Using Bayesian inference to recover the material parameters of a heterogeneous hyperelastic body

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ABSTRACT

We present a method for calculating a Bayesian uncertainty estimate on the recovered material parameters of a heterogeneous geometrically non-linear hyperelastic body. We formulate the problem in the Bayesian inference framework [1]; given noisy and sparse observations of a body, some prior knowledge on the parameters and a parameter-to-observable map the goal is to recover the posterior distribution of the parameters given the observations. In this work we primarily focus on the challenges of developing dimension-independent algorithms in the context of very large inverse problems (tens to hundreds of thousands of parameters). Critical to the success of the method is viewing the problem in the correct infinite-dimensional function space setting [2]. With this goal in mind, we show the use of automatic symbolic differentiation techniques to construct high-order adjoint models [3], scalable maximum a posteriori (MAP) estimators, and efficient low-rank update methods to calculate credible regions on the posterior distribution [4].

References

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