Improving Robustness of Cyclostationary Detectors to Cyclic Frequency Mismatch Using Slepian Basis

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***Abstract*—Spectrum Sensing (SS) is one of the fundamental mechanisms required by a Cognitive Radio (CR). Among several SS techniques, cyclostationary feature detection is considered as an important technique due to its robustness against noise variance uncertainty and its capability to distinguish among different systems on the basis of their cyclostationary features. However, one of the main limitations of this detector in practical scenarios is its performance degradation in the presence of cyclic frequency mismatch, which mainly arises due to the lack of knowledge about the transmitter clock/oscillator errors at the detector. In this context, this paper proposes a novel solution to address the cyclic frequency mismatch problem utilizing the Slepian basis expansion instead of the widely used Fourier basis expansion. It is shown that the proposed approach captures the deviation in the cyclic frequency caused by the aforementioned imperfections and hence provides a significant improvement in the sensing performance in the presence of cyclic frequency mismatch.**

I. INTRODUCTION

Spectrum scarcity has become one of the important challenges faced by today's wireless operators to provide high data rate services to a large number of users. To address this problem, the current research trend is in the direction of finding suitable techniques/architectures for possible future coexistence of licensed (primary) and unlicensed (secondary) wireless systems over the same radio spectrum. In this context, Cognitive Radio (CR) has been considered as a potential candidate to address this problem in the future generation of wireless communications [1]. The main functions of a CR are to be aware of its surrounding radio environment, i.e., spectrum awareness, and to utilize the available spectral opportunities effectively, i.e., spectrum exploitation [2].

Spectrum Sensing (SS) is one of the spectrum awareness mechanisms required by a CR in order to acquire the spectrum occupancy information of the primary systems. Several SS techniques such as Energy Detection (ED), matched filter based detection, autocorrelation based detection, cyclostationary feature detection, and eigenvalue based detection have been studied in the literature [2]–[4]. These methods have their own advantages and disadvantages. Among these techniques, this paper focuses on the cyclostationary feature detection, which has received important attention in the literature due to its robustness against noise variance uncertainty and its capability to distinguish among systems having distinct cyclostationary features.

The cyclostationary detection method basically exploits the cyclostationary features of various parameters such as modulation type, symbol duration, and carrier frequency at different cyclic frequencies [5], [6]. In this method, signal detection is carried out by verifying whether a particular cyclic feature is present or not at certain cyclic frequencies. The main drawback of this approach is that it usually requires the knowledge of the signal's carrier frequency and the symbol rate. However, in practice, the cyclic frequency mismatch problem exists due to the clock/oscillator error or other errors, and the detector may not exactly know the cyclic frequency of the primary signal with the cyclostationary feature. In this context, authors in [6] have shown that even a very small mismatch error can result in the significant performance degradation of the cyclostationary detector. However, contributions towards addressing this problem are quite limited [7], [8]. Therefore, investigating suitable techniques to address the aforementioned mismatch problem is a valid and interesting research problem.

In contrast to the Fourier basis based approach considered in the literature [7], [8], this paper proposes to employ the Discrete Prolate Spheroidal (DPS) or Slepian basis based approach in order to combat the aforementioned mismatch affect. Due to the peculiar feature of this basis over the conventional Fourier Basis that it represents a set of orthogonal sequences which is exactly bandlimited and can simultaneously possess a high time concentration [9], it has been used for several applications such as adaptive beamforming [10], multitaper sensing [11], [12], and time-variant channel estimation [9]. To the best of authors' knowledge, this is the first time in the literature we exploit this approach in order to alleviate the cyclic frequency mismatch problem. In this paper, first, we discuss the cyclic frequency problem in cyclostationary detectors and then present the main features of the Slepian basis. Subsequently, we propose a novel Slepian basis based approach towards addressing the considered mismatch problem. Finally, we evaluate the performance of the proposed approach with the help of numerical results and validate its superiority over the conventional Fourier approach [6] and the block-based Fourier approach proposed in [8].

The remainder of this paper is organized as follows: Section II presents the basic principles of the cyclostationary detectors and describes the cyclic frequency mismatch problem. Section III provides an overview of the Slepian basis along with the mathematical details. Subsequently, Section IV proposes a novel approach to address the cyclic frequency mismatch problem while Section V evaluates the performance of the proposed approach with the help of numerical results. Finally, Section VI concludes the paper.

II. Cyclostationary Detector

Cyclostationary processes can be defined as the random processes for which statistical properties such as the mean and autocorrelation vary periodically with time [13]. Practical communication signals may possess special features such as double sideness, and keying rate in modulated signals [6], cyclostationarity caused by modulation and coding, Cyclic Prefix (CP) in an OFDM signal, etc. In general, a particular cyclostationarity feature can be extracted by utilizing either the Cyclic Autocorrelation (CAC) or the Spectral Correlation Density (SCD) function [13], [14].

A. Basic Principle

A stochastic process is said to be wide-sense cyclostationary if the mean μ_x, and the autocorrelation function, R_x, of a signal $x(t)$ satisfy the following conditions for all integer k: $\mu_x(t+kT) = \mu_x(t)$, and $R_x(t_1+kT, t_2+kT) = R_x(t_1, t_2)$. For a fixed period T, R_x can be expressed as a Fourier series in the following way [13], [14]

$$R_x\left(t+\frac{\tau}{2}, t-\frac{\tau}{2}\right) = \sum_{\alpha} R_x^{\alpha}(\tau)e^{j2\pi\alpha t}, \tag{1}$$

where the Fourier coefficients R_x^{α} are given by

$$R_x^{\alpha}(\tau) = \lim_{Z\to\infty}\frac{1}{Z}\int_{-Z/2}^{Z/2} x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)e^{-j2\pi\alpha t}dt. \tag{2}$$

The above function $R_x^{\alpha}(\tau)$ is called a CAC function and α is called the cyclic frequency parameter. The summation in (1) is taken over all integer multiples of $1/T$ i.e., $\alpha = k/T$ for all $k \in Z$. The value $1/T$ is referred to as the fundamental frequency. A process $x(t)$ is said to exhibit cyclostationarity if there exists a parameter α for which the Fourier coefficient given by (2) is non-zero.

One of the fundamental concepts behind cyclostationary analysis is that certain spectral components of cyclostationary signals are correlated, which is usually measured by the SCD function and is defined as

$$S_x^{\alpha}(f) = \int_{-\infty}^{\infty} R_x^{\alpha}(\tau)e^{-j2\pi\tau}d\tau. \tag{3}$$

Thus, the SCD is the Fourier transform of the CAC.

Let H_0 denote the noise only hypothesis and H_1 denote the signal plus noise hypothesis. Then the Primary User (PU) signal detection problem can be expressed as a binary hypothesis testing problem in the following way

$$\begin{aligned} y(t) &= z(t), && H_0 \\ y(t) &= hx(t) + z(t), && H_1 \end{aligned} \tag{4}$$

where $x(t)$ is the signal transmitted by the PU, $z(t)$ is the Additive White Gaussian Noise (AWGN), and h denotes the channel response.

We assume that $x(t)$ contains cyclostationary features, i.e., there exists at least one non-zero α such that $R_x^{\alpha}(\tau) \neq 0$ for some τ, while the noise $n(t)$ is a purely stationary process, i.e., for any non-zero α, $R_z^{\alpha}(\tau) = 0$, $\forall\tau$. Let α_0 denote the non-zero cyclic frequency such that $R_x^{\alpha_0} \neq 0$ for some τ. Assuming that signal and the noise are mutually independent, the problem (4) in the form of the CAC function can be written as

$$\begin{aligned} R_y^{\alpha_0}(\tau) &= 0, && H_0 \\ R_y^{\alpha_0}(\tau) &= R_x^{\alpha_0}(\tau) \neq 0, \text{for some } \tau, && H_1. \end{aligned} \tag{5}$$

Let T_s and N denote the sampling duration and the number of samples respectively, then the discrete version of the CAC (2) of the received signal $y(t)$ can be written as

$$R_y^{\alpha}(kT_s) = \frac{1}{N}\sum_{n=0}^{N-1} y((n+k)T_s)y^*(nT_s)e^{-j2\pi\alpha nT_s}, \tag{6}$$

where the lag $k = 0, 1, \ldots, M-1$ with $M \ll N$. The test statistic for the decision process is given by [5]

$$D = \sum_{j=0}^{M-1} |R_y^{\alpha_0}(kT_s)|^2. \tag{7}$$

Subsequently, the decision about the presence or absence of the PU signal can be taken by comparing the test statistic D with a decision threshold, which is usually determined based on the noise power considering a target probability of false alarm.

B. Cyclic Frequency Mismatch

One major constraint on the sensing performance of a cyclostationary detector comes due to the limitations on the number of samples N that can be acquired in practice. As N increases, sensing performance increases due to the larger relative difference between the test statistics under the H_1 and H_0 hypotheses, i.e., as $N \to \infty$, the perfect sensing performance is achieved. Besides this limitation, another major constraint specific to a cyclostationary detector is that this detector does not know the cyclic frequency α_0 accurately, and the cyclic frequency $\hat{\alpha}$, at which CAC/SCD is calculated, deviates from the actual α_0. This mismatch is herein referred as cyclic frequency mismatch, denoted by $\triangle\alpha$, and it mainly occurs due to the lack of knowledge of transmitter clock and oscillator errors at the detector. In [6], authors have investigated the effect of this mismatch on the performance of cyclostationary detectors and have shown that performance of the cyclostationary detector is highly susceptible to this mismatch.

Following the analysis in [6] for the case of a single carrier signal $x(t) = \cos(2\pi f_0 t)$, the ratio of test statistics under the H_1 and H_0 hypotheses can approximated as

$$R_D \approx 1 + \frac{M|h|^4}{16(M+1)\sigma_z^4 N}\left(\frac{\sin(\pi\triangle\alpha NT_s)}{\sin(\pi\triangle\alpha T_s)}\right)^2, \tag{8}$$

where $\triangle\alpha = 0$ for no cyclic frequency mismatch. From the signal detection theory, the above ratio R_D determines the sensing performance, i.e., the larger the ratio, the better the performance. From (8), $R_D|\triangle\alpha = 0 > R_D|\triangle\alpha \neq 0$. Thus, it is evident that the mismatch in the value of α degrades the performance of a cyclostationary detector. As stated earlier, one option to enhance the sensing performance is to increase the value of N, i.e., sensing time. However, this is true for the cyclostationary detector only when $\triangle\alpha = 0$ since $N \rightarrow \infty, R_D = \infty$. For the case with cyclic frequency mismatch, this may be untrue since $N \rightarrow \infty, R_D = 1$ for $\triangle\alpha \neq 0$. Therefore, the sensing performance of a cyclostationary detector in the presence of cyclic frequency mismatch does not improve or further degrades even if we increase the sensing time. From practical perspectives, this is the serious limitation of the cyclostationary detection and this limitation motivates us to investigate a robust approach in this paper.

III. Slepian Basis Expansion

As noted in Section II-A, SCD is the Fourier transform of the CAC and hence uses the Fourier Basis Expansion Model (BEM). However, the Fourier BEM has the following drawbacks [15]: (i) the rectangular windowing associated with the Discrete Fourier Transform (DFT) introduces spectral leakage, i.e., the energy from low frequency Fourier coefficients leaks to the full frequency range; (ii) when the DFT is truncated at the Doppler bandwidth, the Gibbs effect together with spectral leakage leads to significant phase and amplitude errors at the beginning and the end of the data block. To overcome these drawbacks, authors in [9] have exploited the features of the Slepian BEM in order to estimate a time-variant wireless channel. In this paper, we utilize this basis in order to address the problem of cyclic frequency mismatch in a cyclostationary detector. In the following, we briefly describe the main features of the Slepian BEM.

The Slepian BEM represents bandlimited sequences with the minimum number of basis functions avoiding the deficiencies of the Fourier BEM. Slepian in [16] demonstrated that time-limited parts of bandlimited sequences span a low-dimensional subspace. The orthogonal basis is spanned by the so-called Discrete Prolate Spheroidal (DPS) sequences. In other words, DPS sequences are a set of orthogonal sequences that is exactly bandlimited, let's say within the frequency range $[f_{\min}, f_{\max}]$ and simultaneously possess a high (but not complete) time concentration in a certain interval with the length N. The sequences $u_i[n]$ with $i \in \{0, \ldots, N-1\}$, $n \in \{-\infty, \ldots, \infty\}$, which maximize the energy concentration in an interval with the length N are the DPS sequences [16]. For example, the DPS sequence $u_0[n]$ is the unique sequence that is band-limited and most time-concentrated in a given interval with length N, $u_1[n]$ is the next sequence having the maximum energy concentration among the DPS sequences orthogonal to $u_0[n]$, and so on.

The DPS sequences have a double orthogonality property, i.e., they are orthogonal over the finite set $n \in \{0, \ldots, N-1\}$ and the infinite set $n \in \{-\infty, \ldots, \infty\}$, simultaneously. This important property helps to overcome the drawbacks of windowing in the Fourier BEM for different applications such as channel estimation [9]. The double orthogonality property of the DPS sequences can be specifically written as

$$\sum_{n=0}^{N-1} u_i[n]u_j[n] = \lambda_i \sum_{n=-\infty}^{\infty} u_i[n]u_j[n] = \delta_{ij}, \qquad (9)$$

where $i, j \in \{0, \ldots, N-1\}$.

In this paper, Slepian sequences are considered as index limited DPS sequences as in [9], [15]. A vector $\mathbf{u}_i \in \mathcal{R}^{N\times 1}$, obtained by index limiting the DPS sequences $u_i(n)$ to the range $n \in [0, N-1]$, is an eigenvector of the matrix $\mathbf{C} \in \mathcal{R}^{N\times N}$ satisfying following condition

$$\mathbf{C}\mathbf{u}_i = \lambda_i \mathbf{u}_i, \qquad (10)$$

where the matrix $\mathbf{C}$ is given by

$$[\mathbf{C}]_{i,k} = \frac{\sin[2\pi f_{\max}(i-k)]}{\pi(i-k)}, \qquad (11)$$

with $i, k \in \{0, \ldots, N-1\}$. A sequence $g[n]$ can be expanded in terms of the Slepian sequences $u_i[n]$ using the Slepian BEM as

$$g[n] \approx \hat{g}[n] = \sum_{i=0}^{L-1} u_i[n]\lambda_i, \qquad (12)$$

where $n \in \{0, \ldots, N-1\}$, L denotes the dimension of the basis expansion and satisfies the following condition: $L' \leq L \leq N$, where the lower limit L' is the signal space dimension of the time-limited snapshots of a bandlimited signal and is given by; $L' = \lceil 2 f_{\max} N \rceil + 1$.

IV. Proposed Approach

The serious limitation of the cyclostationary detector with respect to the cyclic frequency mismatch comes from the incapability of the Fourier basis-based approach to capture the deviation in the cyclic frequency caused by clock and oscillator errors. Similar to the noise uncertainty problem in energy detection, the decision in the presence of this mismatch becomes confusing even if $N \rightarrow \infty$. Let $\triangle\alpha$ denote the mismatch in the cyclic frequency, i.e., $\hat{\alpha_0} = \alpha_0 + \triangle\alpha$. To mitigate the effect of this mismatch, one should be able to track $\triangle\alpha$ in $f_{\min} \leq \triangle\alpha \leq f_{\max}$, however, the Fourier approach can represent only single information at a particular value of α as noted from (1). Thus the Fourier basis can not capture the cyclic frequency mismatch in the traditional approaches.

As mentioned in Section III, Slepian sequences are exactly bandlimited within the frequency range $[f_{\min}, f_{\max}]$ and have a double orthogonality property. These features motivate us to examine the possibility of using Slepian sequences in order to capture the cyclic frequency mismatch. In this context, the proposed idea is to replace the Fourier coefficients R_x^α in (1) by the Slepian sequences $\mathbf{u}_i$ from (10). In our proposed approach, the Slepian BEM with an appropriately chosen set of basis functions represents the variation of the cyclic frequency within the considered band limits.

In (6), the term $e^{-j2\pi\alpha n T_s}$ represents the Fourier basis. To incorporate the Slepian basis into our analysis, we replace this term by $\mathbf{u}_i$ obtained from (10). Then the SCD of a single carrier signal $x(t) = \cos(2\pi f_0 t)$ with the Slepian basis can be written as

$$\begin{aligned} R_x^\alpha(kT_s) &= \frac{1}{N}\sum_{n=0}^{N-1} \cos((n+k)T_s)\cos(nT_s)\mathbf{u}_i, \\ &= \frac{1}{4N}\sum_{n=0}^{N-1} [e^{j2\pi f_0(k+2n)T_s} + e^{-j2\pi f_0(k+2n)T_s} \\ &\qquad + 2\cos(2\pi f_0 k T_s)]\mathbf{u}_i \end{aligned} \qquad (13)$$

The Slepian sequences $\mathbf{u}_i$ are usually generated based on the sequence length N and the value of time half bandwidth product, let us denote by β, given by; $\beta = \frac{NB}{2}$, with $B = f_{\max} - f_{\min}$ being the effective bandwidth of the sequence. The parameters N and B determine how many Slepian sequences will have energy concentration ratios near to 1, and there are usually $(BN-1)$ Slepian sequences having energy concentration ratios approximately near to 1 [17].

From the above description, it can be deduced that the design of the Slepian sequence depends only on the values of β and N [9], and not on the value of the cyclic frequency α. Outside the range $|f| > f_{\max}$, the spectrum of this expansion is zero. Therefore, the SCD function in (13) is completely invariant to the cyclic frequency mismatch within the range $[f_{\min}, f_{\max}]$. For optimal performance using the Slepian basis expansion, the power spectral density must be zero for the range $f_{\max} \leq |f| < \frac{1}{2}$ [9].

For our analysis in this paper, we set the value of β in the following two ways

- Unoptimized approach: In this approach, the value of β is set to the minimum value 0.5 which is equivalent to the time-bandwidth product of 1 .
- Optimized approach: In this approach, the value of β is adapted based on the value of the mismatch. The optimized value of β is selected in the following way

$$\beta = \max\left(\frac{\triangle\alpha N}{4}, 0.5\right), \tag{14}$$

where the factor $\frac{N}{4}$ denotes the scaling factor used to relate β with $\triangle\alpha$.

V. Numerical Results

In this section, we evaluate the performance of the proposed Slepian-based approach and compare its performance with those of the conventional Fourier-based approach [6] and block-based Fourier approach [8]. For this purpose, we consider a test signal $x(t) = \cos(2\pi f_c t)$ and in general, $x(t)$ contains a non-zero CAC value at the cyclic frequency $\pm 2f_c$ [6]. The sampling frequency is set to $f_s = 8f_c$ and $f_c = 10^6$ Hz. Further, we define the Signal to Noise Ratio (SNR) of the transmitted PU signal as $\frac{E[x(t)]^2}{\sigma^2}$ and set this value to -15 dB. We carry out the performance evaluation of the cyclostarionary detector in terms of Receiver Operating Characteristics (ROCs), i.e., probability of detection (P_d) versus probability of false alarm (P_f).

As mentioned in Section IV, we utilize unoptimized and optimized approaches in order to validate the performance of the proposed Slepian-based approach. Figure 1 depicts ROC curves for the Slepian sequence and Fourier based approaches when $\triangle\alpha = 0$, i.e., no cyclic frequency mismatch considering the number of samples $N = 2^{14}$ and $N = 2^{15}$. For generating Slepian sequences in this case, we utilize the unoptimized β, i.e., $\beta = 0.5$. From the figure, we can deduce that Fourier approach achieves better performance than that of the Slepian sequence based approach in the absence of cyclic frequency mismatch. Furthermore, it can be observed that sensing performance for both approaches increases when the value of N is increased as expected.

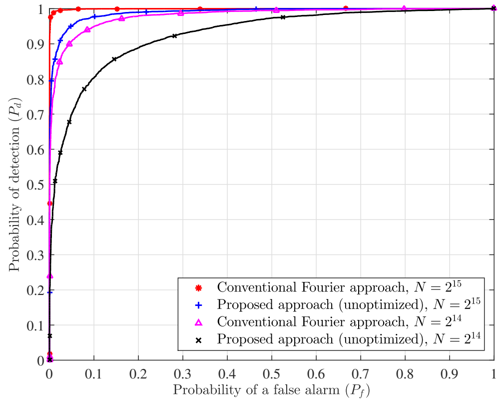

Fig. 1. Comparison of the proposed (unoptimized) approach with Fourier approach when there is no cyclic frequency mismatch ($\triangle\alpha = 0$, SNR $= -15$ dB, $\beta = 0.5$)

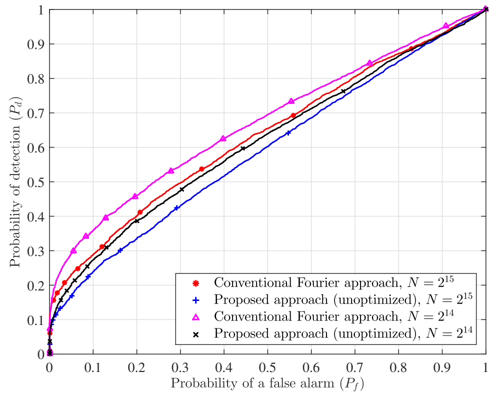

Fig. 2. Comparison of the proposed (unoptimized) approach with Fourier approach when there is cyclic frequency mismatch ($\triangle\alpha = 5 \times 10^{-4} f_c$, SNR $= -15$ dB, $\beta = 0.5$)

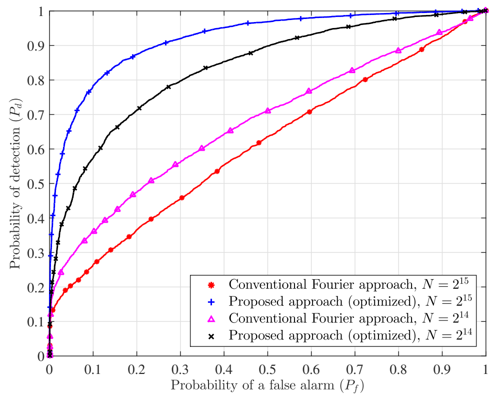

Fig. 3. Comparison of the proposed (optimized) approach with Fourier approach when there is cyclic frequency mismatch ($\triangle\alpha = 5 \times 10^{-4} f_c$, SNR $= -15$ dB)

To analyze the effect of $\triangle\alpha$, we plot ROC curves in Fig. 2 considering the cyclic frequency mismatch of $\triangle\alpha = 5 \times 10^{-4} f_c$ (i.e., 500 ppm). For this result, we use the unoptimized value of β, i.e., $\beta = 0.5$ as in Fig. 1. While comparing this result with the result in Fig. 1, we can observe the significant degradation on the performance of both conventional Fourier and the proposed approach with the unoptimized β in the presence of cyclic frequency mismatch. Furthermore, increasing

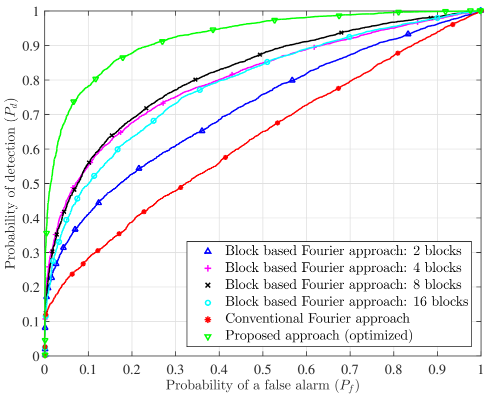

Fig. 4. Comparison of the proposed (optimized) approach with block-based Fourier approach considering different number of blocks ($\triangle\alpha = 5\times10^{-4}f_c$, SNR $= -15$ dB, $N = 2^{15}$)

the value of N further degrades the sensing performances of both Fourier and the proposed approach with unoptimized β.

In Fig. 3, we examine the performance of the proposed algorithm when β is optimized appropriately based on (14) with $N = 2^{14}$ and $N = 2^{15}$ considering the cyclic frequency mismatch of $\triangle\alpha = 5 \times 10^{-4}f_c$. From this figure, it can be noted that the proposed approach with the optimized β achieves better performance than that of the conventional Fourier and unoptimized β scenarios. Furthermore, with the optimized β, the sensing performance for a fixed P_f increases with the increase in the value of N.

The superiority of the Fourier approach over the proposed approach in the absence of cyclic frequency mismatch in Fig. 1 comes from the fact that Fourier basis provides the best frequency resolution. However, it does not provide better resolution in the time domain and its performance is severely affected when $\triangle\alpha \neq 0$ as noted in Fig. 3. More importantly, due to the capability of providing better resolution in both time and frequency domains, the proposed Slepian approach with the optimized value of β provides better performance in the presence of cyclic frequency mismatch.

Next, we provide the comparison of our approach with the block-based Fourier approach [8] in Fig. 4. For implementing the block-based detector, we divide N number of samples into different blocks and then apply the decision process in each block. Subsequently, the final values of P_d and P_f are obtained by considering the maximum and the minimum probabilities over the considered blocks, respectively. From Fig. 4, it can be noted that the block-based Fourier approach performs better than the conventional Fourier approach as also observed in [8]. However, the block-based approach of [8] does not provide performance improvement after 8 blocks, which may be the optimal block number in the considered scenario (assuming the block numbers as the powers of 2). On the other hand, the proposed Slepian based approach performs much better that the best performance of the block-based Fourier based approach in the presence of cyclic frequency mismatch. This is due to the reason that the divide and conquer approach used in block-based detector has low accuracy, i.e., low resolution while the proposed approach provides better time-frequency resolution.

VI. Conclusions

This paper has proposed a novel solution to mitigate the effect of cyclic frequency mismatch in cyclostationary detectors utilizing the Slepian basis expansion in contrast to the widely used Fourier basis expansion. Via numerical studies, it can be concluded that the conventional Fourier approach is the best in the absence of cyclic frequency mismatch while the proposed Slepian based approach provides significantly better performance than the conventional Fourier basis-based and the block-based Fourier approaches in the presence of cyclic frequency mismatch. In our future work, we plan to apply the proposed approach to digitally modulated signals and to extend this work to wideband scenarios using the compressive sensing approach based on the Slepian basis.

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References

[1] A. Goldsmith *et al.*, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894 –914, May 2009.

[2] S. K. Sharma *et al.*, "Cognitive radio techniques under practical imperfections: A survey", *IEEE Commun. Surveys and Tutorials*, July 2015.

[3] T. E. Bogale and L. Vandendorpe, "Multi-cycle cyclostationary based spectrum sensing algorithm for OFDM signals with noise uncertainty in cognitive radio networks," in *IEEE Military Commun. Conf.*, Oct. 2012, pp. 1–6.

[4] S. K. Sharma, S. Chatzinotas, and B. Ottersten, "Eigenvalue based sensing and SNR estimation for cognitive radio in presence of noise correlation," *IEEE Trans. Veh. Technol.*, vol. 62, no. 8, pp. 3671-3684, Sept. 2013.

[5] A. V. Dandawate and G. B. Giannakis, "Statistical tests for presence of cyclostationarity," *IEEE Trans. Signal Process.*, vol. 42, no. 9, pp. 2355–2369, Sep. 1994.

[6] Y. Zeng and Y.-C. Liang, "Robustness of the cyclostationary detection to cyclic frequency mismatch," in *IEEE Int. Symp. PIMRC*, Sept. 2010, pp. 2704–2709.

[7] E. Rebeiz, P. Urriza, and D. Cabric, "Optimizing wideband cyclostationary spectrum sensing under receiver impairments," *IEEE Trans. Signal Process.*, vol. 61, no. 15, pp. 3931–3943, Aug. 2013.

[8] S. Y. Kumar and M. Z. A. Khan, "Robust, block based cyclostationary detection," in *Int. Conf. Telecommun.*, May 2013, pp. 1–5.

[9] T. Zemen and C. F. Mecklenbrauker, "Time-variant channel estimation using discrete prolate spheroidal sequences," *IEEE Trans. Signal Process.*, vol. 53, no. 9, pp. 3597–3607, Sept. 2005.

[10] T. Trump, "A robust adaptive sensor array with Slepian sequences," in *IEEE SAM Workshop*, June 2012, pp. 121–124.

[11] S. Kyperountas *et al.*, "A multitaper hardware core for spectrum sensing," in *IEEE Int. Symp. DySPAN*, Oct. 2012, pp. 42–46.

[12] Gao Hu *et al.*, "An improved multitaper method for spectrum sensing in cognitive radio networks," in *IEEE Int. Conf. Computer Science and Info. Technol.*, July 2010, vol. 2, pp. 393–396.

[13] W. A. Gardner, W. Brown, and C.-K. Chen, "Spectral correlation of modulated signals: Part II–digital modulation," *IEEE Trans. Commun.*, vol. 35, no. 6, pp. 595–601, June 1987.

[14] W. A. Gardner, "Exploitation of spectral redundancy in cyclostationary signals," *IEEE Signal Process. Mag.*, vol. 8, pp. 14–36, Apr. 1991.

[15] T. Zemen and C. F. Mecklenbrauker, "Time-variant channel equalization via discrete prolate spheroidal sequences,"in *Asilomar Conf. on Signals, Systems and Computers*, Nov. 2003, pp. 1288–1292.

[16] D. Slepian, "Prolate spheroidal wave functions, Fourier analysis, and uncertainty-V: The discrete case," *The Bell System Technical J.*, vol. 57, no. 5, pp. 1371–1430, May 1978.

[17] D. B. Percival and A. T. Walden, *Spectral Analysis for Physical Applications*, Cambridge University Press, UK, 1993.