How many bits should be reported in quantized cooperative spectrum sensing?

Nhan Nguyen-Thanh, Philippe Ciblat, Sina Maleki and Van-Tam Nguyen

***Abstract*—We introduce an algorithm for optimizing sensing parameters including the number of sensing samples and the number of reporting bits of a quantization-based cooperative spectrum sensing scheme in cognitive radio networks. This is obtained by maximizing the network throughput subject to a target detection probability. With Rayleigh fading and energy detector, the proposed algorithm simultaneously optimizes the number of sensing samples at a local node, the number of bits for quantizing local sensing data and the global threshold at a fusion center.**

***Index Terms*—cognitive radio, spectrum sensing, cooperative, multibit decision, quantization, sensing-throughput tradeoff.**

I. INTRODUCTION

Cognitive radio (CR), which enables secondary access to licensed bands, is a promising candidate for enhancing the utilization of the scarce spectrum resource in future communication systems. A secondary user can be permitted to use licensed spectrum, provided that it does not interfere with any primary users. This means that CR should be able to exploit spectrum holes by detecting them and using them in a cognitive manner. A widespread approach for characterizing the spectrum usage of primary systems is the so-called spectrum sensing [1], [2].

Spectrum sensing at terminals may not provide sensing results as accurate as required because of deep shadowing or fading. To deal with this problem, a fusion center (FC) collects sensing information from multiple terminals to eventually obtain a more reliable decision. This method is called cooperative spectrum sensing [3]–[5]. Main works related to cooperative sensing dealt with the design of local sensing algorithm, the combination of the local parameters at the FC (see [1] and references therein). In contrast, only a few works have been devoted to the optimization of the whole secondary system, especially by finding the trade-off between the duration of the sensing step and that of the data transmission step [6]–[8]. An efficient way to exhibit this trade-off is to maximize the throughput [6] with respect to the sensing duration with 1-bit hard decision and conventional fusion rules. However, the duration for reporting local information from each CR to the FC, which is linearly related to the quantizer resolution of the local decision, has never been optimized. The reporting step for 1-bit hard decision and even soft decision has been taken into account only through sensing performance [3], [8]–[10]. Obviously, if the reporting time is too short and so carries a degraded version of the local information (in the worst case, 1 bit), the sensing decision at the FC may not be reliable and the whole system may not perform well. In contrast, if the reporting time is too long, the time devoted to data transmission may be too short and the required data rate may not be fulfilled. As a consequence, this paper deals with the number of bits allowed for quantizing local sensing information. This means that our work has a strong connection to the problem of selecting hard decision or soft decision in [5], [8].

The trade-off between the sensing process length and the utilization channel time is investigated by formulating an optimization problem of maximizing the network throughput under the constraint of primary system protection requirement. The algorithm to find the optimal number of sensing samples and the optimal number of reporting bits is proposed.

II. SYSTEM MODEL

We consider a CR network with K users. The CR network utilizes opportunistic spectrum access for sharing spectrum bands with primary systems. Cooperative sensing is adopted to detect primary users. The cooperative sensing scheme includes two steps. The first step consists of spectrum sensing of CR users. The second step is sensing result reporting to the FC, which makes a final decision on the primary user state.

In this work, we consider the energy detection method for the first step because of its simple implementation and its robustness to unknown information of the source signal and channel fading [11], [12]. For the second step, the reporting is done through a control channel with a fixed limited bandwidth [9], [13]. Since every methods of orthogonal multiple access, e.g., Time Division Multiple Access (TDMA), Frequency Division Multiple Access, etc., offer the same spectral efficiency, they can be used equivalently. For the sake of simplicity of the presentation and without lost of generality, we consider TDMA scheme and a soft data fusion rule with multi-bit local decisions at the FC. The structure of the operation frame is illustrated in Fig. 1.

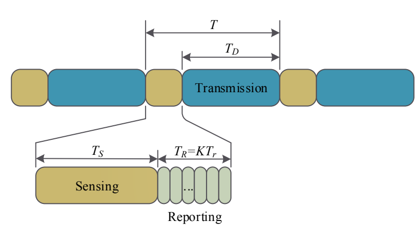

Fig. 1. Frame structure of the CR network.

The local spectrum sensing is a binary hypothesis testing problem as follows

$$y_k[n] = \begin{cases} w_k[n], & H_0 \\ h_k s[n] + w_k[n], & H_1 \end{cases} \quad \begin{matrix} n = 1, 2, ..., N \\ k = 1, 2, \cdots, K \end{matrix} \qquad (1)$$

where $y_k[n]$ is the received signal at time n at the k-th CR user, $w_k[n]$ is the added noise and is assumed to be a zero-mean i.i.d. complex-valued circularly-symmetric Gaussian process with variance $\sigma^2_{w_k}$ per complex dimension, $s[n]$ is the potential unknown deterministic signal coming from the primary user, and h_k is the block-fading channel gain between the primary user and the k-th CR user. N is the number of sensing samples, ($N = f_S T_S$, where f_S is the sampling frequency and T_S is the sensing time). H_0 and H_1 represent the hypotheses of the absence and the presence of primary signal, respectively. We consider that the channel is a slow Rayleigh flat fading with variance $\sigma^2_{h_k}$. The channel realization is generated independently frame by frame as done in [11], [12].

The test statistic of the energy detector is given by $z_k = \sum_{n=1}^{N} |y_k[n]|^2$. Given h_k, it has been shown in [14] that z_k has central and non-central chi-squared distribution under H_0 and H_1, respectively. The test statistic can be then described by

$$z_k \sim \begin{cases} \chi^2_{2N}, & H_0 \\ \chi^2_{2N}(2N\gamma_k), & H_1 \end{cases}$$

where $\gamma_k = |h_k|^2 E_s/\sigma^2_{w_k}$ is the instantaneous Signal-to-Noise Ratio (SNR) of the received signal at the k-th user with the symbol variance E_s. Given h_k, the cumulative density functions (cdf) of the test are thus computed by

$$F_{z_k|H_0}(z|H_0) = P_N(z/2) \quad (2)$$

$$F_{z_k|H_1,h_k}(z|H_1,h_k) = 1 - Q_N\left(\sqrt{2N\gamma_k}, \sqrt{z}\right) \quad (3)$$

where $Q_N(.,.)$ denotes the generalized Marcum Q-function, $P_N(b) = \gamma(N,b)/\Gamma(N)$ with the gamma function $\Gamma(.)$ and the incomplete gamma function $\gamma(.,.)$.

As h_k is a Rayleigh channel, the SNR γ_k follows an exponential probability density function (pdf) given by $f(\gamma_k) = 1/\bar{\gamma}_k \exp(-\gamma_k/\bar{\gamma}_k)$, where $\bar{\gamma}_k = \sigma^2_{h_k} E_s/\sigma^2_{w_k}$ is the average SNR received at the k-th user. Using Eq. (9) in [12] and Section 8.35 in [15], we obtain the cdf and the pdf of z_k under H_1 as follows.

$$F_{z_k|H_1}(z|H_1) = P_\nu\left(\frac{z}{2}\right) - e^{\frac{-z}{2M_k N\bar{\gamma}_k}} {M_k}^{\nu} P_\nu\left(\frac{z}{2M_k}\right) \quad (4)$$

$$f_{z_k|H_1}(z|H_1) = \frac{e^{\frac{-z}{2M_k N\bar{\gamma}_k}}}{2M_k N\bar{\gamma}_k} {M_k}^{\nu} P_\nu\left(\frac{z}{2M_k}\right) \quad (5)$$

where $M_k = 1 + 1/(N\bar{\gamma}_k)$ and $\nu = N - 1$.

The cdf of z_k under H_0 is the same as that of Eq. (2). Since it is independent of the fading, its pdf is given by

$$f_{z_k|H_0}(z|H_0) = \frac{z^{N-1} e^{-\frac{z}{2}}}{2^N \Gamma(N)}. \quad (6)$$

After the sensing period, each energy test is reported to the FC, where a squared-law combining is adopted [12], and the global test is then given by

$$Z = \sum_{k=1}^{K} z_k \underset{H_0}{\overset{H_1}{\gtrless}} \eta \quad (7)$$

where η is the decision threshold.

III. Quantized cooperative sensing

Reporting a raw z_k requires time, bandwidth and energy. It is therefore relevant to communicate with the FC through a quantized version of the test statistic, which corresponds to work with a multi-bit decision at the local nodes. The real-valued (also called raw or soft) energy z_k is replaced with its B-bit quantized version in Eq. (7). The practical test at the FC then becomes

$$Z^{(B)} = \sum_{k=1}^{K} z_k^{(B)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta^{(B)} \quad (8)$$

where $z_k^{(B)} = Q_k^{(B)}(z_k)$ is the quantized version of z_k and $Q_k^{(B)}$ denotes a B-bit quantizer associated with the k-th user. Let M the number of quantization levels, then $M = 2^B$. Let $\{t_{k,i}\}_{i=0}^{M}$ and $\{L_{k,j}\}_{j=1}^{M}$ the set of thresholds and the set of quantization levels for $Q_k^{(B)}$, respectively. As the support of the pdf of z_k is $\mathbb{R}_+$, we have $t_{k,0} = 0$, $t_{k,M} = +\infty$, and $\Re_{k,i} = [t_{k,i-1}, t_{k,i}), i = 1, ..., M$. $\Re_{k,i}$ denotes the i-th quantization region of the k-th user. The quantization level is usually the central point of the quantization region. Hence, we have

$$L_{k,i} = \frac{1}{S_{k,i}} \int_{\Re_{k,i}} z f_{z_k}(z) dz \quad (9)$$

where $S_{k,i} = \int_{\Re_{k,i}} f_{z_k}(z)dz$ and $f_{z_k} = \pi_0 f_{z_k|H_0} + (1-\pi_0) f_{z_k|H_1}$ with π_0 the probability of primary user inactivity.

The following quantizers are hereafter considered:

- *Uniform quantizer:* The quantization thresholds are given by $t_{k,i} = t_{k,i-1} + \Delta_k, i = 1, ..., M-1$, where $\Delta_k = t_{k,\max}/M$. $t_{k,\max}$ is an artificial threshold for defining a maximum support of z_k. Here, it is selected such that $\int_0^{t_{k,\max}} f_{z_k}(z)dz = 1 - 10^{-6}$.
- *Minimum mean square error (MMSE) quantizer* [16]: This quantizer aims at minimizing the quantization error. The levels and thresholds (with $t_{k,i} = (L_{k,i} + L_{k,i+1})/2$) can be found by using Lloyd-Max algorithm.
- *Maximum entropy (ME) quantizer* [17]: The quantization thresholds $t_{k,i}$ are obtained by forcing $S_{k,i} = 1/M, \forall i = 1, ..., M$.

In order to perform the quantization and the dequantization, the local user k needs its quantization thresholds, and the FC needs the pdf of z_k. If the coherence time of the statistics of z_k is large enough, the report of the pdf from the user to the FC will be rarely performed. When the report can not be implemented or when the statistics of z_k can not be archived, z_k can be considered as a uniformly distributed process, and so the uniform quantizer with $L_i = (i - 1/2)\Delta$ (where Δ is a pre-defined term independent of the user) is well adapted.

To determine the threshold $\eta^{(B)}$, the probability mass function (pmf) of Z^B under H_0 and H_1 is needed. Since the test at the FC, given by Eq. (8), is the sum of the K local independent tests, its pdf, denoted by $f_{Z^{(B)}|H_j}$, is obtained by

$$f_{Z^{(B)}|H_j} = f_{z_1^{(B)}|H_j} \star f_{z_2^{(B)}|H_j} \star \cdots \star f_{z_K^{(B)}|H_j} \quad (10)$$

where $\star$ denotes the convolution operator, and $f_{z_k^{(B)}|H_j}$ is the pmf of $z_k^{(B)}$ under H_j and is given by

$$f_{z_k^{(B)}|H_j}(\ell) = \sum_{i=1}^{M} S_{k,i|H_j} \delta(\ell - L_{k,i}) \quad (11)$$

with $S_{k,i|H_j} = \int_{\Re_{k,i}} f_{z_k|H_j}(z)dz$ and $\delta(\bullet)$ is the Dirac delta function. Substituting (11) into (10) leads to

$$f_{Z^{(B)}|H_j}(\ell) = \sum_{i_1...i_K=1}^{M} S_{1,i_1|H_j}...S_{K,i_K|H_j}\delta(\ell - L_{1,i_1}...-L_{K,i_K}).$$

So $f_{Z^{(B)}|H_j}$ is a pmf, where the q-th level is denoted L_q. Thus,

$$f_{Z^{(B)}|H_j}(\ell) = \sum_q \psi_{q|H_j}\delta(\ell - L_q) \quad (12)$$

where $\psi_{q|h_j}$ is the probability of the level L_q and is given by $\psi_{q|h_j} = \sum_{i_1...i_K \in \mathcal{L}_q} S_{1,i_1|H_j}...S_{K,i_K|H_j}$ with $\mathcal{L}_q = \{i_1...i_K | L_{1,i_1} + ... + L_{K,i_K} = L_q\}$. The algorithms for computing $\psi_{q|H_j}$ and L_q are presented in [18].

Given the pmf of $Z^{(B)}$, the false-alarm and the detection probabilities of the test can be expressed by

$$P_F(B, \eta^{(B)}) = \sum\nolimits_{q|L_q \geq \eta^{(B)}} \psi_{q|H_0}, \quad (13a)$$

$$P_D(B, \eta^{(B)}) = \sum\nolimits_{q|L_q \geq \eta^{(B)}} \psi_{q|H_1}. \quad (13b)$$

IV. Optimal quantized cooperative sensing

According to [6], the normalized throughput of a CR network is approximately given by

$$R = \frac{T - T_S - T_R}{T}\pi_0 C_0 (1 - P_F)$$

where C_0 is the data rate per channel used for secondary user when primary user is absent. As shown in Fig. 1, a time frame length T is divided into the sensing time T_S, the reporting time T_R and the data time T_D ($T_D = T - T_S - T_R$). Let f_R the bandwidth devoted to the reporting channel, then $T_R = KB/f_R$. The normalized throughput becomes

$$R(N, B, \eta) \propto \left(1 - \frac{N}{Tf_S} - \frac{KB}{Tf_R}\right)(1 - P_F). \quad (14)$$

For the CR network with K users, the throughput for the secondary user strongly depends on the cooperative sensing process, especially on the following parameters: the number of sensing samples, the number of reported bits and the optimal threshold of the global test. Therefore, optimizing these parameters to maximize the network throughput for a target detection probability $P_D^{(0)}$ is necessary. This optimization is then formulated as

$$[N_*, B_*, \eta_*] = \arg\max_{N,B,\eta} R(N, B, \eta), \text{ s.t. } P_D \geq P_D^{(0)}. \quad (15)$$

For a certain integer value of N, the number of reported bits B, which is also an integer, is necessarily less than $B_{\max}$ with $B_{\max} = \lfloor (T - N/f_S) f_R/K \rfloor$. In addition, $N < N_{\max}$ with $N_{\max} = Tf_S$. Thus, the optimal solution can be obtained by a discrete search along with both N and B. Therefore, for a given pair of $\{N, B\}$, the optimization in (15) leads to

$$\eta_*^{(N,B)} = \arg\min_{\eta^{(N,B)}} P_F(N, B, \eta^{(N,B)}) \text{ s.t. } P_D > P_D^{(0)}. \quad (16)$$

Thanks to Eq. (13), it is equivalent to

$$\eta_*^{(N,B)} = \arg\min_{\eta^{(N,B)}} \sum\nolimits_{q|L_q \geq \eta^{(N,B)}} \psi_{q|H_0} \quad (17a)$$

$$\text{s.t. } \sum\nolimits_{q|L_q \geq \eta^{(N,B)}} \psi_{q|H_1} \geq P_D^{(0)}. \quad (17b)$$

Since the sums in Eqs. (17a) and (17b) decrease with respect to $\eta^{(N,B)}$, the optimal $\eta_*^{(N,B)}$ is equal to the maximum level L_q satisfying Eq. (17b). Consequently, the algorithm for finding $\{N_*, B_*, \eta_*\}$ is given as Algorithm 1.

Algorithm 1 Find $\{N_*, B_*, \eta_*\}$

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for n = 2 to N_max do
    for b = 1 to B_max do
        Compute f_{Z^(b)|H_j} for j = 0, 1 as in Eq. (12)
        Let q_max the number of levels in f_{Z^(b)|H_j}
        q ← q_max
        repeat
            q ← q − 1
        until Σ_q^{q_max} ψ_{q|H_1} ≥ P_D^(0)
        η_*^(n,b) ← L_q, compute R(n, b, η_*^(n,b))
    end for
end for
{N_*, B_*, η_*} ← arg max_{n,b} R(n, b, η_*^(n,b))
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The proposed algorithm should run only when channel statistics (actually the average received SNRs at local users) have changed. Algorithm 1, including the computation of $f_{Z^{(B)}|H_j}$, is performed at the FC. The preliminary parameters for the computation of $f_{Z^{(B)}|H_j}$, i.e., the thresholds $\{t_{k,i}\}_{i=0}^{M}$, the levels $\{L_{k,j}\}_{j=1}^{M}$ and the mass coefficients $\{S_{k,j}\}_{j=1}^{M}$, can be either computed at local users and then sent to the FC, or directly computed at the FC after having received the average SNR from the local users. In both cases, the FC finally sends the optimized quantizer's configuration back to each local user.

Our work is valid for Rayleigh fading and energy detector. The extension for other fading channels is straightforward if $f_{z_k|H_j}$ is available in closed-form (e.g. energy detector along with a Nakagami channel [19]). When $f_{z_k|H_j}$ cannot be derived readily, the proposed algorithm can be adopted if the quantized version $f_{z_k^{(B)}|H_j}$ is achievable, e.g., based on numerical or empirical method, and stored in a lookup table.

V. Numerical results

Unless otherwise stated, the CR network has 6 nodes and the average SNR values are -20, -18, -16, -14, -12, and -10 dB, the target probability of detection $P_D^{(0)}$ is 0.9, the frame length T is 1ms, the sampling frequency f_S is 6MHz, and the reporting channel bandwidth f_R is 100kHz. The variance of the Rayleigh channel is chosen according to the SNR value.

In Fig. 2, we plot the normalized throughput versus B for different SNR configurations and $N = 500$. The normalized throughputs for all considered scenarios and quantization methods have the same shape and exhibit a maximum. When the number of reported bits is too small or too high, the throughput is low, due to the weak accuracy of the sensing or to the increase of the reporting time, respectively. We can see that the gaps between the maximum throughput points of the three quantizers are small, and the optimal numbers of reported bits for the three quantizers are close to each other.

In Fig. 3, we display the normalized throughput versus N and B, when ME quantizer method is employed. The

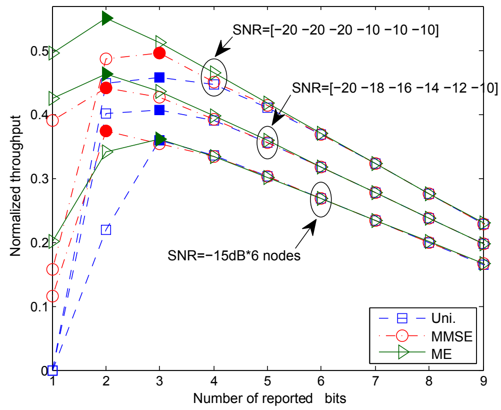

Fig. 2. Normalized throughput versus B.

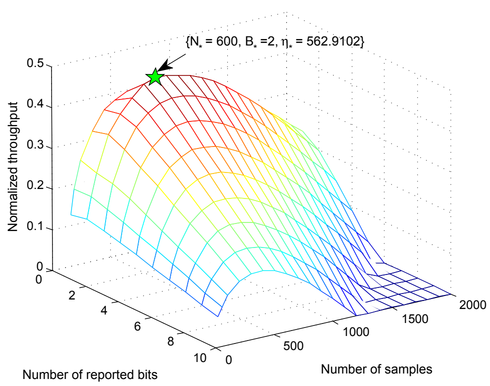

Fig. 3. Normalized throughput versus N and B with ME quantizer.

best combination is $N_* = 600$ and $B_* = 2$, which means that 10% and 2% (resp. 12%) of the frame are devoted to sensing and reporting for each node (resp. for 6 nodes), respectively. Similar optimal combination can be obtained with other quantization methods.

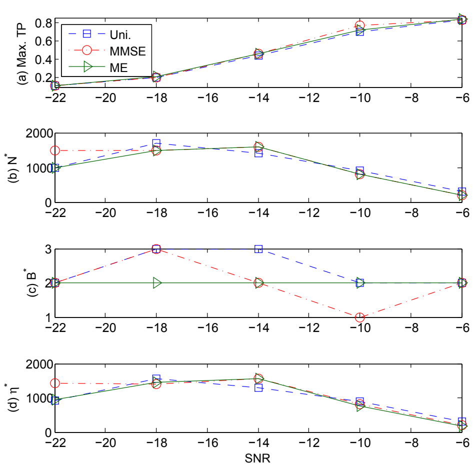

Fig. 4. Maximum normalized throughput, N_*, B_* and η_* versus SNR.

In Fig. 4, we plot (a) the maximum normalized throughput, (b) N_*, (c) B_*, and (d) η_* versus SNR (assuming 6 nodes have identical SNRs). The throughput performance increases with SNR. The sensing time and hence the optimal global threshold depend more strongly on the SNR than on the reporting time.

VI. Conclusion

We maximized the throughput subject to a target detection probability with respect to the number of sensing samples and the number of reported bits. The proposed algorithm provides the method for selecting these parameters optimally. Reporting only a few bits is in general optimal.

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