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DEVELOPING A TOOL FOR ASSESSING ELEMENTARY ALGEBRAIC KNOWLEDGE FOR TEACHING: A TWOFOLD PERSPECTIVE

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This paper reports on the process of developing a questionnaire to evaluate teachers' knowledge of elementary algebra, and presents the initial results of this process. The questionnaire is structured into three dimensions: learning activities, knowledge for teaching and categories of practices, the last one of these highlighting the influence of context on the mobilisation of teachers' knowledge. The initial results show that although teachers acquire a certain knowledge in order to develop effective teaching environments, they do not mobilise this knowledge in certain class contexts. Their resulting decisions sometimes help and sometimes hinder students' understanding of the key algebraic concepts.

INTRODUCTION

Teaching mathematics cannot be confined to imparting knowledge to students: teachers must have both pedagogical knowledge and a knowledge of mathematics. The research defining the various aspects of 'mathematical knowledge for teaching' (Ball, Lubienski, & Mewborn, 2001; Ferrini-Mundy, Floden, McCrory, Burrill, & Sandow, 2005; Hill, Ball, & Schilling, 2008; McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012) demonstrates both the importance and the complexity of defining this knowledge for teaching.

In elementary algebra, researchers currently face a twofold problem. Firstly, although conceptual frameworks help to pinpoint the nature of the knowledge to be evaluated (Kieran, 2007; Ferrini-Mundy et al, 2005; McCrory et al, 2012), there is a regrettable lack of tools for performing a valid diagnosis of such knowledge (McCrory et al, 2012). Secondly, Kieran (2007) is critical of the lack of connections between research into the learning and the teaching of algebra: for example, although effective environments that make it possible to understand the basics of algebra have been developed, teachers continue to focus on memorisation and the immediate application of procedures, without encouraging the development of a thorough understanding of the processes involved in algebra (Ball et al, 2001; Kieran, 2007).

This is the context of our paper. It presents a tool for the evaluation of algebraic knowledge for teaching that takes account of research into both the teaching and the learning of algebra. It also looks at the initial results of the use of this tool with 88 teachers in Luxembourg. The tool enables a twofold look to be taken at algebraic knowledge for teaching: firstly, by quantifying the discrepancies between teachers' positions and the findings discussed in the research literature, and secondly, by taking a more qualitative look at these positions in order to gain a better understanding of the influence of class contexts on the mobilisation of their knowledge.

CONCEPTUAL FRAMEWORK

The questionnaire is structured into three approaches. The first focuses on elementary algebra learning activities. Kieran (2007) summarises these tasks in a model that lists three types of activities. 'Generational activities' cause students to develop algebraic expressions or equations corresponding to situations. 'Transformational activities' correspond to algebraic calculation techniques such as factorisation, developing expressions or solving equations. Finally, the 'Global/meta-level activities' are those in which algebra is used as a tool to solve problems.

The second approach refers to research focusing on knowledge for teaching elementary algebra. Based on the work of Ball et al. (2001), Hill et al. (2007) identify two essential components: the first, 'knowledge of content and students', is about linking together mathematical knowledge and knowledge of students to analyse their emerging and incomplete thinking or understand their mistakes; second, 'knowledge of content and teaching', focuses on the design of instruction and involves knowledge of mathematics and teaching strategies. These components are useful for selecting teaching situations, guiding students' discussions or providing rich examples which are appropriate for their learning level. Based on classroom observations and interviews with teachers, Ferriny-Mundy et al. (2005) revisited these categories, describing the specific characteristics of knowledge in algebra.

The third approach derives from the work of MacCrory et al. (2012), which highlights the importance of taking account of the variety of contexts in which teachers are immersed in the flow of their classroom practice. These may affect the mobilisation of their knowledge for teaching. These authors define three broad categories of practices: 'trimming', referring to the desire to reduce the mathematical complexity of concepts while maintaining their integrity; 'bridging', i.e. the creation of bridges between mathematical themes, and 'decompressing', which reveals factors that may seem unimportant to the experts.

By contrast with the first two approaches, where the categories are mutually exclusive, these categories of practices are closely related: for example, if the teacher takes an action to help a student to understand his or her mistake, he or she may have decided to do so from a desire to trim, to decompress the material or to build bridges with other fields, or even to combine all three dimensions. It can therefore be a highly complex matter to develop items specifically focusing on this third approach. However, this referential framework can prove very useful for analysing how teachers justify their decisions in response to the classroom situations presented in the questionnaire. This is why, although this third approach is present in the background of the design of the situations, more use will be made of it in the analysis of the results in particular.

QUESTIONNAIRE DESIGN METHODOLOGY

The questionnaire consists of ten case studies organised into 21 items. Structured in reference to the three approaches described above, these situations and items relate to themes studied in research focusing firstly on the difficulties of students in elementary algebra (Kieran, 1992; Booth, 1984), and secondly on the characteristics of effective environments for teaching algebra (Moses and Cobb, 2001 in Koellner, et al, 2011; Bednarz et al, 2001; Kieran, 2007; Radford, 2008). These three approaches and two mainlines have been structured in a diagram that space does not permit us to reproduce here, but which will be shown during the presentation.

The coding guide enables the teachers' answers to be considered in two different ways: firstly, the degree of consistency between their decisions and research findings is assessed, and secondly the contextual factors which form the basis for these decisions are identified.

The questionnaire was developed in four distinct phases: the development of items, the construction of two pilot tests and the final validation phase. The first pilot test involved carrying out individual interviews with five teachers on the basis of a first version of the questionnaire. The second pilot test was conducted with 40 teachers from 10 schools. This second pilot was extended with a posteriori interviews with 8 teachers from 2 schools, in which they commented on the degree of realism of the presented case studies and explained their answers. Finally, the last phase, questionnaire validation, was conducted on the basis of the answers provided by 88 teachers, using all 21 items chosen for the final questionnaire. Indices of discrimination (biserial correlations of points, lying between 0.22 and 0.56 with a mean of 0.38), and an index of internal consistency (Cronbach's alpha, 0.73) confirm the questionnaire's empirical validity.

The analysis of the questionnaire results aims to answer two research questions: 1) To what extent are the options chosen by teachers consistent with the findings identified in the research literature? 2) On what basis do the teachers adopt their position in response to the various case analyses presented?

INITIAL RESULTS

In response to the first research question, the mean score of 53% shows that the decisions taken by the teachers are only partially consistent with the research findings. However, significant differences appear from teacher to teacher and from question to question.

With regard to the second research question, the analyses add nuances to this first observation, highlighting the influence of context on the mobilisation of knowledge for teaching, with reference to the three categories of practice, trimming, bridging and decompressing. The major finding that emerges from this second angle of analysis is the lack of consistency in the teachers' responses. For example, while they recognise that students struggle to make sense of the letter in transformational activities, they do not take the view that this aspect could be central to their actions in response to a mistake made by students in reducing an algebraic expression. A majority even believe that the most effective strategy is one that encourages a completely incorrect understanding of the letter. Similarly, although a majority of teachers are aware of the effectiveness of informal approaches to equation solving by students, they do not believe that this aspect should be taken into account when introducing the formal method of equation solving. These results will be discussed further in the paper.

DISCUSSION AND PERSPECTIVE

The study has various limitations: the analyses are based on teachers' statements and not, as it is the case in other studies (McCrory et al, 2012), on direct observations; the sample is small, as it consists of just 88 respondents, and finally, the number of questions and dimensions explored had to be limited, as the time taken to complete the questionnaire could not exceed 45 minutes. Despite these limitations, these rather paradoxical results are also found in other studies of teachers' pedagogical knowledge (Hill et al, 2008): for example, this author notes that many teachers, when faced with multiple-choice questions, succeed in identifying the correct answer without basing their choice on

knowledge validated by research. Basing her research on the knowledge that teachers have about students' thinking and the difficulties they experience, she concludes that this particular type of knowledge actually has several different dimensions.

These initial results lead us to point to the importance of considering teachers' knowledge from multiple angles, so as to both reveal discrepancies between their decisions and the recommendations of the research literature, and gain a better understanding of the paradoxes that should be addressed as part of a professional development programme in order to promote greater consistency in the teaching of elementary algebra.

The next part of this research pursues this direction. The comparison of results obtained by teachers before and after the professional development programme will help identify its impact on the organisation of teachers' knowledge in elementary algebra.

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