The concept of holonomy is of central importance to the modern understanding of geometry and physics. It provides a link between local information on the one hand and global information on the other. For instance, the shift in the phase of an electron, which is moving within an electromagnetic field, is determined by the holonomy of the gauge potential along the path traced out by the particle.

With the advent of physical theories in which the fundamental entities are not particles, but more complicated objects such as strings, and the emergence of the corresponding mathematical structures, the importance of a higher dimensional version of parallel transport became apparent.

Several ways of generalizing ordinary parallel transport are known. Among these, gerbes and various notions of their 2-holonomies form the most prominent group. Another interesting generalization is provided by the concept of "homotopy quantum field theories" (HQFTs). These theories are close relatives of topological quantum field theories, but with maps to a fixed target space added.

Together with Camilo Arias Abad (now Max-Planck Institute in Bonn), I am working on yet another generalization, the theory of higher holonomies for flat superconnections. These holonomies were originally construced by Kiyoshi Igusa, who used them to study higher torsion classes of fibre bundles. Relying on Chen's iterated integrals, one associates holonomies not only to paths, but to arbitrary simplices inside a manifold M. In higher-category-speak, these higher holonomies assemble into a representation of the ∞-groupoid $\pi_\infty(M)$ of M. An advantage of this approach is that it allows one to define holonomies for simplices of arbitrary dimension in a consistent manner.

Camilo Arias Abad and I extended this approach to flat connections with values in L_∞-algebras. These algebras are the natural replacements of Lie algebras in the world of homotopy theory. In fact, flat connections with values in these coefficient systems allow one to make contact with the theory of rational homotopy theory. Using this connection (no pun intended!), we were able to generalize the Drinfeld-Kohno construction of braid group representations to configuration spaces of points in higher dimensional Euclidean space. These findings recently appeared in Homotopy, Homology Appl. Vol 16 (2014), Nr. 1, 89-118.

Another joint project aimed at relating the higher holonomies of a flat superconnection to the 2-holonomies of a flat gerbe as defined by Baez-Schreiber, Schreiber-Waldorf and Martins-Picken. This project recently resulted in a preprint and opens up the possibility to exchange ideas between the two approaches.

In particular, there are hints from the theory of gerbes that it is possible to construct HQFTs, which were mentioned above, from flat superconnections. For instance, Bunke, Turner and Willerton showed that flat gerbes of rank 1 are in bijection with 2-dimensional HQFTs of rank 1. It would be interesting to translate this result into the language of flat superconnections, and to extend it to the non-abelian, as well as to the higher-dimensional, setting.

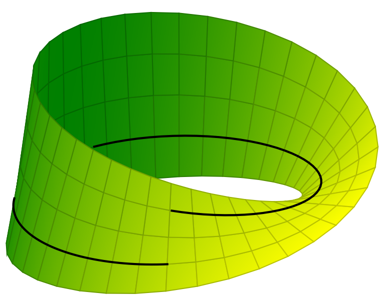

Figure 1: Holonomy of the Möbius strip.