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Transportation Research Procedia 00 (2015) 000–000

Transportation Research Procedia

www.elsevier.com/locate/procedia

18th Euro Working Group on Transportation, EWGT 2015, 14-16 July 2015, Delft, The Netherlands

Assessing the effect of route information on network observability applied to sensor location problems

Marco Rinaldi [a], Francesco Corman [b], Francesco Viti [c] *

[a] Leuven Mobility Research Centre, KU Leuven, Celestijnenlaan 300A, 3001, Heverlee, Belgium
[b] Maritime and Transport Technology Department, Delft University of Technology, Mekelweg 2, 2628 CD Delft, the Netherlands
[c] Research Unit of Engineering Sciences, University of Luxembourg, 6 rue Coudenhove-Kalergi, L-8081, Luxembourg

Abstract

Link flow observability in traffic networks strongly relies on where sensors are installed. Full observability solutions are found by adopting various techniques exploiting existing relationships between links, which are determined either by using link-node relations, or by link-route relations. While the first relations are elegant as they do not require explicit route enumeration and therefore remain tractable for large-sized networks, they do not contain all the information at the route and OD levels. However, in case full route enumeration is not possible, route selection criteria are of paramount importance. In this paper we explore the impact of using k-shortest path algorithms for determining the route sets needed to solve the observability problems using link-route relations. In particular, we show how a limited amount of routes per OD pair is needed in order to incorporate all relevant information. Further, we demonstrate that selection route criteria that consider only linearly independent routes allows to find full observability solutions that need a lower number of sensors. Finally, we show that observability metrics, such as those presented in Viti et al. (2014), describe a direct relation between number of routes considered and degree of information. Through these metrics, we can show that link-route relations selected using only linearly independent links contains systematically more information. Moreover, there is empirical evidence that the marginal increment of information per additional route added decreases. This defines an asymptotic maximum value of information, which is found for a relatively limited amount of routes.

Keywords: Network Sensor Location Problem; Observability; Null-Space metric; k-shortest paths; Linearly-independent routes

* Corresponding author. Tel.: 3524666445352; Fax: +3524666445200.
E-mail address: francesco.viti@uni.lu

1. Introduction

Network Sensor Location Problems (NSLP) aim to find an optimal set of sensors in order to maximize some information on a network. These problems have been dealt with in the past in two ways. A large deal of literature sees the problem as inverse of the origin-destination (OD) estimation (see e.g., Gentili and Mirchandani, 2012 for an overview), while a more recent research direction explores the observability of any state variable (link, route or OD) on the basis of the measured flows, and by explicitly exploiting topological information (see e.g., Viti et al., 2014 for an overview). This paper focuses on the latter class, which is normally referred to as *observability problem*.

In observability problems, the goal is to find the minimum set of observed link (or route, OD) flows that can provide information on the remaining non-observed links. In particular, in *full* observability solutions, all unobserved flows are indirectly observable, as they are function of the observed flows. Closely related to this, a more general goal is to find the tight upper bound of the minimum number of observed flows such that the system becomes fully observable, while, at the same time, the observed flows are linearly independent, i.e. no redundant information is included.

Crucial information needed to solve observability problems are the topological relations expressed in the so-called incidence matrix, which relates link, node, route and possibly OD variables to each other. These relations may be expressed as route-based (Castillo et al., 2008) or node-based (Ng, 2013; He, 2013). The latter approach has the advantage of not requiring route enumeration, which is a very handy property in case of large-sized networks. Node-based solutions, however, lack some important relationships originating at the route level, i.e. those between sequences of links along a specific path. Because of this, route-based approaches may find full observability solutions with a smaller number of sensors. On the other hand, finding how and which routes to enumerate is currently a challenge (Castillo et al., 2014).

In this paper we study this last problem, by proposing and assessing a methodology based on using k-shortest path algorithms for determining the route sets considered in the incidence matrix, when solving the observability problems. In particular, we show how a limited amount of routes per OD pair is needed in order to incorporate all relevant information, and that including routes that are independent with each other yields higher information content. Observability metrics, as those presented in Viti et al. (2014), are able to quantitatively describe a direct relation between number of routes considered and degree of information. Through these metrics, we can show that link-route relations selected using only linearly independent links contains systematically more information. Moreover, there is empirical evidence that the marginal increment of information per additional route added decreases. This defines an asymptotic maximum value of information in a network, which is found for relatively limited amount of routes. We provide such empirical evidence, with the help of different mid- and large size networks, and we compare the aforementioned findings with the node-based solution counterparts.

This paper is structured as follows. The next section introduces the observability problem and a selection of the relevant literature. Section 3 presents a metric previously introduced by the authors to solve partial observability problems and to classify full observability solutions. Section 4 focuses on the impact of k-shortest paths in the definition of the link-route relations and consequently in the efficiency of full observability solutions. The impact of shortest paths is then analyzed using toy networks in Section 5. Section 6 discusses the implication of our findings and provides some direction to explore in terms of k-shortest path heuristics. Finally, Section 7 provides conclusions and recommendations.

List of symbols

$\boldsymbol{h}$	route flows vector
$\boldsymbol{v}$	link flows vector
$\boldsymbol{f}$	OD flows vector
R	route set
L	link set
$\boldsymbol{\delta}$	link-route incidence matrix
$\boldsymbol{\rho}$	OD-route incidence matrix

2. State of the Art

In traffic networks, we generally distinguish three basic flow vectors: route flows h, OD flows f and link flows v. The (static) relations between these flows are simply given by the following equations (see e.g. Cascetta, 2009):

$$v_l = \sum_{r\in R} \delta_{lr} h_r \qquad \forall l \in L \tag{1}$$

$$f_w = \sum_{r\in R} \rho_{wr} h_r \qquad \forall w \in W \tag{2}$$

with f_w the OD flow for an origin-destination pair w in the set of OD pairs W; h_r the route flow on a route r belonging to the routes set R, and v_l the link flow on link l belonging to the link set L. The matrices δ and ρ, consisting of the elements δ_{lr} and ρ_{wr}, have values 0 or 1, depending on whether route r contains or not link l, and whether route r connects OD-pair w, respectively. Alternative and (partly) analogous topological relations can be defined by introducing node-based relations instead of route-based, and by replacing the above eqns (1)-(2) with equivalent conservation of flows equations at nodes. One can refer to e.g. Ng (2012) for the mathematical definition of node-based approaches. For sake of clarity and illustration, but without losing generality, we will refer mainly to the route-based modelling approach specified by eqns (1)-(2). Moreover, we focus on the static case, i.e. the state variables in eqns (1)-(2) are time-independent. Generalizing the results to the dynamic case is straightforward, if functional relationships of matrices δ and ρ are specified (e.g. with a dynamic network loading model or a dynamic traffic assignment model, see e.g. Viti et al., 2008).

In this paper, we focus on traffic counting devices, which are often used to estimate the link flow variables. Moreover, we consider observing a variable equivalent to locating a sensor, i.e. we do not specify which state estimation process is used to translate from sensor information to traffic state. Other sensors may be deployed to represent link flow variables (e.g. fixed cameras, toll gantries) as well as route / OD variables (e.g. scanning devices, tagged vehicles). One can refer to e.g. Viti et al. (2008), Castillo et al. (2008) and Gentili and Mirchandani (2012) for a discussion of how different sensors can be used to estimate each variable.

Eqns (1)-(2) express all linear relations connecting each state variable with other states in the network. Observed states from measurements can therefore be used to infer this information to unobserved links states. When an unobserved variable can be related to only observed variables, then it also becomes observable.

Our main problem is to relate link flows with other link flows only, i.e. to manipulate the matrix δ such that one can find linear relations between a set of link flows with all other link flows. This sub-problem is called *link flow inference*. A *full observability solution* in link flow inference problems then is such that a subset of link flows makes observable all unobserved flows.

Full observability problems have been studied in different disciplines, beyond transportation network applications (e.g., electrical networks, supply chain networks, telecommunications). One approach to solve observability problems is to perform opportune matrix manipulations, i.e. by swapping or sorting rows or columns of δ, while carefully maintaining the original relations between the variables as in the original set of relations (1). This approach (we will often refer to it as pivoting in this paper) has been proposed by Castillo et al. (2000) and applied first to electrical and power networks (Castillo et al. 2006; 2007) and more recently to traffic networks (Castillo et al., 2008). One of the main advantages of the pivoting procedure is that, in any solution, the observed links are linearly independent. Clearly, a disadvantage is that multiple solutions exist, depending on the sequence in which rows and columns are swapped, as well as how δ is defined. Moreover, all route-based approaches cannot guarantee that all information about network topology and connectivity is included, if exhaustive route enumeration is not possible. This may then yield to solutions having different characteristics (or, ranks), i.e. full observability can be found with a different number (and position) of sensors. Hence, a minimum number of links to be observed is likely to exist, but is not guaranteed to be found by the pivoting. This last observation is the main motivation for this paper.

In line with the observation above, Hu et al. (2009) studied the conversion of the matrix δ into its "reduced row echelon form" through the Gaussian elimination method. By performing sensitivity analysis using different toy networks the authors suggested that an upper bound for the number of linearly independent links to be observed is likely to exist, i.e. even if the number is variable, there must be an upper limit, above which any extra link will likely be dependent on the others. Ng (2012) showed that if a node-based approach is adopted, an analytical expression for the upper bound could be found, i.e. it equals the difference between the number of links *m* and the number of non-centroid nodes *n*. He (2013) recently pointed out that the relation found by Ng (2012) does not consider all information contained in a network, as it neglects dependency of routes through the OD relations. The analytical expression provided by Ng (2012) was also discussed by Castillo et al. (2013), who argued that the minimum number of linearly independent links to be observed can be improved if a set of linearly independent paths (in terms of links) is identified first. This also obviates the full route enumeration requirement in link-path matrix-based approaches. Nevertheless, the set of linearly independent paths is not unique so an exact expression using this approach could not be provided yet. Moreover, Castillo et al. (2014) show that solutions from node-based approaches can only provide an upper bound for the minimum number of observed variables, while a lower number can be achieved if link-node information is complemented with information on (linearly independent) paths.

Because full observability solutions may require an exceedingly large amount of sensors to be placed (empirical analysis and analytical methods showed that up to 60-70% of the links in a network would have to be observed), which is infeasible for any real-sized network, problems addressing partial observability, i.e. where part of the information remains unknown, are more suited for realistic applications. Thus, recently the concept of partial observability was introduced, to consider also the information contained in measured flows about the flows that are not fully observed, as well as for assessing partial solutions where only a subset of the variables needed to fully observe the system are considered. For partial observability solutions, some extra information is required, other than the link-route relations. In fact, choosing a subset of links to be considered asks for a ranking of different links in their amount of importance. So far typical ranking functions relate to the maximization of the number of observable variables, but in general there is no straightforward way to consider the amount of information that different sets (possibly of different size) of observed (and observable) variables contain on the unobserved ones. To overcome this problem, Viti et al. (2014) introduced a metric that considers the ensemble of information from both observable and non-observable link flows and relates this to the ‘size’ of the solutions space. A greedy algorithm identifies an efficient sequence of links to measure in order to maximize the information on the network given a budget of sensors to install. This allows categorizing full observability solutions based upon the degree by which information is incrementally acquired across the observed link flows.

By testing the algorithm on a real-sized network, where explicit non-exhaustive route enumeration is used to derive the link-route relations, the authors showed that pivoting solutions may be found with a smaller number of sensors than when using the link-node relations. This suggests that a lower bound for the number of sensors to be installed for full observability can still be sought and strongly depends on the way route-link relations are defined. In the network of Rotterdam, the Netherlands, characterized by 476 links, 239 nodes, 5668 routes (which is obviously a non-exhaustive list) and 1890 OD pairs, Castillo’s pivoting procedure resulted in 281 independent links, Ng’s node-based procedure found 284 independent links while He’s spanning tree procedure determined a solution with 322 independent links. Hence, the route-based approach has apparently room for obtaining a lower number of sensors to install, and it may thus not only be relevant to analytically determine an upper bound of this number, but also its lower limit.

In this paper we start from this observation and study the impact of using conventional k-shortest path algorithms when defining the incidence matrix δ . We do so by first analyzing the impact of considering k shortest paths on different classical toy network examples. The following step is then to look for route selection criteria that can help at defining an efficient incidence matrix where a full observability solution is attained with a lower number of sensors. This analysis is presented after briefly introducing the Null-space metric and the greedy algorithm. For further details one can refer to Viti et al. (2014).

3. The Null-Space Metric and a greedy algorithm

The pivoting procedure may yield multiple solutions given an input incidence matrix δ ; this implies that different partial as well as full observability solutions can be found, depending on the sequence in which rows and columns are swapped. To make a distinction between these pivoting solutions, in Viti et al. (2014) a metric was proposed that enables one to rank full observability solutions based on a limited number of “families”. These families depend on the total amount of information contained by the different full observability solutions, in terms of quantity and quality of independence-dependence relationships. The term "information" used in this paper is the same as the one defined in Viti et al. (2014), and is related to the extent of the full solution space.

In Viti et al. (2014) a second metric was proposed, that allows one to assess the quality of a partial observability solution (in relation to the respective full observability solution) if only a subset of the links of the full observability solution is taken. It does so by quantifying the maximum relative error that can occur if not all sensors characterizing a full observability solution are installed and/or working. This error ranges between 0 (full observability) and 1 (no information about the network) and is related to the Null space of the pivoted matrix, i.e. one obtained by using e.g. the pivoting process by Castillo et al. (2008), or alternatively the node-based solution in Ng (2012). The Null space describes the degrees of freedom resulting from under-determinedness of the subspace of solutions that are possible given the pivoted matrix. In other terms, in case of full observability there are no degrees of freedom. For all other cases, the extent of the solution space is clearly an uncertainty measure for the choice of not measuring one or more independent variables. The goal of the metric is to assign a single scalar value to the vector space defined by any possible subset of the measured variables in the Null space. For a detailed formulation, description and graphical interpretation of the NSP metric we refer to Viti et al. (2014).

In the same paper, greedy algorithms were proposed to determine partial observability solutions (i.e. some set of link flows to be measured). The *Add* heuristic starts from an empty set of observed variables, and adds, as variable to be observed, the one that results in the largest marginal increase of information. Iteratively, the chosen variable is then added in the set of observed variables. This heuristic allows one to identify the most informative links to add to, for example, an existing set of sensors. By contrast, a *Remove* heuristic removes variables from the full observability solution in order of least information contents. This is then useful to identify the most critical links, i.e. those that, if information is missing, will have a strong impact on the reliability of the state estimation solution. The methodology was tested on different toy networks and a real-sized network, showing that even few links might contain a large deal of information, according to the NSP metric, and therefore reduce the estimation uncertainty.

So far, the NSP method has been applied to problems where the initial matrix δ is given, and we investigated the effects of different sequences for pivoting, to show that information on partial observability solutions may be very different. In this paper, we assume that also δ could be different, as is often the case. This implies, as introduced in Section 2, that the starting potential information contained in δ will then be reflected by partial and full observability solutions that are more efficient, i.e. that yield the same (amount of) information, with a smaller number of sensors. In this paper we adopt an exploratory approach, i.e. we study different network sizes and layouts to understand how the specification of δ could affect the observability solution.

4. K-shortest path algorithms and their impact on observability problems

Full and partial observability problems as described so far are solved by taking the incidence matrix δ as input. In case of non-exhaustive route enumeration, explicit route enumeration is usually done following some intuitive criteria. When full route enumeration is not possible, given the risk of combinatorial explosion even for reasonably small-sized networks, the most traditional way of defining routes is by means of k-shortest path algorithms.

K-shortest path algorithms require extra information in addition to the one contained in the purely topological and connectivity relations expressed in (1). Usually this extra information is given in terms of e.g., link lengths, link uncongested travel time, and more generally by the concept of generalized costs (Cascetta, 2009). By using the additive property of links onto routes, simple algorithms allow one to identify the best routes between an origin and a destination. The definition of "best" is dependent on the chosen objective, such as fastest path (i.e. having the minimum travel time), shortest path (minimum distance travelled), most reliable path (i.e. minimum the travel time variance), etc. In this paper we employ shortest distance route concepts, arguing that this choice has no real impact to overall paper conclusions.

Conventional k-shortest path algorithms are usually based on the well-known Dijkstra (1959) algorithm. This method calculates the best (shortest) non-cyclical route connecting an origin node to any other node in the network through a greedy approach, and is shown to yield the exact solution as long as the link costs are non-negative. Starting from the results of the Dijkstra algorithm, Yen's k-shortest path algorithm (Yen, 1971) is the most conventionally employed algorithm to determine the k-shortest paths for a given origin/destination pair. This algorithm determines the second, third, … , k-th shortest alternatives by selecting a deviation node from the original shortest path, and seeking the second-best alternative from said deviation. These deviation nodes are chosen sequentially starting from the first node downstream of the origin node, and any new path that is not already in the set of the k-shortest alternatives found so far is included in the set, until either k paths are found or no new paths can be generated. Several modifications to this simple algorithm have been developed, e.g. focusing on avoiding a too high percentage of overlap with respect to the already existing paths.

4.1. A K-Independent Shortest Path algorithm (KISP)

As introduced earlier, our objective in this work is that of assessing how additional route information can affect the full/partial observability of networks. Following the claims found in Castillo et al. (2014), we go beyond testing only randomly generated k-shortest paths, we develop a simple heuristic variation to Yen's algorithm that allows us to obtain a set of linearly independent paths for each separate OD. In other words, the paths found are independent within each OD, while they might be dependent with respect to paths of other OD pairs. To the best of our knowledge, while research has focused on avoiding or reducing the computational overhead of path enumeration, no contributions in the field of *independent* route enumeration algorithms have been carried.

The k-independent shortest paths algorithm (KISP) extends the algorithm of Yen by performing an additional check: when a new candidate path is generated following Yen's principle of node deviation, the algorithm verifies whether or not it is linearly independent with respect to the other paths already included in the generated set of paths. The linear independence can be checked based on the rank of the matrix containing all routes. If the path is linearly dependent, it is immediately discarded, and the algorithm skips to the next upstream deviation node. The process is repeated until a candidate path that meets the criteria is found, or until no more paths can be found.

K-Independent Shortest Path Algorithm (KISP):
For each Origin/Destination pair
    Compute the initial shortest path
    Do:
        Select a deviation node i from the latest path found
        Remove the the arc(s) connecting i to the latest path found
        Compute the shortest path connecting node i to the destination node d
        If a path is found:
            Check the new path for independence
            Add the new independent path to the set
        Restore the arc(s) connecting i to the previous path
    Until no more paths can be found **or** number of paths found = k

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As introduced in Castillo et al. (2013; 2014), by identifying a set of linearly independent routes and including them in the link-route incidence matrix δ one can guarantee that the pivoting procedure will find the tight upper bound of the number of links that guarantee full observability. However, in analogy to the non-uniqueness of the pivoting procedure's solution, the set of linearly independent routes can also depend on the way routes are selected in sequence. In particular, path enumeration algorithms such as the KISP algorithm identify the shortest path tree given a certain origin, as well as k-shortest paths up to *k* routes given a certain OD pair. The sequence in which the OD pairs are explored might indeed influence which paths are included in the linearly independent path set, as, indeed, some paths that have been included by previously visited OD pairs will influence, due to the independence condition, whether or not a new route can be added.

5. Analysis on toy networks

The impact of including different sources of route information in the observability procedure, and the advantage of the KISP algorithm, is here analyzed on different toy networks found in literature.

We devise our tests in order to test the following hypotheses:

1. full observability matrices based upon independent route sets hold more information than those obtained from randomly generated route sets;
2. full observability matrices based upon link-route relationships hold more information than those obtained from link-node relationships.

To assess this difference in information we perform an a-priori ranking, based on the following metric for ranking full observability solutions, based on Viti et al. (2014):

$$\frac{\|\Omega\|_F}{rk(\Omega)} \tag{3}$$

which assigns a scalar value, which is the highest for the full observability matrix Ω, obtained following Castillo's pivoting procedure, whose information quantity is highest. Afterwards, we analyze the information from the point of view of partial observability, i.e. we perform our *Add* heuristic to the case being studied. In principle, the higher the content of information associated to the full observability solution, the higher information will be associated to some links, and the faster the greedy heuristic will be able to reduce the error according to our NSP metric. Quantitatively, this results in a steeper descent of the metric over the number of observed links.

We present test results comprehensively in two forms. A histogram represents the distribution of different ranks of enumerating 300 full observability solutions based on the Pivoting procedure of Castillo and evaluated with our full observability metric (3). We here call "families" the way those solutions are clustered into a finite amount of sets with given ranks (see again Viti et al., 2014 for more details). We report the distribution of the families for the random routing and the independent routing KISP.

Secondly, we choose the highest and lowest ranking families from the aforementioned results, and apply our partial observability *Add* heuristic, and show the resulting descent in amount of error. We perform this analysis for both full observability solutions based on randomly generated k-shortest paths, as for the KISP generated independent paths.

The first network, which represents a relatively small case, with 4 links, 9 nodes and a total of 24 possible routes, is shown in Fig 1. Enumerating all combinations of independent variables yielding full observability, even for this simple case, through Castillo's pivoting procedure, is however impractical. Given the small size of the network, and the relatively small number of routes to enumerate, there is no difference between random and independent routing and we show instead the variation in number of families when increasing the maximum value of k, and how this growth stabilizes after a number k set high enough.

The fact that no difference ensues in scores between using random routing or independent routing is surely due to the network's simple, highly symmetrical nature. However, as can be clearly seen in the descent graph, there is quite a difference between the cases in which route information (of any kind and nature) is considered, and the case based upon node-link information alone. In other terms, the curve for k =0 is substantially different than all other curves. This agrees with our hypothesis, i.e. that disregarding route information in full observability procedures might yield a loss in total information quantity. Full observability could not be obtained with k<3; instead for k>5 no extra information is contributed by the additional paths in the route set, and k=4 is, for this network, the optimal setting.

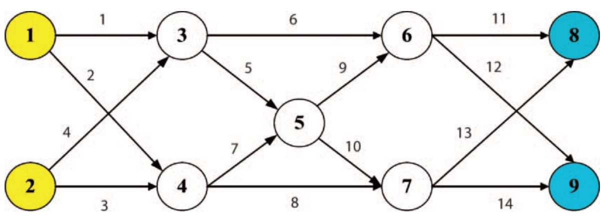

Fig. 1. Parallel Highway Network

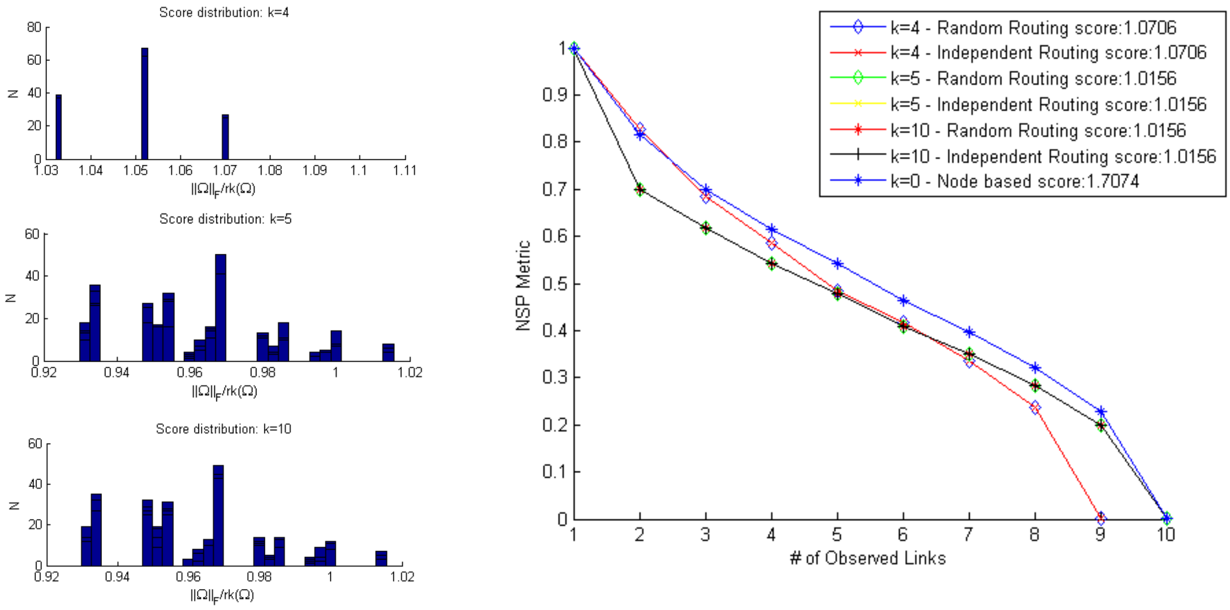

Fig. 2. Results for Parallel Highway network

A slightly more complex network highlighting interesting results in the route selection comparison is the Fishbone network depicted in Fig. 3, and analyzed in Fig. 4 using the same graphical illustrations as before.

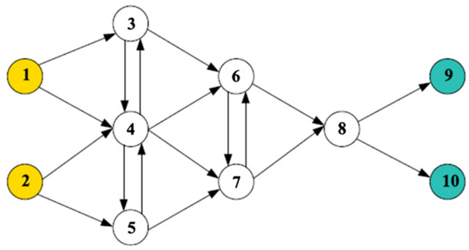

Fig. 3. Fishbone network

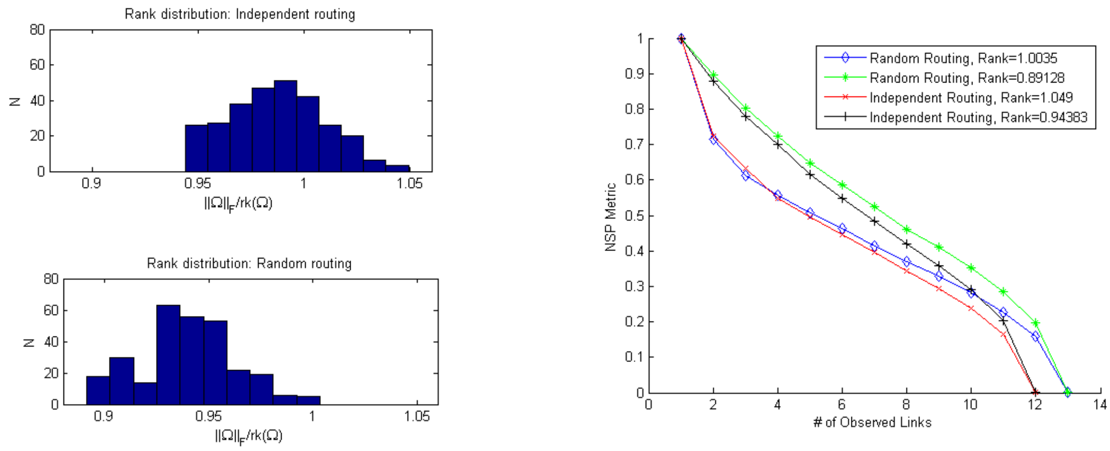

Fig. 4. Results for the Fishbone network

In this network, featuring 10 nodes, 18 links and a total of 64 possible routes, the distribution of randomly generated full observability "pivot" solutions' ranks is strongly influenced by the presence of independent routes in the path set (Figure 4). The amount of k is set to 10. Choosing independent routes yields on average 5% more information than random routes, as one can see from the histograms derived from the random generation of solutions. This again is in line with our first hypothesis.

Generating routes based on the KISP approach appears thus to be beneficial, with respect to both generating routes following the standard k-shortest path approach, as well as including no route information at all in the full observability procedure. Again, as one can observe from the right picture, although it is possible to find random k-shortest path sets that contain a good deal of information in case of partial observability (e.g. the random routing family with highest rank value indicated with blue diamond markers) they require one more sensor for full observability with respect to the ones found with the KISP algorithm. This verifies our second hypothesis.

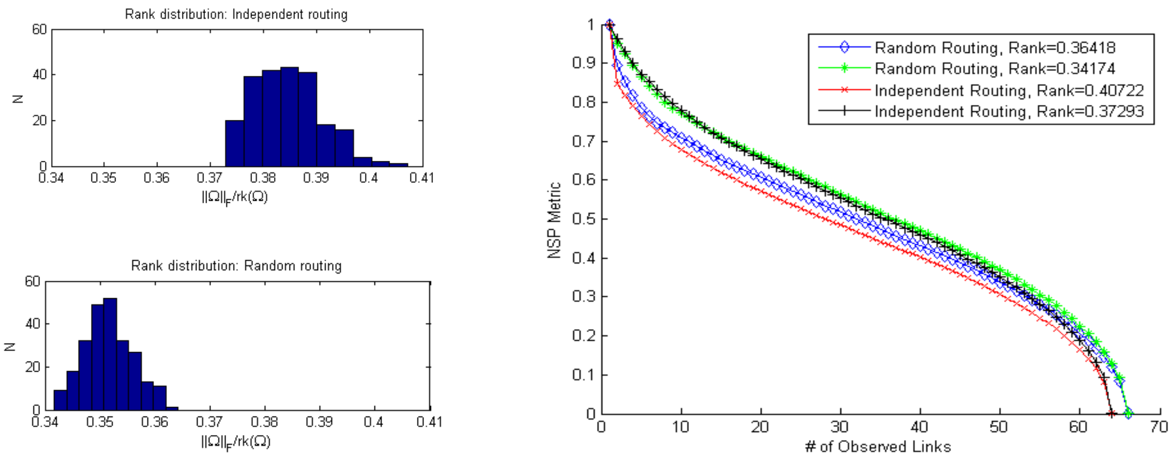

Fig. 5. Results for Sioux Falls network.

We validate this assumption by performing tests on a bigger network, the well-known Sioux Falls network, which is characterized by 76 links, 176 OD pairs and 24 nodes, k=10. The results for this network are shown in Figure 5 and are fully in line with the previous findings. Again, these results quite clearly show how the KISP Algorithm is able to identify independent routes whose impact in terms of information for both full and partial observability metrics is of considerable magnitude.

6. Implications

These first, explorative tests helped us to identify an underlying property of full observability procedures: thanks to the inclusion of route information with particular properties (independence), a lower bound to the number of link variables to be observed can be found. Following what was concluded for node-based approaches in Ng (2012), we expect that this minimum number might be related with the total number of flow variables that need explaining in the given network, i.e. links, nodes and origin/destination pairs. This assumption finds further confirmation when studying the impact of increasing parameter k for the simple k-shortest paths algorithm, where indeed after a given, albeit network-dependent, threshold enumerating additional routes yield no further benefit.

We consider these findings extremely important, as they can guide us in developing purpose-built route enumeration heuristics that explicitly maximize the amount of information contributed by each additional path, so that the most efficient route-set, and therefore full observability solution, can be obtained a-priori. This is however on-going research and will be the main contribution of future papers.

7. Conclusions

Full and partial observability solutions have recently taken the attention, as they allow, through simple topological and supply-related relations, to reduce problem under-determinedness typical of state (e.g. flow) estimation problems.

In this paper we presented an experimental analysis, devised in order to assess the impact that different route set generation methods and parameters have on the quality of solutions for the network observability problem. We extend a well-known route set generation algorithm so to obtain an independent route set and validate the hypothesis that independent routing information greatly contributes to the goodness of full observability solutions.

Acknowledgements

We acknowledge for financing the following grants: (KUL) OT/11/068-project and the SBO project IWT-140433 in the programs 'Richting Morgen' and 'Vlaanderen in Actie Pact 2020'.

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