argumentation framework represents precisely one argumentation framework, and each argumentation framework is represented by at least one but usually several preference-based argumentation frameworks.

Theorem 1. *For each preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ there is precisely one argumentation framework $\langle \mathscr{A}, \mathscr{D} \rangle$ it represents, and for each argumentation framework $\langle \mathscr{A}, \mathscr{D} \rangle$ there is at least one preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ that represents it.*

The following example illustrates how to compute an argumentation framework that represents a preference-based argumentation framework. We use this example as a running example in this paper, and extend it later on with values and value specifications.

Example 1. Let $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ be a preference-based argumentation framework visualized in Fig. 1 with $\mathscr{A} = \{A_0, \ldots, A_7\}$ be a set of arguments, read as follows:

A_0: "The soccer game is going to be very interesting, many people will watch the game because it is Barcelona against Arsenal,"
A_1: "One should not always do the same as ones friends, but also develop an idea oneself what is good to do,"
A_2: "I have to watch the game, because Barcelona has not been in the final for six years,"
A_3: "You better not watch the game, you have an exam tomorrow and you have to prepare it,"
A_4: "I have to watch the game, because I want to discuss it tomorrow with my friends,"
A_5: "You are not going to watch television tonight, because you have been looking television for three evenings in a row, and television is unhealthy, dumb and uninspiring,"
A_6: "The soccer game is going to be boring, because the first half was already won by 6-0,"
A_7: "Children must go to school. Education is important because it gives more options later on."

Moreover, let $\succeq$ be a total pre-order defined as follows:

$$A_0 \simeq A_3 \succ A_1 \simeq A_2 \simeq A_4 \simeq A_5 \succ A_6 \simeq A_7$$

and let $\mathscr{R}$ be an attack relation defined by: A_0 and A_6 attack each other, A_2 and A_3 attack each other, A_2 and A_5 attack each other, A_3 and A_4 attack each other, A_4 and A_5 attack each other. The preference-based argumentation framework is visualized in Fig. 1. This figure should be read as follows. An arrow from one argument to another visualizes that the former argument attacks the latter one.

An argument is preferred to another argument if the former argument is higher than the latter one.

Assume that $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ represents $\langle \mathscr{A}, \mathscr{D} \rangle$, then we have $A_0 \mathscr{D} A_6$, because $A_0 \mathscr{R} A_6$ and $A_6 \succ A_0$ does not hold. Similarly we have $A_3 \mathscr{D} A_2$, $A_2 \mathscr{D} A_5$, $A_5 \mathscr{D} A_2$, $A_3 \mathscr{D} A_4$, $A_4 \mathscr{D} A_5$ and $A_5 \mathscr{D} A_4$, and no other defeat relations hold.

The semantics of a preference-based argumentation framework is simply the semantics of the unique argumentation framework it represents. Due to the representation, we do not need to define new semantic notions.

Definition 8. A set of arguments is an admissible, grounded, preferred, or stable extension of a preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ if it is such an extension of the argumentation framework $\langle \mathscr{A}, \mathscr{D} \rangle$ represented by $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$.

The semantics is illustrated in our running example.

Example 2 (*Continued from Example* 1). The grounded extension of argumentation framework $\langle \mathscr{A}, \mathscr{D} \rangle$ and therefore of preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ is $\mathscr{GE} = \{A_0, A_1, A_3, A_5, A_7\}$.

2.3. Value-based argumentation framework

Bench-Capon [5] notices that in some situations, e.g., for persuasion dialogues, preferences over arguments are derived from points of views, called values, they promote. Atkinson *et al.* [4] discuss an example from political debate, where several arguments to invade Iraq are related to values such as respect for life, human rights, good world relations, and so on. If an argument promoting respect for life attacks an argument promoting good world relations then the attack succeeds only if good world relations is not preferred to respect for life.

Bench-Capon's value-based argumentation framework allows to compare *abstract* arguments without referring to their internal structure, which is an advantage if such information is not available. A set of

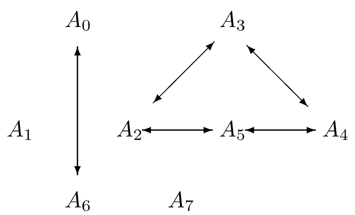

Fig. 1. Example 1: An arrow visualizes an attack, and higher arguments are preferred to lower ones. The represented defeat relation is: $A_0 \mathscr{D} A_6$, $A_3 \mathscr{D} A_2$, $A_2 \mathscr{D} A_5$, $A_5 \mathscr{D} A_2$, $A_3 \mathscr{D} A_4$, $A_4 \mathscr{D} A_5$ and $A_5 \mathscr{D} A_4$. The grounded extension is $\mathscr{GE} = \{A_0, A_1, A_3, A_5, A_7\}$.

audiences is introduced, following Perelman [18], where each audience corresponds to a preference ordering on values.

Definition 9 [5]. A value-based argumentation framework is a 5-tuple $\mathrm{VAF} = \langle \mathscr{A}, \mathscr{R}, V, val, \mathscr{P} \rangle$, where $\mathscr{A}$ is a finite set of arguments, $\mathscr{R}$ is an irreflexive binary relation on $\mathscr{A}$, V is a non-empty set of values, *val* is a function which maps elements of $\mathscr{A}$ to elements of V and $\mathscr{P}$ is the set of possible audiences. An audience specific argumentation framework is a tuple $\mathrm{VAF}_a = \langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$, where $a \in \mathscr{P}$ is an audience and $>_{\mathrm{a}}$ is a partial order on V.

Whereas Bench-Capon is primarily interested in the notion of value-based argumentation framework, in this paper we consider only the notion of audience specific argumentation frameworks.

Definition 10. $\langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$ represents $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ if and only if ($\forall A, B \in \mathscr{A}$, we have $A \succeq B$ if and only if $val(A) >_a val(B)$ or $val(A) = val(B)$).

Concerning existence of audience specific value-based argumentation frameworks representing a preference-based argumentation framework, the situation is the same as between preference-based argumentation frameworks and argumentation frameworks. So there exist audience specific value-based argumentation frameworks representing each preference-based argumentation framework and vice versa.

Lemma 3. *For each audience specific value-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$ there is a preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ it represents, and for each preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ there is an audience specific value-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$ that represents it.*

On the one hand each audience specific value-based argumentation framework represents only one preference-based argumentation framework, but on the other hand each preference-based argumentation framework can be represented by various audience specific value-based argumentation frameworks. However, each of these audience specific value-based argumentation frameworks has the same topology, that is, each of them is a renaming of the others.

Lemma 4. *If $\langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$ represents $\langle \mathscr{A}, \mathscr{R}, \succeq_1 \rangle$ and $\langle \mathscr{A}, \mathscr{R}, \succeq_2 \rangle$, then $\succeq_1 = \succeq_2$. If both $\langle \mathscr{A}, \mathscr{R}, V_1, val_1, >_{a,1} \rangle$ and $\langle \mathscr{A}, \mathscr{R}, V_2, val_2, >_{a,2} \rangle$ represent $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$, then we have:*

(1) *for all $A, B \in \mathscr{A}$ we have $val_1(A) = val_1(B)$ if and only if we have $val_2(A) = val_2(B)$*
(2) *for all $A, B \in \mathscr{A}$ we have $val_1(A) >_{a,1} val_1(B)$ if and only if we have $val_2(A) >_{a,2} val_2(B)$.*

Summarizing, each audience specific value-based argumentation framework represents precisely one preference-based argumentation framework, and each preference-based argumentation framework is represented by an equivalence class of alphabetic variants of audience specific value-based argumentation frameworks.

Theorem 2. *For each audience specific value-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$ there is precisely one preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ it represents, and for each preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ there is a set of audience specific value-based argumentation frameworks $\langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$ that represents it, satisfying the property in Lemma 4.*

The relation between the three kinds of argumentation frameworks shown in Theorems 1 and 2 is visualized in Fig. 2. An argumentation framework can be represented by various preference-based argumentation frameworks, which can again be represented by various audience specific value-based argumentation frameworks. Given an audience specific value-based argumentation framework, there is a unique preference-based argumentation framework it represents, and a unique argumentation framework it represents.

The semantics of an audience specific value-based argumentation framework is again simply the semantics of the unique preference-based argumentation framework it represents.

Definition 11. A set of arguments is an admissible, grounded, preferred or stable extension of an audience specific value-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$ if it is such an extension of the preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ represented by $\langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$.

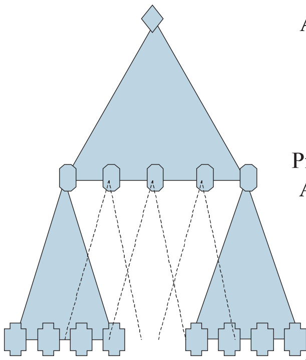

Fig. 2. Hierarchy of representation relations.

The semantics is illustrated in our running example. It is important to note that $\mathscr{R}$ is independent of *val*, in the sense that two arguments promoting the same value may conflict.

Example 3 (*Continued from Example 2*). Let $\langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$ be a value-based argumentation theory with $\mathscr{A}$ and $\mathscr{R}$ as before, and $V = \{red, yellow, blue\}$, $val(A_0) = val(A_3) = red$, $val(A_1) = val(A_2) = val(A_5) = val(A_4) = yellow$ and $val(A_6) = val(A_7) = blue$, and $red >_a yellow >_a blue$. We leave it to the reader that this value-based argumentation theory is represented by the preference-based argumentation theory $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ in Example 1.

The grounded extension of $\langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$ is the grounded extension of $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ and therefore, as shown in Example 2, $\mathscr{GE} = \{A_0, A_1, A_3, A_5, A_7\}$.

Moreover, Bench-Capon defines the notions of objective acceptance for all possible audiences and subjective acceptance for a particular audience. The main insight from the theory is that some arguments can be objectively acceptable regardless of the preference ordering of the audiences, simply due to the structure of the values. These cases are formally characterized in [5].

2.4. Total pre-orders

In this paper we are in particular interested in preference-based argumentation frameworks in which the preference relation is a total pre-order, i.e., in which the preference relation is connected. In such a case it immediately follows from Definition 7 that we have $A \mathscr{D} B$ if and only if $A \mathscr{R} B$ and $A \succeq B$. At a given stage of the reasoning, $>_a$ may be expressed as a partial order, since it is not necessary to commit to all preferences. However, if arguments need to be compared, $>_a$ must be a total order in the sense that commitment must be made, the audience becomes more specific. A total order on values corresponds directly to a total pre-order on arguments, since an argument is preferred to another one if the value promoted by the former is preferred to the value promoted by the latter argument, and the two arguments are equally preferred if they promote the same value.

A natural question to ask now is what happens with the representation results in Theorems 1 and 2 as visualized in Fig. 2 when we consider total pre-orders only. For example, elsewhere we have shown that the theorems no longer hold when we consider only symmetric attack relations [16] (The attack relation is symmetric in the running example, but obviously this is not necessarily always the case). The question can thus be raised whether the same consequence holds for the restriction to total pre-orders. However, the following two theorems show that this is not the case.

Theorem 3. *For each preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ with $\succeq$ a total pre-order there is precisely one argumentation framework $\langle \mathscr{A}, \mathscr{D} \rangle$ it represents, and for each argumentation framework $\langle \mathscr{A}, \mathscr{D} \rangle$ there is at least one preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ with $\succeq$ a total pre-order that represents it.*

Theorem 4. *For each audience specific value-based argumentation framework with $>_a$ a total order there is precisely one preference-based argumentation framework with $\succeq$ a total pre-order it represents, and for each preference-based argumentation framework with $\succeq$ a total pre-order there is a set of audience specific value-based argumentation frameworks with $>_a$ a total order that represents it, satisfying the property in Lemma 4.*

Summarizing, the restriction to total pre-orders is not a limitation of the expressive power of audience specific value-based argumentation, in the sense that the same class of argumentation frameworks can be represented. Moreover, audience specific value-based argumentation frameworks as defined by Bench-Capon may be seen as an equivalent representation as preference-based argumentation frameworks, as each preference-based argumentation framework is represented by precisely one audience specific value-based argumentation framework (under renaming of the values). We emphasize that this property holds only for audience specific argumentation frameworks, and there is no analogue to the general value-based argumentation frameworks defined by Bench-Capon in preference-based argumentation theory (and therefore no analogue to objective and subjective acceptance).

3. A new value-based argumentation theory and value specification

We introduce two extensions for value-based argumentation: arguments promoting multiple values, and various kinds of preferences among values.

3.1. Arguments promoting multiple values

To represent that arguments can promote multiple values, we replace the *val* function in Definition 9 by the *arg* function in Definition 12 below.

Definition 12 (*Value-based argumentation framework*). An audience specific value-based argumentation framework in which arguments can promote multiple values is a 5-tuple $\langle \mathscr{A}, \mathscr{R}, V, arg, >_a \rangle$ where $\mathscr{A}$ is a set of arguments, $\mathscr{R}$ is an attack relation on $\mathscr{A} \times \mathscr{A}$, V is a set of values, arg is a function from V to $2^{\mathscr{A}}$ such that $arg(v)$ is the set of arguments promoting the value v, and $>_a$ is a partial order on V.

Note that, in contrast to Bench-Capon's framework, it may be the case that an argument does not promote any value.

The following example illustrates that we can no longer say that an argument A defeats an argument B if and only if argument A attacks argument B and the value promoted by argument B is not preferred to the value promoted by argument A, as follows from Definitions 7 and 10, since there may be several values promoted by argument A and B.

Example 4. We first consider an example of Bench-Capon [5] where each argument promotes exactly one value. Let $\mathscr{A} = \{A, B, C\}$, $V = \{red, blue\}$ with $val(A) = red$ and $val(B) = val(C) = blue$. In our value-based argumentation framework, we have $arg(red) = \{A\}$ and $arg(blue) = \{B, C\}$. Let $A\mathscr{R}B$ and $B\mathscr{R}C$, as visualized in Fig. 3a and b. If $red > blue$, then $A\mathscr{D}B$ and $B\mathscr{D}C$ and the grounded extension is $\{A, C\}$. If $blue > red$, as visualized in Fig. 3b, then $B\mathscr{D}C$ and the grounded extension is $\{A, B\}$. Note that A is always in the grounded

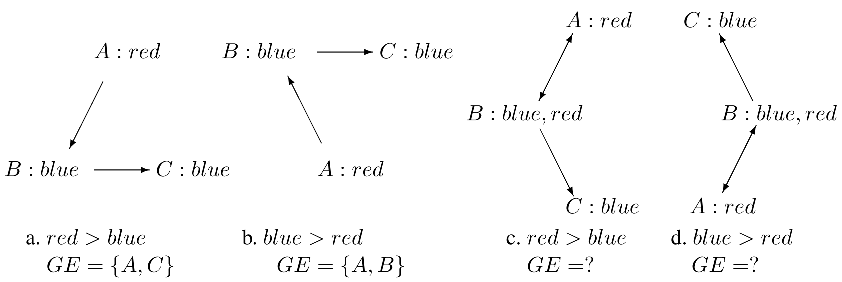

Fig. 3. Example 4.

extension, regardless of the value ordering. It is objectively accepted in Bench-Capon's theory, because he does not consider cases in which values are incomparable or equally preferred.

Now consider the case in which $arg(red) = \{A, B\}$, and moreover that argument B attacks argument A, as visualized in Fig. 3c and d, such that argument B promotes both values. Do we have that argument A is in all grounded extensions? Then the question is how to define the defeat relation since argument B promotes both *red* and *blue*. Should we give a preference to argument B since its promotes both values while argument A promotes only one value?

The problem is that the ordering on values is now an ordering on sets of arguments. There are many ad hoc solutions to the problem, such as taking the preferred value promoted by an argument. Before we consider our solution to this problem, we first consider another generalization of Bench-Capon's theory.

3.2. Preference specification

Since most languages for preferences and non-monotonic reasoning algorithms are based on total pre-orders, from here on we restrict ourselves to total pre-orders. As shown in Theorems 3 and 4, this does not restrict the expressive power of preference-based argumentation in the sense that still all argumentation frameworks can be represented. Instead of a partial order $>$ on values we use a relation $\gg$ which is neither necessarily transitive nor irreflexive. We therefore call it a preference *specification* rather than a preference *relation*. $\gg$ is to be seen as a set of preference statements of the kind $v_1 \gg v_2$ for value v_1 is preferred to value v_2. In Bench-Capon's framework, a preference of a value v_1 over a value v_2 implicitly means that *each* argument promoting v_1 is preferred to *each* argument promoting v_2. However, this is not the only way to compare arguments promoting v_1 and arguments promoting v_2. For example, an agent may be satisfied if at least one argument promoting human rights is preferred to all arguments promoting good world relations, or if at least one argument promoting respect for life is preferred to at least one argument promoting good world relations. The following definition presents three kinds of preference discussed in the literature [17,8,10,7].

Definition 13 (*Preference types*). Let $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ be a preference-based argumentation framework, V a set of values, *arg* a function from values to sets of arguments, and v_1 and v_2 two values.

- $\succeq$ satisfies $v_1 \gg_{mM} v_2$, v_1 is minmax preferred to v_2, if and only if $\forall A \in \min(arg(v_1), \succeq), \forall B \in \max(arg(v_2), \succeq)$ we have $A \succ B$.
- $\succeq$ satisfies $v_1 \gg_{MM} v_2$, v_1 is maxmax preferred to v_2, if and only if $\forall A \in \max(arg(v_1), \succeq), \forall B \in \max(arg(v_2), \succeq)$ we have $A \succ B$.
- $\succeq$ satisfies $v_1 \gg_{mm} v_2$, v_1 is minmin preferred to v_2, if and only if $\forall A \in \min(arg(v_1), \succeq), \forall B \in \min(arg(v_2), \succeq)$ we have $A \succ B$.

Definition 14 (*Preference consistency*). A set of preferences $\gg$ is *consistent* only if there is a total pre-order $\succeq$ which satisfies each $v_i \gg_{\rhd} v_j$ in $\gg$, with $\rhd \in \{mM, MM, mm\}$. $\succeq$ is a *model* of $\gg$ if and only if $\succeq$ satisfies each $v_i \gg_{\rhd} v_j$ in $\gg$.

In natural language, the above preferences are interpreted as follows:

- $\succeq$ satisfies $v_1 \gg_{mM} v_2$ if and only if *each* argument in v_1 is preferred to *each* argument in v_2 with respect to $\succeq$,
- $\succeq$ satisfies $v_1 \gg_{MM} v_2$ if and only if *at least* one argument in v_1 is preferred to *each* argument in v_2 with respect to $\succeq$,
- $\succeq$ satisfies $v_1 \gg_{mm} v_2$ if and only if *each* argument in v_1 is preferred to *at least* one argument in v_2 with respect to $\succeq$.

We do not consider maxmin preferences of the form $v_1 \gg_{Mm} v_2$, defined in the obvious way, since we are interested in distinguished total pre-orders on $\mathscr{A}$ according to the specificity principle (see the next section), and thus far no extensions of the specificity principle have been proposed which cover preferences of the type $v_1 \gg_{Mm} v_2$ (see [15] for a discussion).

Minmax preferences can be reduced to either only maxmax or only minmin preferences.

Lemma 5. *Assume without loss of generality that for each argument A there is a value v with $arg(v) = \{A\}$, which we call v_A. We have that $\succeq$ satisfies $v_1 \gg_{mM} v_2$ if and only if for each $A \in arg(v_1)$ ($A \in arg(v_2)$), $\succeq$ satisfies $v_A \gg_{MM} v_2$ ($v_1 \gg_{mm} v_A$).*

$\gg$ is interpreted as a compact specification of $\succeq$, and we therefore call our framework a *value-specification argumentation framework*. Since minmax preferences can be reduced into either maxmax or minmin preferences, we focus on maxmax and minmin preferences. They may be interpreted as two audiences in Bench-capon's framework, though we do not further discuss this interpretation in this paper, because we do not consider notions like objective and subjective acceptance.

Definition 15 (*Value-specification*). A value-specification argumentation framework is a 5-tuple $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd} \rangle$ where $\mathscr{A}$ is a set of arguments, $\mathscr{R}$ is an attack relation on $\mathscr{A} \times \mathscr{A}$, V is a set of values, arg is a function from V to $2^{\mathscr{A}}$, and $\gg_{\rhd} \subseteq V \times V$ is a set of preference statements over V with $\rhd \in \{MM, mm\}$.

We now address the problem discussed in Example 4, that we can no longer say that an argument A defeats an argument B if and only if A attacks B and the value promoted by B is not preferred to the value promoted by argument A. Our framework requires to *first* order the set of all arguments before we determine the defeat relation. We construct a total pre-order on $\mathscr{A}$ given a set of preferences over values using the minimal/maximal specificity principle [23].

Definition 16 (*Minimal/Maximal specificity principle*). Let $\succeq = (E_0, \ldots, E_n)$ and $\succeq' = (E'_0, \ldots, E'_m)$ be two total pre-orders on $\mathscr{A}$. $\succeq$ is less specific than or equally specific as $\succeq'$, written as $\succeq \sqsubseteq \succeq'$, if and only if $\forall A \in \mathscr{A}$, if $A \in E_i$ and $A \in E'_j$ then $i \leqslant j$.

$\succeq$ belongs to the set of the least (respectively most) specific pre-orders among a set of pre-orders $\mathscr{O}$ if there is no $\succeq'$ in $\mathscr{O}$ such that $\succeq' \sqsubset \succeq$ (respectively $\succeq \sqsubset \succeq'$), i.e., $\succeq' \sqsubseteq \succeq$ holds but $\succeq \sqsubseteq \succeq'$ does not (respectively $\succeq \sqsubseteq \succeq'$ holds but $\succeq' \sqsubseteq \succeq$ does not).

It is well known that there exists a unique least specific model for maxmax preferences and a unique most specific model for minmin preferences [17,7]. A unique most (respectively least) specific model for the former (respectively latter) does not necessarily exist.

We use a value-specification argumentation framework to represent a preference-based argumentation framework rather than a value-based argumentation framework, because of the close relationship between preference-based and value-based argumentation as shown in Theorems 2 and 4. Note that the following definition consists of two steps. First we define when a preference-based argumentation framework satisfies a value-specification argumentation framework. Second, for a value-based argumentation framework, we define the represented preference-based argumentation framework as the most specific one among the ones that satisfy the value-based argumentation framework.

Definition 17. $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ satisfies $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd} \rangle$ if and only if $\succeq$ satisfies each $v_i \gg_{\rhd} v_j$ in $\gg_{\rhd}$. $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd} \rangle$ represents $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ if and only if $\rhd = MM$ ($\rhd = mm$) and $\succeq$ is the least (most) specific relation among the $\succeq'$ such that $\langle \mathscr{A}, \mathscr{R}, \succeq' \rangle$ satisfies $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd} \rangle$.

Concerning existence of value-specification argumentation frameworks representing a preference-based argumentation framework, the situation is different from before. There exist value-specification argumentation frameworks representing each preference-based argumentation framework, but not necessarily vice versa.

Lemma 6. *For each preference-based argumentation framework there is a value-specification argumentation framework that represents it.*

There are value-specification argumentation frameworks without a preference-based argumentation framework it represents. Any value-specification argumentation framework with an inconsistent set of preferences cannot be represented by a preference-based argumentation framework.

As before, on the one hand each value-specification argumentation framework represents at most one preference-based argumentation framework, but on the other hand each preference-based argumentation framework can be represented by various value-specification argumentation frameworks.

Lemma 7. *If $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd} \rangle$ represents $\langle \mathscr{A}, \mathscr{R}, \succeq_1 \rangle$ and $\langle \mathscr{A}, \mathscr{R}, \succeq_2 \rangle$, then we have $\succeq_1 = \succeq_2$.*

Note that if both $\langle \mathscr{A}, \mathscr{R}, V_1, arg_1, \gg_{\rhd,1}\rangle$ and $\langle \mathscr{A}, \mathscr{R}, V_2, arg_2, \gg_{\rhd,2}\rangle$ represent $\langle \mathscr{A}, \mathscr{R}, \succeq\rangle$, then we do not necessarily have that the property of Lemma 4 holds. Consider the set of preferences $\{\{A,B\} \gg_{MM} \{C\}\}$ and the set of preferences $\{\{A\} \gg_{MM} \{C\}, \{B\} \gg_{MM} \{C\}\}$. They have the same minimal model, and therefore they represent the same preference-based argumentation theory.

Summarizing, each value-specification argumentation framework represents at most one preference-based argumentation framework, whereas each preference-based argumentation framework is represented by at least one but usually several value-specification argumentation frameworks. These value-specification argumentation frameworks are not necessarily renamings of each other.

Theorem 5. *For each value-specification argumentation framework there is at most one preference-based argumentation framework it represents, and for each preference-based argumentation framework there is at least one value-specification argumentation framework that represents it.*

In other words, the hierarchy of representation relations in Fig. 2 also holds for value specification.

Example 5 (*Continued from Example* 3). Let the tuple $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd}\rangle$ be a value-specification argumentation framework defined by the set of arguments $\mathscr{A} = \{A_0, A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$, and the attack relation $\mathscr{R}$ given in Example 1, $V = \{v_1, v_2, v_3, v_4, v_5, v_6\} = \{health, short\ term\ pleasure, education, enjoy, social, alone\}$ with $arg(v_1) = \{A_5\}$, $arg(v_2) = \{A_6, A_7\}$, $arg(v_3) = \{A_3, A_7\}$, $arg(v_4) = \{A_2, A_4\}$, $arg(v_5) = \{A_0, A_4\}$, $arg(v_6) = \{A_1, A_5\}$, and $\gg_{MM} = \{v_1 \gg_{MM} v_2, v_3 \gg_{MM} v_4, v_5 \gg_{MM} v_6\}$.

Let $\succeq$ be a total pre-order on $\mathscr{A}$ defined by

$$A_0 \simeq A_3 \succ A_1 \simeq A_2 \simeq A_4 \simeq A_5 \succ A_6 \simeq A_7$$

We can check that $\succeq$ satisfies each preference in $\gg_{MM}$. Then $\langle \mathscr{A}, \mathscr{R}, \succeq\rangle$ satisfies $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM}\rangle$. We can also check that $\succeq$ is the least specific relation among $\succeq'$ such that $\langle \mathscr{A}, \mathscr{R}, \succeq'\rangle$ satisfies $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM}\rangle$ so $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM}\rangle$ represents $\langle \mathscr{A}, \mathscr{R}, \succeq\rangle$.

Note that a value-specification argumentation framework that represents a preference-based argumentation framework exists only if $\gg_{\rhd}$ is consistent.

The semantics of a value-specification argumentation framework is simply the semantics of the unique preference-based argumentation framework it represents, if such a framework exists. Otherwise its semantics is discussed in Section 5.

Definition 18. A set of arguments is an admissible, grounded, preferred or stable extension of a value-specification argumentation framework $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd}\rangle$ if it is such an extension of the preference-based argumentation framework represented by $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd}\rangle$.

The new theory is illustrated in the following section using new algorithms for the framework.

4. Algorithms

We can compute the grounded extension of a value-specification argumentation framework in the following two steps. Given a value-specification argumentation framework, in the first step we compute the preference-based argumentation framework it represents using algorithms for minimal and maximal specificity developed in non-monotonic logic (Definition 17). Since the existence of a preference-based argumentation framework that is represented by a value-specification argumentation framework requires the consistency of $\gg_{\rhd}$, we suppose that $\gg_{\rhd}$ is consistent. We discuss the case of inconsistent $\gg_{\rhd}$ in Section 5.

Given the preference-based argumentation framework, in the second step we compute the represented argumentation framework using Definition 7. Finally we can compute the grounded extension of the argumentation framework. In the remainder of this paper we study whether this process can be optimized.

Let us analyze how least and most specific models are computed. Let $v_1 \gg_{\rhd} v_2$. The least specific model associated with $\{v_1 \gg_{MM} v_2\}$ is $\succeq = (E_1, E_2)$ with $E_1 = \mathscr{A} \setminus arg(v_2)$ and $E_2 = arg(v_2)$. The most specific model associated with $\{v_1 \gg_{mm} v_2\}$ is $\succeq' = (E'_1, E'_2)$ with $E'_1 = arg(v_1)$ and $E'_2 = \mathscr{A} \setminus arg(v_1)$. Both models prefer $arg(v_1)$ over $arg(v_2)$. However they treat irrelevant arguments with respect to these preferences, i.e. $\mathscr{A} \setminus (arg(v_1) \cup arg(v_2))$, in an opposite way. The least specific model prefers such arguments to $arg(v_2)$; this is an optimistic way of reasoning, while the most specific models considers that such arguments cannot be put in the same level as preferred arguments, i.e. $arg(v_1)$, so they are put in the same level as $arg(v_2)$; this is a pessimistic way of reasoning.

In general, given a set of preferences, in the least specific model each argument is put in the highest possible rank while in the most specific model each argument is put in the lowest possible rank in the pre-order. It is worth noticing that optimistic and pessimistic adjectives refer to the way the total pre-orders are computed and by no way mean that the most specific pre-order (corresponding to a pessimistic reasoning) gives less output than the least specific pre-order (corresponding to an optimistic reasoning). This is shown in the following example.

Example 6. Let $\mathscr{A} = \{A, B, C\}$, $V = \{v_1, v_2, v_3\}$ with $arg(v_1) = \{A\}$, $arg(v_2) = \{B\}$ and $arg(v_3) = \{C\}$. Let $C\mathscr{R}B$ and $B\mathscr{R}C$. Let $v_1 \gg_{\rhd} v_2$. Following the optimistic reasoning the least specific pre-order satisfying $\{v_1 \gg_{MM} v_2\}$ is $\succeq = (\{A, C\}, \{B\})$, as visualized in Fig. 4a. We can check that each argument is put in the highest possible rank in $\succeq$ such that $v_1 \gg_{MM} v_2$ is satisfied. So we have $C\mathscr{D}B$. The grounded extension is $\{A, C\}$. Now following the pessimistic reasoning the most specific pre-order satisfying $\{v_1 \gg_{mm} v_2\}$ is $\succeq' = (\{A\}, \{B, C\})$, as visualized in Fig. 4b. Here also we can check that each argument is put in the lowest possible rank in $\succeq'$ such that $v_1 \gg_{mm} v_2$ is satisfied. In this case we have $B\mathscr{D}C$ and $C\mathscr{D}B$. The grounded extension is $\{A\}$.

In addition to the above attack relations we give $A\mathscr{R}C$ and $C\mathscr{R}A$. The optimistic reasoning gives an empty grounded extension, as visualized in Fig. 4c, while the pessimistic reasoning gives $\{A, B\}$, as visualized in Fig. 4d.

Let us now consider the following attack relations $A\mathscr{R}C$ and $C\mathscr{R}A$ *only*. Then the grounded extension following the optimistic reasoning is $\{B\}$, as visualized in Fig. 4e, while the grounded extension following the pessimistic reasoning is $\{A, B\}$, as visualized in Fig. 4f.

Thus, the two kinds of reasonings are incomparable, in the sense that in some cases optimistic reasoning gives a larger grounded extension, in other cases it gives a smaller extension.

4.1. Grounded extension in optimistic reasoning

Algorithms of optimistic reasoning obey System Z [17] and compute the total pre-order $\succeq$ starting from the *best* arguments with respect to $\succeq$. This is a very nice property as it allows to compute incrementally the grounded extension during the computation of this pre-order. Let $\succeq = (E_0, \ldots, E_n)$. We first compute E_0. Then some arguments of E_0 will belong to the grounded extension and the remaining arguments of E_0 will not.

Lemma 8. *Let $\mathscr{GE}$ be the grounded extension of $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$, and let $\succeq = (E_0, \ldots, E_n)$ be the least specific model of $\gg_{MM}$.*

(1) *$\forall A \in E_0$ if $\nexists B \in E_0$ such that $B\mathscr{R}A$ then $A \in \mathscr{GE}$.*
(2) *Let S_1 be a subset of $\mathscr{GE}$. $\forall A \in E_0$, if $\exists B \in E_0$ such that $B\mathscr{R}A$ and A is defended by S_1 then $A \in \mathscr{GE}$.*
(3) *Let $S_2 \subseteq \mathscr{GE}$ be the minimal subset of E_0 satisfying the conditions in items* 1 *and* 2. *Then $\forall A \in E_0 \setminus S_2$, $A \notin \mathscr{GE}$.*

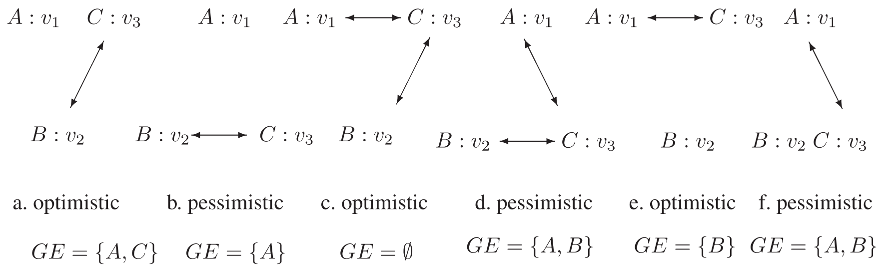

Fig. 4. Example 6.

Item 1 requires that $\mathscr{GE}$ contains all arguments of E_0 that are non-attacked at all. It also puts in $\mathscr{GE}$ arguments of E_0 which are attacked only by arguments from $\mathscr{A} \setminus E_0$. Such attacks do not succeed since the attackers are strictly less preferred than the attacked arguments, that is, these attackers are not defeaters in the represented argumentation theory. S_1 corresponds to $Safe(E_0)$ in Algorithm 1; $\Sigma = \emptyset$ for E_0. It is computed in line 4.

Item 2 requires that $\mathscr{GE}$ contains elements of E_0 that are attacked by elements of E_0 but defended by acceptable arguments of E_0, those already put in $\mathscr{GE}$ following item 1. This is done in loop 6 where $Acceptable_{\mathscr{GE}}(E_0)$ computes the set of arguments of E_0 defended by acceptable arguments; $\Sigma = \emptyset$ for E_0.

Finally, item 3 states that arguments of E_0 which are defeated and not defended do not belong to $\mathscr{GE}$ and can be removed from $\mathscr{A}$. This is treated in line 7 of Algorithm 1.

Lemma 9 states that arguments of $\mathscr{A} \setminus E_0$ attacked by arguments of $\mathscr{GE}$ do not belong to the grounded extension. This is treated in line 8.

Lemma 9. *Let $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$ be a value-specification argumentation framework and $\mathscr{GE}$ be its grounded extension. Let $\succeq = (E_0, \ldots, E_n)$ be the least specific model of $\gg_{MM}$. Then,*

$$\forall A \in \mathscr{A} \setminus E_0,\ \text{if } \exists B \in \mathscr{GE} \text{ such that } B\mathscr{R}A \text{ then } A \notin \mathscr{GE}$$

Following Pearl's algorithm [17], once $\mathscr{A}$ updated we remove satisfied preferences. $v_1 \gg_{MM} v_2$ is satisfied if at least one argument in $arg(v_1)$ belongs to E_0. This is treated in line 9.

The next step is to compute the set of immediately preferred arguments in $\succeq$, i.e., E_1. We repeat the same steps as for E_0. The only difference is that the set Σ may be non-empty. In the first loop of the algorithm (after computing E_0 in line 2), Σ is composed of arguments of E_0 which do not belong to $\mathscr{GE}$. Suppose now that we computed E_1 and are looking to which arguments of E_1 will belong to $\mathscr{GE}$. Since $Safe(E_1)$ is the set of arguments of E_1 which are not defeated, we should check whether some arguments of E_0 attack arguments of E_1. If it is the case, then $Acceptable_{\mathscr{GE}}(E_1)$ determines which arguments of E_1 are defended by the current grounded extension. Σ is computed in line 10 of the algorithm. The role of Σ is illustrated in the following example.

Example 7. Let $\mathscr{A} = \{A, B, C, D\}$ and $v_1 \gg_{MM} v_2$ with $arg(v_1) = \{A, B, C\}$ and $arg(v_2) = \{D\}$. Let $B\mathscr{R}C$, $C\mathscr{R}B$ and $B\mathscr{R}D$. We have $E_0 = \{A, B, C\}$. A belongs to the grounded extension while B and C do not (since they defeat each other). Following the algorithm we update $\mathscr{A}$ and get $\mathscr{A} = \{D\}$. At this stage it is important to keep B and C in a set, let's say Σ. The reason is that in the second iteration of the algorithm we should not put D in the grounded extension just because it is not defeated by A. In fact D is defeated by B and not defended by A. This justifies why we consider $E_l \cup \Sigma$ when computing $Safe(E_l)$ and $Acceptable_{\mathscr{GE}}(E_l)$.

The above reasoning is repeated until the set of arguments is empty.

Lemma 10. *Let $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$ be a value-specification argumentation framework and $\mathscr{GE}$ be its grounded extension.*

For each E_k, $(k \neq 0)$:

Let $\mathscr{A}$ be the set of arguments after iteration $k-1$ and $\Sigma = (E_0 \cup \cdots \cup E_{k-1}) \setminus \mathscr{GE}$. Then,

- *$\forall A \in E_k$, if $\nexists B \in (E_k \cup \Sigma)$ such that $B\mathscr{R}A$ then $A \in \mathscr{GE}$.*
- *Let S_1 be a subset of $\mathscr{GE}$. $\forall A \in E_k$, if $\exists B \in (E_k \cup \Sigma)$ such that $B\mathscr{R}A$ and A is defended by S_1 then $A \in \mathscr{GE}$.*
- *Let $S_2 \subseteq \mathscr{GE}$ be the minimal subset of E_k computed following items 1 and 2. Then $\forall A \in E_k \setminus S_2$, $A \notin \mathscr{GE}$.*
- *$\forall A \in \mathscr{A} \setminus E_k$, if $\exists B \in \mathscr{GE}$ such that $B\mathscr{R}A$ then $A \notin \mathscr{GE}$.*

Algorithm 1 gives a formal description of our procedure to compute progressively the grounded extension. If we ignore items 4, 5 (except $\mathscr{B} = E_l$), 6, 8 and 10 in Algorithm 1, then we recover Pearl's algorithm [17] to compute the least specific model of $\gg_{MM}$. Let

- $Safe(E_l) = \{B : B \in E_l$ such that $\nexists B' \in (E_l \cup \Sigma)$ with $B'\mathscr{R}B\}$,
- $Acceptable_{\mathscr{GE}}(E_l) = \{B : B \in E_l,$ for each $B' \in (E_l \cup \Sigma)$ such that $B'\mathscr{R}B, \exists C \in \mathscr{GE}$ such that $C\mathscr{R}B'\}$,
- $non\text{-}Safe_{\mathscr{GE}}(\mathscr{A}) = \{B : B \in \mathscr{A}$ such that $\exists B' \in \mathscr{GE}$ with $B'\mathscr{R}B\}$.

Algorithm 1. Interleaved computation of the grounded extension in optimistic reasoning.

Data: $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$, $\gg_{MM} = \{v_i \gg_{MM} v_j\}$.
Result: The grounded extension.

begin
1. $l = 0$, $\mathscr{GE} = \emptyset$, $\Sigma = \emptyset$

while $\mathscr{A} \neq \emptyset$ **do**

2. $E_l = \{B : B \in \mathscr{A}, \forall v_i \gg_{MM} v_j, B \notin arg(v_j)\}$
3. **if** $E_l = \emptyset$ **then**
   Stop (inconsistent preferences)
4. $\mathscr{GE} = \mathscr{GE} \cup Safe(E_l)$
5. $\mathscr{B} = E_l$, $E_l = E_l \setminus Safe(E_l)$, $x = 1$
6. **while** $E_l \neq \emptyset$ and $x = 1$ **do**
   – $\mathscr{GE} = \mathscr{GE} \cup Acceptable_{\mathscr{GE}}(E_l)$

   if $Acceptable_{\mathscr{GE}}(E_l) = \emptyset$ **then** $x = 0$

   – $E_l = E_l \setminus Acceptable_{\mathscr{GE}}(E_l)$
7. $\mathscr{A} = \mathscr{A} \setminus \mathscr{B}$,
8. $\mathscr{A} = \mathscr{A} \setminus non\text{-}Safe_{\mathscr{GE}}(\mathscr{A})$
9. remove $v_i \gg_{MM} v_j$ where $arg(v_i) \cap \mathscr{B} \neq \emptyset$
10. $\Sigma = \Sigma \cup (\mathscr{B} \setminus \mathscr{GE})$,
11. $l = l + 1$.

return $\mathscr{GE}$

end

Example 8 (*Continued from Example 5*). Let $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$ as before. We have $E_0 = \{A_0, A_3\}$. Both E_0 and E_3 are attacked by arguments of $\mathscr{A} \setminus E_0$ (strictly less preferred). So both belong to $\mathscr{GE}$, i.e. $\{A_0, A_3\} \subseteq \mathscr{GE}$ following item 1 of Lemma 8. Now A_2, A_4 and A_6 are attacked by A_0 and A_3. So they do not belong to $\mathscr{GE}$ following Lemma 9. Then $\mathscr{A} = \{A_1, A_5, A_7\}$. We remove *education* $\gg_{MM}$ *enjoy* and *social* $\gg_{MM}$ *alone* since they are satisfied. Now $E_1 = \{A_1, A_5\}$. A_1 and A_5 are not defeated so they are added to $\mathscr{GE}$ i.e. $\{A_0, A_3, A_1, A_5\} \subseteq \mathscr{GE}$. $\mathscr{A} = \{A_7\}$. *health* $\gg_{MM}$ *short term pleasure* is satisfied; it is removed. Lastly $E_2 = \{A_7\}$. A_7 is not attacked so it is added to $\mathscr{GE}$. In sum $\mathscr{GE} = \{A_0, A_1, A_3, A_5, A_7\}$.

Theorem 6. *Let $\mathscr{F} = \langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$ be a value-specification argumentation framework. Algorithm 1 computes the grounded extension of $\mathscr{F}$.*

The algorithm is an anytime algorithm, in the sense that when it is stopped in the middle of the execution, it has built part of the grounded extension. The computation of the grounded extension following Algorithm 1 is achieved in a polynomial time in the number of arguments and preferences in $\gg_{MM}$. So there is no extra cost when using our approach with respect to Dung's framework in which the computation of the grounded extension is also polynomial [13].

4.2. Grounded extension in pessimistic reasoning

In this section the argumentation framework is defined by $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{mm} \rangle$. A particularity of pessimistic reasoning is that it computes the most specific model of $\gg_{mm}$ starting from the *lowest* rank-ordered arguments in this model. Indeed it is no longer possible to compute progressively the grounded extension. So we have to compute the most specific model of $\gg_{mm}$, let's say $\succeq'$, then to define defeat relations on the basis of $\succeq'$ and $\mathscr{R}$ and lastly to compute the grounded extension. Algorithm 2 computes the most specific model of $\gg_{mm}$.

Algorithm 2. Pessimistic reasoning – A model of $\gg_{mm}$

Data: $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{mm} \rangle$, $\gg_{mm} = \{v_i \gg_{mm} v_j\}$.
Result: The most specific model of $\gg_{mm}$.

begin
$l = 0$
while $\mathscr{A} \neq \emptyset$ **do**
$E_l = \{B : B \in \mathscr{A}, \forall v_i \gg_{mm} v_j, B \notin arg(v_i)\}$
if $E_l = \emptyset$ **then**
Stop (inconsistent preferences)
– $\mathscr{A} = \mathscr{A} \setminus E_l$
– Remove $v_i \gg_{mm} v_j$ where $arg(v_j) \cap E_l \neq \emptyset$
– $l = l + 1$.
return $\succeq' = (E'_0, \ldots, E'_{l-1})$ with $\forall\ 0 \leqslant h \leqslant l-1, E'_h = E_{l-h-1}$
end

Example 9 (*Continued from Example 8*). We have $E_0 = \{A_1, A_2, A_6\}$ and $E_1 = \{A_0, A_3, A_4, A_5, A_7\}$. Indeed $\succeq' = (\{A_0, A_3, A_4, A_5, A_7\}, \{A_1, A_2, A_6\})$. We have $A_0\mathscr{D}A_6, A_3\mathscr{D}A_2, A_5\mathscr{D}A_2, A_3\mathscr{D}A_4, A_4\mathscr{D}A_3, A_4\mathscr{D}A_5$ and $A_5\mathscr{D}A_4$. Then $\mathscr{GE} = \{A_0, A_1, A_7\}$.

Here also the computation of the total pre-order following Algorithm 2 is achieved in a polynomial time. Once defeat relations are computed, we know from [13] that the computation of the grounded extension is polynomial in time.

As said before, the grounded extension in pessimistic reasoning cannot be computed progressively as in the optimistic reasoning. Nevertheless we can ensure in some cases whether an argument will belong to $\mathscr{GE}$ or not. Let us consider our running example. Following Algorithm 2, the worst arguments are A_1, A_2 and A_6. At this stage we can ensure that A_1 belongs to $\mathscr{GE}$ since it is not attacked. We can also ensure that A_6 does not belong to $\mathscr{GE}$ since its defeater A_0 is not attacked and, indeed, will belong to $\mathscr{GE}$. However we cannot say anything about A_2 since its defeaters A_3 and A_5 are attacked by A_4. At this stage the order over A_3, A_4 and A_5 is not stated yet.

Lemma 11. *Let $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{mm}\rangle$ be an argumentation framework and $\mathscr{GE}$ be its associated grounded extension.*

- *If $A \in E_k$ such that $\nexists B \in \mathscr{A}$ with $B\mathscr{R}A$ then $A \in \mathscr{GE}$.*
- *If $A \in E_k$ such that $\exists B \in \mathscr{A}$ with $B\mathscr{R}A$ and $\nexists C \in \mathscr{A}$ with $C\mathscr{R}B$ then $A \notin \mathscr{GE}$.*

Note that in both optimistic and pessimistic reasonings, Algorithms 1 and 2 consider *all* arguments of $\mathscr{A}$ when computing E_l. The set $\gg_{\rhd}$ is used to order arguments and to determine defeat relations later. Indeed in our approach an argument does not necessarily promote a value. This is intuitively meaningful since it is possible to use arguments, in a persuasion dialogue for example, which their only role is to attack other arguments without promoting any value.

5. Value-specification argumentation framework with inconsistent preferences

In the previous sections we focused on consistent preferences. The consistency of $\gg_{\rhd}$ is a necessary condition to compute a preference-based argumentation framework that is represented by the value-specification argumentation framework.

The consistency of a set of preferences $\gg_{\rhd}$ in our framework means that there exists a total pre-order $\succeq$ satisfying each preference $v_i \gg_{\rhd} v_j$ in $\gg_{\rhd}$. Otherwise $\gg_{\rhd}$ is inconsistent.

Example 10. Let $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM}\rangle$ be a value-specification argumentation framework with $\mathscr{A} = \{A_0, A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$, $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $arg(v_1) = \{A_1, A_3\}$, $arg(v_2) = \{A_6, A_7\}$, $arg(v_3) = \{A_3, A_5\}$, $arg(v_4) = \{A_0, A_1, A_2\}$, $arg(v_5) = \{A_0, A_1\}$ and $arg(v_6) = \{A_3, A_4, A_5\}$. $\mathscr{R}$ is the attack relation given in Example 1. Let $\gg_{MM} = \{v_1 \gg_{MM} v_2, v_3 \gg_{MM} v_4, v_5 \gg_{MM} v_6\}$. Given $\gg_{MM}$ we would like to compute a total pre-order $\succeq = (E_0, \ldots, E_n)$ satisfying each preference in $\gg_{MM}$. Following the minimal specificity principle E_0 is composed of arguments not falsifying any preference in $\gg_{MM}$. However $E_0 = \emptyset$ which means that $\gg_{MM}$ is inconsistent.

In such a case we propose to put all arguments in the same equivalence class and reduce $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$ to a preference-based argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ with $\succeq = (E_0) = (\mathscr{A})$ which is Dung's argumentation framework $\langle \mathscr{A}, \mathscr{D} \rangle$ with $\mathscr{D} = \mathscr{R}$.

Note however that we do not associate $\succeq = (E_0) = (\mathscr{A})$ to $\gg_{\rhd}$ as soon as $\gg_{\rhd}$ is inconsistent. In some cases, a more refined total pre-order can be computed when some preferences are not responsible of inconsistency. We take our inspiration from the algorithm computing the least specific model of $\gg_{MM}$.

Example 11 Example 10continued. We consider the argumentation framework given in the previous example with the following set of preferences $\gg_{MM} = \{v_2 \gg_{MM} v_1, v_3 \gg_{MM} v_4, v_5 \gg_{MM} v_6\}$.

We have $E_0 = \{A_6, A_7\}$. We remove $v_2 \gg_{MM} v_1$. The remaining preferences are inconsistent. So $\succeq = (E_0, E_1)$ with $E_0 = \{A_6, A_7\}$ and $E_1 = \mathscr{A} \setminus E_0$.

Algorithm 3 gives a total pre-order associated with an inconsistent set of preferences $\gg_{MM}$. Algorithm 2 can also be extended to treat inconsistent preferences by adding $E_l = \mathscr{A}$ to **<u>If</u>** box.

Algorithm 3. A total pre-order associated with $\gg_{MM}$.

Data: $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$, $\gg_{MM} = \{v_i \gg_{MM} v_j\}$.
Result: $\succeq = (E_0, \ldots, E_n)$.

begin
  $l = 0$
  while $\mathscr{A} \neq \emptyset$ **do**
    $E_l = \{B : B \in \mathscr{A}, \forall v_i \gg_{MM} v_j, B \notin arg(v_j)\}$
    if $E_l = \emptyset$ **then**
      inconsistent preferences,
      $E_l = \mathscr{A}$
    $\mathscr{A} = \mathscr{A} \setminus E_l$
    $l = l + 1$
  return $\succeq = (E_0, \ldots, E_{l-1})$
end

As we already said, once $\succeq$ is computed, $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd} \rangle$ is reduced to $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$. Of course we cannot say that $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd} \rangle$ represents $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ since $\gg_{\rhd}$ is inconsistent and $\succeq$ does not satisfy all preferences of $\gg_{\rhd}$. In the best case $\succeq$ is composed of more than one equivalence class, in which case it satisfies some preferences of $\gg_{\rhd}$. Such a way to treat inconsistent preferences is in accordance with our basic idea, namely to use a specificity principle. Clearly one can imagine other ways to tackle this problem as suggested by the following example.

Example 12. Let $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$ be a value-specification argumentation framework such that $\mathscr{A} = \{A_1, A_2, A_3\}$, $V = \{v_1, v_2, v_3\}$ with $arg(v_1) = \{A_1, A_2\}$, $arg(v_2) = \{A_1, A_3\}$ and $arg(v_3) = \{A_2, A_3\}$. Let preferences be specified by $\gg_{MM} = \{v_1 \gg_{MM} v_2, v_2 \gg_{MM} v_3, v_1 \gg_{MM} v_3\}$ and $\mathscr{R}$ be defined by: $A_1 \mathscr{R} A_3$, $A_2 \mathscr{R} A_3$, $A_3 \mathscr{R} A_1$ and $A_3 \mathscr{R} A_2$ (see Fig. 5).

At first sight we may think that preferences of $\gg_{MM}$ are not conflicting since we have $v_1 \gg_{MM} v_2 \gg_{MM} v_3$. However following our approach, we use minimal specificity principle to reduce $v_1 \gg_{MM} v_2 \gg_{MM} v_3$ to a total pre-order $\succeq = (E_0, \ldots, E_n)$ on $\mathscr{A}$. E_0 is composed of arguments that do not appear in any right-hand side of

$$
\begin{array}{ccc}
A_1 & \longleftrightarrow & A_3 \\
 & & \updownarrow \\
 & & A_2
\end{array}
$$

Fig. 5. Example 12.

any preference. E_0 is empty which means that there is no total pre-order which satisfies $\gg_{MM}$, i.e. $\gg_{MM}$ is inconsistent.

We reduce $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$ to $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ with $\succeq = (\{A_1, A_2, A_3\})$. Then A_1 and A_3 defeat each other, A_2 and A_3 defeat each other. The grounded extension is empty. Let us now compute preferred extensions. We have $S_1 = \{A_1, A_2\}$ and $S_2 = \{A_3\}$. Then we may prefer to accept S_1 but not S_2 since the former promote v_1 while the latter promotes v_2 and v_3 which are less preferred than v_1. However this way (satisfactory and intuitive in the above value-specification argumentation framework) may be debatable in some cases. First we have to determine which values are promoted by a set of arguments. Does S_1 promote v_1 only or v_1, v_2 and v_3? Intuitively we would say v_1 only because the whole set S_1 is accepted so we should look for values promoted by all arguments of S_1. However comparing sets of arguments on the basis of values they promote may lead to an extreme situation where each preferred extension promotes an empty set of values; which leads to incomparable preferred extensions. Moreover as an argument does not necessarily promote a value, this way to compare preferred extensions cannot be used in our approach.

The above examples suggest that dealing with inconsistent preferences is an interesting issue that should be further discussed and explored. This is left for further research.

6. Related work

Concerning the extensive work of Amgoud and colleagues on preference-based argumentation theory, our value-based argumentation theory seems closest to the argumentation framework based on contextual preferences given in [3]. A context may be an agent, a criterion, a viewpoint, *etc.*, and they are ordered. For example, in law earlier arguments are preferred to later ones, arguments of a higher authority are preferred to arguments of a lower authority, more specific arguments are preferred over more general arguments, and these three rules are ordered themselves too. However our approach is more general since we compare *sets* of arguments.

Specificity principle has been also used in much other work [20–22]. However in that work, the preference relation over arguments is defined on the basis of specificity of their *internal* structure. In our framework specificity concerns *abstract* arguments without referring to their internal structure. The use of specificity principle in our framework suggests that we have default knowledge about preferences over the arguments and use algorithms to derive the most likely preference order on arguments.

7. Summary

We distinguish three kinds of argumentation frameworks in the literature, as far as relevant for the new theory developed in this paper:

Dung's abstract argumentation framework is a tuple $\langle \mathscr{A}, \mathscr{D} \rangle$ where $\mathscr{A}$ is a finite set of arguments and $\mathscr{D}$ is a binary defeat relation defined on $\mathscr{A} \times \mathscr{A}$.

Amgoud and Cayrol's preference-based argumentation framework is a triplet $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ where $\mathscr{A}$ is a set of arguments, $\mathscr{R}$ is a binary attack relation defined on $\mathscr{A} \times \mathscr{A}$ and $\succeq$ is a (total or partial) pre-order (preference relation) defined on $\mathscr{A} \times \mathscr{A}$.

Bench-Capon's value-based argumentation framework is a 5-tuple VAF $= \langle \mathscr{A}, \mathscr{R}, V, val, \mathscr{P} \rangle$, where $\mathscr{A}$ is a finite set of arguments, $\mathscr{R}$ is an irreflexive binary relation on $\mathscr{A}$, V is a non-empty set of values, val is a function which maps from elements of $\mathscr{A}$ to elements of V and $\mathscr{P}$ is the set of possible audiences. An audience specific argumentation framework is a tuple $VAF_a = \langle \mathscr{A}, \mathscr{R}, V, val, >_a \rangle$, where $a \in \mathscr{P}$ is an audience and $>_a$ is a partial order on V.

Dung's semantics for argumentation frameworks can be used for preference-based argumentation and audience specific value-based argumentation frameworks too, using the following relations between the frameworks:

$\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ *represents* $\langle \mathscr{A}, \mathscr{D} \rangle$ if and only if ($\forall A, B \in \mathscr{A}$, we have $A \mathscr{D} B$ if and only if $A \mathscr{R} B$ and it is not the case that $B \succ A$), and

$\langle \mathscr{A}, \mathscr{R}, V, arg, >_a \rangle$ *represents* $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ if and only if ($\forall A, B \in \mathscr{A}$, we have $A \succeq B$ if and only if $val(A) >_a val(B)$ or $val(A) = val(B)$).

Composing both relations we define also what it means for an audience specific value-based argumentation framework to represent an argumentation framework. We show that an argumentation framework is represented by a set of preference-based argumentation frameworks, and that a preference-based argumentation framework is represented by a set of audience specific value-based argumentation frameworks, though the latter are renamings of each other. We show also that these relations hold when we restrict ourselves to total pre-orders.

In this paper we consider a generalization of Bench-Capon's audience specific value-based argumentation framework, in which arguments can promote multiple values, and various kinds of preferences among values can be expressed. We call our structure a value-specification argumentation framework. To calculate the preference-based argumentation theory represented by the value-specification framework, we use techniques from non-monotonic reasoning about preferences.

An audience specific value-based argumentation framework is a 5-tuple $\langle \mathscr{A}, \mathscr{R}, V, arg, >_a \rangle$ where $\mathscr{A}$ is a set of arguments, $\mathscr{R}$ is an attack relation on $\mathscr{A} \times \mathscr{A}$, V is a set of values, arg is a function from V to $2^{\mathscr{A}}$ such that $arg(v)$ is the set of arguments promoting the value v, and $>_a$ is a partial order on V.

A value-specification argumentation framework is a 5-tuple $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd} \rangle$ where $\mathscr{A}$ is a set of arguments, $\mathscr{R}$ is an attack relation on $\mathscr{A} \times \mathscr{A}$, V is a set of values, arg is a function from V to $2^{\mathscr{A}}$, and $\gg_{\rhd} \subseteq V \times V$ is a set of preferences over V with $\rhd \in \{MM, mm\}$.

The problem of reducing ordered values to a preference relation comes down to reducing a preference relation over sets of arguments to a preference relation over single arguments. To reason about ordered values and to compute the preference relation over arguments, we are inspired by insights from the non-monotonic logic of preference known as minimal specificity, System Z, gravitation to normality, and otherwise, and we use both so-called optimistic and pessimistic ways to define the preference relation.

$\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd} \rangle$ *represents* $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ if and only if $\rhd = MM$ ($\rhd = mm$) and $\succeq$ is the least (most) specific relation among the $\succeq'$ such that $\langle \mathscr{A}, \mathscr{R}, \succeq' \rangle$ satisfies $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd} \rangle$, where $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ satisfies $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{\rhd} \rangle$ if and only if $\succeq$ satisfies each $v_i \gg_{\rhd} v_j$ in $\gg_{\rhd}$.

We have that a preference-based argumentation framework can be represented by several value-specification frameworks, but in this case we do not have that the value-specification frameworks are renamings of each other. This illustrates one aspect in which our way of reasoning with values is distinct from the way values are treated in Bench-Capon's framework.

The set of acceptable arguments can be calculated by combining algorithms from non-monotonic reasoning with algorithms for calculating extensions in argumentation theory. We introduce an algorithm for Dung's grounded semantics. It shows that the computation of the set of acceptable arguments can be combined with the optimistic reasoning to incrementally define the set of acceptable arguments, because in this construction for each equivalence class we can deduce which arguments are not attacked by other arguments. This property does not hold for pessimistic reasoning.

The present work can be extended in various ways, for example:

- The elicitation of preference specification $\gg_{\rhd}$. For example, Bench-Capon et al. [6] define an ordering on values through a dialogue between two players. The basic item in this dialogue are arguments that an agent likes to see accepted and others to be rejected,
- To study the case where both maxmax and minmin preferences are provided at the same time i.e. $\gg_{MM} \cup \gg_{mm}$. A possible solution would be to compute acceptable arguments with respect to $\gg_{MM}$ and $\gg_{mm}$ given individually and then use the notions of objective and subjective acceptance as in [5],
- To study the reinforcement among different arguments promoting the same value [5],
- A theory of value-based extension of Bochman's generalization of Dung's theory called collective argumentation [11], where the attack relation is defined over sets of arguments instead of single arguments. It seems natural to develop a unified framework where both attack and preference relations are defined over sets of arguments.

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Appendix

Proof of Lemma 1. For each preference-based argumentation framework $\langle\mathscr{A},\mathscr{R},\succeq\rangle$ there is an argumentation framework $\langle\mathscr{A},\mathscr{D}\rangle$ it represents. By construction.

For each argumentation framework $\langle\mathscr{A},\mathscr{D}\rangle$ there is a preference-based argumentation framework $\langle\mathscr{A},\mathscr{R},\succeq\rangle$ that represents it. Take $\mathscr{R}=\mathscr{D}$ and $\succeq$ is the universal relation.

Proof of Lemma 2. If $\langle\mathscr{A},\mathscr{R},\succeq\rangle$ represents $\langle\mathscr{A},\mathscr{D}_1\rangle$ and $\langle\mathscr{A},\mathscr{D}_2\rangle$, then $\mathscr{D}_1=\mathscr{D}_2$. By construction.

Proof of Theorem 1. Follows directly from Lemmas 1 and 2.

Proof of Lemma 3. For each audience specific value-based argumentation framework there is a preference-based argumentation framework it represents. By construction. For each preference-based argumentation framework $\langle\mathscr{A},\mathscr{R},\succeq\rangle$ there is an audience specific value-based argumentation framework $\langle\mathscr{A},\mathscr{R},V,val,>_a\rangle$ that represents it. Assume a unique value for each equivalence class of $\succeq$, and let $val(A)$ be this unique value for argument A. Then $val(A)>_a val(B)$ if $A\succ B$ and $val(A)=val(B)$ if $A\simeq B$.

Proof of Lemma 4. Suppose that $\langle\mathscr{A},\mathscr{R},V,val,>_a\rangle$ represents $\langle\mathscr{A},\mathscr{R},\succeq_1\rangle$ and $\langle\mathscr{A},\mathscr{R},\succeq_2\rangle$. By definition this means that ($\forall A,B\in\mathscr{A}$, we have $A\succeq_1 B$ iff $val(A)>_a val(B)$ or $val(A)=val(B)$) and ($\forall A,B\in\mathscr{A}$, we have $A\succeq_2 B$ iff $val(A)>_a val(B)$ or $val(A)=val(B)$). Indeed $\forall A,B\in\mathscr{A}$ we have $A\succeq_1 B$ iff $val(A)>_a val(B)$ or $val(A)=val(B)$ iff $A\succeq_2 B$. Indeed $\succeq_1=\succeq_2$ since $\succeq_1$ and $\succeq_2$ are defined in the same way.

Suppose that $\langle\mathscr{A},\mathscr{R},V_1,val_1,>_{a,1}\rangle$ and $\langle\mathscr{A},\mathscr{R},V_2,val_2,>_{a,2}\rangle$ represent $\langle\mathscr{A},\mathscr{R},\succeq\rangle$. By definition the first hypothesis means that $\forall A,B\in\mathscr{A}$ we have $A\succeq B$ iff $val_1(A)>_{a,1}val_1(B)$ or $val_1(A)=val_1(B)$ (*). Also $\langle\mathscr{A},\mathscr{R},V_2,val_2,>_{a,2}\rangle$ represents $\langle\mathscr{A},\mathscr{R},\succeq\rangle$ means that $\forall A,B\in\mathscr{A}$ we have $A\succeq B$ iff $val_2(A)>_{a,2}val_2(B)$ or $val_2(A)=val_2(B)$ (**).

(1) Suppose that $val_1(A)=val_1(B)$. Following hypothesis (*) this means that $A\succeq B$. Moreover, following hypothesis (**) we have ($val_2(A)>_{a,2}val_2(B)$ or $val_2(A)=val_2(B)$). Now $val_1(A)=val_1(B)$ also implies $val_1(B)=val_1(A)$ due to symmetry. Then we have $B\succeq A$ following hypothesis (*). Following hypothesis (**) we also have ($val_2(B)>_{a,2}val_2(B)$ or $val_2(B)=val_2(A)$). Indeed we have $val_2(A)=val_2(B)$. Similar reasoning when $val_2(A)=val_2(B)$.
(2) Suppose that $val_1(A)>_{a,1}val_1(B)$. This means that $A\succeq B$ following hypothesis (*). $A\succeq B$ is equivalent to $val_2(A)>_{a,2}val_2(B)$ or $val_2(A)=val_2(B)$ following hypothesis (**). Suppose that we have $val_2(A)=val_2(B)$. This implies $val_2(B)=val_2(A)$. Then we have $B\succeq A$ following hypothesis (**) which is equivalent to $val_1(B)>_{a,1}val_1(A)$ or $val_1(A)=val_1(B)$. Contradiction. Similar reasoning when $val_2(A)>_{a,2}val_2(B)$.

Proof of Theorem 2. Follows directly from Lemmas 3 and 4.

Proof of Theorem 3. Analogous to the proof of Theorem 1.

Proof of Theorem 4. Analogous to the proof of Theorem 2.

Proof of Lemma 5. Let $v_1\gg_{mM}v_2$ be a minmax preference and $\succeq$ be a total pre-order on $\mathscr{A}$ such that $\succeq$ satisfies $v_1\gg_{mM}v_2$.

Assume that for each argument A there is a value v with $arg(v)=\{A\}$, which we call v_A.

(1) Let us first prove that $\succeq$ satisfies $v_1\gg_{mM}v_2$ if and only if for each $A\in arg(v_1)$, $\succeq$ satisfies $v_A\gg_{MM}v_2$.
   - $\Rightarrow$: If $\succeq$ satisfies $v_1\gg_{mM}v_2$ then for each $A\in arg(v_1)$, $\succeq$ satisfies $v_A\gg_{MM}v_2$.
     Suppose that $\succeq$ satisfies $v_1\gg_{mM}v_2$. Following Definition 13 this means that each argument A in $arg(v_1)$ is preferred to each argument B in $arg(v_2)$ with respect to $\succeq$. So the argument A in v_A is preferred to each argument B in $arg(v_2)$. Since $arg(v_A)$ is composed of A and no other arguments, we have $\max(arg(v_A),\succeq)=\{A\}$. Then $\succeq$ satisfies $v_A\gg_{MM}v_2$.
   - $\Leftarrow$: If for each $A\in arg(v_1)$, $\succeq$ satisfies $v_A\gg_{MM}v_2$ then $\succeq$ satisfies $v_1\gg_{mM}v_2$.
     Suppose that $\forall A\in arg(v_1)$. We have that $\succeq$ satisfies $v_A\gg_{MM}v_2$. Since $arg(v_A)$ is composed of argu-

ment A and no other arguments, we have $\max(arg(v_A), \succeq) = \{A\}$. Indeed $\forall A \in arg(v_1)$, $\succeq$ satisfies $v_A \gg_{MM} v_2$ means that for each A in $arg(v_1)$ we have $A \succ B$, $\forall B \in \max(arg(v_2), \succeq)$. So the least preferred argument in $arg(v_1)$ with respect to $\succeq$ is preferred to B, and we have $\forall B \in \max(arg(v_2), \succeq)$. Indeed $\forall A \in \min(arg(v_1), \succeq)$, $\forall B \in \max(arg(v_2), \succeq)$ we have $A \succ B$ which is equivalent to state that $\succeq$ satisfies $v_1 \gg_{mM} v_2$.

(2) We now prove that $\succeq$ satisfies $v_1 \gg_{mM} v_2$ if and only if for each $A \in arg(v_2)$, $\succeq$ satisfies $v_1 \gg_{mm} v_A$.
$\Rightarrow$: If $\succeq$ satisfies $v_1 \gg_{mM} v_2$ then $\forall A \in arg(v_2)$, $\succeq$ satisfies $v_1 \gg_{mm} v_A$.
Suppose that $\succeq$ satisfies $v_1 \gg_{mM} v_2$. Following Definition 13 this means that each argument in $arg(v_1)$ is preferred to each argument in $arg(v_2)$ with respect to $\succeq$. Formally $\forall B \in \min(arg(v_1), \succeq)$, $\forall A \in \max(arg(v_2), \succeq)$ we have $B \succ A$. Then $\forall B \in \min(arg(v_1), \succeq)$, $\forall A \in \min(arg(v_A), \succeq)$ we have $B \succ A$ since $\min(arg(v_A), \succeq) = \{A\}$ (Recall that $arg(v_A)$ is composed of only one argument namely A). Indeed $\succeq$ satisfies $v_1 \gg_{mm} v_A$ for each $A \in arg(v_2)$.
$\Leftarrow$: If $\forall A \in arg(v_2)$, $\succeq$ satisfies $v_1 \gg_{mm} v_2$ then $\succeq$ satisfies $v_1 \gg_{mM} v_2$.
Suppose that $\forall A \in arg(v_2)$, $\succeq$ satisfies $v_1 \gg_{mm} v_A$. This means that $\forall B \in \min(arg(v_1), \succeq)$, $\forall A \in \min(arg(v_A), \succeq)$ we have $B \succ A$. Note that $arg(v_A) = \{A\}$ so $\min(arg(v_A), \succeq) = \{A\}$. Indeed each argument B in $arg(v_1)$ is preferred to each argument A in $arg(v_2)$ with respect to $\succeq$ i.e. $\succeq$ satisfies $v_1 \gg_{mM} v_2$.

Proof of Lemma 6. The proof is given by construction from Definition 10. Given $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$, we associate a value to each argument in $\mathscr{A}$ such that $\forall A, B \in \mathscr{A}$, $val(A) = val(B)$ iff $A \simeq B$. Then we define $>_a$ as follows: $\forall A, B \in \mathscr{A}$, we have $val(A) >_a val(B)$ iff $A \succ B$.

Proof of Lemma 7. If $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_a \rangle$ represents $\langle \mathscr{A}, \mathscr{R}, \succeq_1 \rangle$ and $\langle \mathscr{A}, \mathscr{R}, \succeq_2 \rangle$, then $\succeq_1 = \succeq_2$. By construction.

Proof of Theorem 5. follows directly from Lemmas 6 and 7.

Proof of Lemma 8. Let $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$ be a value-specification argumentation framework and $\succeq = (E_0, \ldots, E_n)$ be the least specific model of $\gg_{MM}$.

(1) Let $A \in E_0$. Suppose that $\nexists B \in E_0$ such that $B\mathscr{R}A$. We distinguish two cases:
Case 1: there is no C in $E_1 \cup \cdots E_n$ such that $C\mathscr{R}A$. This means that A is not attacked at all and then not defeated. So it belongs to the grounded extension.
Case 2: A is attacked by an argument B from $E_1 \cup \cdots \cup E_n$. This attack does not succeed since A belongs to E_0 so it is strictly preferred to B. Then A is not defeated and should belong to the grounded extension.

(2) Let S_1 be a subset of $\mathscr{GE}$. Let A and B be two arguments of E_0 such that $B\mathscr{R}A$ and A is defended by S_1. Note that S_1 is a subset of $\mathscr{GE}$ so it is composed of acceptable arguments. Following the definition of the grounded extension, each argument defended by acceptable arguments also belongs to $\mathscr{GE}$. So we have $A \in \mathscr{GE}$.

(3) Let $S_2 \subseteq \mathscr{GE}$ be the minimal subset of E_0 computed following items 1 and 2. This means that S_2 is composed of arguments in E_0 which are not defeated and all arguments of E_0 which are defeated but defended by acceptable arguments. So each argument A of E_0 outside S_2 is defeated and non-defended by arguments already put in $\mathscr{GE}$. So A does not belong to $\mathscr{GE}$.

Proof of Lemma 9. By definition of the grounded extension each argument defeated by acceptable arguments (those already put in the grounded extension) does not belong to $\mathscr{GE}$. So once the subset of $\mathscr{GE}$ computed in the first iteration of the algorithm (when computing E_0), each argument outside E_0 and defeated by $\mathscr{GE}$ does not belong to $\mathscr{GE}$.

Proof of Lemma 10. Let $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$ be a value-specification argumentation framework. For each $k \neq 0$, let $\mathscr{A}$ be the set of arguments after iteration $k-1$. Moreover, let $\Sigma = (E_0 \cup \cdots \cup E_{k-1}) \setminus \mathscr{GE}$. Note that Σ is composed of arguments of $E_0 \cup \cdots \cup E_{k-1}$ that do not belong to the grounded extension.

(1) Let $A \in E_k$. Suppose that $\nexists B \in (E_k \cup \Sigma)$ such that $B\mathscr{R}A$. We distinguish two cases:
Case 1: A is attacked by an argument in $E_0 \cup \cdots \cup E_{k-1}$. Since we have that $\Sigma = E_0 \cup \cdots \cup E_{k-1} \setminus \mathscr{GE}$, this means that A is defeated by an argument in the current $\mathscr{GE}$ then it should has been removed from $\mathscr{A}$ in earlier iteration, line 8. Indeed A belongs to $\mathscr{GE}$.
Case 2: A is attacked by an argument in $E_{k+1} \cup \cdots \cup E_n$. This attack does not succeed since A is strictly preferred to its attackers. Then A is not defeated and then belongs to $\mathscr{GE}$.
(2) The proof is similar to the one of item 2, Lemma 8. A belongs to $\mathscr{GE}$ as soon as it is defended by acceptable arguments.
(3) The proofs of item (3) and (4) are similar to the ones of item (3) and Lemma 9 respectively.

Proof of Theorem 6. By definition the grounded extension is composed of non-defeated arguments, defeated arguments but defended by the current grounded extension and so on until we reach a fixpoint. Note that at each iteration the algorithm puts in $\mathscr{GE}$ arguments that are not defeated (line 4). This is stated in item 1 of Lemmas 8 and 9. So each argument which is not defeated belongs to the grounded extension. Then following item 2 of Lemmas 8 and 9, at each iteration of the algorithm defeated arguments that are defended by arguments already accepted (i.e. already put in the grounded extension) belong also to the grounded extension. Following Algorithm 1 this reasoning is repeated until all arguments are treated. Indeed Algorithm 1 computes the grounded extension of $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{MM} \rangle$.

Proof of Lemma 11. Let $\langle \mathscr{A}, \mathscr{R}, V, arg, \gg_{mm} \rangle$ be a value-specification argumentation framework and $\mathscr{GE}$ be its grounded extension. Let $\succeq = (E_0, \ldots, E_n)$ be the most specific pre-order satisfying $\gg_{mm}$.

(1) Suppose that $A \in E_k$ such that $\nexists B \in \mathscr{A}$ with $B\mathscr{R}A$. This means that B is not defeated. Then it belongs to $\mathscr{GE}$.
(2) Suppose that $A \in E_k$ such that $\exists B \in \mathscr{A}$ with $B\mathscr{R}A$ and $\nexists C \in \mathscr{A}$ with $C\mathscr{R}B$. This means that A is defeated and its defeater is not attacked at all. So A is not defended. Then A does not belong to $\mathscr{GE}$.

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