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# A preliminary investigation of the growth of an aneurysm with a multiscale monolithic Fluid-Structure interaction solver

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**Abstract.** In this work we investigate the potentialities of multi-scale engineering techniques to approach complex problems related to biomedical and biological fields. In particular we study the interaction between blood and blood vessel focusing on the presence of an aneurysm. The study of each component of the cardiovascular system is very difficult due to the fact that the movement of the fluid and solid is determined by the rest of system through dynamical boundary conditions. The use of multi-scale techniques allows us to investigate the effect of the whole loop on the aneurysm dynamic. A three-dimensional fluid-structure interaction model for the aneurysm is developed and coupled to a mono-dimensional one for the remaining part of the cardiovascular system, where a point zero-dimensional model for the heart is provided. In this manner it is possible to achieve rigorous and quantitative investigations of the cardiovascular disease without losing the system dynamic. In order to study this biomedical problem we use a monolithic fluid-structure interaction (FSI) model where the fluid and solid equations are solved together. The use of a monolithic solver allows us to handle the convergence issues caused by large deformations. By using this monolithic approach different solid and fluid regions are treated as a single continuum and the interface conditions are automatically taken into account. In this way the iterative process characteristic of the commonly used segregated approach, it is not needed any more.

## 1. Introduction

In this work the potentialities of a monolithic FEM fluid-structure interaction (FSI) approach for the computational investigation of an abdominal aortic aneurysm are investigated. The development of abdominal aortic aneurysms (AAA) is associated with alterations of the tissue in the aortic wall, in particular collagen degradation, which leads to the loss of elasticity of the vessel. This process can lead to the rupture of the aneurysm with consequent internal bleeding. Accurate numerical simulations could provide large benefits in this field avoiding unnecessary and dangerous surgical removal operations. The simulation of the interaction between blood and blood vessel is not a trivial task because of the softness of the tissues, which leads to very large deformations. As a consequence of this highly non-linear behavior the stability of the numerical problem is not easily ensured. A monolithic approach, which solves the fluid and the structure problems in a coupled way, can bring significant advantages concerning the stability issues. This approach is, however, CPU time expensive and not suitable to simulate a large portion of the circulatory system. The use of mono-dimensional models, for all the components that are not of

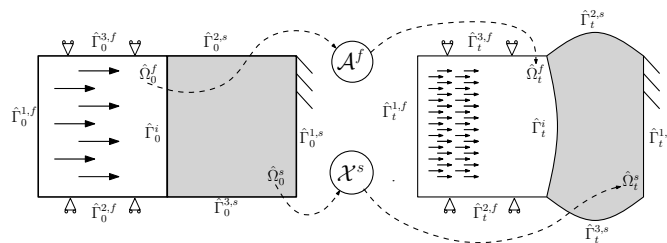


specific interest, and three-dimensional ones to describe the most critical structures (multi-scale approach) can reduce the computational costs of simulations. Furthermore one can achieve accurate studies of the specific component without losing the system dynamic. The paper is organized as follows. In a first section the FSI equation of the problem are presented with particular reference to the ALE formulation, the monolithic approach, and the mono-dimensional model for the multi-scale approach. Finally some numerical examples are shown.

## 2. FSI EQUATIONS

In this section we give a description of the equations that are taken into account in order to solve the fluid structure interaction problem.

### 2.1. ALE Reference



**Figure 1.** Reference configuration, on the left, and current configuration, on the right, and the operators for the Arbitrary Lagrangian-Eulerian (ALE) formulation.

In this work a Lagrangian reference is adopted for the solid domain while an ALE (Arbitrary Lagrangian Eulerian) reference is introduced in the fluid domain. The Lagrangian viewpoint consists of following the material particles of the continuum in their motion. To this purpose, one can introduce a computational grid, which follows the solid domain. The grid nodes are permanently connected to the same material points. The Lagrangian reference is therefore useful to describe deforming materials, but can only handle (in grid-based approaches) the investigation of systems where the displacement field is limited, such as the solid domain. For this reason in the fluid description the Eulerian reference is very popular. The basic idea of the Eulerian formulation consist on examining, as time evolves, the physical quantities associated with the fluid particles flowing through a fixed region of space. In the Eulerian description, the domain is fixed and the fluid moves and deforms with respect to the computational grid. Obviously these two references can not be coupled in a straightforward way. As previously stated, in this work a mixed Lagrangian-Eulerian description is used in order to combine Lagrangian and Eulerian references. For this purpose let us consider a mechanical system composed by a fluid and solid domain  $\Omega_t$  as shown in Figure 1. Let  $\Omega_t^f$  and  $\Omega_t^s$  be the region occupied by the fluid and the solid respectively at the time  $t \in ]0, T]$ . In the initial configuration fluid and solid regions are therefore defined by  $\Omega_0^f$  and  $\Omega_0^s$ , respectively. Let  $\Gamma_t^i$  be the interface between the solid and the fluid and  $\Gamma_t^k$  all the rest of the boundary. The evolution of the domain  $\Omega_t$  can be described by considering the motion of the solid and fluid domains  $\Omega_0^f$  and  $\Omega_0^s$  defined by the mapping

$$\begin{aligned} \chi^s : \Omega_0^s \times \mathbb{R} &\rightarrow \mathbb{R}^3, \\ A_f : \Omega_0^f \times \mathbb{R} &\rightarrow \mathbb{R}^3. \end{aligned} \quad (1)$$

The range of the mapping  $\chi^s$  is  $\Omega_t^s$ .  $\chi^s$  maps the position of the material point  $\mathbf{x}_0^s$  to the current solid material configuration  $\Omega_t^s$ . The solid displacement is then defined as

$$\mathbf{d}^s(\mathbf{x}_0^s, t) = \chi(\mathbf{x}_0^s, t) - \mathbf{d}_0^s. \quad (2)$$

In a similar way for the application  $A^f$  we define

$$\mathbf{d}^f(\mathbf{x}_0^f, t) = \chi(\mathbf{x}_0^f, t) - \mathbf{d}_0^f, \quad (3)$$

where  $\mathbf{d}^f(\mathbf{x}_0^f, t)$  is as an arbitrary extension over the fluid domain  $\Omega_0^f$ . This arbitrary extension could be written as

$$\mathbf{d}^f(\mathbf{x}_0^f, t) = Ext(\mathbf{d}^s|_{\Gamma_0^i}) \quad in \quad \Omega_0^f. \quad (4)$$

The harmonic or Laplace operator, which is also adopted in this work, is the most common extension operator. It is now possible to define the velocity  $\mathbf{w}^f$  of the points of the fluid domain in the current configuration as

$$\mathbf{w}^f = \frac{\partial \mathbf{d}^f}{\partial t} \circ \mathbf{d}_0^f \quad in \quad \Omega_0^f, \quad (5)$$

where  $\partial \mathbf{d}^f / \partial t$  is the velocity in the reference coordinates system.

## 2.2. Monolithic Approach

In order to couple the fluid and solid equations the liquid-solid interface conditions must be imposed. Two main approaches can be adopted: partitioned and monolithic. The first one aims to solve the fluid and the solid equations with two different dedicated solvers, The stress and displacement functions at the interface are exchanged through the two solvers and imposed as boundary conditions. The second approach solves the solid and fluid equations simultaneously via an unique solver. As we will see in with this approach the equivalence of stresses and displacement at the interface is therefore automatically taken into account. A monolithic approach is generally more computationally expensive than the partitioned ones, but can give some important advantages in terms of stability. In the ALE reference system, the fluid mass and momentum balance equations reads

$$\nabla \cdot \mathbf{u}^f = 0, \quad \frac{\partial \mathbf{u}^f}{\partial \tau} + (\mathbf{u}^f - \mathbf{w}^f) \cdot \nabla \mathbf{u}^f = -\nabla \cdot p^f \mathbf{I} + \nu \nabla^2 \mathbf{u}^f + f_b, \quad (6)$$

where  $\tau$  is the time variable,  $f_b$  represents a generic body-force,  $\mathbf{u}^f$  is the fluid velocity vector,  $\mathbf{w}^f$  is the previously defined ALE velocity,  $p^f$  is the specific pressure, and  $\nu$  is the dynamic viscosity which is here taken to be constant. The FSI equation system in strong form then becomes

$$\begin{aligned} \nabla \cdot \mathbf{u}^f &= 0, & \frac{\partial \mathbf{u}^f}{\partial \tau} + (\mathbf{u}^f - \mathbf{w}^f) \cdot \nabla \mathbf{u}^f &= -\nabla \cdot p^f \mathbf{I} + \nu \nabla^2 \mathbf{u}^f + f_b \quad in \quad \Omega_t^f, \\ \mathbf{w}^f &= \frac{\partial \mathbf{d}^f}{\partial t}, & \frac{\partial \mathbf{d}^f}{\partial t} - k \nabla^2 \mathbf{d}^f &= 0 \quad in \quad \Omega_t^f, \\ \rho \frac{\partial \mathbf{u}^s}{\partial t} + \rho(\mathbf{u}^s \cdot \nabla) \mathbf{u}^s &= \nabla \cdot \sigma(\mathbf{d}^s) + \mathbf{b}^s, & \mathbf{u}^s &= \frac{d\mathbf{d}^s}{dt} \quad in \quad \Omega_t^s, \end{aligned} \quad (7)$$

where  $\mathbf{u}^s$  is the solid velocity vector,  $\sigma(\mathbf{d}^s)$  the solid strain tensor as a function of the solid displacement ( $\mathbf{d}^s$ ), and  $\mathbf{b}^s$  the external force.

The system (7) must be provided with the following boundary and initial conditions

$$\sigma^s \cdot \mathbf{n} = \sigma^f \cdot \mathbf{n}, \quad \mathbf{d}^f = \mathbf{d}^s \quad on \quad \Gamma_t^i, \quad \mathbf{u}(\mathbf{x}_0, 0) = \mathbf{u}_0, \quad \mathbf{d}(\mathbf{x}_0, 0) = \mathbf{d}_0 \quad in \quad \Omega_0. \quad (8)$$

In particular the first two conditions set the continuity of stress and velocity at the fluid-solid interface. Other suitable boundary conditions must be set on  $\Gamma_t^k$ . The weak formulation of

the problem is obtained by multiplying each equation for a proper test function  $\phi$  and then integrating over the computational domain. For a rigorous treatment of the subject one can refer for example to [2]. For the specific subject of fluid structure interaction one can refer to [1], as well as [5],[8]. For the fluid and solid momentum equations we obtain

$$\int_{\Omega_t^s} \left( \rho \frac{\partial \mathbf{u}^s}{\partial t} \phi + \rho (\mathbf{u}^s \cdot \nabla) \mathbf{u}^s \phi \right) d\Omega + \int_{\Omega_t^s} \sigma^s : \nabla \phi d\Omega = \int_{\Gamma_{t,N}^s} \mathbf{g}_N^s \cdot \phi d\gamma - \int_{\Gamma_t^i} (\sigma^f \cdot \mathbf{n}) \cdot \phi d\gamma \quad (9)$$

$$\forall \phi \in V_{\Gamma_D}^s, \quad V_{\Gamma_D}^s = [H_{\Gamma_{Dv}}^1(\Omega)^d] = \{ \phi \in [H^1(\Omega)^d] : \phi|_{\Gamma_{Dv}} = 0 \}.$$

$$\int_{\Omega_t^f} \left( \rho \frac{\partial \mathbf{u}^f}{\partial t} \phi + \rho (\mathbf{u}^f - \mathbf{w}) \cdot \nabla \mathbf{u}^f \phi \right) d\Omega + \int_{\Omega_t^f} \sigma^f : \nabla \phi d\Omega = \int_{\Gamma_{t,N}^f} \mathbf{g}_N^f \cdot \phi d\gamma + \int_{\Gamma_t^i} (\sigma^f \cdot \mathbf{n}) \cdot \phi d\gamma \quad (10)$$

$$\phi \in V_{\Gamma_D}^f, \quad V_{\Gamma_D}^f = [H_{\Gamma_{Dv}}^1(\Omega)^d] = \{ \phi \in [H^1(\Omega)^d] : \phi|_{\Gamma_{Dv}} = 0 \}.$$

$H^1(\Omega)^d$  denotes the Sobolev space. It is easy to notice that the last surface integrals of (9) and (10) are equal but with opposite sign due to the equivalence of the stresses at the interface. Such terms are implicitly taken into account in the monolithic approach, while they must be evaluated and substituted in the segregated approach.

### 2.3. 1D model for the multi-scale approach

The mono-dimensional model is a spatially reduced FSI model of a straight cylinder surrounded by a linear membrane structure. We remark that this simplified model is only used to evaluate the flow rate distributions and pressure wave propagation in order to save computational resources. The physical consistency of the model has been investigated by many authors. One can see reference [1] and citations therein. In particular, referring to a straight cylinder, the integration of the continuum balance equation over a generic section leads to the simplified system which reads

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0, \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \frac{\alpha Q^2}{A} + \frac{A}{\rho_f} \frac{\partial P}{\partial x} + K_r \frac{Q}{A} = 0, \quad (11)$$

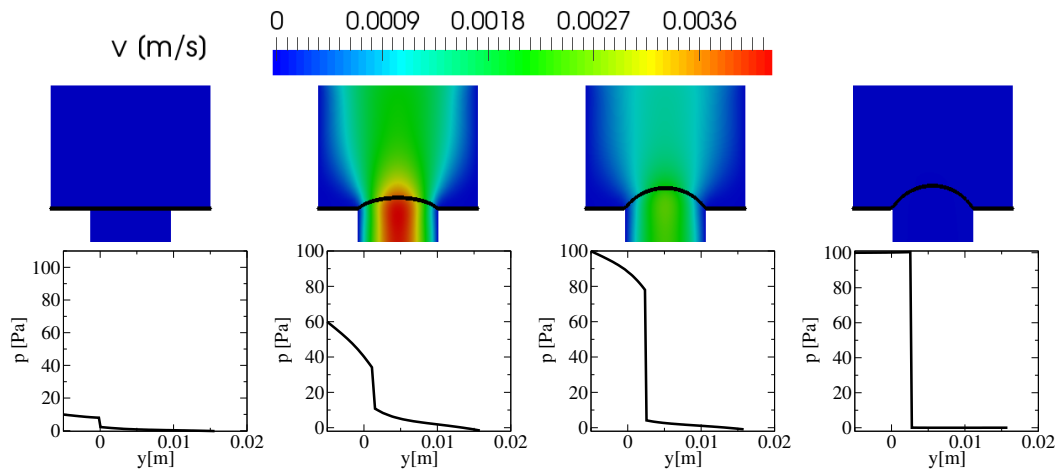
where  $Q$  is the flow rate over the generic section  $S$ ,  $A$  is its surface and  $P$  the average pressure.  $K_r$  is a resistance parameter which takes into account the fluid viscosity. The parameter  $\alpha$  defines the shape of the velocity profile over the section. The (11) is a system of two equations with three unknowns  $P$ ,  $Q$ ,  $A$ . For its closure a third equation is needed. For this purpose we use a suitable wall model relating the radial displacement to the mean pressure  $P$ . In particular we set

$$P = P_{ext} + \psi(A), \quad \frac{\partial \psi}{\partial A} > 0, \quad \psi(A_0) = 0, \quad (12)$$

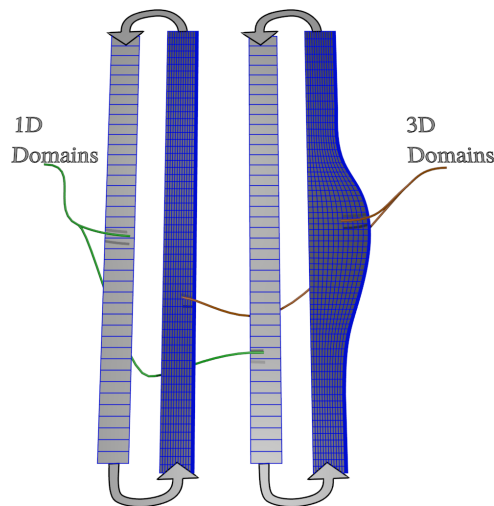
where  $A_0$  is the area of the surface  $S$  at  $t = 0$ . A classical choice is to consider an algebraic law of the type

$$\psi(A) = \beta(A) \frac{\sqrt{A} - \sqrt{A_0}}{\sqrt{\pi}}, \quad \beta = \frac{H_s E \pi}{(1 - \nu^2) A}, \quad (13)$$

where  $H$  is the thickness of the structure,  $E$  and  $\nu$  are the Young modulus and Poisson ratio, respectively. It is possible to take  $\beta$  constant as a mean value to simplify the treatment.



**Figure 2.** Benchmark 1. Velocity field (on the top) and pressure profile along the axis (on the bottom), from left to right at  $t = 0.1, 0.6, 1$  and  $2.3$  s.

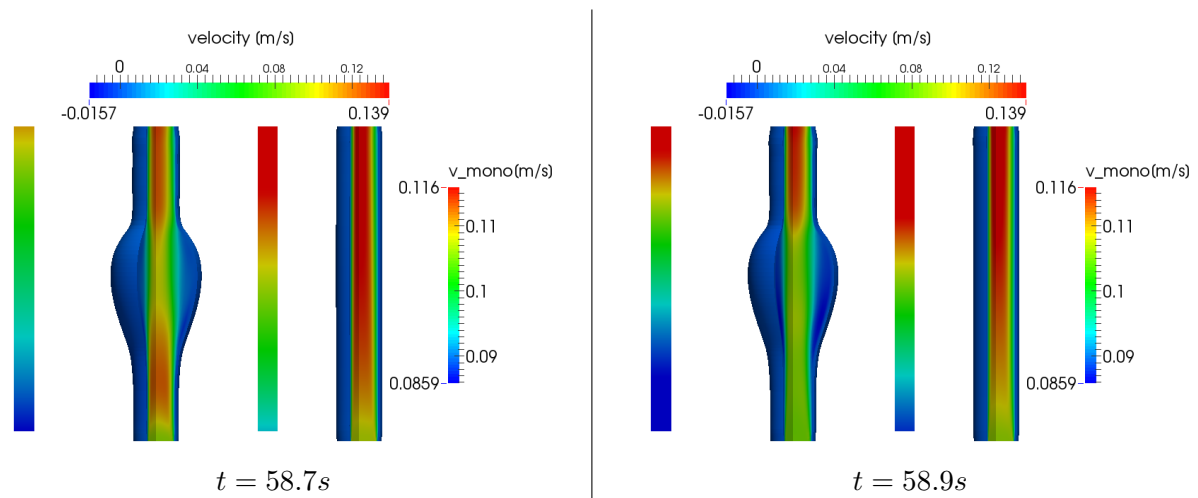


**Figure 3.** Case A. Configuration of the aneurysm (right) and healthy aorta (left) together with the mono dimensional domain.

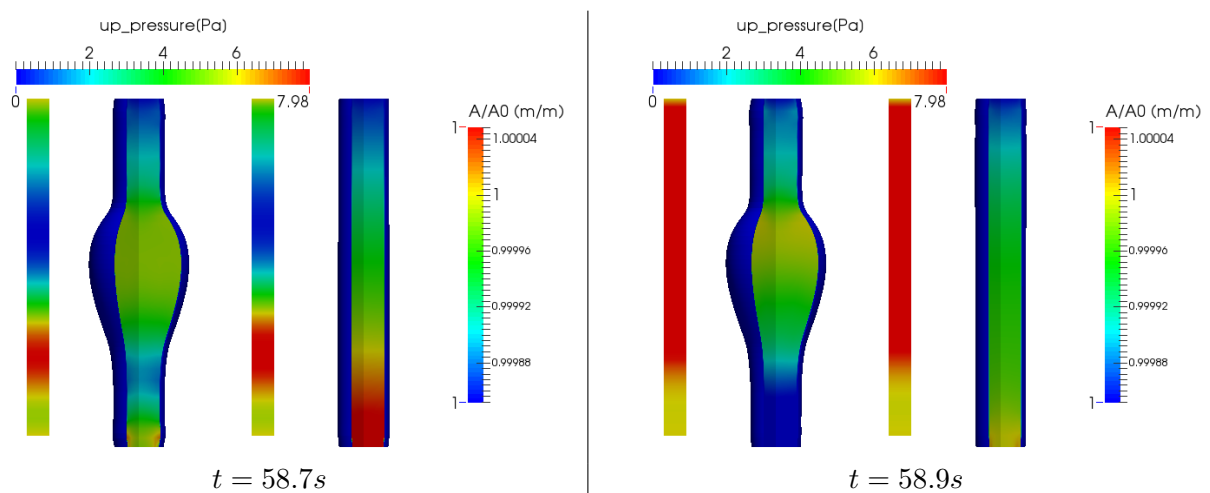
### 3. Numerical Results

#### 3.1. FSI Benchmark

In the first test we validate the fluid structure interaction model. We reproduce the benchmark originally proposed in [4] for large deformation problems and test the stability of the monolithic fluid structure model. In particular the benchmark consider two different fluid regions, divided by a thin soft solid. The lower chamber is loaded with a pressure and the soft material undergoes to large deformation. The interested reader could see all the specific geometrical dimensions of the benchmark in [4]. In figure 2 we show an overview of the obtained results, in particular on the top we can appreciate the continuity of the velocity field at different time steps together with the deformation of the solid structure. On the bottom one can see the evolution of the pressure field at the same time steps.



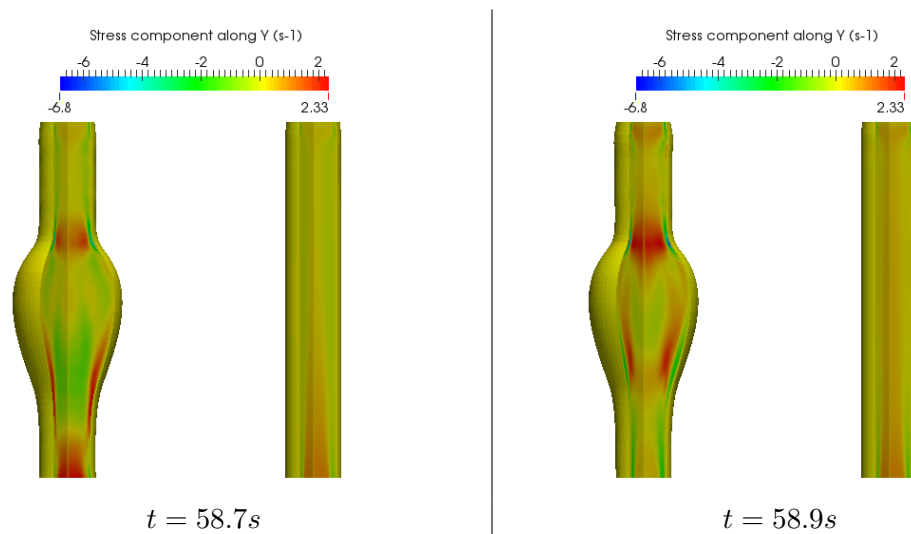
**Figure 4.** Case A. Velocity along the y-axis in the case of presence of an aneurysm (on the left) and for an healthy aorta (on the right) at  $t = 58.7$  (on the left) and  $58.9s$  (on the right).



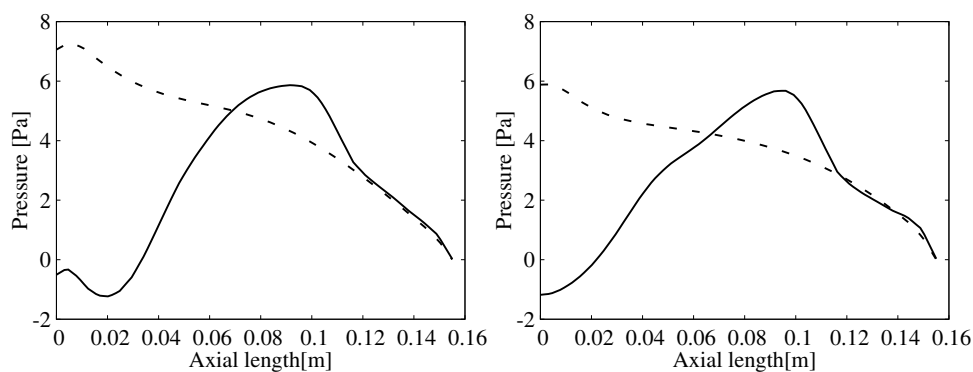
**Figure 5.** Case A. Pressure in the 3D domain and area variation on the 1D domain in the case of presence of an aneurysm (on the left) and for an healthy aorta (on the right) at  $t = 58.7$  (on the left) and  $58.9s$  (on the right).

### 3.2. Stable Aneurysm

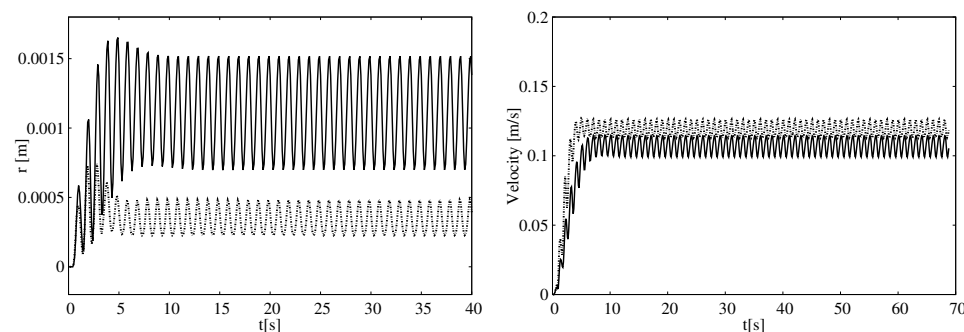
A multi-scale, axial symmetric, multi-dimensional simulation is performed with the purpose of studying the behavior of a stable aneurysm inserted in the whole cardiovascular system. We couple the inlet and the outlet surface of the domain with the outlet and the inlet boundary of a mono-dimensional pipe as it is shown in Figure 3. The inlet velocity of the multi-dimensional domain is the one coming from the the simplified system and the outflow of the complex domain is imposed at the inlet region of the 1D system. This coupling represents the first step for a detailed multi-scale approach that could lead to obtain realistic simulations of the whole cardiovascular loop with a reasonable computational cost. The same simulation were performed considering a non-deformed blood vessel in order to compare the obtained results. The Young modulus, as is common in literature for this type of problems, is set to  $10^3 Pa$  and the Poisson ratio to 0.4. We also take into account the hearth contraction by adding a periodic momentum



**Figure 6.** Case A. Stress component in the axial direction in the 3D domain in the case of presence of an aneurysm (on the left) and for an healthy aorta (on the right) at  $t = 58.7$  (on the left) and  $58.9s$  (on the right).



**Figure 7.** Case A. Pressure along the axis for the 3D domain in the case of presence of an aneurysm (on the left) and for an healthy aorta (on the right) at  $t = 58.7$  (on the left) and  $58.9s$  (on the right).

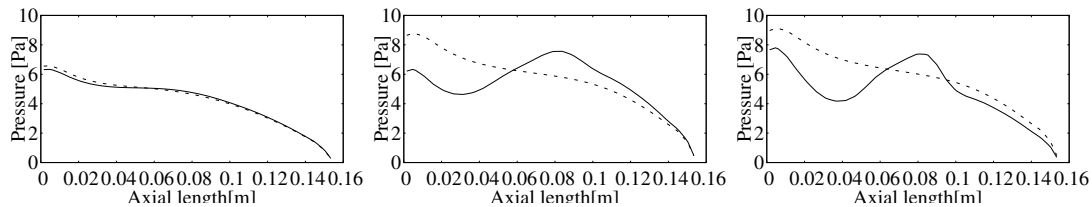


**Figure 8.** Case A. Evolution of the displacement (on the left) and of the velocity (on the right) in the presence of an aneurysm (solid line) and an healthy aorta (dashed line).



source in the simplified system in the form  $H\sin(\pi t)^4$ . The multi-scale approach allows us the study of the effects of the presence of the aneurysm on a simplified closed loop. When the flow becomes periodic stabilized a period of 1 s is observed. In Figure 4 the velocity field in the axial direction, for a normal and a deformed domain, is shown at different time steps. In particular we can observe that in the deformed case the inlet velocity undergoes large variations. Such an effect could not be taken into account without using a multidimensional approach. In Figure 5, the pressure field in the three-dimensional domain and the area distribution in the mono-dimensional pipe are shown at different time steps. In Figure 6 one can appreciate the increasing of the axial stress in the presence of an aneurysm at different time steps. In the healthy aorta interface stresses are negligible and, in the presence of an aneurysm, the stresses increase near the inlet and the outlet region of the aneurysm itself with high probability to cause the blood vessel rupture. In Figure 7 the pressure field along the axis at different time steps is shown. We plot, in a dashed line, the values for the healthy aorta and in a solid line the values for the aneurysm. We remark that several differences can be observed between these two cases. Such differences are mostly due to the recirculation phenomena that occur in the presence of the aneurysm. In Figure 8 the radial displacement, at  $y = 0.07\text{ m}$  is shown together with the axial velocity. We plot, in a dashed line, the values for the healthy aorta and, in a solid line, the values for the aneurysm. One can notice that in the presence of an aneurysm displacements becomes greater than the ones evaluated in a straight blood vessel. The large deformation that occurs in the presence of the aneurysm reduces the overall magnitude of velocity field and the characteristic frequency of the structure, as we can see on the right of Figure 8.

### 3.3. Aneurysm formation and growing



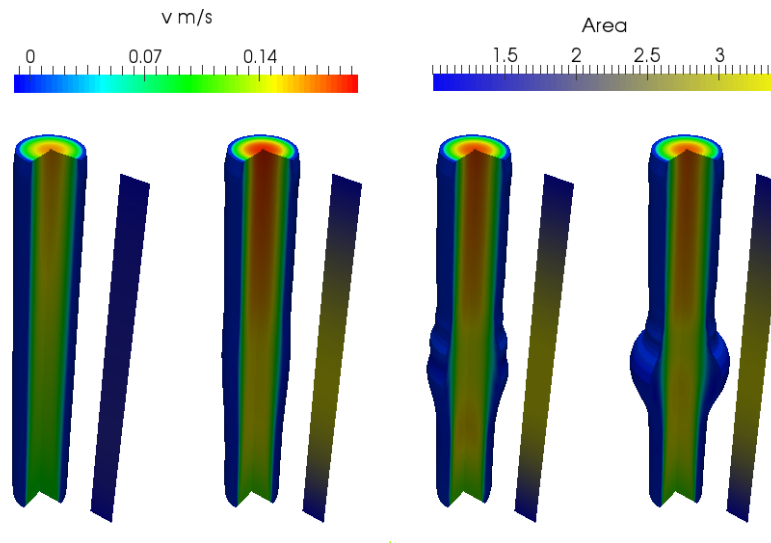
**Figure 9.** Case B. Formation of the aneurysm pressure as a function of the axial coordinate for the three-dimensional domain. Aneurysm formation (solid line) and healthy aorta (dashed line) for  $t = 5, 35$  and  $80\text{ s}$ .

In this test we simulate the growth of an aneurysm and we show the evolution of the velocity field. The formation of the aneurysm is caused by a progressive, localized loss of elasticity in the aorta wall. The mono-dimensional loop is treated with the model described previously. The boundary and the coupling conditions are the same imposed in the previous section for the healthy aorta in order to compare the velocity profile obtained. A progressive localized loss of elasticity is simulated with a decreasing of the Young modulus in time by using the following model

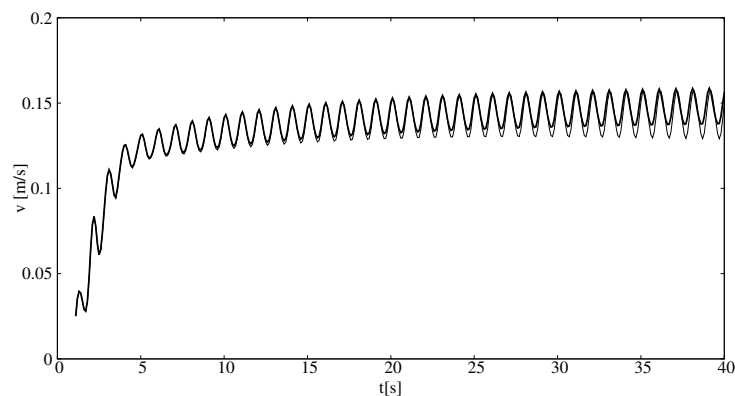
$$E = E_0(1 - 80t(x - 0.04)(0.1 - x))^4 \quad \forall x \in [0.04; 0.1], \quad E = E_0 \quad \forall x \notin [0.04; 0.1]. \quad (14)$$

The simulation is performed in a time interval of 400 s in which the aneurysm is formed and reaches the steady state. As in the previous example a multi-scale approach is used in order to study the effects of an aneurysm development in the framework of a closed connected loop. This setup may be used to study the evolution of an aneurysm and investigate the behavior of the whole circulatory system when an aneurysm is growing in a certain blood vessel. In Figure

10 the velocity profile and the cross section dimension of the multidimensional and the mono-dimensional domain are shown respectively at different time steps. In Figure 9 we show the pressure distribution along the axis of the multi dimensional domain at different time steps. In Figure 11 we can observe the variation of the inlet velocity in the mono-dimensional domain in the normal and the deformed case. We can notice that the average velocity field is not constant over time. This is a consequence of the huge deformation of the multidimensional domain. Such a feedback could not be taken into account without considering the simplified coupled domain.



**Figure 10.** Case B. Formation of the aneurysm, velocity along the axis in the three-dimensional domain and area variation in the mono-dimensional domain for  $t = 5, 35, 65$  and  $80$  s.



**Figure 11.** Case B. Formation of the aneurysm. Mono-dimensional velocity as a function of time at the inlet of the mono-dimensional domain. Aneurysm formation (thick line) and healthy aorta (thin line).

#### 4. Conclusion

In this work we have presented a preliminary investigation of an highly unsteady and transient fluid-structure interaction system using a multi-scale, monolithic approach. We have shown how

this technique can be used in order to study a complex problem such as an aneurysm growth in a blood vessel. In particular we have used a full scale multi-dimensional model to simulate the growing aneurysm region where and a mono-dimensional simplified model for the rest of the system. Such a coupling scheme allow us to use CFD methodology and still keep into account the whole system response with reasonable computational costs.

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