

Internal Report

Bilateral comparison
between
the FG5X-216 and the FG5-242



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1. Introduction

From June 25th to June 29th 2014, a comparison between the absolute gravimeters of the University of Luxembourg and the Federal Office of Metrology and Surveying (BEV) was hosted in the Underground Laboratory for Geodynamics in Walferdange (WULG). Both absolute gravimeters have been manufactured by Microg-LaCoste Inc. and are working on the same principle [1] even if their construction concepts differ from each other. The aim of the project was to ensure that there is a good agreement between these two devices and to link the result to the CCM.G-K2 Key Comparison held in the WULG in November 2013. During the Key Comparison, only one site could be measured by the FG5-242. In the meantime, this gravimeter went through a major maintenance. It was important to re-estimate the degree of equivalence of the FG5-242.

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2. Instrument description

The instruments used in the context of this project are ballistic free fall absolute gravimeters manufactured by Microg-LaCoste Inc. The absolute gravimeter FG5-216 of University of Luxembourg is in operation since 2001 but was upgraded to the FG5X-216 in 2011. The FG-242 is operating since 2010 and was serviced a few month before the comparison.



FG5X-216, UL



FG5-242, BEV

Figure 2.1. Absolute gravimeters used during this comparison.

3. Test procedure

The test procedure applied during this comparison is quiet simple. From June 25th 2014 to June 30th 2014, each gravimeter was placed alternatively on the reference stations B1 and B3 and C5 of the WULG. For each station, the absolute value of gravity was measured by each gravimeter for at least 12 hours. Additionally the FG5-242 measured at pillar A3. This latter measurement was not used for the comparison as the FG5X-216 did not occupy that pillar.

4. Measurement results

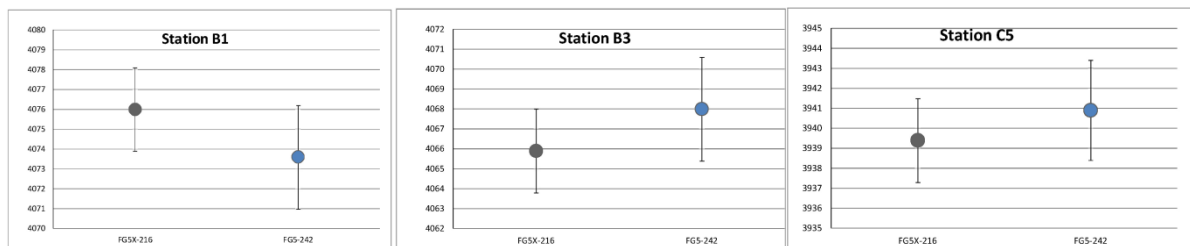
The results obtained during the comparison at the stations B1, B3 and C5 by the two gravimeters are summarized in Table 4.1. The values are given in μGal ($\mu\text{Gal} = 10^{-8} \text{ ms}^{-2}$).

Table 4.1. Gravity value measured by the two gravimeters at stations B1, B3, C5 and A3 ($g_{\text{offset}} = 980960000 \mu\text{Gal}$)

| Stations | FG5X-216 (UL) | | FG5-242 (BEV) | |
|-----------|--|----------------------------|--|----------------------------|
| | $g - g_{\text{offset}}$ (μGal) | u (μGal) | $g - g_{\text{offset}}$ (μGal) | u (μGal) |
| B1 | 4076.0 | 2.1 | 4073.6 | 2.6 |
| B3 | 4065.9 | 2.1 | 4068.0 | 2.6 |
| C5 | 3939.4 | 2.1 | 3940.9 | 2.5 |
| A3 | | | 4206,5 | 2.6 |

The uncertainty given in the Table 4.1 corresponds to the square root of the quadratic sum of the typical uncertainty of an FG5, and the site dependent uncertainty [3].

The gravity values measured by each gravimeter at each station are graphically represented in the Figure 4.1.



Figures 4.1. Gravity value measured by the two gravimeters at the stations B1, B3 and C5.

5. Evaluation of the measurement results

5.1 Compatibility index E_n

The compatibility index E_n being defined by,

$$E_n = \frac{x_i - x_j}{\sqrt{U^2(x_i) + U^2(x_j)}} \quad (1)$$

defines the ratio between the difference of two estimated values and the expanded uncertainty (k=2) of the difference. An E_n factor larger than one means that the difference between the values cannot be covered by the uncertainty. With a perfectly repeatable transfer standard, this implies that either at least one of the two values is corrupted, or the claimed uncertainties are too small.

For the measured values at the three stations B1, B3 and C5, the E_n factors are listed in Table 5.1.

Table 5.1. E_n factors at the gravity stations B1, B3 and C5 evaluated with the extended uncertainty.

| | B1 | | B3 | | C5 | |
|-----------------|-----------------|----------------|-----------------|----------------|-----------------|----------------|
| | FG5X-216 | FG5-242 | FG5X-216 | FG5-242 | FG5X-216 | FG5-242 |
| FG5X-216 | - | 0.36 | - | -0.31 | - | -0.23 |
| FG5-242 | -0.36 | - | 0.31 | - | 0.23 | - |

The values of the E_n factors, given in table 5.1, shows that all measures are in equivalence.

5.2 Evaluation of the reference values

As described by M. G. Cox [2], the reference values at each gravity station are determined by the weighted mean y_i :

$$y_i = \frac{x_{i1}/u^2(x_{i1}) + \dots + x_{iN}/u^2(x_{iN})}{1/u^2(x_{i1}) + \dots + 1/u^2(x_{iN})} \quad (2)$$

$$\frac{1}{u^2(y_i)} = \frac{1}{u^2(x_{i1})} + \dots + \frac{1}{u^2(x_{iN})} \quad (3)$$

where:

- y_i : weighted mean value for the gravity value at the station i
- $u(y_i)$: uncertainty of the weighted mean value for the gravity value at the station i
- x_{ij} : estimated gravity value by the gravimeter j at the station i
- u_{ij} : uncertainty of the gravimeter j at the station i

The evaluated reference values for each station are given in Table 5.2.

Table 5.2. Reference gravity value at the stations B1, B3 and C5 ($g_{\text{offset}} = 980964000 \mu\text{Gal}$).

| Station | Weighted mean | |
|-----------|---|------------------------|
| | $g_{\text{ref}} - g_{\text{offset}}$ μGal | u μGal |
| B1 | 4075.1 | 1.6 |
| B3 | 4066.7 | 1.6 |
| C4 | 3940.0 | 1.6 |

5.3 Degree of equivalence

For both gravimeters, the degree of equivalence of the measured gravity values has been evaluated.

Equivalence between institute i and the reference value

The degree of equivalence between the institute and the reference value is given by:

$$d_i = x_i - x_{\text{ref}} \quad (4)$$

$$U(d_i) = 2u(d_i) \quad u^2(d_i) = u^2(x_i) - u^2(x_{\text{ref}}) \quad (5)$$

The obtained values are given in table 5.4.

Table 5.4. Degree of equivalence between the different institutes and the reference values

| Station | UL | | BEV | |
|-------------|------------------------------|---------------------------------|------------------------------|---------------------------------|
| | d_{216} μGal | $U(d_{216})$ μGal | d_{242} μGal | $U(d_{242})$ μGal |
| B1 | 0.9 | 2.6 | -1.5 | 4.0 |
| B3 | -0.8 | 2.6 | 1.3 | 4.0 |
| C5 | -0.6 | 2.7 | 0.9 | 3.8 |
| Mean | -0.2 | 2.7 | 0.2 | 4.0 |

Equivalence between institute i and institute j

The degree of equivalence between the institutes is given by:

$$d_{ij} = x_i - x_j \quad (6)$$

$$U(d_{ij}) = 2u(d_{ij}) \quad u^2(d_{ij}) = u^2(x_i) + u^2(x_j) \quad (7)$$

The obtained values are given in table 5.5.

Table 5.6. Degree of equivalence between the gravimeter at the three stations B1, B3 and C5.

| | B1 μGal | | B3 μGal | | C5 μGal | |
|--|--|-----|--|-----|--|-----|
| D₂₄₂-d₂₁₆ | -2.4 | 6.7 | 2.1 | 6.7 | 1.5 | 6.5 |

6. Link to CCM.G-K2 Key Comparison

To link this comparison to the results of the CCM.G-K2 Key Comparison held in the WULG in November 2013 [5], the Key Comparison Reference Values (KCRVs) at the stations B1, B3 and C5 have been adjusted for the gravity change observed with the relative superconducting gravimeter OSG-CT040 between November 2013 and June 2014. An uncertainty of 1 microgal has been assigned to the observed gravity change to take the uncertainty on the SG instrumental drift estimate into account (Table 6.1).

Table 6.1. KCRVs from the Key comparison in November 2013, CRVs of the present comparison and the KCRVs corrected for the observed gravity change with the OSG-CT040.

| Station | KCRV 11/2013 | | CRV 06/2014 | | Gravity changes from the OSG- CT040 | | KCRV cor- rected for gravity change | |
|---------|-------------------------------|--------------------------|------------------------------|--------------------------|--|--------------------------|--|--------------------------|
| | G /μGal | U /μGal | g /μGal | U /μGal | g /μGal | U /μGal | g /μGal | U /μGal |
| B1 | 4076.7 | 3.4 | 4075.1 | 3.3 | 1.5 | 1 | 4078.2 | 3.5 |
| B3 | 4068.4 | 3.2 | 4066.7 | 3.3 | 1.5 | 1 | 4069.9 | 3.4 |
| C5 | 3942.5 | 3.3 | 3940.0 | 3.2 | 1.5 | 1 | 3944.0 | 3.4 |

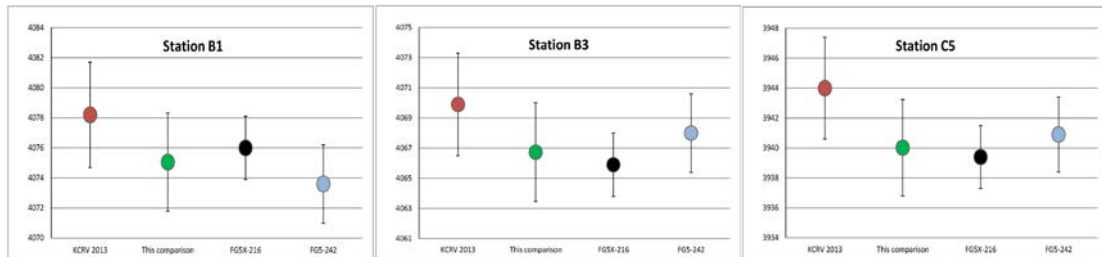


Figure 6.1. Comparison between the KCRVs in 2013 corrected for the observed gravity change observed with the superconducting gravimeter OSG-CT040, the CRVs of this comparison and the g-values measured with the FG5X-216 and FG5-242, respectively.

It demonstrates that the KCRVs and the CRVs of the present comparison and the measured g values by the FG5X-216 and FG4-242 are coherent. Based on this, we can consider that the link between these two comparisons is acceptable.

7. Conclusion

The measurements of the FG5X-216 and FG5-242 are in agreement (or in equivalence) between themselves and with the Key Comparison Reference Values of the CCM.G-K2 Key Comparison.

8. Literature

- [1] Niebauer T M, Sasagawa G S, Faller J E, Hilt R and Klopping F 1995 *Metrologia* **32** 159-180.
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- [5] Francis, O., Baumann, H., Ullrich, C., Castelein, S., Camp, M. V., Sousa, M. A. D., Melhorato, R. L., Li, C., Xu, J., Su, D., Wu, S., Hu, H., Wu, K., Li, G., Li, Z., Hsieh, W.-C., Pálinkás, P. V., Kostecký, J., Mäkinen, J., Näränen, J., Merlet, S., Santos, F. P. D., Gillot, P., Hinderer, J., Bernard, J.-D., Moigne, N. L., Fores, B., Gitlein, O., Schilling, M., Falk, R., Wilmes, H., Germak, A., Biolcati, E., Origlia, C., Iacovone, D., Baccaro, F., Mizushima, S., De Plaen, R., Klein, G., Seil, M., Radinovic, R., Sekowski, M., Dykowski, P., Choi, I.-M., Kim, M.-S., Borreguero, A., Sainz-Maza, S., Calvo, M., Engfeldt, A., Agren, J., Reudink, R., Eckl, M., Westrum, D. V., Billson, R., & Ellis, B., CCM.G-K2 key comparison, *Metrologia*, 52(1A), 07009, 2015.