# Well Conditioned and Optimally Convergent Extended Finite Elements and Vector Level Sets for Three-Dimensional Crack Propagation

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# Outline

# Global enrichment XFEM

An XFEM variant (Agathos, Chatzi, Bordas, & Talaslidis, 2015) is introduced which:

- Enables the application of geometrical enrichment to 3D.
- Extends dof gathering to 3D through global enrichment.
- Employs weight function blending.
- Employs enrichment function shifting.

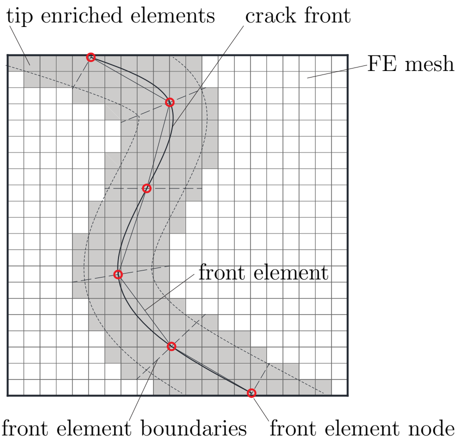
# Front elements

A superimposed mesh is used to provide a p.u. basis.

Desired properties:

- Satisfaction of the partition of unity property.
- Spatial variation only along the direction of the crack front.
- No variation on the plane normal to the crack front.

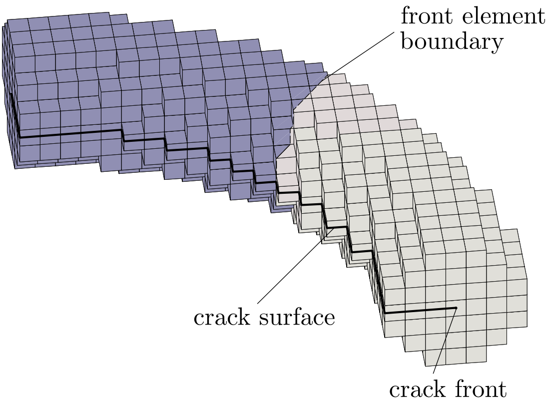
# Front elements




- A set of nodes along the crack front is defined.
- Each element is defined by two nodes.
- A good starting point for front element thickness is $h$.

# Front elements

Volume corresponding to two consecutive front elements.




Different element colors correspond to different front elements.

# Front element shape functions

Linear 1D shape functions are used:

$$\mathbf{N}^g(\xi) = \left[\frac{1-\xi}{2} \quad \frac{1+\xi}{2}\right]$$

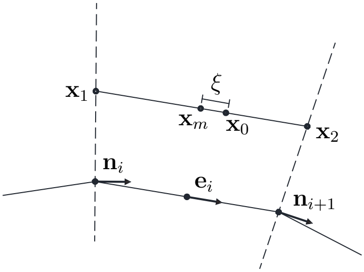
where $\xi$ is the local coordinate of the superimposed element.

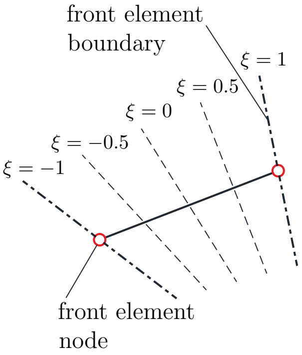
Those functions:

- form a partition of unity.
- are used to weight tip enrichment functions.

# Front element shape functions

Definition of the front element parameter used for shape function evaluation.



# Tip enrichment functions

Tip enrichment functions used:

$$F_j(\mathbf{x}) \equiv F_j(r,\theta) = \left[\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right]$$

Tip enriched part of the displacements:

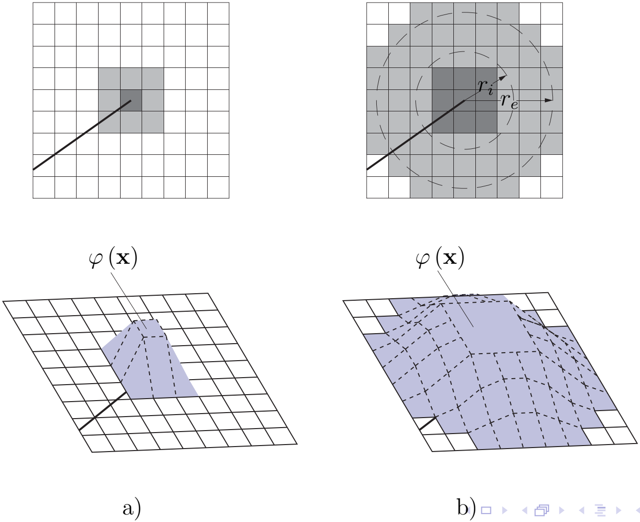
$$\mathbf{u_t}(\mathbf{x}) = \sum_{K\in\mathcal{N}^s} N_K^g(\mathbf{x}) \sum_j F_j(\mathbf{x})\,\mathbf{c}_{Kj}$$

where

- $N_K^g$ are the global shape functions
- $\mathcal{N}^s$ is the set of superimposed nodes

# Weight functions

Weight functions for a) topological (Fries, 2008) and b) geometrical enrichment (Ventura, Gracie, & Belytschko, 2009).



# Displacement approximation

$$\mathbf{u}(\mathbf{x}) = \sum_{I \in \mathcal{N}} N_I(\mathbf{x}) \mathbf{u}_I + \bar{\varphi}(\mathbf{x}) \sum_{J \in \mathcal{N}^j} N_J(\mathbf{x}) (H(\mathbf{x}) - H_J) \mathbf{b}_J +$$

$$+ \varphi(\mathbf{x}) \left( \sum_{K \in \mathcal{N}^s} N_K^g(\mathbf{x}) \sum_j F_j(\mathbf{x}) - \right.$$

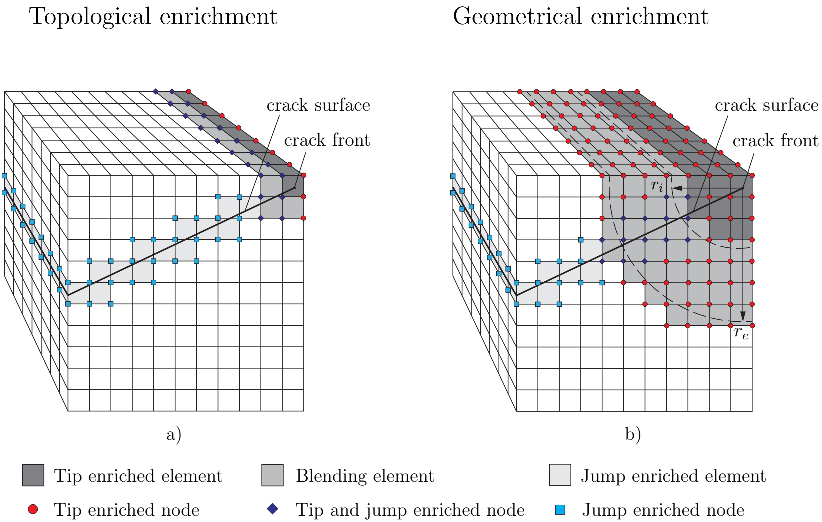
$$\left. - \sum_{T \in \mathcal{N}^t} N_T(\mathbf{x}) \sum_{K \in \mathcal{N}^s} N_K^g(\mathbf{x}_T) \sum_j F_j(\mathbf{x}_T) \right) \mathbf{c}_{Kj}$$

where:

- $\mathcal{N}$ is the set of all nodes in the FE mesh.
- $\mathcal{N}^j$ is the set of jump enriched nodes.
- $\mathcal{N}^t$ is the set of tip enriched nodes.
- $\mathcal{N}^s$ is the set of nodes in the superimposed mesh.

# Weight functions

Enrichment strategies used for tip and jump enrichment.



# Vector Level Sets

A method for the representation of 3D cracks is introduced which:

- Produces level set functions using geometric operations.
- Does not require integration of evolution equations.

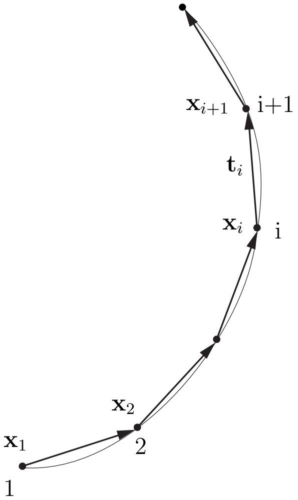
Similar methods:

- 2D Vector level sets (Ventura, Budyn, & Belytschko, 2003).
- Hybrid implicit-explicit crack representation (Fries & Baydoun, 2012).

# Crack front

Crack front at time $t$:

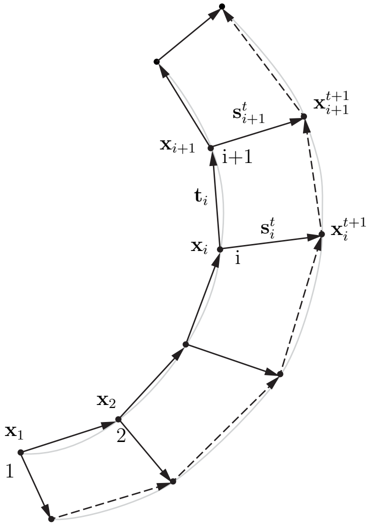
- Ordered series of line segments $\mathbf{t}_i$
- Set of points $\mathbf{x}_i$



# Crack front advance

Crack front at time $t + 1$:

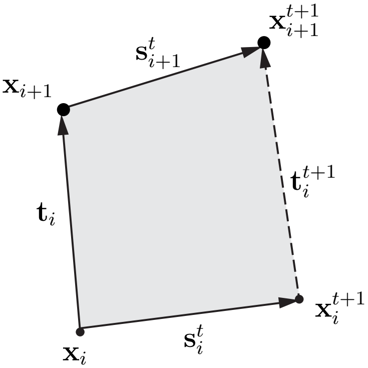
- Crack advance vectors $\mathbf{s}_i^t$ at points $\mathbf{x}_i$
- New set of points $\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{s}_i^t$



# Crack surface advance

Crack surface advance:

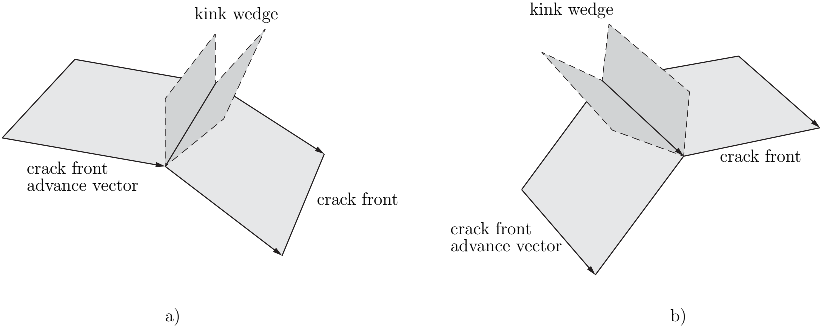
- Sequence of four sided bilinear segments.
- Vertexes: $\mathbf{x}_i^t$, $\mathbf{x}_{i+1}^t$, $\mathbf{x}_{i+1}^{t+1}$, $\mathbf{x}_i^{t+1}$



# Kink wedges

Discontinuities (*kink wedges*) are present:

- Along the crack front (a).
- Along the advance vectors (b).

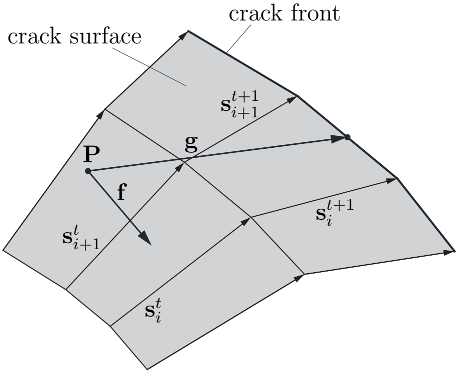


# Level set functions

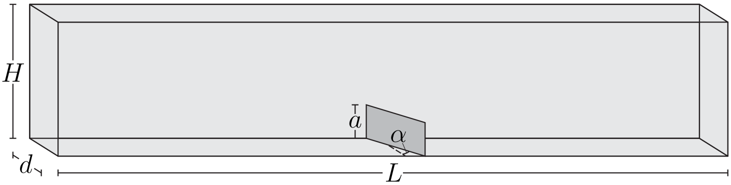
Definition of the level set functions at a point **P**:

**f** distance from the crack surface.

**g** distance from the crack front.



# Edge crack in a beam




Geometry:

$L$ = 2 unit

$H$ = 0.4 units

$d$ = 0.2 units

$a$ = 0.1 units

$\alpha$ = 45°

Mesh:

- $h = 0.02$ units

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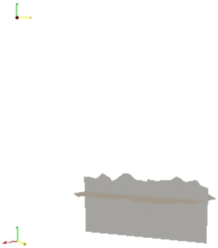

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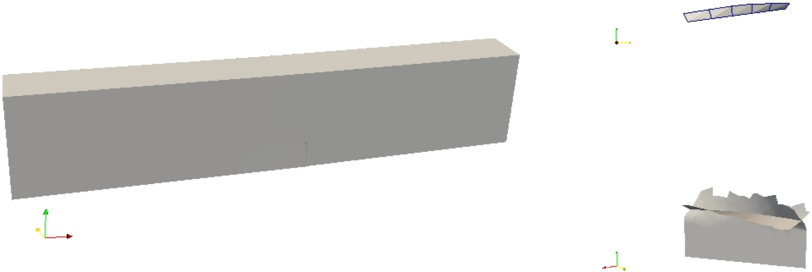

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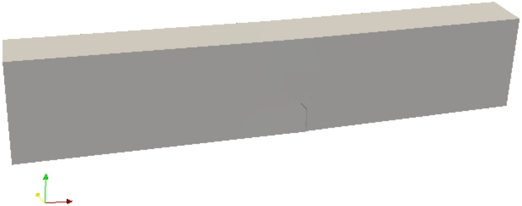

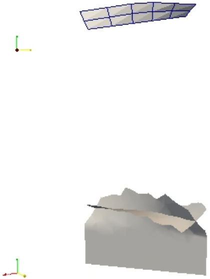

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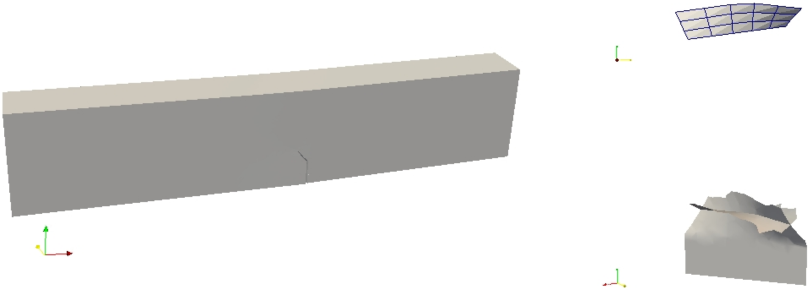

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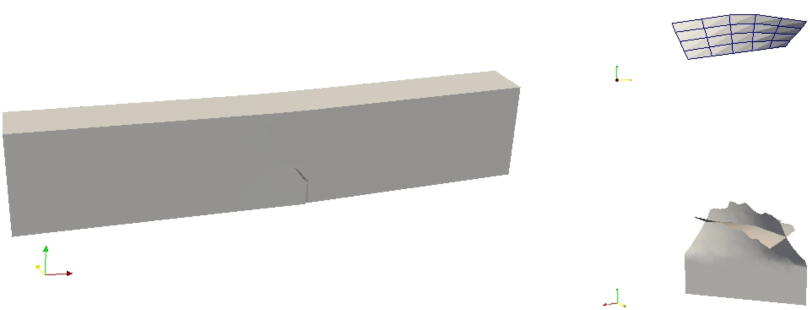

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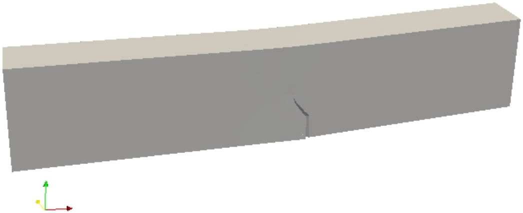

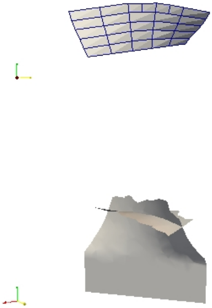

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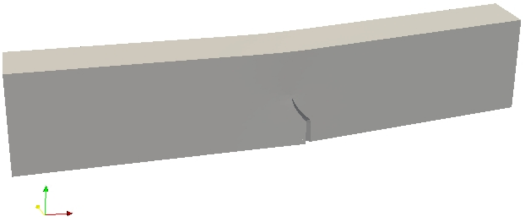

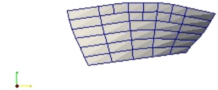

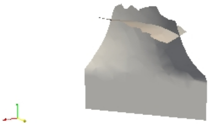

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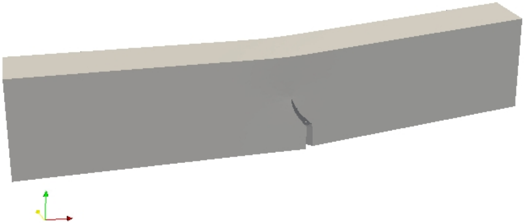

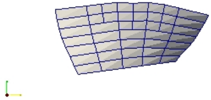

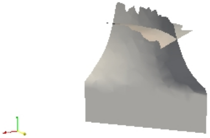

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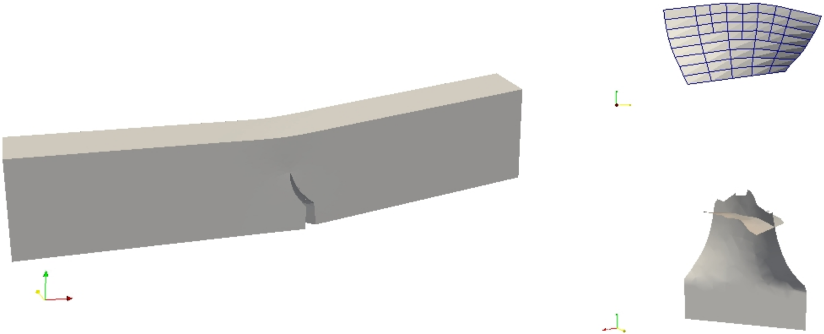

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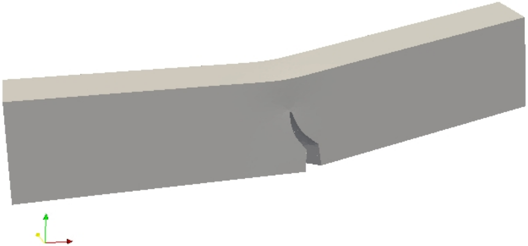

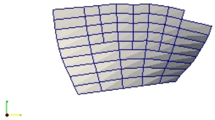

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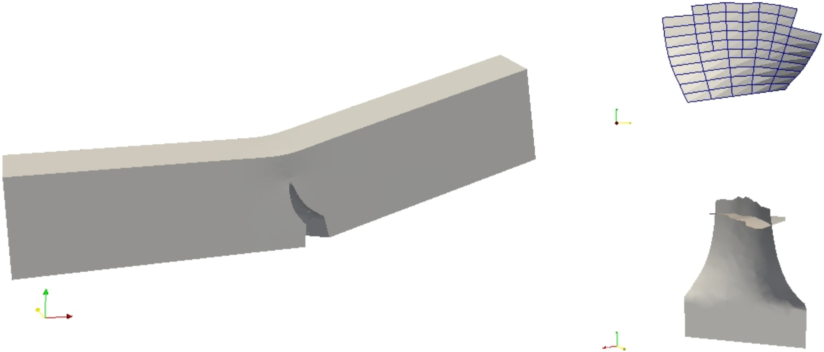

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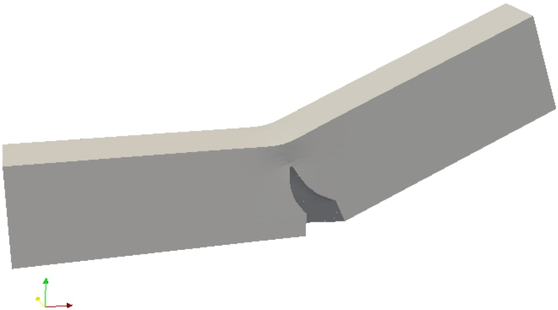

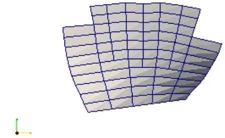

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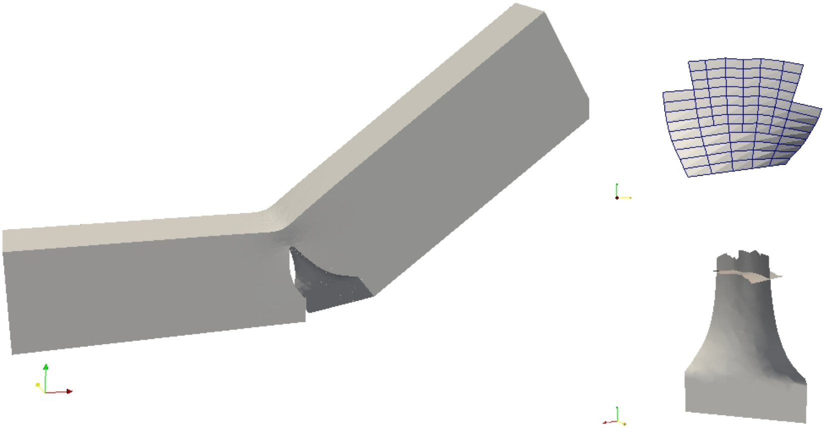

# Conclusions

A method for 3D fracture mechanics was presented which:

- Enables the use of geometrical enrichment in 3D.
- Eliminates blending errors.

A method for the representation of 3D cracks was presented which:

- Avoids the solution of evolution equations.
- Utilizes only simple geometrical operations.

The methods were combined to solve 3D crack propagation problems.

# Future work

Possibilities for future work:

- Strain smoothing-error estimation.
- Alternative enrichment functions.
- Dynamic crack propagation.

# Bibliography


Agathos, K., Chatzi, E., Bordas, S., & Talaslidis, D. (2015). A well-conditioned and optimally convergent xfem for 3d linear elastic fracture. *International Journal for Numerical Methods in Engineering*.

Fries, T. (2008). A corrected XFEM approximation without problems in blending elements. *International Journal for Numerical Methods in Engineering*.

Fries, T., & Baydoun, M. (2012). Crack propagation with the extended finite element method and a hybrid explicit-implicit crack description. *International Journal for Numerical Methods in Engineering*.

Ventura, G., Budyn, E., & Belytschko, T. (2003). Vector level sets for description of propagating cracks in finite elements. *International Journal for Numerical Methods in Engineering*.

Ventura, G., Gracie, R., & Belytschko, T. (2009). Fast integration and weight function blending in the extended finite element method. *International journal for numerical methods in engineering*.