

# ROBUST PRECODING DESIGN FOR MULTIBEAM DOWNLINK SATELLITE CHANNEL WITH PHASE UNCERTAINTY

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## ABSTRACT

In this work, we study the design of a precoder on the user downlink of a multibeam satellite channel. The variations in channel due to phase noise introduced by on-board oscillators and the long round trip delay result in outdated channel information at the transmitter. The phase uncertainty is modelled and a robust design framework is formulated based on availability and power constraints. The optimization problem is cast into the convex paradigm after approximations and the benefits of the resulting precoder are highlighted.

**Index Terms**— Robust Precoding, Satellite Channel, Phase Uncertainty, Convex optimization, SDR

## 1. INTRODUCTION

Downlink precoding techniques have been widely studied in the context of multiuser terrestrial communication systems for their ability to enhance communication efficiency [1]. An objective of these techniques is to minimize the transmitted power, towards meeting certain Quality of Service (QoS) considerations under a variety of system constraints. Following terrestrial trends, satellite communications have moved from the traditional TV broadcasting to providing interactive broadband services even to urban customers (see Viasat's Exede service in the US). Such a development is triggered by the emergence of multiple spot beam satellites where the frequency reuse provides a trade-off between available bandwidth and co-channel interference (CCI). Since precoding techniques have been studied in terrestrial systems to mitigate CCI, of late, they have also attracted much research interest among academia and industry for application in multibeam satellite systems with low frequency reuse factors [2–4].

A key requirement of effective precoding is the availability of accurate channel state information at the transmitter (CSIT) [5]. This has been assumed to be the case in [2, 3]. Unlike the terrestrial counterparts, GEO satellite communication systems result in long round trip delays (RTD) between the gateway (GW) and User Terminal (UT). In particular, the two hop propagation delay in the GEO orbit is about 250 ms [4] compared to the few ms in cellular systems. Towards incorporating the use of precoding in next generation satellite systems, it becomes essential to investigate the effect of artifacts, if any, induced by RTD. Pursuing this activity, we recognize that the main component affecting the channel amplitude is the rain attenuation whose variations are slow [6]. Hence, we can assume that the amplitude of the channel is fixed during the feedback interval. On the other hand, there is a significant variation in the channel phase component arising out of time varying phase noise introduced by oscillators on-board the satellite [7]. Thus time varying nature of

the channel and a high RTD result in outdated CSIT, caused by uncertainty in the channel phase. Thus the performance of the system becomes unpredictable when the GW uses the CSIT (specifically, uncertainty in phase) at  $t_0$ , to design a precoder for a channel at  $t_1 = t_0 + 250$  ms.

The robust precoding design paradigms have been considered in literature towards mitigating the sensitivity of the precoding techniques on CSIT [8–11]. In general, there are several different strategies to obtain robustness against channel uncertainty [12]: (a) Worst-case design using a deterministic uncertainty model, where the true value is known to be within a certain interval, and optimizing the performance of the worst case situation. This is sometimes called maxim robustness [8]. (b) Expectation-based design using a stochastic uncertainty model but optimizing the average performance. (c) Probabilistic design using a stochastic uncertainty model and optimizing the performance at a certain outage level [9–11].

The purpose of this work is to design a robust precoder under the existence of the phase uncertainty due to the outdated feedback in a multibeam satellite system. We consider a precoder design aiming to minimize the transmitted power under per antenna power constraints though other system aspects can be easily incorporated. The robustness is imparted to the design by modelling the phase uncertainty as a random process and ensuring that the outage probability is maintained at desired levels. In particular, the probabilistic approach will be used for the robust design since it can guarantee a certain level of user availability ( $1 - \text{outage probability} \times 100\%$ ) which is a very important QoS measure for satellite operators. While literature focuses on additive uncertainty model for robust designs [8–13], the current study employs a multiplicative model for phase induced channel uncertainty. The use of such an uncertainty model and the ensuing analysis are novel, especially in the satellite communication literature.

The remainder of this paper is organised as follows. Section 2 discusses the time varying satellite channel and introduces a phase uncertainty model. Section 3 presents a robust precoding formulation and shows how it can be transformed to a convex optimization problem. In section 4, the satellite channel model is described and system parameters are provided. Also, numerical results are presented and discussed. Finally, concluding remarks are provided in section 5.

## 2. MULTIBEAM SATELLITES : SYSTEM MODEL AND CHANNEL PHASE UNCERTAINTY

We consider a typical Ka-band multibeam satellite system with  $K$  beams [2] employing a full frequency reuse where all the beams operate at the same frequency. We further assume a single feed per

beam scenario with  $K$  antenna feeds at the satellite used to form the  $K$  fixed beams. Further, each feed has a constraint on the maximum power. Towards focussing on the precoder design, a single GW is assumed to manage  $K$  adjacent beams and the feeder link (link between GW and the satellite) is considered ideal; such assumptions are commonplace in related literature [2, 3]. Time Division Multiple Access (TDMA) is employed on the user downlink (link between satellite and user) whereby a single user is served in a beam for every time slot. The resulting system then resembles the traditional terrestrial multiuser MISO downlink, thereby facilitating further analysis.

In the equivalent MISO system, let  $h_{i,j}(t) = |h_{i,j}|e^{j\theta_{i,j}(t)}$  denote the baseband time varying sub-channel between user  $i$  and the antenna  $j$  at time  $t$ . In Ka-band fixed satellite systems, the channel is typically modelled as being frequency flat. The amplitude of the user downlink channel is dominated by the rain attenuation and is modelled using the log normal distribution [6]. Further, the temporal variations of the amplitude are considered to be negligible over the intervals of interest and hence time dependence is omitted from  $|h_{i,j}|$ . On the other hand, each feed can be affected by a different phase noise component arising from the use of different local oscillators (LO) on-board the satellite for different feeds. Further, phase changes to the signal are also effected by the movement of the satellite within its *station keeping box*. Finally, the receiver front end at the UT contributes additional phase noise. Among these time varying phase components, the only contribution affecting the SINR is the phase noise from LOs as other components are same for all sub-channels of  $i$ th user and don't change the received SINR. The interested reader is referred to [4] for further details on the various contributors to the time varying phase. Thus, the imperfections result in independent time varying phase components that are incorporated in the channel model as  $\theta_{i,j}(t)$ .

The  $i$ th UT estimates the amplitude and the phase of each of the sub-channels at time  $t_0$  and feeds them back to the GW. For ease of notation, we let  $\boldsymbol{\theta}_i(t) = [\theta_{i,1}(t), \theta_{i,2}(t), \dots, \theta_{i,K}(t)]^T$ . As mentioned earlier, due to the long delay and time varying LO phase noise, the phase of the channel when precoding is applied at  $t_1 \approx t_0 + 250$  ms will be different than  $\boldsymbol{\theta}_i(t_0)$ . Since  $\theta_{i,k}(t_1)$  is the actual phase for the sub-channel  $h_{i,k}(t_1)$  and further using  $\boldsymbol{\theta}_i(t_1)$ , we model the temporal variations as,

$$\boldsymbol{\theta}_i(t_1) = \boldsymbol{\theta}_i(t_0) + \mathbf{e}_i, \quad (1)$$

where  $\mathbf{e}_i = [e_{i,1}, e_{i,2}, \dots, e_{i,K}]^T$  is the phase error vector with i.i.d Gaussian random entries,  $e_{k,i} \sim \mathcal{N}(0, \sigma^2)$ . For ease of notation, we define  $\mathbf{h}_i = [|h_{i,1}|e^{j\theta_{i,1}(t_1)}, |h_{i,2}|e^{j\theta_{i,2}(t_1)}, \dots, |h_{i,K}|e^{j\theta_{i,K}(t_1)}]^T$  and  $\hat{\mathbf{h}}_i$  as the corresponding channel at  $t_0$ . Under these notations, the  $K \times 1$  channel fading coefficients from all antenna feeds towards the  $i$ th UT at instance  $t_1$  are then given by

$$\mathbf{h}_i = \hat{\mathbf{h}}_i \odot \mathbf{q}_i, \quad (2)$$

where  $\mathbf{q}_i = [\exp(je_{i,1}), \exp(je_{i,2}), \dots, \exp(je_{i,K})]^T$ . In (2),  $\hat{\mathbf{h}}_i$  is a known channel vector at  $t_0$ , but  $\mathbf{q}_i$  is a random vector representing the uncertainty. We further assume that the correlation matrix of  $\mathbf{q}_i$ , denoted by  $\mathbf{C}_i$ , is known and takes the form,

$$\mathbf{C}_i = \mathbb{E}\{\mathbf{q}_i \mathbf{q}_i^\dagger\} = \mathbb{E}\{\mathbf{Q}_i\}. \quad (3)$$

The diagonal elements of  $\mathbf{C}_i$  are one and off-diagonal entries can be found as follows,

$$[\mathbf{C}_i]_{lm} = \mathbb{E}\{[\mathbf{Q}_i]_{lm}\} = \mathbb{E}\{e^{-je_{i,l}}\} \mathbb{E}\{e^{je_{i,m}}\} = e^{-\sigma^2}. \quad (4)$$

Note that we can also express  $\mathbf{q}_i$  as

$$\begin{aligned} \mathbf{q}_i &= \mathbf{C}_i^{1/2} \mathbf{v}_i, \\ \mathbf{Q}_i &= \mathbf{C}_i^{1/2} \mathbf{V}_i \mathbf{C}_i^{1/2}, \end{aligned} \quad (5)$$

where  $\mathbf{V}_i = \mathbf{v}_i \mathbf{v}_i^*$ ,  $\mathbf{C}_i^{1/2} \succcurlyeq \mathbf{0}$  is the positive semidefinite square root of  $\mathbf{C}_i$  and  $\mathbf{v}_i$  is a decorrelated vector (correlation matrix,  $\mathbb{E}\{\mathbf{v}_i \mathbf{v}_i^*\} = \mathbf{I}_i$ ).

While the current paper considers phase uncertainty in the satellite channel, the ensuing analysis can be applied to any downlink channel with phase uncertainty.

### 3. ROBUST PRECODING FORMULATION

Let  $s_i(t)$  denote the complex signal intended for user  $i$  with  $\mathbb{E}[|s_i(t)|^2] = 1$ . Prior to transmission, each  $s_i(t)$ ,  $i \in [1, K]$  is weighted at GW by the corresponding precoding vector  $\mathbf{w}_i \in \mathbb{C}^K$ , and the resulting transmitted signal is given by,

$$\mathbf{x}(t) = \sum_{i=1}^K \mathbf{w}_i s_i(t). \quad (6)$$

At time instance  $t_1$ , the signal  $\mathbf{x}(t_1)$  is acted upon by the channel vectors  $\{\mathbf{h}_i\}$  and the signal received by user  $i$  is (time index is dropped),

$$r_i = \mathbf{h}_i^\dagger \mathbf{w}_i s_i + \mathbf{h}_i^\dagger \sum_{j \neq i} \mathbf{w}_j s_j + n_i, \quad (7)$$

where  $n_i$  is the additive white Gaussian noise with zero mean and variance  $\sigma_n^2$ . This additive noise is independent and identically distributed (i.i.d.) across the users. The received SINR at  $i$ th user then takes the form,

$$\text{SINR}_i = \frac{\text{Tr}(\mathbf{R}_i \mathbf{W}_i)}{\sum_{j \neq i} \text{Tr}(\mathbf{R}_i \mathbf{W}_j) + \sigma_n^2}, \quad (8)$$

$$\mathbf{R}_i \triangleq \mathbf{h}_i \mathbf{h}_i^\dagger = \text{diag}(\hat{\mathbf{h}}_i) \mathbf{Q}_i \text{diag}(\hat{\mathbf{h}}_i^\dagger), \quad (9)$$

where  $\mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^\dagger$  and  $\mathbf{R}_i \in \mathbb{C}^{K \times K}$  is the instantaneous channel correlation matrix at  $t_1$ . In (9),  $\hat{\mathbf{h}}_i$  is the known (estimated) channel vector at  $t_0$  and  $\mathbf{Q}_i = \mathbf{q}_i \mathbf{q}_i^\dagger$  which is a random matrix.

A meaningful formulation of the downlink precoding problem leads to,

$$\begin{aligned} \mathcal{P} : & \text{minimize} && \tau \\ & \mathbf{W}_1, \dots, \mathbf{W}_K, \tau \\ \text{subject to} & && \Pr\left\{\text{SINR}_i \geq \gamma_p\right\} \geq \alpha_i, \\ & && \forall i \\ & && \left[\sum_{j=1}^K \mathbf{W}_j\right]_{i,i} \leq \tau P_i, \\ & && \mathbf{W}_i \succcurlyeq \mathbf{0}, \text{rank}(\mathbf{W}_i) = 1 \end{aligned}$$

where  $\gamma_p$  is the required SINR for an availability requirement of  $\alpha_i$  for  $i$ th user. Equation (8) indicates that  $\text{SINR}_i$  is a random variable due to  $\mathbf{q}_i$ . A probabilistic approach is then pursued towards evaluating the availability over these random variables and  $\alpha_i$  is chosen to be greater than 0.5. Further,  $P_i$  denotes the transmit power constraint of the  $i$ th antenna feed and  $\tau$  is the power factor similar to one introduced in [14] that is same for all  $i$ . The objective of this optimization problem is to satisfy the required availability for each user ( $\alpha_i$ ) with the lowest possible transmit power from each antennal feed ( $\tau P_i$ ). In

optimization problem  $\mathcal{P}$ , the rank one constraint is non-convex but it can be relaxed by retaining only the semidefiniteness constraint,  $\mathbf{W}_i \succcurlyeq 0$ , which is convex [15]. Under this relaxation, the variables of optimization problem would then be  $\{\mathbf{W}_i\}_1^K$  and the per antenna power constraints will also be convex. In general, the first constraint is difficult to tackle and in the following we will study it in more detail.

We denote the availability of the  $i$ th user in the first constraint by  $f_i(\mathbf{W})$ , where  $\mathbf{W} = \{\mathbf{W}_i\}_1^K$ . So, it can be defined as

$$\begin{aligned} f_i(\mathbf{W}) &\triangleq \Pr \left\{ \text{SINR}_i \geq \gamma_p \right\} \\ &= \Pr \left\{ \text{Tr}(\mathbf{R}_i \mathbf{W}_i) \geq \gamma_p \sum_{j \neq i} \text{Tr}(\mathbf{R}_i \mathbf{W}_j) + \gamma_p \sigma_n^2 \right\}. \end{aligned} \quad (10)$$

By defining  $\mathbf{Z}_i = \mathbf{W}_i - \gamma_p \sum_{j \neq i} \mathbf{W}_j$  [9], we can write  $f_i(\mathbf{W}) = \Pr \left\{ \text{Tr}(\mathbf{R}_i \mathbf{Z}_i) \geq \gamma_p \sigma_n^2 \right\}$ . Clearly,  $y_i \triangleq \text{Tr}(\mathbf{R}_i \mathbf{Z}_i)$  is a random variable and using (9), it can be simplified to

$$y_i = \text{Tr} \left( \text{diag}(\hat{\mathbf{h}}_i) \mathbf{Q}_i \text{diag}(\hat{\mathbf{h}}_i^\dagger) \mathbf{Z}_i \right) = \text{Tr}(\mathbf{A}_i \mathbf{Q}_i), \quad (11)$$

where  $\mathbf{A}_i = \text{diag}(\hat{\mathbf{h}}_i^\dagger) \mathbf{Z}_i \text{diag}(\hat{\mathbf{h}}_i)$ . It can be shown that  $y_i$  can be expressed as the sum of uncorrelated random variables. Therefore, based on the central limit theorem,  $y_i$  can be approximated by a Gaussian random variable with the following mean and variance,

$$\mu_i = \text{E}\{y_i\} = \text{Tr}(\mathbf{A}_i \text{E}\{\mathbf{Q}_i\}) = \text{Tr}(\mathbf{A}_i \mathbf{C}_i), \quad (12)$$

$$\sigma_i^2 = \text{E}\{\text{Tr}(\mathbf{A}_i \mathbf{Q}_i) \text{Tr}(\mathbf{A}_i \mathbf{Q}_i)\} - \mu_i^2, \quad (13)$$

where  $\text{E}\{\text{Tr}(\mathbf{A}_i \mathbf{Q}_i \mathbf{A}_i \mathbf{Q}_i)\} = \text{vec}(\mathbf{A}_i^T)^T \text{E}\{\mathbf{Q}_i^T \otimes \mathbf{Q}_i\} \text{vec}(\mathbf{A}_i)$ . We define  $\mathbf{G}_i = \text{E}\{\mathbf{Q}_i^T \otimes \mathbf{Q}_i\}$  and after some calculation, the entry of  $\mathbf{G}_i$  in the  $r$ th row and the  $c$ th column can be found as,

$$[\mathbf{G}_i]_{rc} = \text{E}\{e^{j\mathbf{e}_i, r_1} e^{-j\mathbf{e}_i, c_1} e^{j\mathbf{e}_i, r_2} e^{-j\mathbf{e}_i, c_2}\}, \quad (14)$$

$$r = (r_1 - 1)K + r_2, \quad 1 \leq r_1, r_2 \leq K, \quad (15)$$

$$c = (c_1 - 1)K + c_2, \quad 1 \leq c_1, c_2 \leq K. \quad (16)$$

Therefore,  $\sigma_i^2 = \text{vec}(\mathbf{A}_i^T)^T \mathbf{G}_i \text{vec}(\mathbf{A}_i) - \text{Tr}(\mathbf{A}_i \mathbf{C}_i)^2$ . With the Gaussian assumption for  $y_i$  and knowing  $\mu_i$  and  $\sigma_i^2$ , we can evaluate  $f_i(\mathbf{W})$  in (10). Since acceptable user availability is  $\alpha_i > 0.5$ ,  $f_i(\mathbf{W})$  should be greater than 0.5. Therefore, we can write the first constraint in  $\mathcal{P}$  as,

$$f_i(\mathbf{W}) \approx 0.5 + 0.5 \text{erf} \left( \frac{\mu_i - \gamma_p \sigma_n^2}{\sqrt{2} \sigma_i^2} \right) \geq \alpha_i. \quad (17)$$

After some manipulation and using  $\mu_i$  and  $\sigma_i^2$  defined in (12) and (13), above constraint can be expressed as,

$$\text{Tr}(\mathbf{A}_i \mathbf{C}_i) - a \geq b_i \sqrt{\text{vec}(\mathbf{A}_i^T)^T \mathbf{G}_i \text{vec}(\mathbf{A}_i) - \text{Tr}(\mathbf{A}_i \mathbf{C}_i)^2}, \quad (18)$$

where  $a = \gamma_p \sigma_n^2$  and  $b_i = \sqrt{2} \text{erf}^{-1}(2\alpha_i - 1)$ . Finally, this can be rewritten as,

$$\begin{aligned} \|\sqrt{\mathbf{G}_i} \text{vec}(\mathbf{A}_i)\|^2 &\leq \\ \frac{1}{b_i^2} &\left( \sqrt{b_i^2 + 1} \text{Tr}(\mathbf{A}_i \mathbf{C}_i) - \frac{a}{\sqrt{b_i^2 + 1}} \right)^2 + \frac{a^2}{1 + b_i^2}, \end{aligned} \quad (19)$$

where  $\|\cdot\|$  is the Euclidean norm. It can be seen that (19) is not a convex constraint but can be easily converted into one if we drop the

last fixed term,  $a^2/(1+b_i^2)$ . The resulting constraint, after aforementioned approximations and bounding, is convex since it is an inverse image of the second-order cone [16],

$$S = \{(\mathbf{x}, z) | \mathbf{x}^T \mathbf{x} \leq (\lambda z + \nu)^2, \lambda z + \nu \geq 0\}, \quad (20)$$

under the affine functions  $\mathbf{x} = \sqrt{\mathbf{G}_i} \text{vec}(\mathbf{A}_i)$  and  $z = \text{Tr}(\mathbf{A}_i \mathbf{C}_i)$ . Note that  $\text{Tr}(\mathbf{A}_i \mathbf{C}_i) - a/(b_i^2 + 1) \geq 0$ . Replacing the all constraints in  $\mathcal{P}$  by their convex approximations, a new but related optimization problem can be written as,

$$\begin{aligned} \mathcal{P}_r : \text{minimize} \quad & \tau \\ \text{subject to} \quad & \|\sqrt{\mathbf{G}_i} \text{vec}(\mathbf{A}_i)\| \leq \\ & \frac{1}{b_i} \left( \sqrt{b_i^2 + 1} \text{Tr}(\mathbf{A}_i \mathbf{C}_i) - \frac{a}{\sqrt{b_i^2 + 1}} \right), \\ & \left[ \sum_{j=1}^K \mathbf{W}_j \right]_{i,i} \leq \tau P_i, \mathbf{W}_i \succcurlyeq 0 \end{aligned}$$

In the new formulation, all constraints are convex and the problem can be solved by using standard convex solvers like CVX [17]. It should be noted that the convex set resulting from the new upper bounded constraint in problem  $\mathcal{P}_r$  is a subset of the feasible set of the original constraint in (19).

We denote the resulting precoding matrix of  $\mathcal{P}_r$  by  $\mathbf{W}^* = [\mathbf{w}_1^*, \dots, \mathbf{w}_K^*]$  where  $\mathbf{w}_i^*$  is the precoding vector for the  $i$ th user. Based on the numerical results, problem  $\mathcal{P}_r$  mostly yields rank one solutions; else, approximation technique like Gaussian Randomization [18] can be used.

#### 4. NUMERICAL RESULTS AND DISCUSSION

In this section, we discuss the performance of the proposed robust precoder through numerical evaluations. As mentioned in section 2, a multibeam structure with single color (all beams have the same frequency) and  $K = 7$  beams is considered. In general, the downlink channel vector between  $K$  satellite transmit antennas and  $i$ th user can be expressed as,

$$\hat{\mathbf{h}}_i = \sqrt{C} \mathbf{r}_i \odot \mathbf{b}_i^{\frac{1}{2}}, \quad (21)$$

where  $\mathbf{r}_i$  models the rain fading effect and the beam pattern is represented by  $\mathbf{b}_i$ . Rain attenuation in dB is commonly modeled as lognormal random variable; refer to [2] and [6] for a detailed description of the rain fading model. Given the  $i$ th user position, we define the angle subtended by the chord between  $i$ th user and the  $k$ th beam center at the satellite by  $\theta_{k,i}$  and the 3 dB angle for the  $k$ th beam by  $\theta_{k,3\text{dB}}$  which is a constant. Then the beam gain from  $k$ th antenna to  $i$ th user is approximated by [19],

$$\mathbf{b}_i(k) = G_{s,k} \left( \frac{J_1(u_k)}{2u_k} + 36 \frac{J_3(u_k)}{u_k^3} \right)^2, \quad (22)$$

where  $G_{s,k}$  is the satellite transmit antenna gain for the  $k$ th beam and  $u_k = 2.07123 \frac{\sin(\theta_{k,i})}{\sin(\theta_{k,3\text{dB}})}$ . Here,  $J_1$  and  $J_3$ , respectively, are the first and third order Bessel functions of first kind. In (21), the coefficient  $C$  is defined as,

$$C = \left( \frac{\nu}{4\pi f d_0} \right)^2 \frac{G_{r,i}}{\kappa B T}, \quad (23)$$

**Table 1.** Link Budget and System Parameters

Parameter	Value
Orbit	GEO, $d_0 = 35786$ (Km)
Downlink Band	Ka-Band, $f = 20$ (GHz)
Number of beams	$K = 7$
Beam radius	250 (Km)
Boltzmann's constant	$\kappa = 1.38 \times 10^{-23}$ (J/m)
Noise bandwidth	$B = 50$ (MHz)
Satellite antennae gain	$G_{s,k} = 35$ (dBi)
Receiver gain to noise temperature	$G_{r,i}/T = 34$ (dB/K)
3dB Angle	$\theta_{3dB} = 0.4^\circ$
TWTA RF Power @ Saturation	$P_i = 20$ (W)
Downlink Clear Sky SNR	24 (dB)

which includes effects of the free space loss,  $(\nu/(4\pi fd_0))^2$ , UT's receive gain,  $G_{r,i}$ , and noise power at receiver,  $\kappa BT$ . In (23),  $\nu$  is the speed of light,  $f$  is operating frequency of the downlink,  $\kappa$  is Boltzmann's constant,  $B$  is noise bandwidth and  $T$  is the receive noise temperature. Table 1 provides the link budget and the system parameters. Note that we normalized the noise power by  $\kappa BT$ , so in (7) it can be assumed that  $\sigma_n^2 = 1$ .

We compare the performance of our designed robust precoder with the precoder optimized for  $\hat{\mathbf{h}}_i$ , but employed for  $\mathbf{h}_i$ . In particular, we compare the solution of  $\mathcal{P}_r$  with that obtained from the following optimization problem:

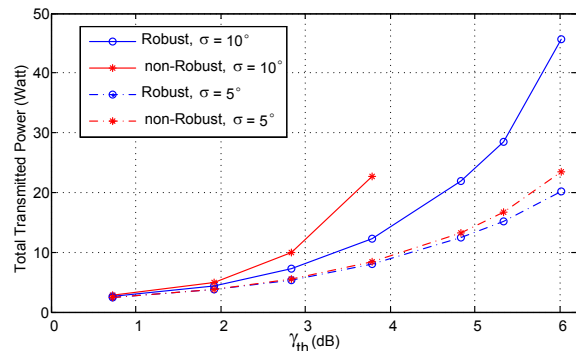
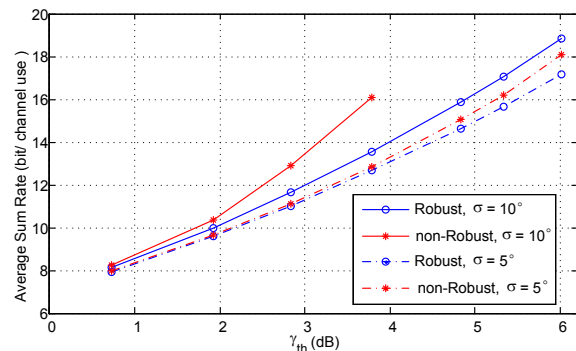
$$\begin{aligned} \mathcal{Q} : & \text{minimize} && \tau \\ & \mathbf{W}_i, \dots, \mathbf{W}_K, \tau \\ & \text{subject to} && \text{SINR}_i \geq \gamma_q \\ & && \forall_i \\ & && \left[ \sum_{j=1}^K \hat{\mathbf{W}}_j \right]_{i,i} \leq \tau P_i, \hat{\mathbf{W}}_i \succeq 0 \end{aligned}$$

In this formulation,  $\gamma_q$  is a design variable that is adjusted to achieve the required availability. In  $\mathcal{Q}$ ,  $\text{SINR}_i$  is defined similarly to (8) with  $\mathbf{R}_i$  replaced by  $\hat{\mathbf{R}}_i = \hat{\mathbf{h}}_i \hat{\mathbf{h}}_i^H$ . The resulting precoding matrix is denoted by  $\hat{\mathbf{W}}^* = [\hat{\mathbf{w}}_1^*, \dots, \hat{\mathbf{w}}_K^*]$  where  $\hat{\mathbf{w}}_i^*$  is the precoding vector for the  $i$ th user. Here, we assume that there is no phase uncertainty and design the precoder for  $\hat{\mathbf{h}}_i$ ; it is then applied to the channel that has phase uncertainty,  $\mathbf{h}_i = \hat{\mathbf{h}}_i \odot \mathbf{q}_i$ . Due to mismatch in uncertainty models, comparison with other robust designs is not pursued.

After generating a random realization of the channel based on the model described earlier, both optimization problems,  $\mathcal{P}_r$  and  $\mathcal{Q}$ , are solved. We define availability of the users as  $\Pr\{\text{SINR}_i \geq \gamma_{th}\}$  and  $\Pr\{\text{SINR}_i \geq \gamma_{th}\}$  respectively for problem  $\mathcal{P}_r$  and  $\mathcal{Q}$  where  $\gamma_{th}$  is the outage threshold. From its definition, it is clear that  $\gamma_p$  of  $\mathcal{P}_r$  equals  $\gamma_{th}$ . However, for  $\mathcal{Q}$ , it can be argued that  $\gamma_{th} < \gamma_q$  to achieve an availability same as  $\mathcal{P}$ .

Fig. 1 shows the total transmit power for the two precoders ( $\|\mathbf{W}^*\|_F$  and  $\|\hat{\mathbf{W}}^*\|_F$ ) resulting from  $\mathcal{P}_r$  and  $\mathcal{Q}$  to achieve an availability of 90% for different uncertainty levels. As prevalent in robust designs, we treated  $\alpha_i$  as a tuning parameter to obtain the desired empirical availability level for  $\mathcal{P}_r$ . It can be seen that for the same  $\gamma_{th}$  and availability of 90%, the robust precoder requires lesser power compared to the non-robust precoder. As phase uncertainty increases,  $\sigma = 10^\circ$ , formulation  $\mathcal{P}_r$  shows better performance than  $\mathcal{Q}$ . It also should be noted that non-robust precoder can not provide availability of 90% when  $\gamma_{th} > 3.8$  dB and  $\sigma = 10^\circ$ . Fig. 2 shows the average sum rate of users versus outage threshold,  $\gamma_{th}$ . The average sum rate is calculated as  $E\{\sum_{i=1}^K \log_2(1 + \text{SINR}_i)\}$ , where

expectation is over  $\mathbf{q}_i$ . It can be seen that non-robust approach in  $\mathcal{Q}$  achieves higher sum rate for the same  $\gamma_{th}$  and availability of 90%. This is because of the fact that  $\gamma_q > \gamma_p$  for the same  $\gamma_{th}$  indicating the traditional sum-rate availability trade-off.

**Fig. 1.** Required transmit power for an availability of 90%**Fig. 2.** Average Sum Rate versus  $\gamma_{th}$  for an availability of 90%

## 5. CONCLUSION

We follow a probabilistic approach to the design of a precoder which is robust against phase uncertainty. Phase uncertainty resulting from independent on-board oscillator is modelled as a random process and constraints are imposed on the resulting outage probability. An optimization of the precoder to meet the design constraints while minimizing per antenna power is formulated and the problem is relaxed into a convex formulation. The performance of the robust precoder is compared with a precoder optimized for outdated CSIT. The results show that the proposed precoder guarantees the desired availability for a lower power. Thus the designed precoder is a strong candidate for use in future satellite systems since on-board power and availability are key aspects for a satellite service operator.

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