# On some issues in trajectory modeling with finite mixture models

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joint work with
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- finite : sums across a finite number of groups

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# The Likelihood Function (2)

Aim of the analysis: Find $r$ groups of trajectories of a given kind (for instance polynomials of degree 4, $P(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \beta_4 t^4$.

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We try to estimate a set of parameters $\Omega = \left\{\beta_0^j, \beta_1^j, \beta_2^j, \beta_3^j, \beta_4^j, \pi_j\right\}$ which allow to maximize the probability of the measured data.

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- binary data $\Rightarrow$ Binary logit distribution
- censored data $\Rightarrow$ Censored normal distribution

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$$L = \frac{1}{\sigma} \prod_{i=1}^{N} \sum_{j=1}^{r} \pi_j \prod_{t=1}^{T} \phi \left( \frac{y_{i_t} - \beta^j t_{i_t}}{\sigma} \right). \tag{2}$$

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It is too complicated to get closed-forms equations.

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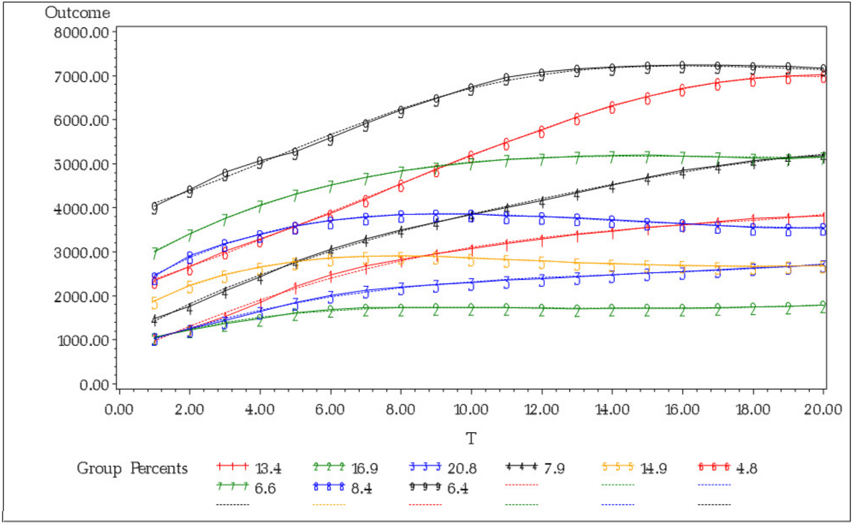

# Results for 9 groups (dataset 1)

Maximum Likelihood Estimates
Model: Censored Normal (CNORM)

| Group | Parameter | Estimate | Standard Error | T for H0: Parameter=0 | Prob > \|T\| |
|---|---|---|---|---|---|
| 1 | Intercept | 589.03067 | 18.46813 | 31.894 | 0.0000 |
| | Linear | 387.72145 | 11.31617 | 34.263 | 0.0000 |
| | Quadratic | -14.36621 | 2.12997 | -6.745 | 0.0000 |
| | Cubic | -0.01563 | 0.15109 | -0.103 | 0.9176 |
| | Quartic | 0.00856 | 0.00358 | 2.395 | 0.0166 |
| 2 | Intercept | 784.79156 | 15.75939 | 49.798 | 0.0000 |
| | Linear | 277.63602 | 9.78078 | 28.386 | 0.0000 |
| | Quadratic | -28.36731 | 1.83236 | -15.481 | 0.0000 |
| | Cubic | 1.17739 | 0.12972 | 9.076 | 0.0000 |
| | Quartic | -0.01635 | 0.00307 | -5.330 | 0.0000 |
| 3 | Intercept | 709.28728 | 15.90545 | 44.594 | 0.0000 |
| | Linear | 318.88029 | 8.97949 | 35.512 | 0.0000 |
| | Quadratic | -21.54540 | 1.69611 | -12.703 | 0.0000 |
| | Cubic | 0.62010 | 0.12002 | 5.167 | 0.0000 |
| | Quartic | -0.00440 | 0.00284 | -1.554 | 0.1203 |

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Multinomial logit model:

$$\pi_j(x_i) = \frac{e^{x_i\theta_j}}{\sum_{k=1}^{r} e^{x_i\theta_k}}, \tag{3}$$

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$$y_{i_t} = \beta_0^j + \beta_1^j Age_{i_t} + \beta_2^j Age_{i_t}^2 + \beta_3^j Age_{i_t}^3 + \beta_4^j Age_{i_t}^4 + \alpha_1^j z_1 + ... + \alpha_M^j z_M + \varepsilon_{i_t}, \quad (5)$$

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Unfortunately the influence of the covariates in this model is limited to the intercept of the trajectory.

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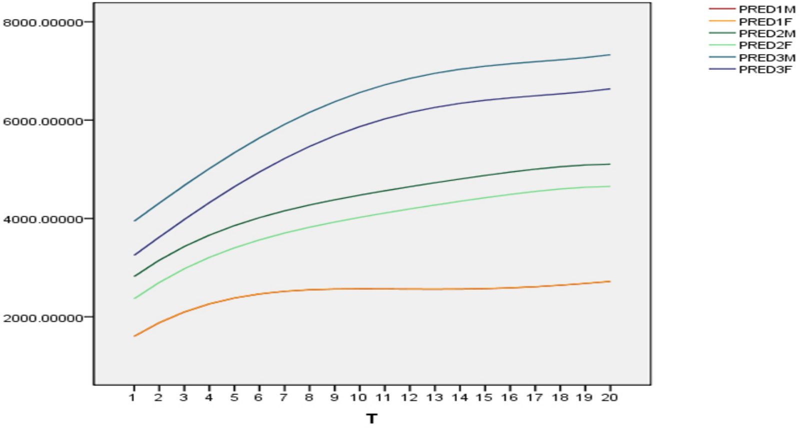

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We propose the following model:

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$$+ \left(\beta_4^j + \sum_{l=1}^{M} \alpha_{4l}^j x_l + \gamma_4^j z_{i_t}\right) Age_{i_t}^4 + \varepsilon_{i_t}^j,$$

where $\varepsilon_{i_t} \sim \mathcal{N}(0, \sigma^j)$, $\sigma^j$ being the standard deviation, constant in group $j$.

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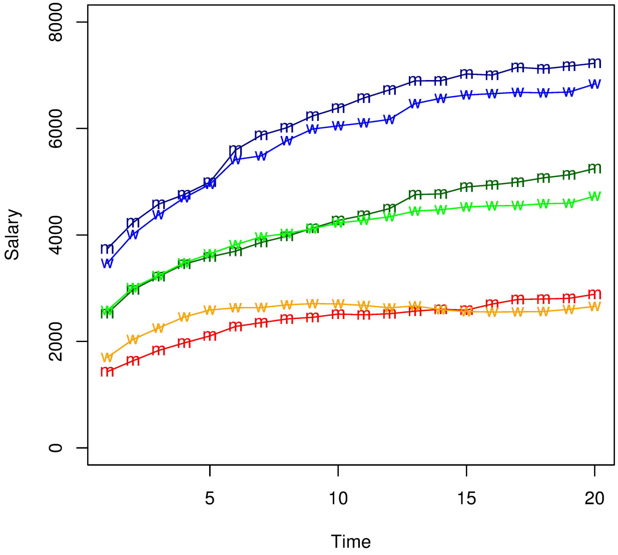
# Men versus women



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Confidence intervals of level $\alpha$ for the disturbance factor $\sigma_j$:

$$CI_\alpha(\sigma_j) = \left[\sqrt{\frac{(N-(2+M)s-1)\hat{\sigma}_j{}^2}{\chi^2_{1-\alpha/2;N-(2+M)s-1}}}; \sqrt{\frac{(N-(2+M)s-1)\hat{\sigma}_j{}^2}{\chi^2_{\alpha/2;N-(2+M)s-1}}}\right]. \tag{7}$$

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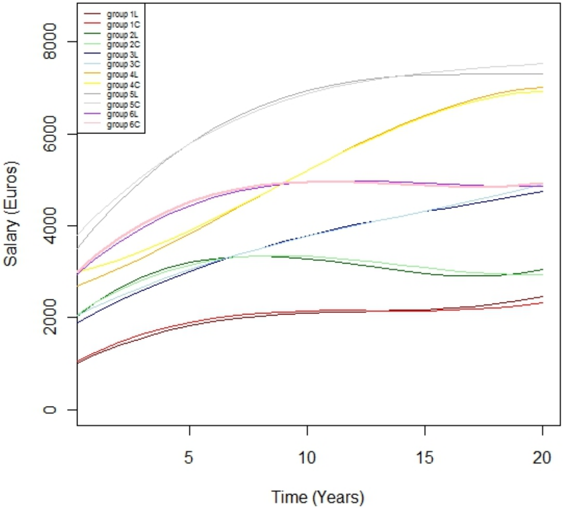
We calibrated the model

$$S_{it} = (\beta_0^j + \alpha_0^j x_i) + (\beta_1^j + \alpha_1^j x_i)t + (\beta_2^j + \alpha_2^j x_i)t^2 + (\beta_3^j + \alpha_3^j x_i)t^3, \quad (8)$$

where $S$ denotes the salary and $x$ the country of residence variable (The Luxembourg resident are coded by 1 and the commuters by 0).

# Resident versus non resident workers



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- Use the latest version of proc.traj to test if the covariates have indeed an influence on the trajectories.
- Apply proc.traj to the data without covariates do the clustering and obtain the number of groups and the constitution of the groups.
- Use your favorite regression model software to get the trajectories separately for each group.

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Because of obvious multicolinearity problems, almost none of the parameters would be significant.

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CPI and time have a correlation of 0.995.

Hence a model like

$$S_{it} = (\beta_0^j + \gamma_0^j z_{it}) + (\beta_1^j + \gamma_1^j z_{it})t + (\beta_2^j + \gamma_2^j z_{it})t^2 + (\beta_3^j + \gamma_3^j z_{it})t^3, \quad (9)$$

where $S$ denotes the salary and $z_t$ is Luxembourg's CPI in year $t$ of the study, makes no sense.

Because of obvious multicolinearity problems, almost none of the parameters would be significant.

Therefore, we simplify the model and calibrate

$$S_{it} = (\beta_0^j + \gamma_0^j z_{it}) + \gamma_1^j z_{it} t + \gamma_2^j z_{it} t^2 + \gamma_3^j z_{it} t^3. \quad (10)$$

# Attention to multicolinearity issues!

We observe a significant influence of the CPI for all six groups, which is not astonishing, since by law, the salaries are coupled with the CPI.

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For groups two, three and six all parameters are significant. The trajectories in groups one and five do not have any constant term, nor a linear dependency on the CPI but depend only on the interaction of CPI and time. Group four, finally, exhibits only linear behaviour with respect to CPI, as well as the interaction of CPI and time.

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The disturbance terms for the six groups are $\sigma_1 = 41.49$, $\sigma_2 = 33.18$, $\sigma_3 = 68.48$, $\sigma_4 = 64.84$, $\sigma_5 = 111.83$ and $\sigma_6 = 39.74$

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