On boosting KOHLER's ranking-by-choosing rule with a quantiles preordering

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Abstract

Several ranking methods, like TIDEMAN's ranked pairs, progressive RUBIS best choices or KOHLER's rule, have been proposed for ranking decision actions with multiple incommensurable performance criteria. Of these, KOHLER's rule is by far the fastest ranking heuristic, yet also the less satisfactory in terms of ordinal correlation with a corresponding pairwise outranking relation. We show that preordering the decision actions to be ranked along quantiles, and then locally applying KOHLER's rule on each equivalence class, greatly enhances this correlation without adding any essential algorithmic complexity. The combination of both, hence, becomes efficient for tackling large scale multiple criteria ranking problems.

Keywords: Multiple criteria ranking, Kohler's ranking-by-choosing rule, Outranking approach, Quantiles sorting

Aggregating multiple incommensurable criteria when following a bipolarly valued outranking approach leads to a reflexive and weakly complete binary preference relation which is reflexive and which satisfies the coduality principle (Bisdorff 2013 [1]). Exploiting such an outranking in order to build a ranking consists in computing a transitive and anti-symmetric (TAS) closure, a non trivial computational problem due to the potential existence of circular preferences (Lamboray & Dias 2010 [2]) .

An efficient heuristic TAS-closure, in terms of computational complexity, is given by KOHLER's ranking-by-choosing rule [3]. It consists of progressively choosing the decision alternative which shows the maximal minimal outranking degree. In case of ties, the selection follows the lexicographic order of the alternatives' identifier. This rule is codual to ARROW-RAYNAUD's prudent rule [2], which consist in progressively rejecting the alternative which shows the minimal maximal outranked degree. Being of $\mathcal{O}(n^2)$ complexity, we may today easily compute on a standard laptop a KOHLER TAS-closure of an outranking digraph of order 20 in less than ten microseconds.

When comparing now KOHLER's ranking-by-choosing rule with more complex heuristic TAS-closures, like the progressive RUBIS best choice extraction [5] or TIDEMAN's ranked pairs rule [6] , we notice that the operational efficiency of KOHLER's rule goes together with a potental loss of ordinal quality of the ranking result. We assess this correlation weakness with a Kendal τ index extended to valued bipolarly valued outranking relations [4]. When the RUBIS and TIDEMAN TAS-closures show very high correlations (usually more than +0.7), KOHLER's rule may not seldomly result, however, in a ranking that is only poorly correlated with the given outranking relation (less than +0.5). Unfortunately, both the RUBIS as well as the TIDEMAN TAS-closure, being not of polynomial complexity, quickly become inefficient when tackling digraphs of order 30 and more.

Only KOHLER's rule is readily scalable to larger outranking digraph orders. However, the ordinal correlation with the underlying outranking relation may get also weaker and weaker. Hence the idea to compute a TAS-closure in two steps: First, compute a preordering by sorting the decision actions into k multiple criteria quantile equivalence classes. And, secondly, apply KOHLER's ranking-by-choosing rule locally to each such equivalence class. Monte Carlo simulations show indeed that the ordinal correlation with the outranking relation now always exceeds +0.5, which represents a great enhancement of the ordinal quality of the TAS-closure without loosing runtime efficiency. Effectively, the run-time compexity of this two step TAS-closure is $\mathcal{O}(kn + (n/k)^2)$. If we choose $k = n$, we may keep the same run-time complexity as before. Furthermore, independence of irrelevant alternatives of the k quantiles sorting procedure, as well as the restriction of KOHLER's ranking-by-choosing rule to each one of the k preordered quantile classes, gives way to an efficient multiprocessing implementation of the two step TAS-closure. Thus it is actually possible to construct a TAS-closure of outranking digraphs of very large orders (up to several thousands of decision alternatives).

References

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