Three-Dimensional Crack Propagation with Global Enrichment XFEM and Vector Level Sets

K. Agathos[1] G. Ventura[2] E. Chatzi[3] S. P. A. Bordas[4,5]

[1]Institute of Structural Analysis and Dynamics of Structures
Aristotle University Thessaloniki

[2]Department of Structural and Geotechnical Engineering
Politecnico di Torino

[3]Institute of Structural Engineering
ETH Zürich

[4]Research Unit in Engineering Science
Luxembourg University

[5]Institute of Theoretical, Applied and Computational Mechanics
Cardiff University

September 9, 2015

Outline

Global enrichment XFEM

An XFEM variant (Agathos, Chatzi, Bordas, & Talaslidis, 2015) is introduced which:

- Enables the application of geometrical enrichment to 3D.
- Extends dof gathering to 3D through global enrichment.
- Employs weight function blending.
- Employs enrichment function shifting.

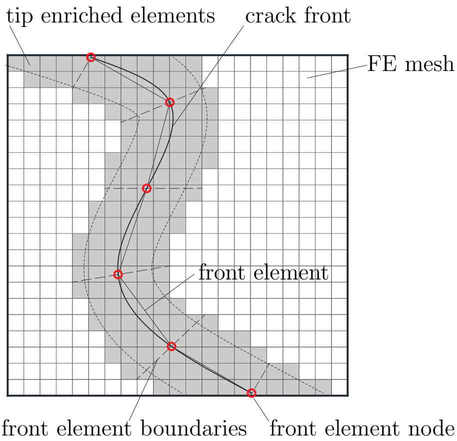
Front elements

A superimposed mesh is used to provide a p.u. basis.

Desired properties:

- Satisfaction of the partition of unity property.
- Spatial variation only along the direction of the crack front.

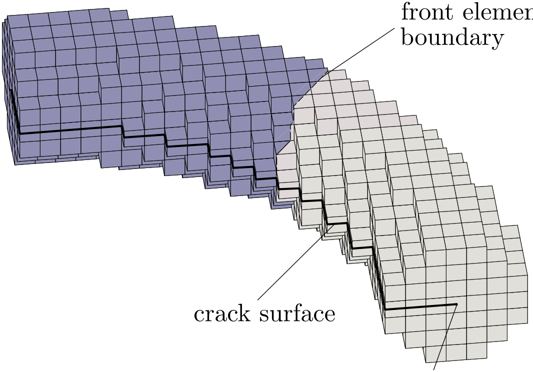
Front elements



- A set of nodes along the crack front is defined.
- Each element is defined by two nodes.
- A good starting point for front element thickness is h.

Front elements

Volume corresponding to two consecutive front elements.



Different element colors correspond to different front elements.

Front element shape functions

Linear 1D shape functions are used:

$$\mathbf{N}^g(\xi) = \begin{bmatrix} \frac{1-\xi}{2} & \frac{1+\xi}{2} \end{bmatrix}$$

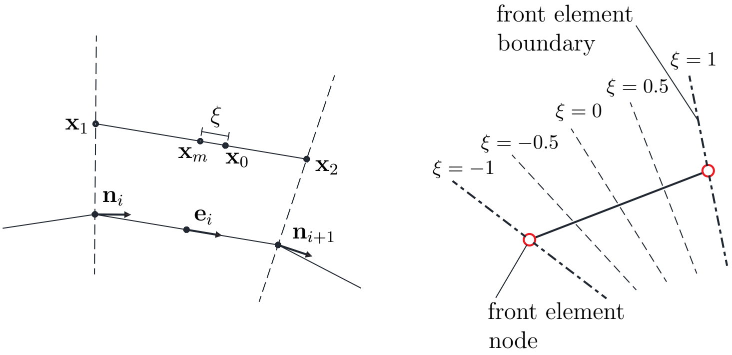
where ξ is the local coordinate of the superimposed element.

Those functions:

- form a partition of unity.
- are used to weight tip enrichment functions.

Front element shape functions

Definition of the front element parameter used for shape function evaluation.



Tip enrichment functions

Tip enrichment functions used:

$$F_j(\mathbf{x}) \equiv F_j(r,\theta) = \left[\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right]$$

Tip enriched part of the displacements:

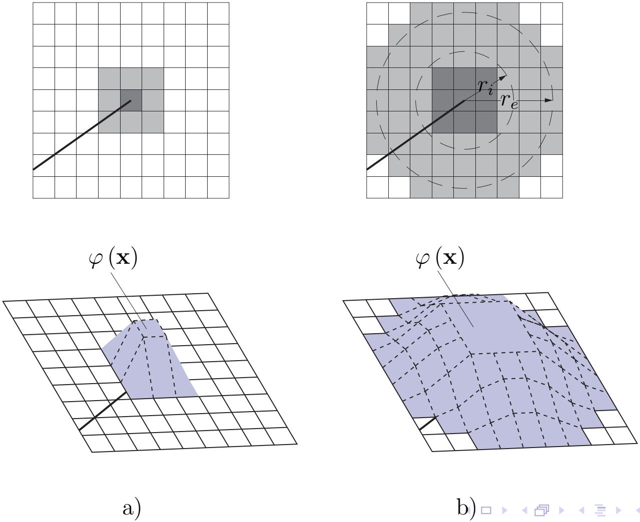
$$\mathbf{u_t}(\mathbf{x}) = \sum_{K\in\mathcal{N}^s} N_K^g(\mathbf{x}) \sum_j F_j(\mathbf{x})\, \mathbf{c}_{Kj}$$

where

- N_K^g are the global shape functions
- $\mathcal{N}^s$ is the set of superimposed nodes

Weight functions

Weight functions for a) topological (Fries, 2008) and b) geometrical enrichment (Ventura, Gracie, & Belytschko, 2009).



Displacement approximation

$$\mathbf{u}(\mathbf{x}) = \sum_{I \in \mathcal{N}} N_I(\mathbf{x})\,\mathbf{u}_I + \bar{\varphi}(\mathbf{x}) \sum_{J \in \mathcal{N}^j} N_J(\mathbf{x})\,(H(\mathbf{x}) - H_J)\mathbf{b}_J +$$

$$+ \varphi(\mathbf{x}) \left(\sum_{K \in \mathcal{N}^s} N_K^g(\mathbf{x}) \sum_j F_j(\mathbf{x}) - \right.$$

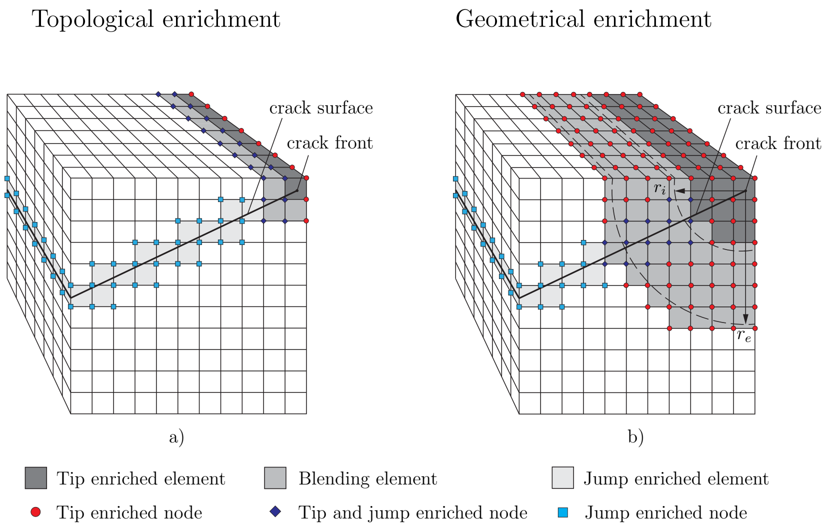
$$\left. - \sum_{T \in \mathcal{N}^t} N_T(\mathbf{x}) \sum_{K \in \mathcal{N}^s} N_K^g(\mathbf{x}_T) \sum_j F_j(\mathbf{x}_T) \right) \mathbf{c}_{Kj}$$

where:

- $\mathcal{N}$ is the set of all nodes in the FE mesh.
- $\mathcal{N}^j$ is the set of jump enriched nodes.
- $\mathcal{N}^t$ is the set of tip enriched nodes.
- $\mathcal{N}^s$ is the set of nodes in the superimposed mesh.

Weight functions

Enrichment strategies used for tip and jump enrichment.



Vector Level Sets

A method for the representation of 3D cracks is introduced which:

- Produces level set functions using geometric operations.
- Does not require integration of evolution equations.

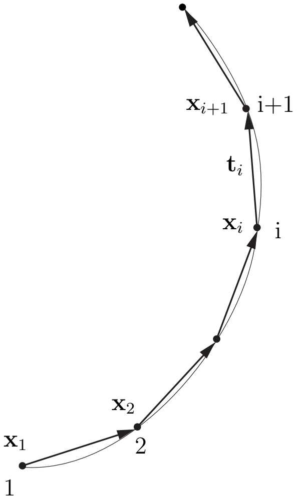
Similar methods:

- 2D Vector level sets (Ventura, Budyn, & Belytschko, 2003).
- Hybrid implicit-explicit crack representation (Fries & Baydoun, 2012).

Crack front

Crack front at time t:

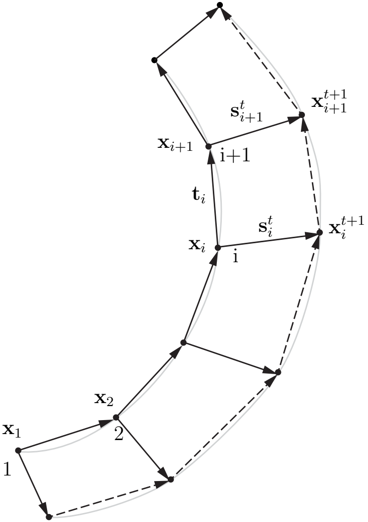
- Ordered series of line segments $\mathbf{t}_i$
- Set of points $\mathbf{x}_i$



Crack front advance

Crack front at time $t + 1$:

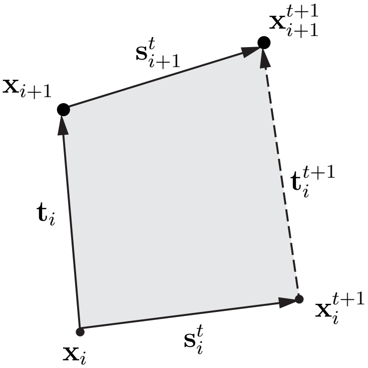
- Crack advance vectors $\mathbf{s}_i^t$ at points $\mathbf{x}_i$
- New set of points $\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{s}_i^t$



Crack surface advance

Crack surface advance:

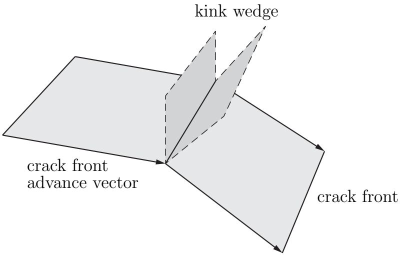
- Sequence of four sided bilinear segments.
- Vertexes: $\mathbf{x}_i^t$, $\mathbf{x}_{i+1}^t$, $\mathbf{x}_{i+1}^{t+1}$, $\mathbf{x}_i^{t+1}$



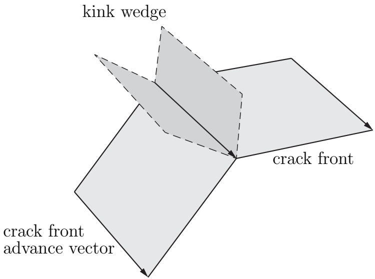
Kink wedges

Discontinuities (*kink wedges*) are present:

- Along the crack front (a).
- Along the advance vectors (b).



a)



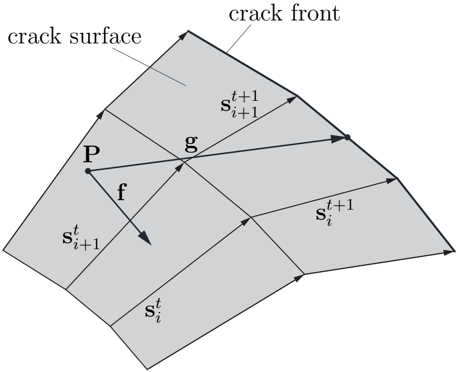
b)

Level set functions

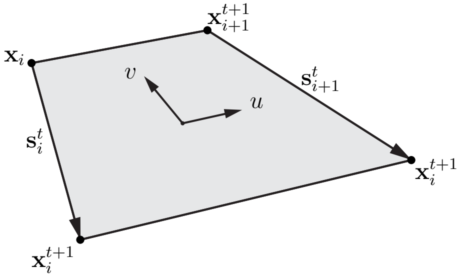
Definition of the level set functions at a point **P**:

f distance from the crack surface.

g distance from the crack front.



Point projection



Element parametric equations $\phi(u, v)$, $u, v \in [-1, 1]$:

$$\begin{cases} \phi_x &= g_1(u,v)\, x_i^t + g_2(u,v)\, x_{i+1}^t + g_3(u,v)\, x_{i+1}^{t+1} + g_4(u,v)\, x_i^{t+1} \\ \phi_y &= g_1(u,v)\, y_i^t + g_2(u,v)\, y_{i+1}^t + g_3(u,v)\, y_{i+1}^{t+1} + g_4(u,v)\, y_i^{t+1} \\ \phi_z &= g_1(u,v)\, z_i^t + g_2(u,v)\, z_{i+1}^t + g_3(u,v)\, z_{i+1}^{t+1} + g_4(u,v)\, z_i^{t+1} \end{cases}$$

where $g_i(u, v)$, $u, v \in [-1, 1]$ are linear shape functions.

Point projection

Equation of the tangent plane Π_0 at (u_0, v_0):

$$\det \begin{bmatrix} x - \phi_x(u_0, v_0) & y - \phi_y(u_0, v_0) & z - \phi_z(u_0, v_0) \\ \phi_{x,u}(u_0, v_0) & \phi_{y,u}(u_0, v_0) & \phi_{z,u}(u_0, v_0) \\ \phi_{x,v}(u_0, v_0) & \phi_{y,v}(u_0, v_0) & \phi_{z,v}(u_0, v_0) \end{bmatrix} = 0$$

Normal vector to the parametric surface at (u_0, v_0):

$$\mathbf{n}(u_0, v_0) = (A, B, C)$$

where A, B, C are the minors of the previous matrix at (u_0, v_0).

Point projection

Point **P** can be expressed as:

$$\mathbf{P} = \mathbf{P}'(u, v) + \lambda \mathbf{n}(u, v)$$

where:

$\mathbf{P}'$ the projection of the point to the surface.

λ unknown parameter.

The above is solved for u, v and λ to obtain the projection.

Evaluation of the level set functions

At each step t:

- For each point all crack advance segments are tested.
- If for a certain element $u, v \in [-1, 1]$ then the point is projected on that element.
- If $u \notin [-1, 1]$ for all elements then the projection lies on the advance vector.
- If $v \notin [-1, 1]$ for all elements then the projection lies either:
  - $\rightarrow$ at a previous crack advance segment
  - $\rightarrow$ at the crack front at time $t-1$ or t

Evaluation of the level set functions

Level set function $\mathbf{f}$:

$$\mathbf{f} = \mathbf{P} - \mathbf{P}'$$

where $\mathbf{P}'$ is either:

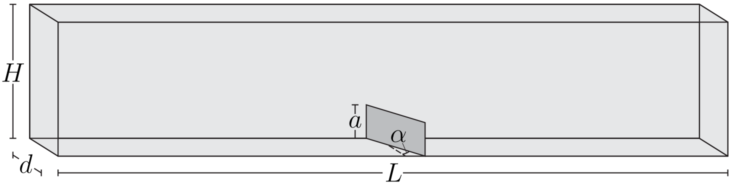
- Projection to an element of the crack surface
- Closest point projection to a kink wedge

Level set function $\mathbf{g}$:

$$\mathbf{g} = \mathbf{P} - \mathbf{P}'$$

where $\mathbf{P}'$ is a closest point projection to the crack front

Edge crack in a beam



Geometry:

L = 1 unit

H = 0.2 units

d = 0.1 units

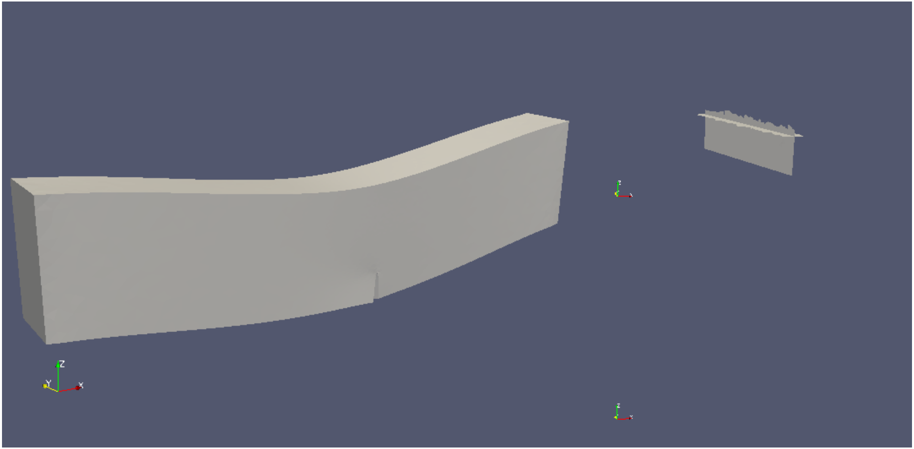
a = 0.05 units

$\alpha = 45^{\circ}$

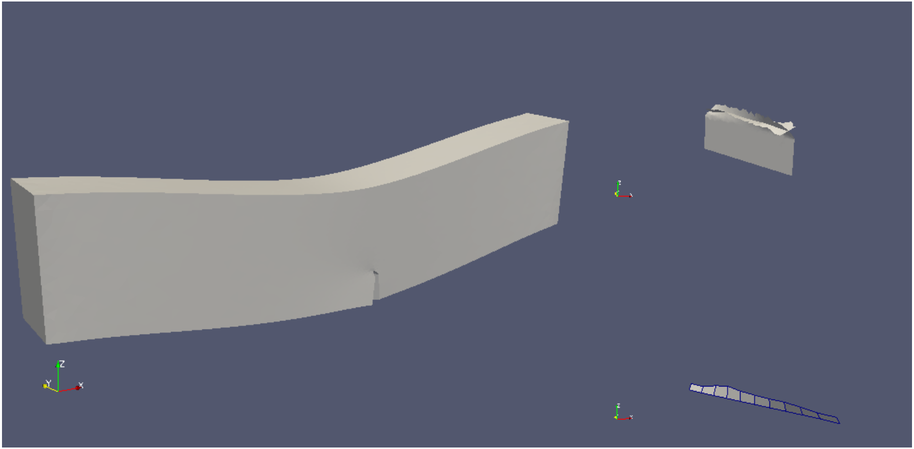
Mesh:

- Far from the crack $h = 0.02$ units
- In the vicinity of the crack $h = 0.005$ units

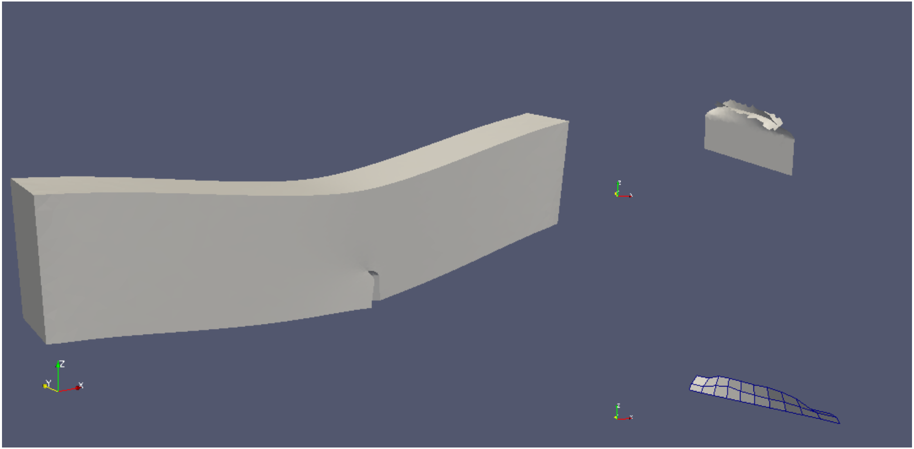
Edge crack in a beam



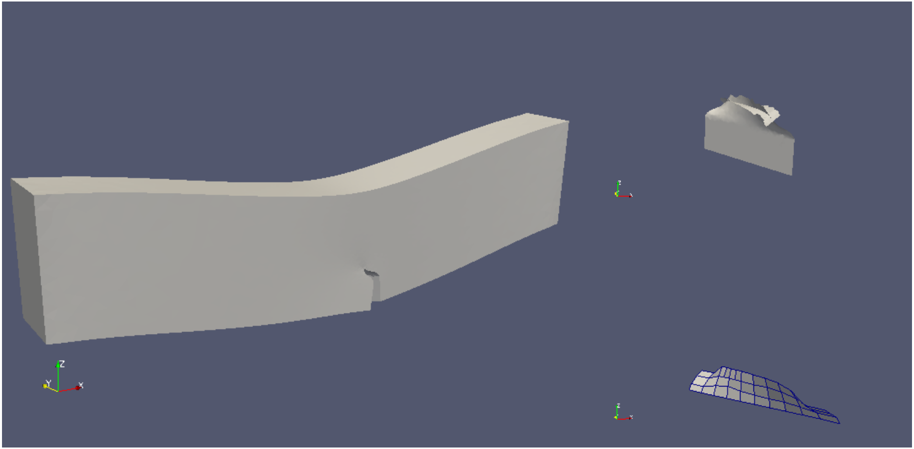
Edge crack in a beam



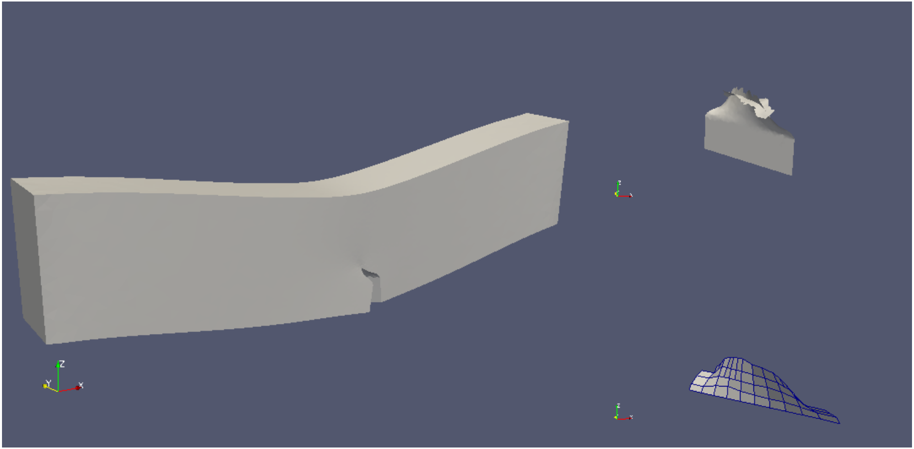
Edge crack in a beam



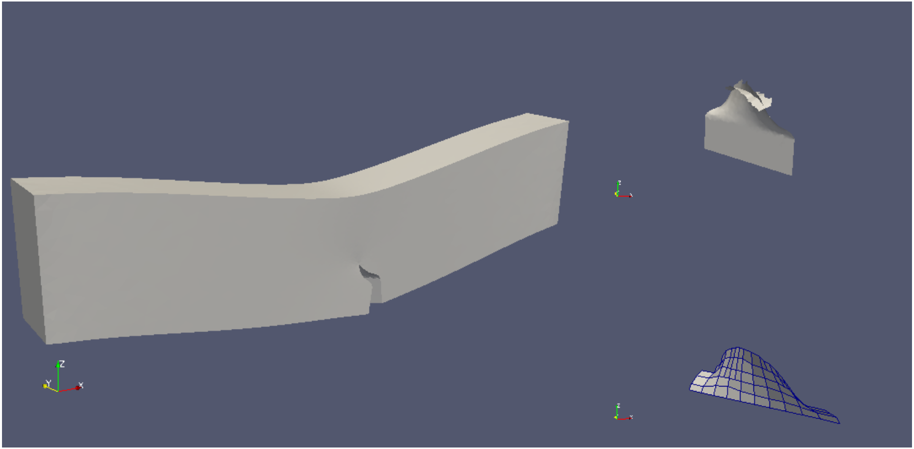
Edge crack in a beam



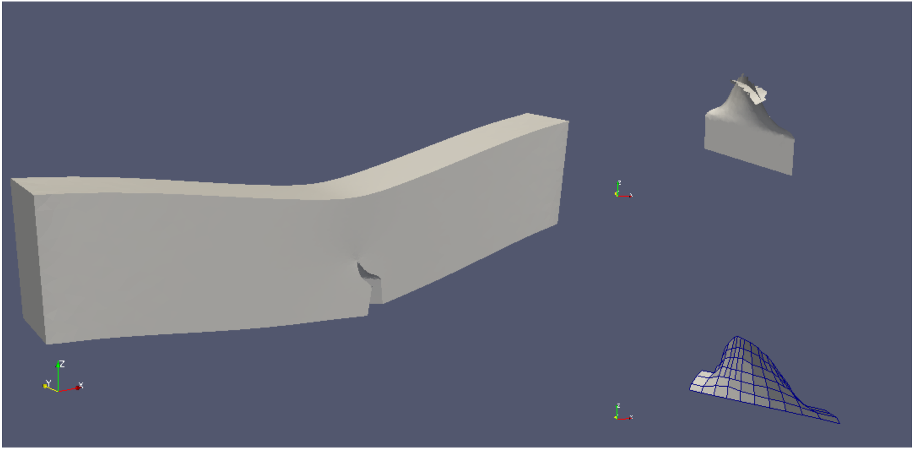
Edge crack in a beam



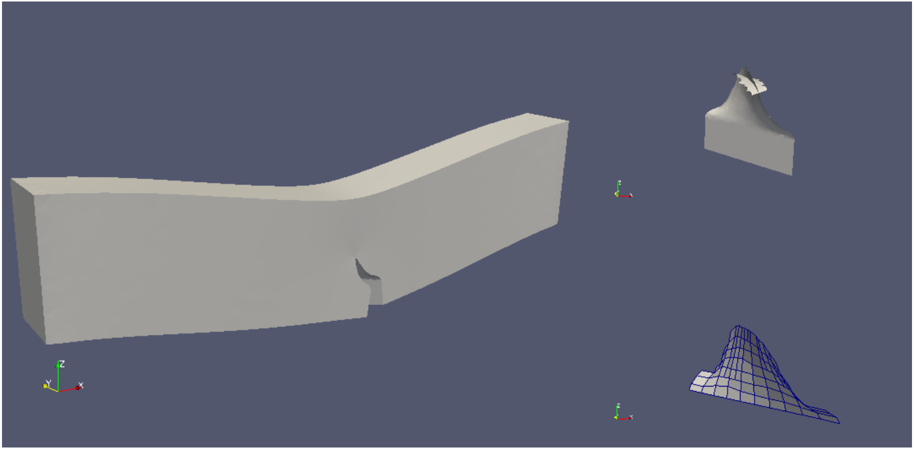
Edge crack in a beam



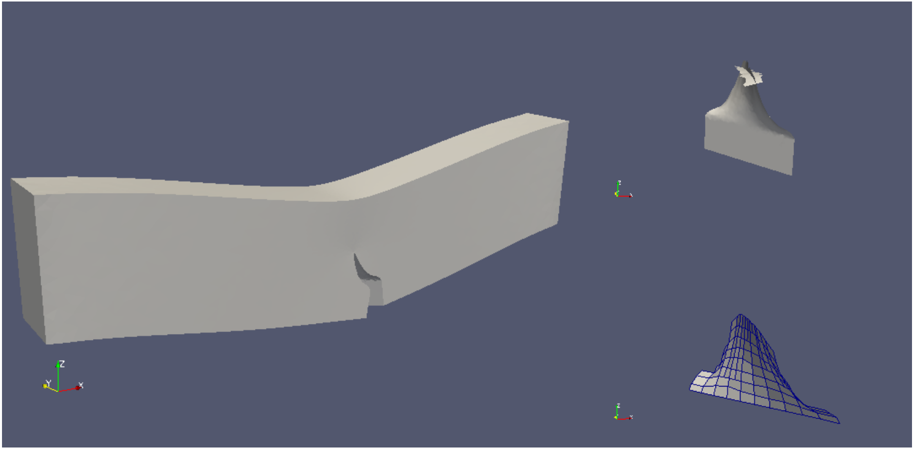
Edge crack in a beam



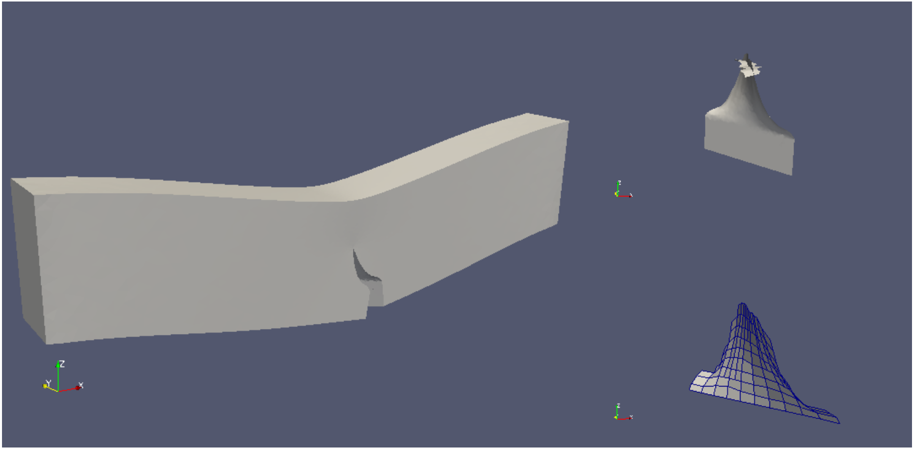
Edge crack in a beam



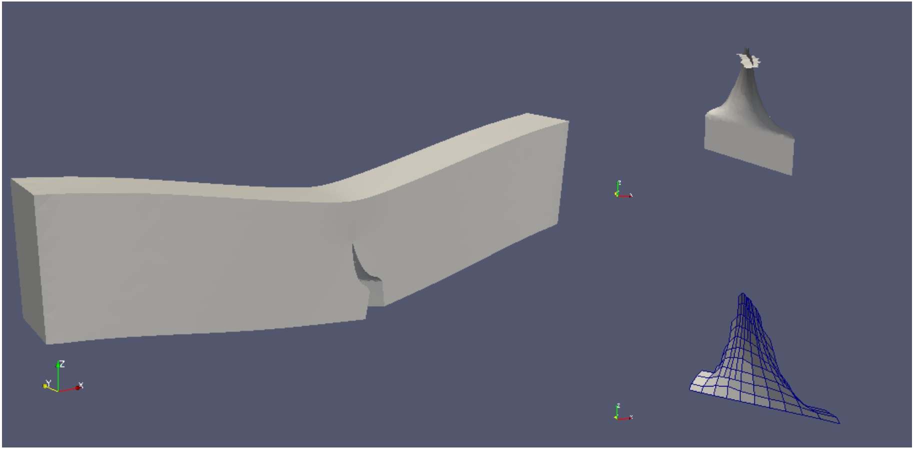
Edge crack in a beam



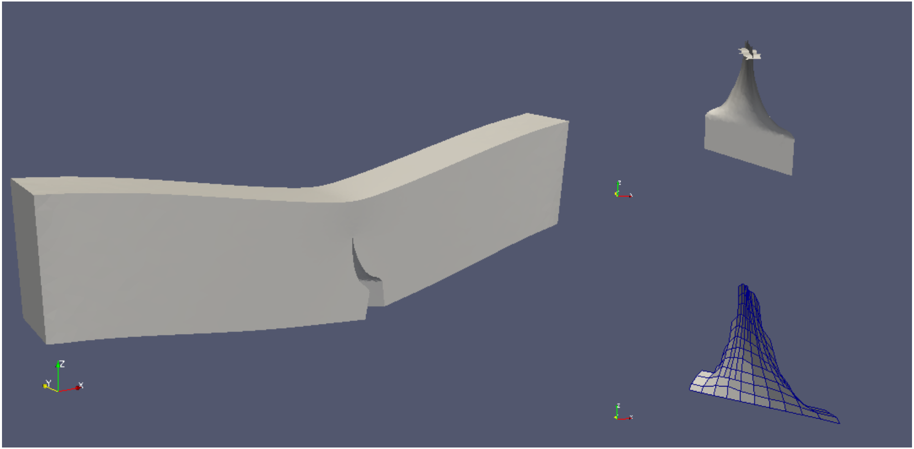
Edge crack in a beam



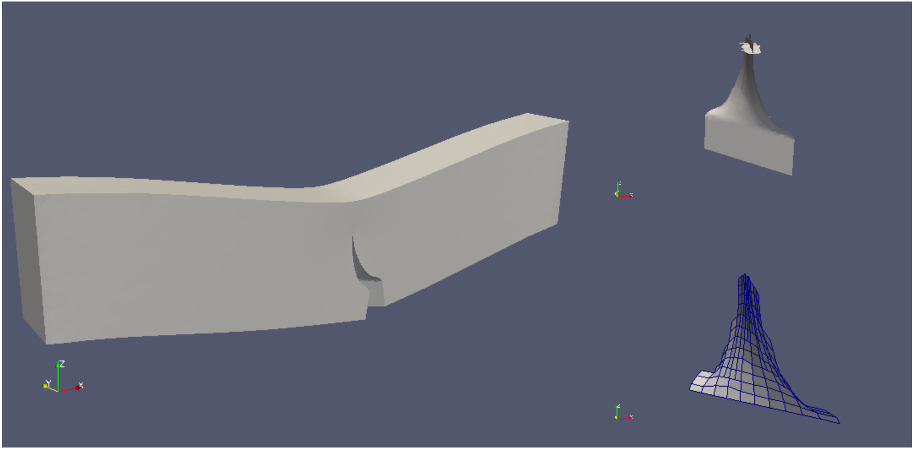
Edge crack in a beam



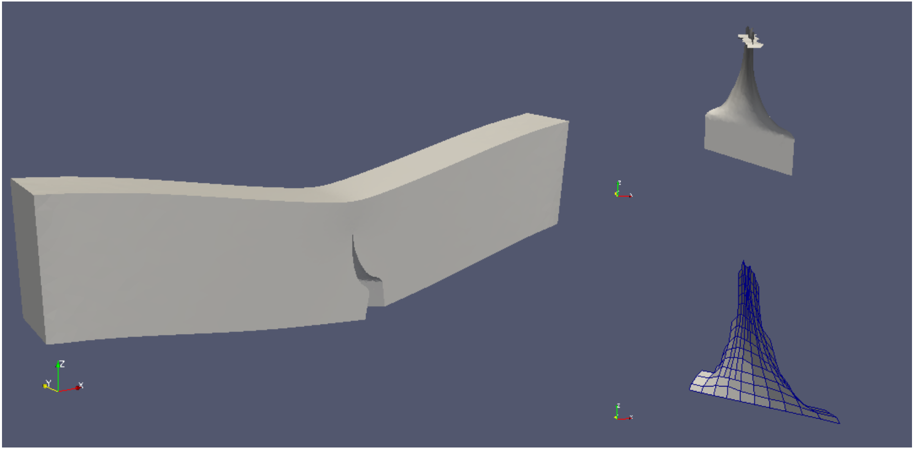
Edge crack in a beam



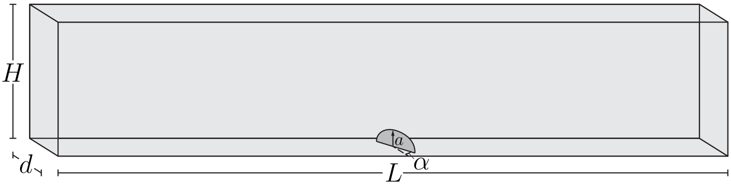
Edge crack in a beam



Edge crack in a beam



Semi circular crack in a beam



Geometry:

L = 1 unit

H = 0.2 units

d = 0.1 units

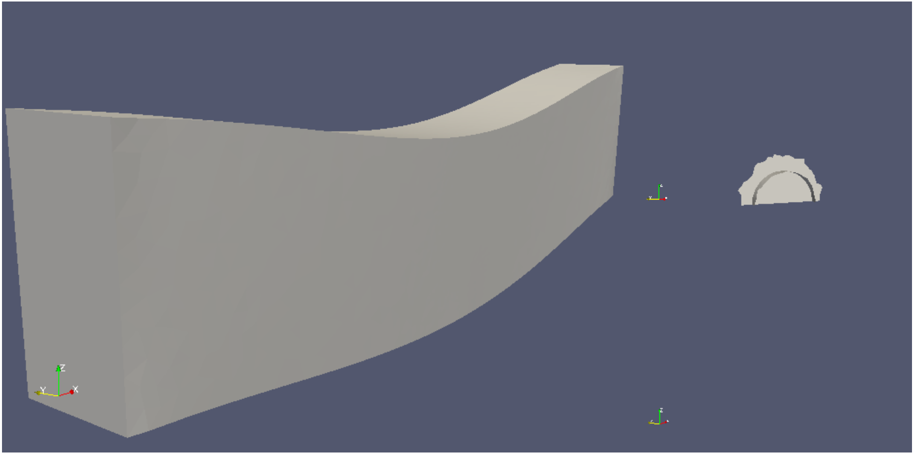
a = 0.025 units

α = 45°

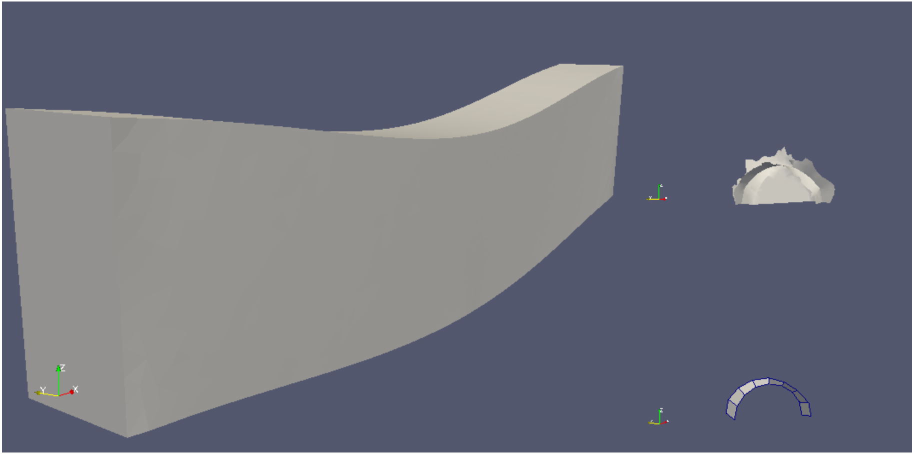
Mesh:

- Far from the crack $h = 0.02$ units
- In the vicinity of the crack $h = 0.005$ units

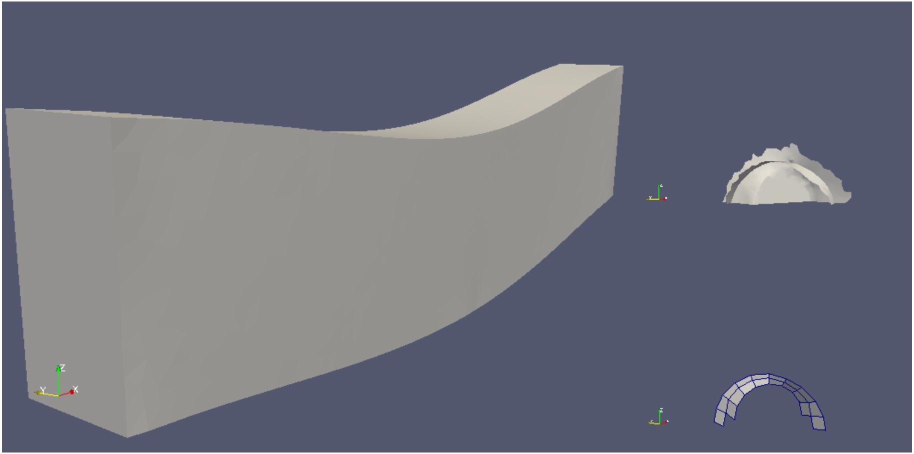
Semi circular crack in a beam



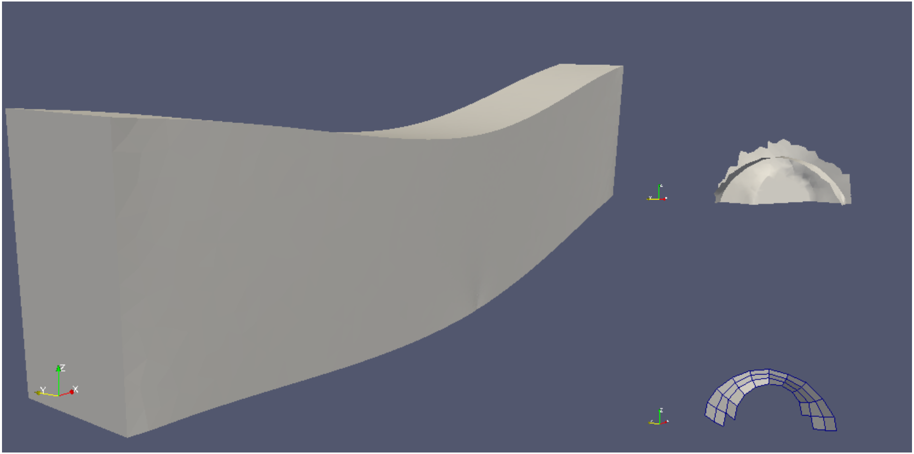
Semi circular crack in a beam



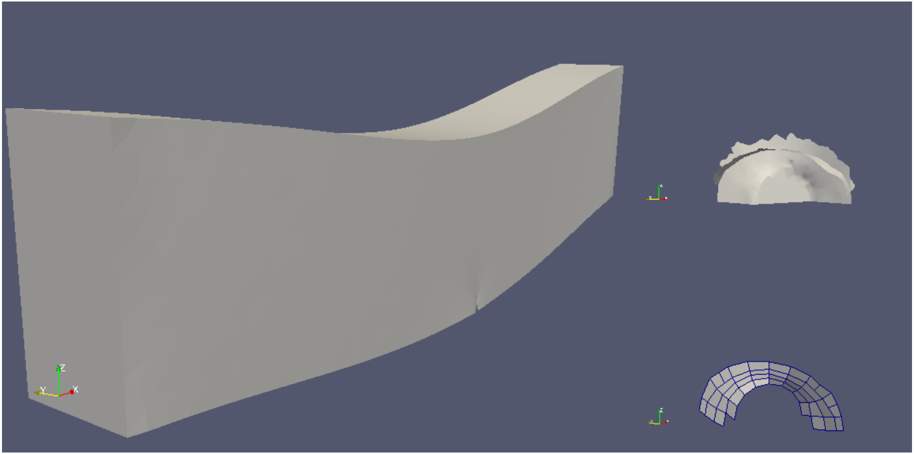
Semi circular crack in a beam



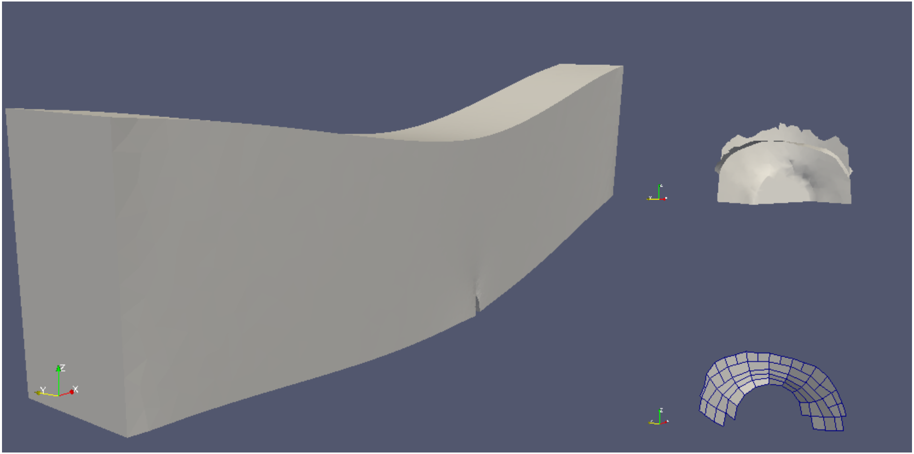
Semi circular crack in a beam



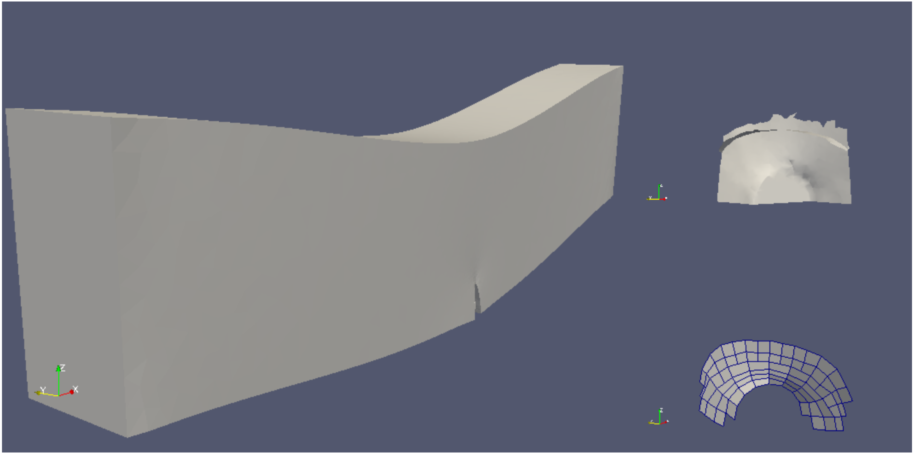
Semi circular crack in a beam



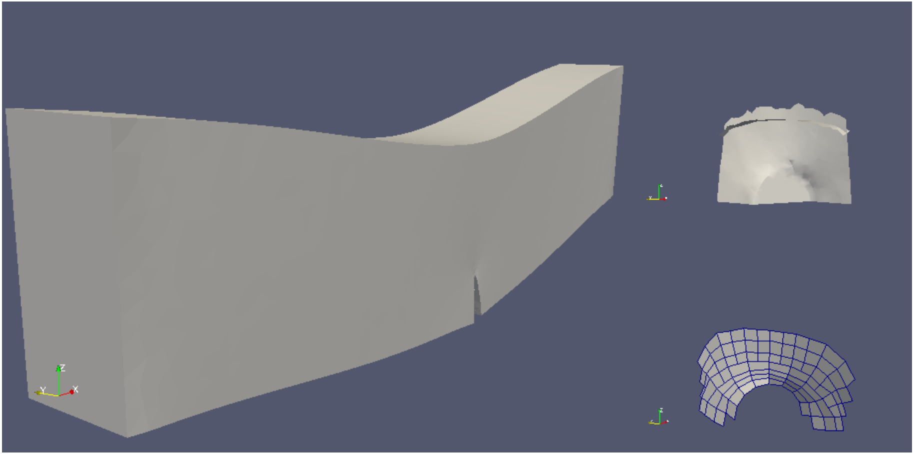
Semi circular crack in a beam



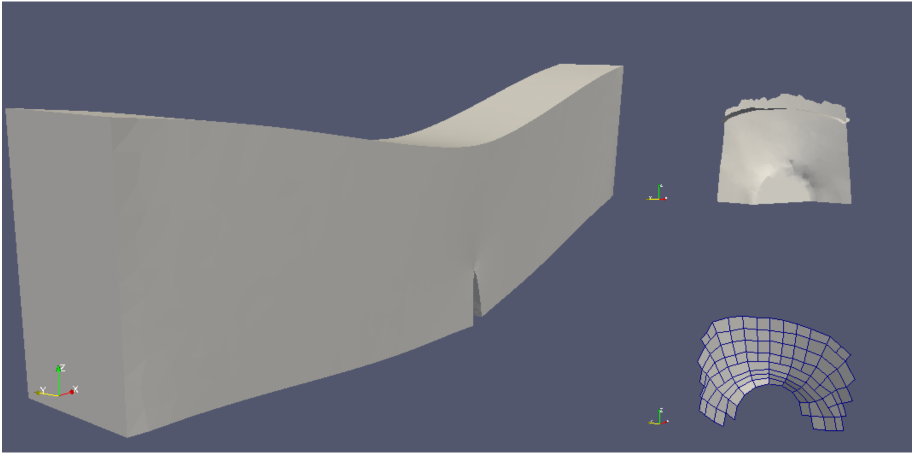
Semi circular crack in a beam



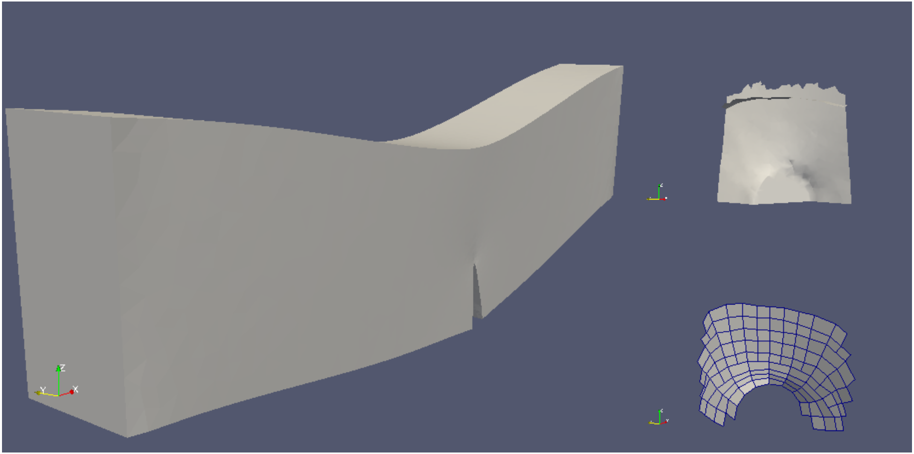
Semi circular crack in a beam



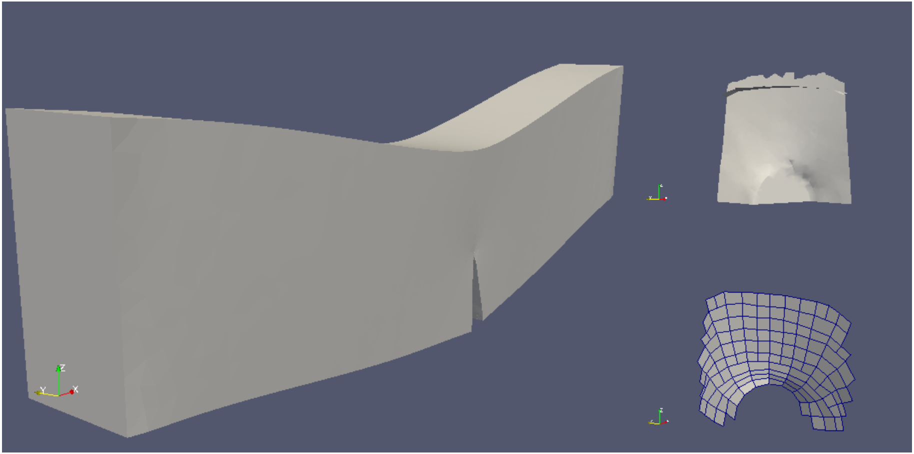
Semi circular crack in a beam



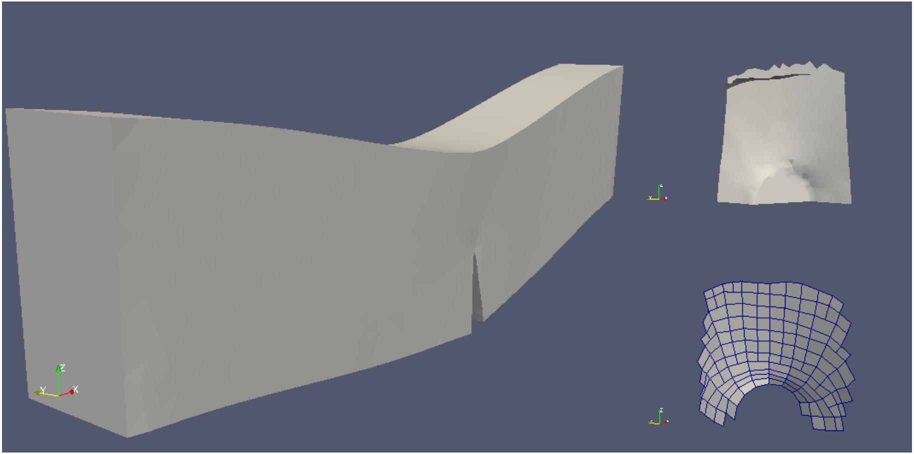
Semi circular crack in a beam



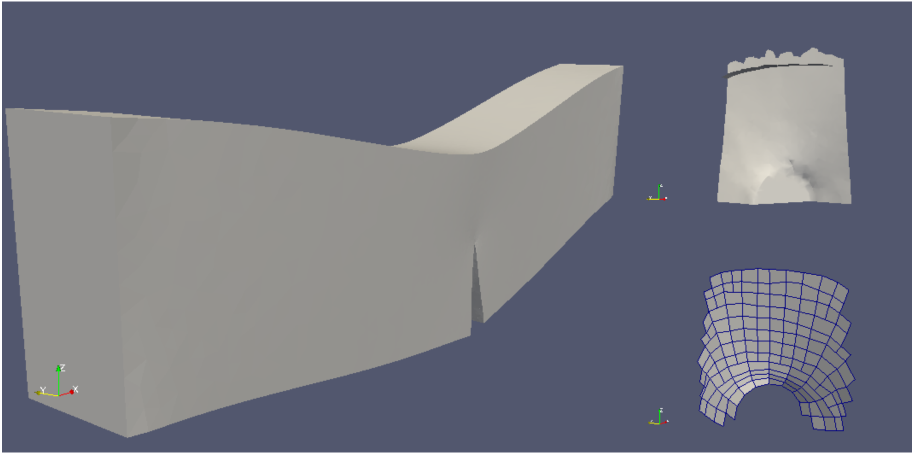
Semi circular crack in a beam



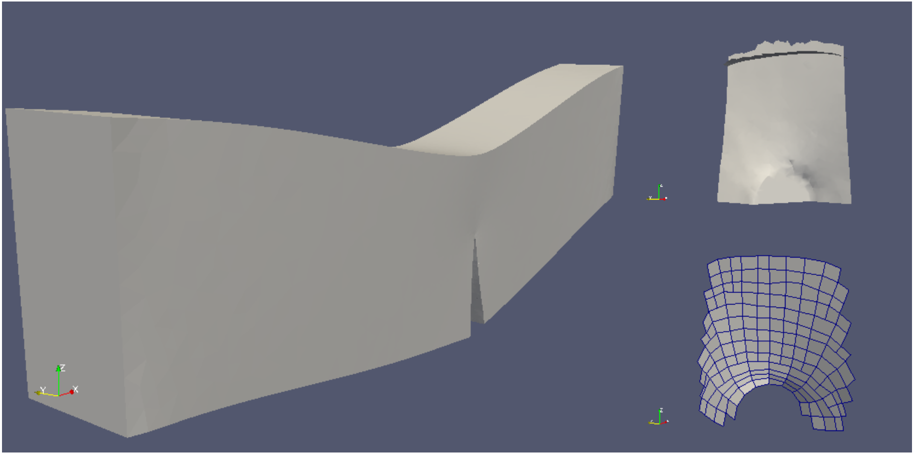
Semi circular crack in a beam



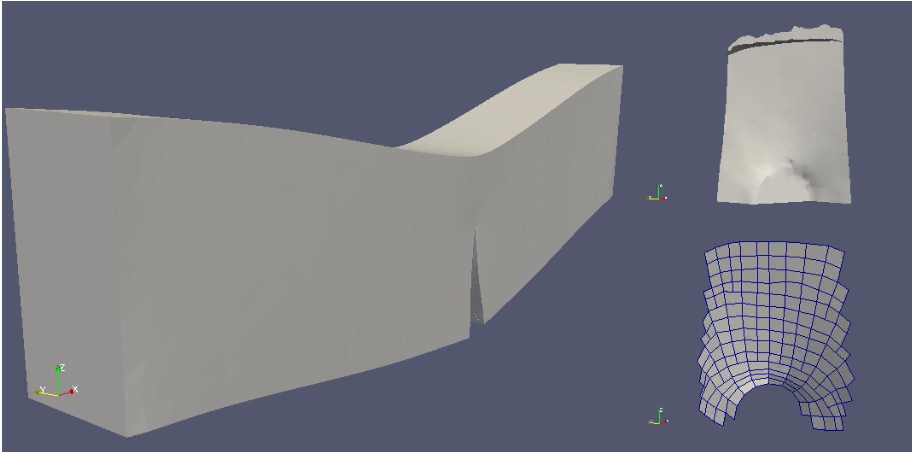
Semi circular crack in a beam



Semi circular crack in a beam



Semi circular crack in a beam



Conclusions

A method for 3D fracture mechanics was presented which:

- Enables the use of geometrical enrichment in 3D.
- Eliminates blending errors.

A method for the representation of 3D cracks was presented which:

- Avoids the solution of evolution equations.
- Utilizes only simple geometrical operations.

The methods were combined to solve 3D crack propagation problems.

Bibliography

Agathos, K., Chatzi, E., Bordas, S., & Talaslidis, D. (2015). A well-conditioned and optimally convergent xfem for 3d linear elastic fracture. *International Journal for Numerical Methods in Engineering*.

Fries, T. (2008). A corrected XFEM approximation without problems in blending elements. *International Journal for Numerical Methods in Engineering*.

Fries, T., & Baydoun, M. (2012). Crack propagation with the extended finite element method and a hybrid explicit-implicit crack description. *International Journal for Numerical Methods in Engineering*.

Ventura, G., Budyn, E., & Belytschko, T. (2003). Vector level sets for description of propagating cracks in finite elements. *International Journal for Numerical Methods in Engineering*.

Ventura, G., Gracie, R., & Belytschko, T. (2009). Fast integration and weight function blending in the extended finite element method. *International journal for numerical methods in engineering*.