Constructive Interference through Symbol Level Precoding for Multi-level Modulation

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***Abstract*—The constructive interference concept in the downlink of multiple-antenna systems is addressed in this paper. The concept of the joint exploitation of the channel state information (CSI) and data information (DI) is discussed. Using symbol-level precoding, the interference between data streams is transformed under certain conditions into useful signal that can improve the signal to interference noise ratio (SINR) of the downlink transmissions. In the previous work, different constructive interference precoding techniques have been proposed for the MPSK scenario. In this context, a novel constructive interference precoding technique that tackles the transmit power minimization (min-power) with individual SINR constraints at each user's receivers is proposed assuming MQAM modulation. Extensive simulations are performed to validate the proposed technique.**

I. Introduction

Interference is one of the crucial factors that degrades the performance in wireless networks. Exploiting the spatial dimension empowers the wireless system with additional dimension by adding multiple antennas at the communication terminals. In the literature, utilizing the time and frequency resources has been proposed to allow different users to share the resources without inducing harmful interference. The concept of exploiting the users' spatial separation has been a fertile research domain for more than one decade [1]. This can be implemented by adding multiple antennas at one or both communication sides. Multiantenna transceivers provide the communication systems with more degrees of freedom that can boost the performance if the multiuser interference is mitigated properly. Exploiting the space dimension, to serve different users simultaneously in the same time slot and the same frequency band through spatial division multiplexing (SDMA), has been investigated in [1].

In this paper, the main idea is to constructively correlate the interference among the spatial streams rather than fully decorrelate them as in the conventional schemes [2]. In [9], the interference in the scenario of BPSK and QPSK is classified into types: constructive and destructive. Based on this classification, a selective channel inversion scheme is proposed to eliminate the destructive interference while it preserves the constructive one to be received at the users' terminal. A more advanced scheme is proposed in [10], which rotates the destructive interference to be received as useful signal with the constructive one. These schemes outperform the conventional precodings [2] and show considerable gains. However, the anticipated gains come at the expense of additional complexity at the system design level. Assuming that the channel coherence time is τ_c, and the symbol period is τ_s, with $\tau_c \gg \tau_s$ for slow fading channels, the user precoder has to be recalculated with a frequency of $\frac{1}{\tau_c}$ in comparison with the symbol based precoder $\frac{1}{\min(\tau_c,\tau_s)} = \frac{1}{\tau_s}$. Therefore, faster precoder calculation and switching is needed in the symbol-level precoding which can be translated to more expensive hardware.

In [13]- [14], we have set the foundation for a symbol based precoding which opens new possibilities for exploiting the interference by establishing the connection between the constructive interference precoding and multicast. Moreover, several constructive interference precoding schemes have been proposed in [14], including Maximum ratio transmission (MRT)-based algorithm and objective-driven constructive interference techniques. The MRT based algorithm, titled as Constructive interference MRT (CIMRT), exploits the singular value decomposition (SVD) of the concatenated channel matrix. This enables the decoupled rotation using Givens rotation matrices between the users' channels subspaces to ensure that the interference is received constructively at the users. On the other hand, the objective- driven optimization formulates the constructive interference problem by considering its relation to PHY-multicasting. However, all the previous contributions focus on utilizing the constructive interference assuming MPSK modulation, The contributions of this paper can be summarized in the following points:

- The previous works have discussed the constructive interference for M-PSK modulation. In this paper, we extend the constructive interference approach for MQAM modulation. The solution depends on the relation between the constructive interference precoding and PHY-layer multicasting.
- Energy efficiency analysis is discussed to select the optimal SNR target for each modulation. Based on symbol error rate analysis and the power consumption, we find the SNR target that optimizes the energy efficiency.

Notation: We use boldface upper and lower case letters for matrices and column vectors, respectively. $(\cdot)^H$, $(\cdot)^*$ stand for Hermitian transpose and conjugate of $(\cdot)$. $\mathbb{E}(\cdot)$ and $\|\cdot\|$ denote the statistical expectation and the Euclidean norm, $\otimes$ denotes the kronecker product, and $\mathbf{A} \succeq \mathbf{0}$ is used to indicate

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the positive semidefinite matrix. $\angle(\cdot)$, $|\cdot|$ are the angle and magnitude of $(\cdot)$ respectively. $\mathcal{R}(\cdot)$, $\mathcal{I}(\cdot)$ are the real and the imaginary part of $(\cdot)$.

II. System and Signal Models

We consider a single-cell multiple-antenna downlink scenario, where a single BS is equipped with M transmit antennas that serves K user terminals, each one of them equipped with a single receiving antenna. The adopted modulation technique is M-QAM. We assume a quasi static block fading channel $\mathbf{h}_j \in \mathbb{C}^{1\times M}$ between the BS antennas and the j^{th} user, where the received signal at jth user is written as

$$y_j[n] \quad = \quad \mathbf{h}_j\mathbf{x}[n] + z_j[n]. \tag{1}$$

$\mathbf{x}[n] \in \mathbb{C}^{M\times 1}$ is the transmitted symbol sampled signal vector at time n from the multiple antennas transmitter and z_j denotes the noise at j^{th} receiver, which is assumed independent and identically distributed complex Gaussian distributed variable $\mathcal{CN}(0,1)$. A compact formulation of the received signal at all users' receivers can be written as

$$\mathbf{y}[n] \quad = \quad \mathbf{H}\mathbf{x}[n] + \mathbf{z}[n]. \tag{2}$$

Let $\mathbf{x}[n]$ be written as $\mathbf{x}[n] = \sum_{j=1}^{K}\sqrt{p_j[n]}\mathbf{w}_j[n]d_j[n]$, where $\mathbf{w}_j$ is the $\mathbb{C}^{M\times 1}$ unit power precoding vector for the user j. The received signal at j^{th} user y_j in n^{th} symbol period is given by

$$y_j[n] = \sqrt{p_j[n]}\mathbf{h}_j\mathbf{w}_j[n]d_j[n] + \sum_{k\neq j}\sqrt{p_k[n]}\mathbf{h}_j\mathbf{w}_k[n]d_k[n] + z_j[n] \tag{3}$$

where p_j is the allocated power to the j^{th} user. Notice that the transmitted signal $\mathbf{d} \in \mathbb{C}^{K\times 1}$ includes the uncorrelated data symbols d_k for all users with $\mathbb{E}[|d_k|^2] = 1$. It should be noted that both CSI and data information (DI) are available at the transmitter side. From now on, we assume that the precoding design is performed at each symbol period and accordingly we drop the time index for the sake of notation.

III. Conventional Multiuser Precoding Techniques

The main goal of transmit beamforming is to increase the signal power at the intended user and mitigate the interference to non-intended users. This can be mathematically translated to a design problem that targets beamforming vectors to have maximal inner products with the intended channels and minimal inner products with the non-intended ones. Several approaches have been proposed including minimizing the sum power while satisfying a set of SINR constraints [5] and maximizing the jointly achievable SINR margin under a power constraint [6]. In any scenario, the generic received signal can be formulated as

$$\begin{aligned}
\mathbf{y}[n] &= \mathbf{H}\mathbf{x}[n] + \mathbf{z}[n] = \mathbf{H}\mathbf{W}[n]\mathbf{P}^{\frac{1}{2}}[n]\mathbf{d}[n] + \mathbf{z}[n] \\
&= \begin{bmatrix} \underbrace{a_{11}}_{\text{desired}} & \underbrace{a_{12}}_{\text{interference}} & \cdots & \underbrace{a_{1K}}_{\text{interference}} \\ a_{21} & \underbrace{a_{22}}_{\text{desired}} & \cdots & a_{2K} \\ \vdots & \vdots & \vdots & \vdots \\ \underbrace{a_{K1}}_{\text{interference}} & \underbrace{a_{K2}}_{\text{interference}} & \cdots & \underbrace{a_{KK}}_{\text{desired}} \end{bmatrix} \begin{bmatrix} d_1 \\ \vdots \\ d_K \end{bmatrix} + \mathbf{z}.
\end{aligned} \tag{4}$$

The corresponding SINR of user j can be expressed as

$$\gamma_j = \frac{p_j\|\mathbf{h}_j\mathbf{w}_j\|^2}{\sum_{i=1,i\neq j}^{K} p_i\|\mathbf{h}_j\mathbf{w}_i\|^2 + \sigma^2} = \frac{|a_{jj}|^2}{\sum_{i=1,i\neq j}^{K}|a_{ji}|^2 + \sigma^2}. \tag{5}$$

Symbol-level precoding tries to go beyond this conventional look at the interference. This precoding can under certain conditions convert the inner product with the non-intended channels into useful power by maximizing them but with the specific directions to which constructively add-up at each user receivers. Taking into account the I/Q plane of the symbol detection, the constructive interference is achieved by using the interfering signal vector to move the received point deeper into the correct detection region. Considering that each user receives a constructive interference from other users' streams, the received signal can be written as

$$y_j[n] = \sum_{i=1}^{K}\underbrace{\sqrt{p_j[n]}\mathbf{h}_j\mathbf{w}_i[n]d_i[n]}_{a_{ji}[n]d_j[n]} + z_j[n]. \tag{6}$$

This yields the SINR expression for M-PSK symbols as

$$\gamma_j[n] = \frac{\|\sum_{i=1}^{K}\sqrt{p_j[n]}\mathbf{h}_j\mathbf{w}_i[n]\|^2}{\sigma^2} = \frac{|\sum_{i=1}^{K}a_{ji}|^2}{\sigma^2}. \tag{7}$$

Different precoding techniques that redesign the terms $a_{ji}, j \neq i$ to constructively correlate them with a_{jj} are proposed in the next sections (IV).

A. Power constraints for user based and symbol based precodings

In the conventional user based precoding, the transmitter needs to precode every τ_c which means that the power constraint has to be satisfied along the coherence time $\mathbb{E}_{\tau_c}\{\|\mathbf{x}\|^2\} \leq P$. Taking the expectation of $\mathbb{E}_{\tau_c}\{\|\mathbf{x}\|^2\} = \mathbb{E}_{\tau_c}\{tr(\mathbf{W}\mathbf{d}\mathbf{d}^H\mathbf{W}^H)\}$, and since $\mathbf{W}$ is fixed along τ_c, the previous expression can be reformulated as $tr(\mathbf{W}\mathbb{E}_{\tau_c}\{\mathbf{d}\mathbf{d}^H\}\mathbf{W}^H) = tr(\mathbf{W}\mathbf{W}^H) = \sum_{j=1}^{K}\|\mathbf{w}_j\|^2$, where $\mathbb{E}_{\tau_c}\{\mathbf{d}\mathbf{d}^H\} = \mathbf{I}$ due to uncorrelated symbols over τ_c. However, in symbol level precoding the power constraint should be guaranteed for each symbol vector transmission namely for each τ_s. In this case the power constraint equals to $\|\mathbf{x}\|^2 = \mathbf{W}\mathbf{d}\mathbf{d}^H\mathbf{W}^H = \|\sum_{j=1}^{K}\mathbf{w}_j d_j\|^2$. In the next sections, we characterize the constructive interference and show how to exploit it in the multiuser downlink transmissions[2].

[2]From now on, we assume that the transmission changes at each symbol and we drop the time index for the ease of notation

IV. Constructive interference for power minimization

The interference among the simultaneous spatial streams leads to deviation of the received symbols from their detection region. However, this interference can be designed to push the received symbols further into the correct detection region assuming MPSK modulation and, as a consequence it enhances the system performance [13]- [16]. However the case is different for MQAM, the constructive interference can be exploited to push the outer constellation symbols deeper in their detection regions. For the inner constellation symbols, this cannot be applied directly. Assuming both DI and CSI are available at the transmitter, the cross correlation between the k^{th} data stream and the j^{th} user can be formulated as:

$$\rho_{jk} = \frac{\mathbf{h}_j \mathbf{h}_k^H}{\|\mathbf{h}_j\| \|\mathbf{h}_k\|}. \tag{8}$$

A. Constructive Interference Power Minimization Precoding for MQAM modulation (MCIPM)

Based on the definition of constructive interference, we should design the constructive interference precoders by guaranteeing that the sum of the precoders and data symbols pushes the received signal deeper in the correct detection region for outer constellation symbols and achieves the exact symbols for the inner constellation ones. Therefore, the optimization that minimizes the transmit power and grants the constructive reception of the transmitted data symbols can be written as

$$\begin{aligned} \mathbf{w}_j(d_j, \mathbf{H}, \boldsymbol{\zeta}) &= \arg \min_{\mathbf{w}_1,\dots,\mathbf{w}_K} \quad \|\sum_{k=1}^{K} \mathbf{w}_k d_k\|^2 \\ s.t. & \qquad \mathcal{C}_1, \mathcal{C}_2 \end{aligned} \tag{9}$$

For the received signal at j^{th} user, we denote α_j^r,α_j^i as the in-phase and the quadrature components respectively. α_j^r, α_j^i can be mathematically formulated as

$$\alpha_j^r = \frac{\mathbf{h}_j \sum_k \mathbf{w}_k d_k + (\mathbf{h}_j \sum_k \mathbf{w}_k d_k)^H}{2} \tag{10}$$

$$\alpha_j^i = \frac{\mathbf{h}_j \sum_k \mathbf{w}_k d_k - (\mathbf{h}_j \sum_k \mathbf{w}_k d_k)^H}{2i} \tag{11}$$

$\mathcal{C}_1, \mathcal{C}_2$ can be formulated to guarantee that the received signal lies in the correct detection region, which depends on the data symbols. A detailed formulation for $\mathcal{C}_1$, $\mathcal{C}_2$ can be expressed as

- For the inner-constellation symbols, the constraints $\mathcal{C}_1$, $\mathcal{C}_2$ should guarantee that the received signals achieve the exact constellation point. For 16-QAM as depicted in Fig. (1), the symbols marked by 1 should be received with the exact symbols. The constraints can be written as

$$\begin{aligned} \mathcal{C}_1: &\quad \alpha_j^r = \sqrt{\zeta_j}\mathcal{R}\{d_j\} \\ \mathcal{C}_2: &\quad \alpha_j^i = \sqrt{\zeta_j}\mathcal{I}\{d_j\} \end{aligned}$$

- Outer constellation symbols, the constraints $\mathcal{C}_1, \mathcal{C}_2$ should guarantee the received signals lie in the correct detection. For 16-QAM as depicted in Fig. (1), the symbols marked by 2 should be received with the exact symbols. The constraints can be written as

$$\begin{aligned} \mathcal{C}_1: &\quad \alpha_j^r \gtreqless \sqrt{\zeta_j}\mathcal{R}\{d_j\} \\ \mathcal{C}_2: &\quad \alpha_j^i = \sqrt{\zeta_j}\mathcal{I}\{d_j\} \end{aligned}$$

$$\begin{aligned} \mathcal{C}_1: &\quad \alpha_j^r = \sqrt{\zeta_j}\mathcal{R}\{d_j\} \\ \mathcal{C}_2: &\quad \alpha_j^i \gtreqless \sqrt{\zeta_j}\mathcal{I}\{d_j\}, \end{aligned}$$

- Outermost constellation symbols, the constraints $\mathcal{C}_1$, $\mathcal{C}_2$ should guarantee the received signals lie in the correct detection. For 16-QAM as depicted in Fig. (1), the symbols marked by 3 should be received with the exact symbols. The constraints can be written as

$$\begin{aligned} \mathcal{C}_1: &\quad \alpha_j^r \gtreqless \sqrt{\zeta_j}\mathcal{R}\{d_j\} \\ \mathcal{C}_2: &\quad \alpha_j^i \gtreqless \sqrt{\zeta_j}\mathcal{I}\{d_j\}. \end{aligned}$$

  The sign $\gtreqless$ indicates that the symbols should locate in the correct detection region, for the symbols in the first quadrant $\gtreqless$ mean $\geq$.

where ζ_j is the SNR target for the j^{th} user that should be granted by the transmitter, and $\boldsymbol{\zeta} = [\zeta_1, \dots, \zeta_K]$ is the vector that contains all the SNR targets that should be guaranteed by BS to each user. This way, each receiver can correctly scale its margins during the symbol detection. The set of constraints $\mathcal{C}_1$ $\mathcal{C}_2$ guarantees that each user receives its corresponding data symbol d_j correctly. If we assume that all users' data symbols lie at the outer constellation points at certain instant, the optimization in (9) can be expressed in details as:

$$\begin{aligned} \mathbf{w}_j = &\quad \arg \min_{\mathbf{w}_1,\dots,\mathbf{w}_k} \quad \|\sum_{k=1}^{K} \mathbf{w}_k\|^2 \\ & s.t. \quad \begin{cases} \mathcal{C}1: \alpha_j^r \geq \sigma\sqrt{\zeta_j}\mathcal{R}\{d_j\}, \forall j \in K \\ \mathcal{C}2: \alpha_j^i \geq \sigma\sqrt{\zeta_j}\mathcal{I}\{d_j\}, \forall j \in K. \end{cases} \end{aligned} \tag{12}$$

If we denote $\mathbf{x} = \sum_{k=1}^{K} \mathbf{w}_k d_k$, a formulation of (12) can be expressed as

$$\begin{aligned} \mathbf{x} = &\quad \arg\min_{\mathbf{x}} \quad \|\mathbf{x}\|^2 \\ s.t. &\quad \begin{cases} \mathcal{C}1: \frac{\mathbf{h}_j\mathbf{x} + (\mathbf{h}_j\mathbf{x})^H}{2} \geq \sigma\sqrt{\zeta_j}\mathcal{R}\{d_j\}, \forall j \in K \\ \mathcal{C}2: \frac{\mathbf{h}_j\mathbf{x} - (\mathbf{h}_j\mathbf{x})^H}{2i} \geq \sigma\sqrt{\zeta_j}\mathcal{I}\{d_j\}, \forall j \in K. \end{cases} \end{aligned} \tag{13}$$

The solution for (13) can be found by writing the Lagrangian function as follows

$$\begin{aligned} \mathcal{L} &\quad (\mathbf{x}) = \|\mathbf{x}\|^2 \\ + &\quad \sum_j \mu_j \Big(-0.5i \frac{(\mathbf{h}_j\mathbf{x} - \mathbf{x}^H \mathbf{h}_j^H)}{\sigma\sqrt{\zeta_j}} - \mathcal{I}\{d_j\} \Big) \\ + &\quad \sum_j \alpha_j \Big(0.5 \frac{(\mathbf{h}_j\mathbf{x} + \mathbf{x}^H \mathbf{h}_j^H)}{\sigma\sqrt{\zeta_j}} - \mathcal{R}\{d_j\} \Big) \end{aligned} \tag{14}$$

where μ_j and α_j are the Lagrangian dual variables. It should be noted that the Lagrange function is dependent on the set of constraints related to the symbols. For example, the Lagrange function changes with the symbol set that should be sent to

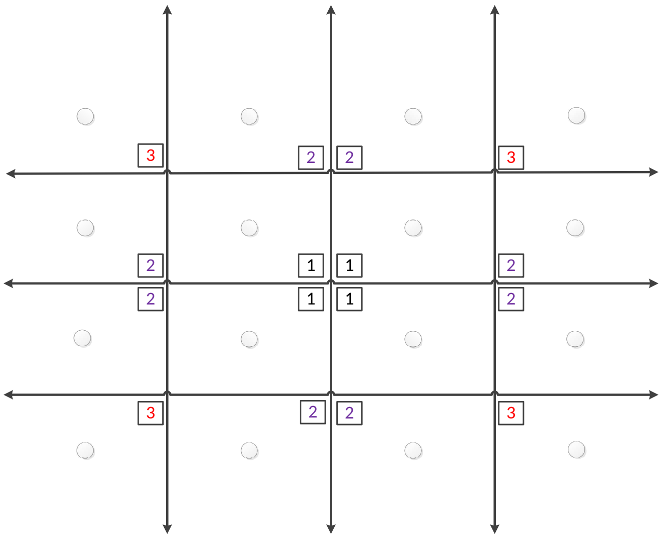

Fig. 1. Constellation for 16-QAM modulation

each user. The derivative for the Lagrangian function can be written as

$$\frac{d\mathcal{L}(\mathbf{x})}{d\mathbf{x}^{*}} = \mathbf{x} + 0.5i\sum_{j}\mu_j\mathbf{h}_j^H + 0.5\sum_{j}\alpha_j\mathbf{h}_j^H. \tag{15}$$

By equating this term to zero, $\mathbf{x}$ can be written as

$$\begin{aligned}\mathbf{x} &= i\sum_{j=1}^{K}\mu_j\mathbf{h}_j^H + 0.5\sum_{j}\alpha_j\mathbf{h}_j^H \\ &= \sum_{j=1}^{K}\nu_j\mathbf{h}_j^H, \forall j \in K \end{aligned} \tag{16}$$

where $\nu_j \in \mathbb{C}$. The optimal values of the Lagrangian variables μ_j and α_j can be found by substituting $\mathbf{w}$ in the constraints (13) which result in solving the set of $2K$ equations (18). The final precoder can be found by substituting all μ_j and α_j in (16). A generic solution for any set of simultaneous data symbols can be formulated as

$$\mathbf{x} = \sum_{j=1}^{K}\nu_j\mathbf{h}_j^H. \tag{17}$$

It can be noted that the precoding is a summation of maximum ratio transmissions precoding for all users.

V. Energy efficiency analysis

Due to the noise at the receiver, the detected symbols can deviate from the correct detection region. The effective rate $\bar{R}_j$ (i.e. goodput) for j^{th} user can be expressed as

$$\bar{R}_j \approx R_j \times \left(1 - SER_j(\zeta_j, z_j)\right) \tag{19}$$

R_j is the j^{th} user target rate of the employed modulation and SER_j denotes the symbol error rate of the j^{th} users. From (19), it can be noticed that increasing the SNR targets ζ_j reduces the probability of errors resulted from the noise, and as a result it enhances the effective rate.

A. Energy efficiency analysis

Increasing the SNR target reduces the SER while it increases the power consumption to achieve the SNR target. To find the optimal balance between these two aspects, the system energy efficiency metric is proposed to find how many bits can be conveyed correctly to the receivers per energy unit. The system energy efficiency can be defined as

$$\eta(\boldsymbol{\zeta}) = \frac{\sum_{j=1}^{K}\bar{R}_j\left(SER_j(\zeta_j)\right)}{P(\boldsymbol{\zeta})} \tag{20}$$

where $P(\boldsymbol{\zeta}) = \|\mathbf{x}(\mathbf{H}, \mathbf{d}, \boldsymbol{\zeta})\|^2$. It should be noted that the energy efficiency is a function of the SNR target ζ_j since it increases the transmit power amount required to achieve the target rate. Changing the SNR target affects both the numerator and the denominator in (20) by increasing the effective rate and transmit power respectively.

VI. Numerical Results

The channel between the base station and j^{th} user terminal is characterized by $\mathbf{h}_j = \sqrt{\gamma_\circ}\mathbf{h}_j^{'}$ where $\mathbf{h}_j^{'} \sim \mathcal{CN}(0, \mathbf{1})$, and $\gamma_\circ$ is the average channel power. For the sake of comparison, we plot the performance the physical layer multicasting as a bound [7]

$$\mathbf{Q} = \arg\min_{\mathbf{Q}} \quad \text{trace}(\mathbf{Q}), s.t. \quad \mathbf{h}_j\mathbf{Q}\mathbf{h}_j^H \geq \zeta_j, \forall j \in K. \tag{21}$$

For the sake of comparison with an achievable user-level precoding method, we use the power minimization objective for user-level linear beamforming which is defined as [5]:

$$\begin{aligned}\mathbf{w}_K = \quad & \arg\min \quad \sum_{j=1}^{K}\|\mathbf{w}_k\|^2 \\ & s.t. \quad \frac{\|\mathbf{h}_j\mathbf{w}_j\|^2}{\sum_{k\neq j,k=1}^{K}\|\mathbf{h}_j\mathbf{w}_k\|^2 + \sigma_z^2} \geq \zeta_j, \forall j \in K \end{aligned} \tag{22}$$

It should be noted for the sake of comparison between 4-QAM, 8-QAM, and 16QAM, the constellations are scaled mathematically to have average power of all constellation symbols should equal to 1. The scaling factor equals to 1, $\frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{5}}$ for 4-QAM, 8-QAM and 16-QAM respectively.

For 8-QAM, the constraints $\mathcal{C}_1$, $\mathcal{C}_2$ for each symbol can be written in details as

$$\mathcal{C}_1 = \begin{cases} \alpha_r = \sigma\sqrt{\frac{\zeta_j}{3}}\mathcal{R}\{d_j\}, d_j = \frac{\pm1\pm i}{\sqrt{2}} \\ \alpha_r \geq \sigma\frac{\sqrt{\zeta_j}}{\sqrt{3}}\mathcal{R}\{d_j\}, d_j = \frac{3+i}{\sqrt{2}}, \frac{3-i}{\sqrt{2}} \\ \alpha_r \leq \sigma\frac{\sqrt{\zeta_j}}{\sqrt{3}}\mathcal{R}\{d_j\}, d_j = \frac{-3+i}{\sqrt{2}}, \frac{-3-i}{\sqrt{2}} \end{cases}$$

$$\mathcal{C}_2 = \begin{cases} \alpha_i \geq \sigma\sqrt{\frac{\zeta_j}{3}}\mathcal{I}\{d_j\}, d_j = \frac{\pm1+i}{\sqrt{2}}, \frac{\pm3+i}{\sqrt{2}}, \\ \alpha_i \leq \sigma\sqrt{\frac{\zeta_j}{3}}\mathcal{I}\{d_j\}, d_j = \frac{\pm1-i}{\sqrt{2}}, \frac{\pm3-i}{\sqrt{2}} \end{cases}$$

$$
\begin{array}{lcl}
0.5\|\mathbf{h}_1\|(\sum_k(-\mu_k+\alpha_k i)\|\mathbf{h}_k\|\rho_{1k} & - & \sum_k(-\mu_k+\alpha_k i)\|\mathbf{h}_k\|\rho_{1k}^*)\geq\sigma\sqrt{\zeta_1}\mathcal{I}(d_1)\\
0.5\|\mathbf{h}_1\|(\sum_k(-\mu_k i-\alpha_k)\|\mathbf{h}_k\|\rho_{1k} & + & \sum_k(-\mu_k i-\alpha_k)\|\mathbf{h}_k\|\rho_{1k}^*)\geq\sigma\sqrt{\zeta_1}\mathcal{R}(d_1)\\
 & \vdots & \\
0.5\|\mathbf{h}_K\|(\sum_k(-\mu_k+\alpha_k i)\|\mathbf{h}_k\|\rho_{Kk} & - & \sum_k(-\mu_k+\alpha_k i)\|\mathbf{h}_k\|\rho_{Kk}^*)\geq\sigma\sqrt{\zeta_K}\mathcal{I}(d_K)\\
0.5\|\mathbf{h}_K\|(\sum_k(-\mu_k i-\alpha_k)\|\mathbf{h}_k\|\rho_{Kk} & + & \sum_k(-\mu_k i-\alpha_k)\|\mathbf{h}_k\|\rho_{Kk}^*)\geq\sigma\sqrt{\zeta_K}\mathcal{R}(d_K)
\end{array} \quad (18)
$$

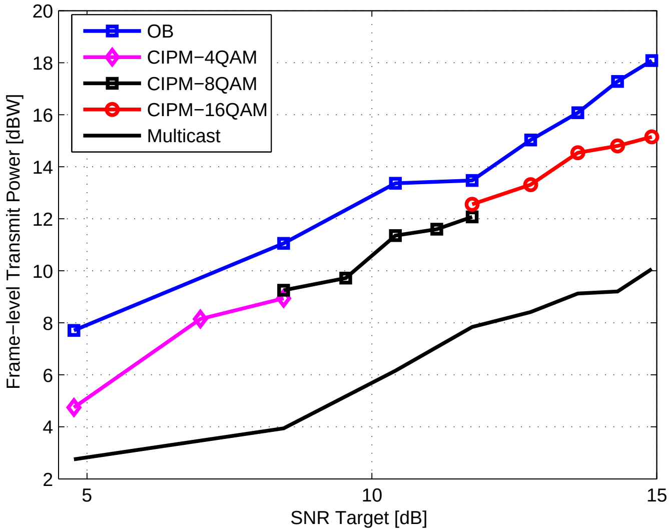

Fig. 2. *Transmit power vs. the target SNR, $M = 2$,$K = 2$, $\sigma^2 = 0dB$, $\gamma_\circ = 0dB$ $\zeta_j = \zeta_{th}, \forall j \in K$.*

For the 16-QAM modulation, the constraints $\mathcal{C}_1$, $\mathcal{C}_2$ can be expressed as

$$
\mathcal{C}_1 = \begin{cases} \alpha_r = \sigma\sqrt{\frac{\zeta_j}{5}}\mathcal{R}\{d_j\}, d_j = \frac{\pm1+\pm i}{\sqrt{2}}, \frac{\pm1+\pm3i}{\sqrt{2}} \\ \alpha_r \geq \sigma\sqrt{\frac{\zeta_j}{5}}\mathcal{R}\{d_j\}, d_j = \frac{3+i}{\sqrt{2}}, \frac{3-i}{\sqrt{2}}, \frac{3+3i}{\sqrt{2}}, \frac{3-3i}{\sqrt{2}} \\ \alpha_r \leq 2\sigma\sqrt{\frac{\zeta_j}{5}}\mathcal{R}\{d_j\}, d_j = \frac{-3+i}{\sqrt{2}}, \frac{-3-i}{\sqrt{2}}, \frac{-3+3i}{\sqrt{2}}, \frac{-3-3i}{\sqrt{2}} \end{cases}
$$

$$
\mathcal{C}_2 = \begin{cases} \alpha_i = \sigma\sqrt{\frac{\zeta_j}{5}}\mathcal{I}\{d_j\}, d_j = \frac{\pm1+\pm i}{\sqrt{2}}, \frac{\pm3+\pm i}{\sqrt{2}}, \\ \alpha_i \geq \sigma\sqrt{\frac{\zeta_j}{5}}\mathcal{I}\{d_j\}, d_j = \frac{\pm1+3i}{\sqrt{2}}, \frac{\pm3+3i}{\sqrt{2}} \\ \alpha_i \leq \sigma\sqrt{\frac{\zeta_j}{5}}\mathcal{I}\{d_j\}, d_j = \frac{\pm1-3i}{\sqrt{2}}, \frac{\pm3-3i}{\sqrt{2}} \end{cases}
$$

Fig. (2) depicts the amount of the required transmit power $\|\mathbf{x}\|^2$ to achieve certain target SNR exploiting symbol-level precoding CIPM. For 2×2 scenario, it can be noted that conventional beamforming (OB) needs more power to satisfy the same SNR targets. For the symbol-level precoding, the PHY-multicasting presents a lower-bound. It can be deduced that the performance of different modulations a continuous pattern with increasing the modulation order. Moreover, the power consumption increases linearly in dB with increasing the SNR target.

Fig. (3) depicts the comparison between the energy efficiency of 8-QAM and 4-QAM respectively. In this figure, we assume that the SNR targets for 8-QAM and 4-QAM

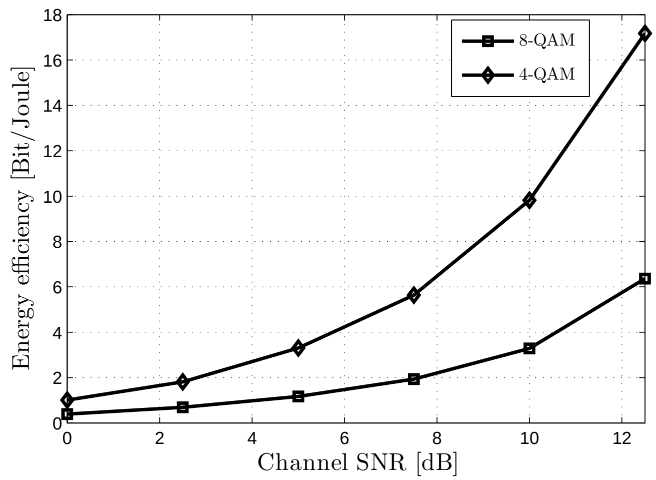

Fig. 3. *Energy efficiency η vs. the channel SNR σ^2, $M = 2$,$K = 2$, $\sigma^2 = 0dB$, $\zeta_j = \zeta_{th}, \forall j \in K$, ζ_{th} for 8-QAM equals to 9 dB and ζ_{th} for 4-QAM equals to 6 dB.*

equal to 9 dB 6 dB respectively to fit the requirement of having higher SNR targets. Although 4-QAM has lower rate with increasing the channel SNR, It can be noted that it has higher energy efficiency. This can be explained by the fact that SER in 8-QAM is higher which makes the numerator in the energy efficiency more sensitive to the SER. Moreover, the power consumption in 8-QAM is higher due to higher SNR requirement, which results in higher energy efficiency.

Fig. (4) depicts the energy efficiency performance of 16-QAM and 8-QAM with respect to SNR target ζ_{th}. It can be noted that the energy efficiency decreases with increasing SNR target ζ_{th}, we assume that 8-QAM and 16-QAM have the same ζ_{th} to see the impact of SER. It can be noted that constructive interference for 8-QAM has higher energy efficiency due to lower SER in comparison to 16-QAM.

VII. CONCLUSIONS

In this paper, we utilized jointly CSI and DI in symbol based precoding to exploit received interfering signal as useful energy in constructive interference precoding. In these cases, the precoding design exploits the overlap in users' subspace instead of mitigating it. This fact enabled us to find the connection between the constructive interference precoding and multicast precoding wherein no interference should be mitigated. In this work, we propose precoding techniques that extends the concept of constructive interference to multi-level constellation. Therefore, we found the solution for power minimization considering two inputs scenario: the optimal input and the

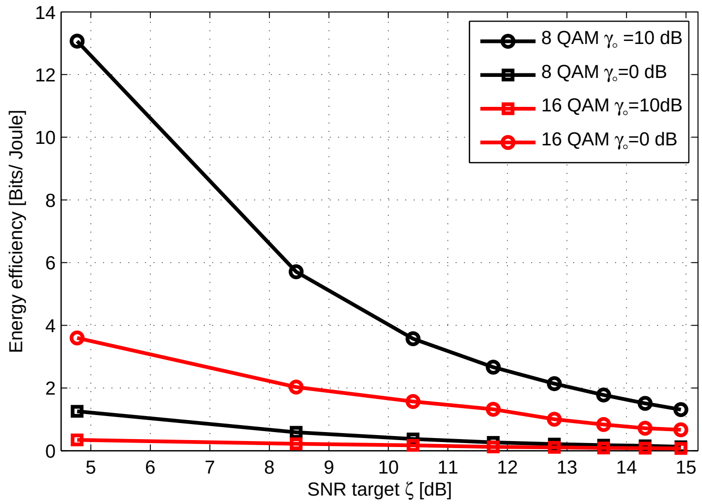

Fig. 4. *Energy efficiency* η *vs. the target SNR,* $M = 3, K = 2$, $\zeta_j = \zeta_{th}, \forall j \in K$, $\sigma^2 = 0dB$.

constrained constellation. From their closed formulations, we concluded that their transmissions should span the subspaces of each user. From the numerical results, it can be concluded that the energy efficiency is higher for lower modulation order.

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