What we can learn about ocean tides from tide gauge and gravity loading measurements?

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Abstract

The harmonic constants from tide gauge stations and from gravity loading measurements of the ICET data bank are separately inverted to produce M2 cotidal maps on a global scale. The optimal interpolation of these data, based on the Total Inverse Theory in L2 norm, makes use of a set of a priori stationary and isotropic covariance functions deduced from a global hydrodynamical model and from its corresponding gravity loading map. The preliminary results presented here with formal error estimates are surprisingly good, in particular the M2 maps obtained from the numerous tide gauge data. These results are briefly discussed and further lines of work are drawn.

1. Introduction

A highly accurate knowledge of the global ocean tides is today necessary in particular to study the response of the solid earth to this oceanic forcing, or to precisely correct the satellite altimeter data before the extraction of the ocean dynamics signals. The modelling of the ocean tides may be "theoretical" or empirical (or mixed in the so-called semi-empirical models). The theoretical models solve the hydrodynamical equations of the tides when bathymetry, coast lines geometry and proper boundary conditions are known (e.g. Vincent & Le Provost 1988). Until now, global empirical tidal models have been mainly deduced from the analysis of satellite altimetry which provides the necessary spatial data coverage (see the review by Cartwright 1988). Nevertheless, the recovery of the tides from these measurements raises several difficulties (tidal aliasing, radial orbit error; e.g. Mazzega 1988) so that the best purely empirical tidal mapping will be probably achieved by the analysis of mixed data sets.

Indeed, data from tide gauges (TG) and gravity loading (GL) measurements provide valuable information on the ocean tides with high signal to noise ratio. But the former are poorly distributed, the bulk of TG data coming from coastal stations, and the latter have a sparse coverage -we retain 233 worldwide stations of the International Center for Earth Tides (ICET) data bank . It seems that nobody has tried to recover the global tides from solely the TG data since the late attempts by Dietrich (1944) and Vilain (1952) of subjectively drawing cotidal maps from the coastal harmonic constants. Moreover the inverse problem of mapping the ocean tides from measurements of the gravity loading

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effects on the solid earth has not been revisited since the regional inversion (based on a linear programming algorithm) made by Kuo & Jachens (1977), though the GL measurements are now far more numerous and accurate, thanks to the work of the ICET staff (Ducarme 1984).

Today, the Total Inverse Theory (Tarantola & Valette 1982, Tarantola 1987) provides a powerful mathematical frame to objectively interpolate and invert the TG and GL data for the ocean tides as well as to mix these two heterogeneous data sets in a consistent way. After a general comment on the inverse method applied to tidal mapping (§2; mathematics are developped in papers given in reference) we explain how the a priori tidal covariance functions -an information central in our inverse scheme- are estimated (§3). Then, preliminary results of separate inversions of TG and GL data sets for the M2 ocean tide are presented with formal error estimates (§4). The good quality of these purely empirical maps shows that far more can be learned from these measurements about ocean tides, even in the deep oceans, than could be previouly suspected. The necessary tests and comparisons of these maps with independant data or models are not yet carried out so their actual accuracy is not quantified at this moment. The fundamental assumptions of this inverse method are briefly discussed and the future work to be done to improve the present partial results is suggested (§5).

2. Inverting TG & GL Measurements

Only a brief introduction to our inverse method specifically developped for the analysis of TG and GL measurements is presented in this short report. An extended version of this work, with improved and new results, will be soon submitted for publication. A similar inverse method designed to extract the tides from satellite altimetry is also given with a detailed and complete set of equations in Mazzega (1988,1989) and Mazzega & Jourdin (1988).

The statistical background of the Total Inverse Theory (cf Tarantola 1987) leads us to consider the unknowns of a given constituent of the ocean tides as stochastic functions of the geographical coordinates. The data base being constituted by harmonic constants of the oceanic partial tide itself -e.g. M2- or of its gravity loading effect, the time is excluded from the observation equations. So, for the TG data we simply have:

$$\bar{A}(r^i)\cos(\bar{P}(r^i)) = \int_\sigma A(r)\cos(P(r))\delta(r^i - r)dr \quad (1)$$

where $\bar{A}$ and $\bar{P}$ are the observed amplitude and phase of the M2 tide at the geographical location $r^i(\varphi^i,\lambda^i)$ while A and P are the unknown amplitude and phase considered as continuous functions of r. The integral over the oceanic domain σ with the Dirac-delta as kernel stands for the "discretization" of the continous tide at the TG location. The observation equations for the gravity loading measurements are written as:

$$\bar{\Delta}(r^i)\cos(\bar{\Psi}(r^i)) = \rho_w \int_\sigma A(r)\cos(P(r))\gamma(r^i,r)dr \quad (2)$$

where $\bar{\Delta}$ and $\bar{\Psi}$ are respectively the observed amplitude and phase of the gravity loading effect of the partial tide at station r^i, ρ_w the mean sea water density and γ the Green's function for gravity acceleration (the integration is also performed over the whole oceanic domain). We further have equations similar to (1-2) but for the in-quadrature component of the partial tide.

From eq.1 we see that the mapping of the tide from the TG data is a problem of interpolation while we also have to invert the operator of eq.2 to deduce the ocean tide from the gravity loading measurements. The Total Inverse Method adapted to our problem allows to perform optimally and in a consistent way these two operations by combining all the a priori available informations. Among them we first have the data sets - the TG harmonic constants from the International Hydrographic Office (1979) and the GL measurements of the ICET data bank-, and the corresponding observation equations (1-2).

The statistical properties of the errors of these data are a priori specified by covariance functions. In the absence of decisive informations about possible correlations of the data errors between the different instruments or data sets (gauges or gravimeters), we must assume that the errors embeded in the data are white noises, say processes independant from an observation station to an other one. The corresponding covariance functions are Dirac distributions with variances of $(2\ \mathrm{cm})^2$ for the TG data and $(0.5\ \mu\mathrm{gal})^2$ for the GL measurements (note that our inversion scheme could be based on refined error covariance models -if available- without any damage).

A complementary information is supplied to the inversion algorithm through a priori covariance functions on the tidal signals (see next section). They can be interpreted as mathematical "measures" of the smoothness of the tidal heights or gravity loading effects. Indeed the tidal fields are long wavelength in nature so that two measurements (e.g. from TG stations) performed at a spherical distance of a few degrees are statistically "correlated" by an amount quantified by the a priori tidal covariances. In the same way, the "correlation" of the harmonic constants $(\bar{A},\bar{P})$ and $(\bar{\Delta},\bar{\Psi})$ as given by two nearby TG and gravimeter stations respectively, is given by the a priori cross-covariances of the tidal heights versus gravity loading effects.

The inversion algorithm, based on the generalized Least Squares criterion, optimally combines all these a priori informations to produce the more likely (in a statistical sense) tidal solution (maps of the in-phase and in-quadrature tidal components to convert into maps of the amplitude and phase). A posteriori tidal covariance functions are also computed which quantify the spatial "correlation" of the errors associated with the inverse solution. From these a posteriori covariances we can further deduce the standard deviation of the solution at each prediction point as well as the power spectra of the associated errors. In this preliminary work only a few illustrations are given (§4) of the various possibilities of the inversion algorithm.

3. A Priori Tidal Covariances

The a priori defined auto- and cross- covariances of the tidal fields are central concepts of the inverse method. In the geographical regions where the data are numerous and accurate, the inverse solution is only very weakly sensitive to the choice of the a priori tidal covariances. But where data are sparse or much noisy these covariances significantly constraint the solution (see §4). So they must be carefully computed and completely independantly from the data to invert. Hereafter we only consider the M2 partial tide.

We first need the spatial covariance of the in-phase AcosP tidal function. It is computed from a global hydrodynamical model by a classical method of statistical geodesy (Moritz 1980). Starting from the Schwiderski's M2 model (1980a-b), we expand the global map of the AcosP tidal component into surface spherical harmonics up to degree and order 180 (the Nyquist wavenumber corresponding to the resolution of the Schwiderski's model). From the expansion coefficients we deduce the variance by degree or auto-power spectrum of AcosP (Fig.1a). This truncated spectrum is prolongated by continuity with an empirical rule of thumb up to a higher degree L.Then the degree variances are used as coefficients of a series of Legendre polynomials with argument ψ, which constitute the needed covariance as a function of the spherical distance ψ (Fig.1b). The same procedure is used to compute the spectrum and covariance of the AsinP tidal component.

The pair of power spectra and covariances of the gravity loading effect (Fig.1a-b) are deduced in the same way. The starting maps are now those obtained by a Farrell's convolution of the same Schwiderski's tidal maps (Francis & Mazzega 1989), thus ensuring the consistency of the informations provided to the inversion algorithm. Cross-covariances between the gravity loadings and the tidal heights are also necessary to invert the ICET data. They are simply deduced from the cross-power spectra of these two fields constructed from their respective expansion coefficients.

An important assumption underlying these computations is the stationarity and isotropy of the tidal signals. Indeed, from the global maps of the tidal heights and gravity loadings we estimate only <u>one</u> set of covariances which will be indifferently used in the various oceanic basins. Moreover these covariances do not depend on the azimuth of the relative locations of any pair of data or prediction points under consideration. As we shall see in the next section on the basis of the results, this crude assumption seems quite consistent with the observed tidal fields, even over the continental shelves.

4.Preliminary M2 Cotidal Maps & Associated Errors

The cotidal maps obtained by the inversion of TG data are given in Figure 2 for the Pacific ocean, in Figure 3 for the Atlantic and the Indian oceans. In each basin only a subset of TG data is selected over the complete IHO bank in order to have a maximum density of 1 datum per square degree. The subset of one basin is inverted independently from the other ones and, in this preliminary work, we have not tried to impose any continuity condition of the solutions at the basins boudaries. The a priori covariance functions have been deduced from M2 auto-power spectra expanded up to L=360.

As can be seen from Fig.2 & 3, all the major features of the M2 tide are very well recovered. The co-amplitude and cotidal lines obtained by the inversion are highly similar to those previously published by ocean tide modellers. The main discrepancies occur in wide regions where no data at all are available (South-East Pacific, southern Atlantic & Indian oceans).

Except in the North Atlantic where numerous deep sea gauges have recorded the tides, it is worth noting that though the inverse solutions of the deep ocean are mainly "driven" by the harmonic constants at coastal sites (e.g. the central Indian ocean) they are higly realistic. Numerical comparisons with independant data have still to be made to assess the actual accuracy of the inverse solutions.

The a posteriori covariance of the in-phase M2 solution has been evaluated in two 10°x10° boxes in the Atlantic (the locations of the boxes are drawn in Fig.3). The first covariance (Fig.4a) is characteritic of the statistical properties of the errors associated to the M2 solution where no data are available (in the South Atlantic). At the center of the box the standard deviation (s.d.) of the solution (21cm) almost equals its a priori value (24.3cm). Moreover the a posteriori covariance of the solution errors is very similar to the a priori covariance of the tidal signal (see Fig.1b). This fact means that in this region the far data bring only marginal information about the tides and the inverse solution is mainly constrained by the tidal a priori covariances. On the contrary, the s.d. at the center of the North Atlantic box which contains 3 TG stations (Fig.4b), is only 6.5 cm and the solution errors are decorrelated over 1 degree or even less, depending on the direction we consider (non-isotropy of the errors).

The GL harmonic constants have also been inverted separately. The a priori M2 power spectra are now truncated at the expansion degree L=180 in order to produce smooth tidal maps in a first attempt. Though less realistic than the previous map, this M2 inverse solution exhibits well known features of the oceanic tide (Fig.5). The best results are probably obtained in the Atlantic because a lot of gravimeter stations are situated in Western Europe and in South America. The information content of the ICET data concerning the tidal phases seems particularly reliable, the main amphidromic points being well positioned (see e.g. the Indian ocean) with the right phase rotations. These preliminary results are very promising owing to the facts that the gravimeter stations are very sparse and the measurements related to the tidal heights via an integral operator (see eq.2).

The a posteriori covariances of this inverse M2 solution in the two boxes of the Atlantic ocean are drawn in Fig.4c & 4d. Compared to Fig.4a & 4b, they reflect the structures of the errors on the M2 tide resulting from the inversion of different functionnals of the tidal heights with different data coverage. The dissimilarity between these two pairs of covariances (or equivalently in the wavenumber domain, between the corresponding error power spectra) suggests that these heterogeneous data sets, when inverted conjointly in a consistent and optimal way, should considerably improve the present M2 tidal maps.

5. Discussion & Perspectives

The inversion of TG data and GL measurements based on the a priori knowledge of the spatial covariances of the ocean tide, provides very useful information on that tide, even in the deep ocean. The deliberate choice to simply consider the tide as a stationary and isotropic stochastic process seems quite consistent with the measurements. Moreover the inverse solutions do not sem too sensitive to the description of the data error properties through their covariance models, probably because the tidal signals are long wavelength in nature.

The present preliminary M2 cotidal maps could be improved in several ways. In particular, we shall invert conjointly the TG and GL measurements. We also plan to add satellite altimeter data in the oceanic regions poorly sampled by the other data sets. The covariance models of the data errors should be refined and the inversion scheme should take into account the uncertainties due to the simplified models of load Love numbers entering the definition of the Green's function in gravity γ (see eq.2).

Nevertheless the computation of purely empirical tidal maps is just a step toward the production of high accuracy global ocean tide models. Centimetric accuracies on the tidal predictions will be only obtained with an hydrodynamical tidal model assimilating in the proper way the informations provided by the various data sets (see Vincent et al. 1989, this symposium). In the assimilation process, the a posteriori covariances or error power spectra associated with the inverse tidal solutions will play a central role in the relative "weighting" of hydrodynamics and empirical informations.

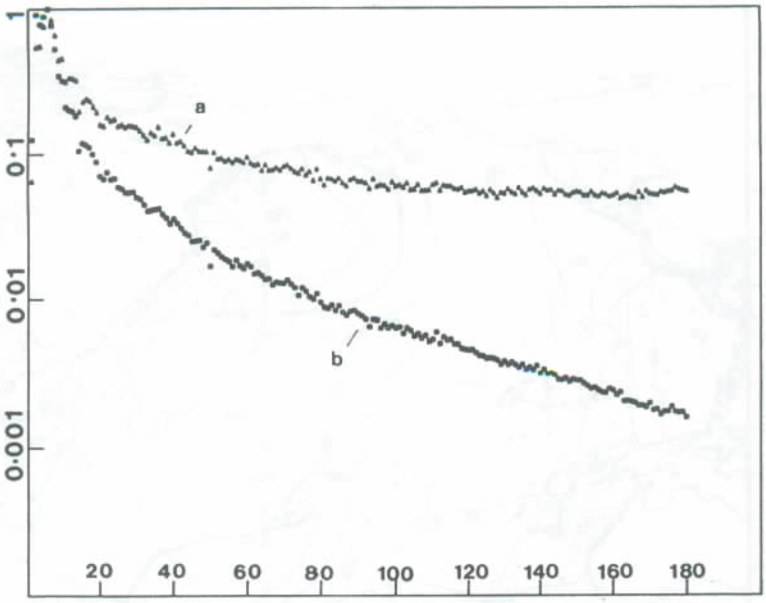

Figure 1a: Auto-power spectra of the in-phase component of the M2 tidal heights (a) and gravity loadings (b) normalized by the higher degree variances at l=4. The total variances are $(22.3\ cm)^2$ for the tidal heights and $(3.3\ gal)^2$ for the gravity loadings. (x-axis:degree l of the expansion; y-axis: a-dimensional logarithmic scale).

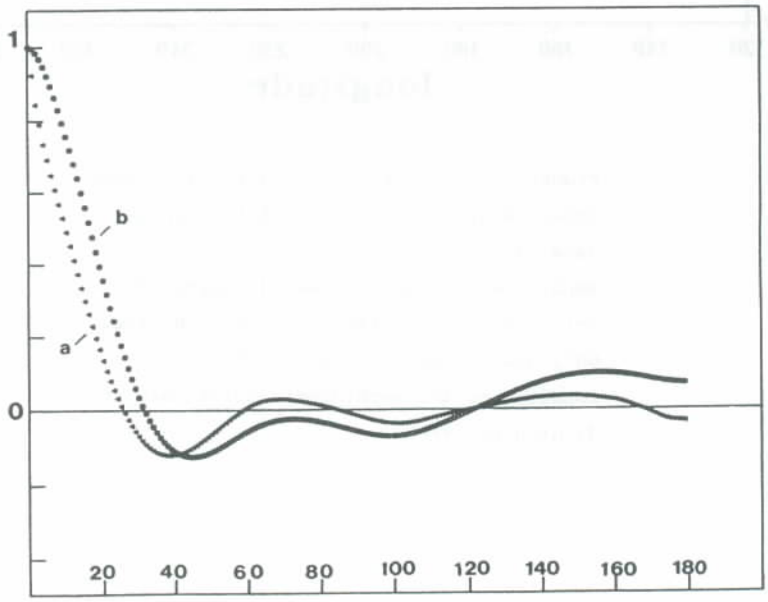

Figure 1b: Auto-correlation functions of the M2 tidal heights (a) and M2 gravity loadings (b) correponding to the previous power spectra. (x-axis: spherical distance).

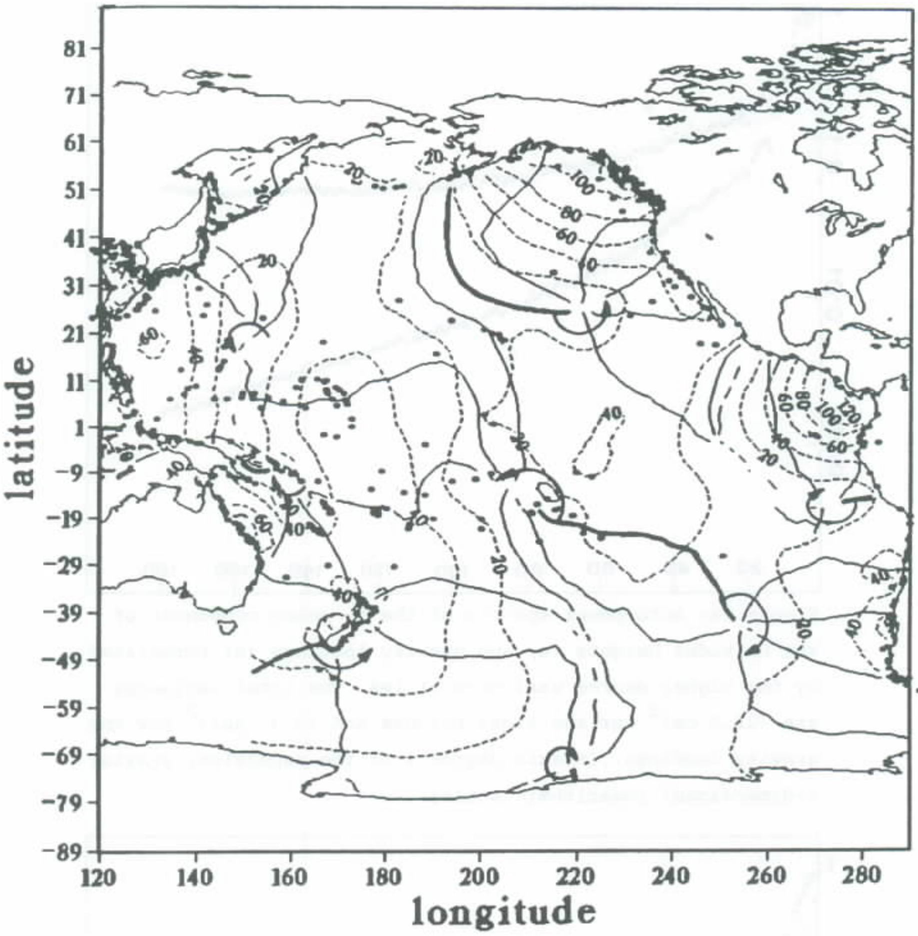

FIGURE 2: M2 solution in the Pacific ocean obtained by the inversion of tide gauge data (dots);
Amplitudes in dashed lines (interval 20 cm); Phases in continuous lines (interval 60°, heavy line: 0° phase). The senses of rotation of the amphidromic points are indicated by arrows.

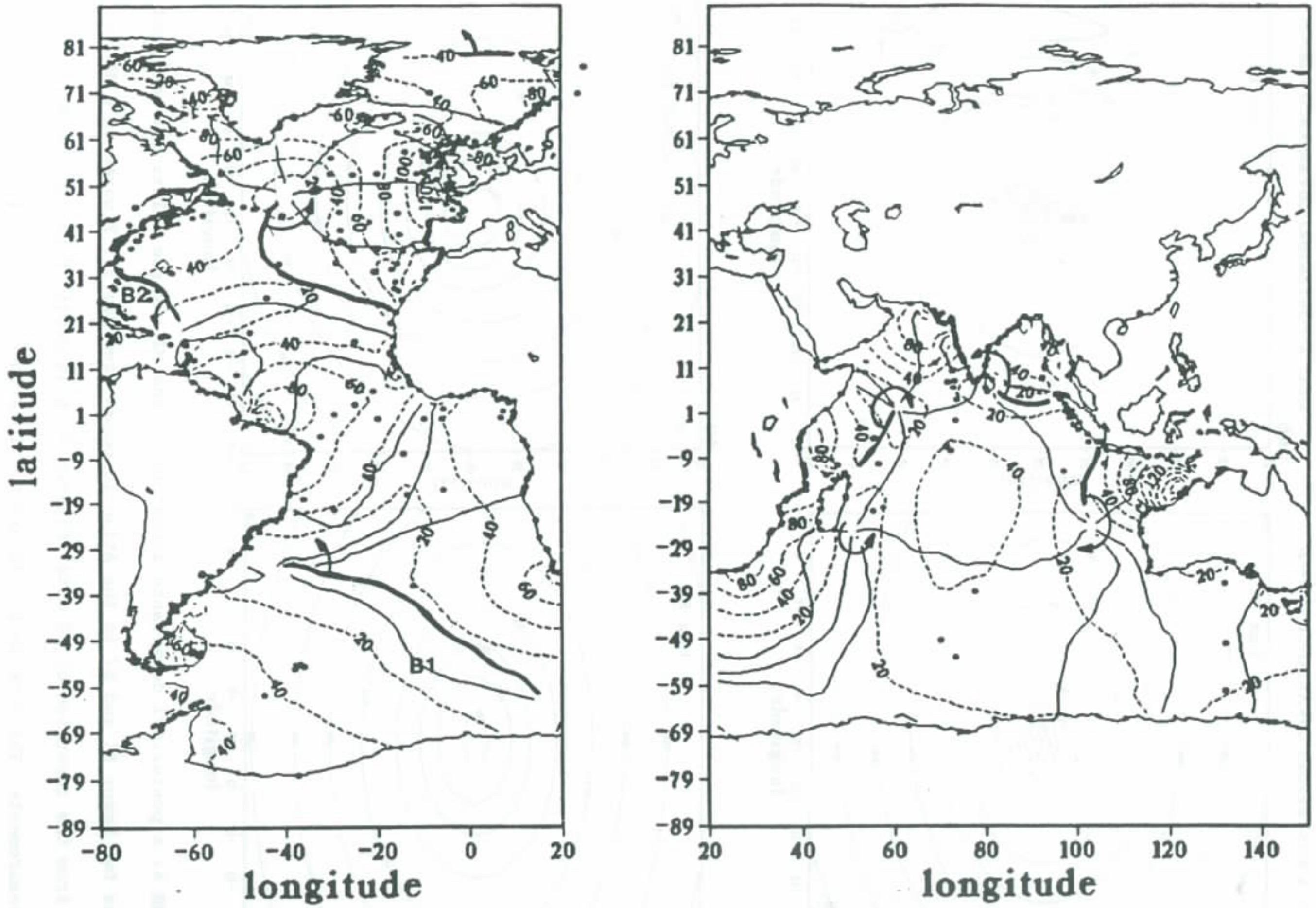

FIGURE 3: Same as Fig.2 but for the
Atlantic & Indian oceans.

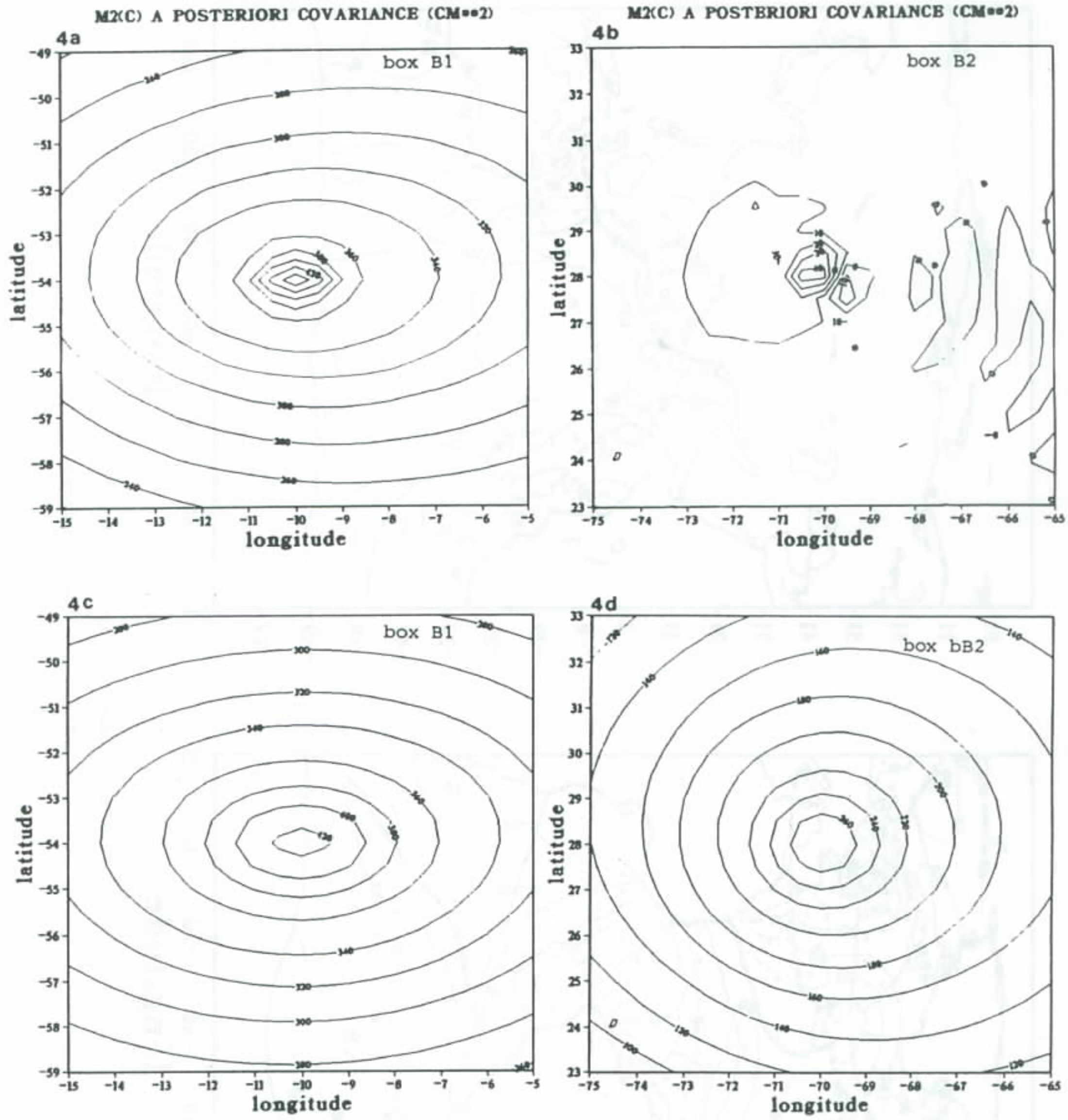

FIGURE 4: A posteriori covariance functions of the M2 inverse solutions, computed in the two boxes B1 and B2 in the Atlantic ocean (see Fig.3). Figures 4a & 4b result from the inversion of TG data; Figures 4c & 4d result from the inversion of GL measurements. The dots in Fig.4b are the locations of the 3 TG stations.

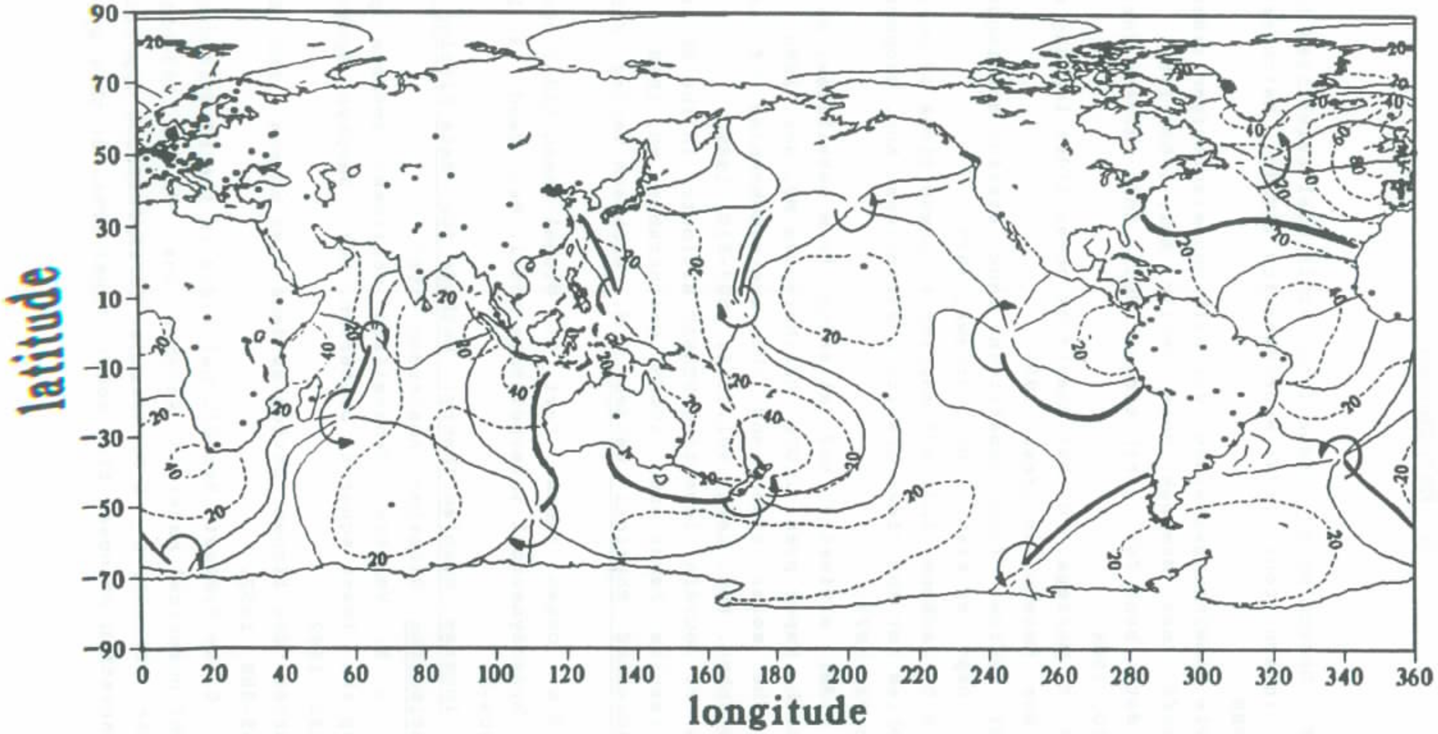

FIGURE 5: Same as Fig.2 but for the global M2 solution obtained by the inversion of gravity loading measurements (dots).

6. References

Cartwright,D.E. Detection of tides from artificial satellites (Review). Paper pres. Intern. Conf. Tidal Hydrodynamics (ICOTH), Gaithersburg MD, USA, Nov. 1988.

Dietrich,G. Die schwingungssysteme der halb-und eientagigen tiden in den ozeanen. Veroff. Inst. Meeresk. Univ. Berlin, A41, 1-68, 1944.

Ducarme,B. A data bank for earth tides. Bull. Inf. Marées Terrestres, 91, 5963-5980, 1984.

Françis, O. & P. Mazzega. Global charts of ocean tides loading effects. J. Geophys. Res. Oceans, in press, 1989.

Inter. Hydrogr. Office. Tidal constituent bank, station catalogue. Oc. & Aquatic Sci., Dept. of Fish. & Oc., Ottawa, 1979.

Kuo, J.T. & R.C. Jachens. Indirect mapping of ocean tides by solving the inverse problem for the tidal gravity observations. Ann. Geophys., t33, fasc.1/2, 73-82, 1977.

Mazzega, P. Tidal altimetry and aliasing from satellites: an inverse theory approach. Paper pres. ICOTH, Gaithersburg MD, Nov. 1988.

Mazzega, P. The solar tides and the sun-synchronism of satellite altimetry. Geophys. Res. Lett.,vol.16(6), 507-510, 1989.

Mazzega, P. & F. Jourdin. Inverting SEASAT altimetry in the NE Atlantic: preliminary results. Paper pres. ICOTH, Gaithersburg MD, 1988.

Moritz, H. Advanced Physical Geodesy, H. Wichmann Verlag, Karlsruhe, FRG, 1980.

Schwiderski, E.W. Ocean tides, Part I: Global ocean tidal equations; Part II: A hydrodynamical interpolation model. Mar. Geod., 3, 161-217, 219-255, 1980a-b.

Tarantola, A. Inverse Problem Theory: Methods for Data Fitting & Model Parameter Estimation. Elsevier, Amsterdam, 1987.

Tarantola, A. & B. Valette. Generalized nonlinear inverse problems solved using the least squares criterion. Rev. Geophys. Space Phys., 20(2), 219-232, 1982.

Vilain, C. Cartes des lignes cotidales dans les oceans. Ann. Hydrog., Paris, 3, 269-388, 1952.

Vincent, P. & C. Le Provost. Semidiurnal tides in the NE Atlantic from a finite element numerical model. J. Geophys. Res., 93, C1, 543-555, 1988.

Vincent, P., Le Provost, C. & P. Mazzega. TOPEX/POSEIDON opportunity to get a new generation of ocean tide model. Paper pres. at this symposium.