(Un)stable vertical collusive agreements*

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Abstract

In this paper, we extend the concept of stability to vertical collusive agreements, involving downstream and upstream firms, using a setup of successive Cournot oligopolies. We show that stable vertical collusive agreements exist even for market structure in which horizontal cartels would be unstable. Furthermore, Stigler statement according to which the only ones who benefit from a collusive agreement are the outsiders need not be valid in vertical agreements.

Keywords: collusion, stability, vertical agreement.

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1 Introduction

In the present paper, we study the stability of *vertical collusive agreements* in the context of successive oligopolies. Such collusive agreements simultaneously embody downstream and upstream firms. Collusion is represented as an agreement through which the insiders act in unison, reducing thereby the total number of decision units operating in the downstream and upstream markets and, thus, the corresponding number of oligopolists in each of them. Collusive outcomes are the Cournot equilibria corresponding to these reduced numbers of oligopolists, which are then compared with those arising when downstream and upstream firms act independently from each other in their respective markets.

More than half a century ago, Stigler (1950) has stressed the main difficulty encountered by a cartel promoter: "the major difficulty in forming a merger is that it is more profitable to be outside a merger than to be a participant. The outsider sells at the same price but at the much larger output at which marginal cost equals price. Hence, the promoter of a merger is likely to receive much encouragement from each firm, almost every encouragement, in fact, except participation". This sentence clearly illustrates the need for analyzing carefully under which conditions a cartel is expected to resist to the forces acting against its stability.

A definition of cartel stability, relying on two natural requirements, namely, external stability and internal stability, has been proposed by d'Aspremont *et al.*, 1983, in the context of horizontal mergers. A cartel with n firms in an industry embodying K firms ($n > K$) is said *internally stable* when the profits realized by each firm member of the cartel exceeds the profits obtained when being outside of it, *taking into account the change in the profits of an outsider resulting from this exit.* Similarly, a cartel with K members is *externally stable* when the profits of a firm member of a cartel of size $K + 1$ are smaller than the profits realized by an outsider when the cartel is of size K. A cartel of size K is stable when it is both internally and externally stable. Formally, assuming that all firms are identical, and defining $\Pi^f(K)$ (resp. $\Pi^c(K)$) the payoff received by each outsider (resp. by each cartel member), a cartel of size K is stable if both the inequalities

$$\Pi^c(K) \geq \Pi^f(K-1)$$

(internal stability) and

$$\Pi^f(K+1) \geq \Pi^c(K)$$

(external stability) hold simultaneously.

This definition of stability is rather abstract since it does not state how the profits of an insider or an outsider are defined or, equivalently, to which market structure it corresponds. As a consequence, the above abstract definition can be applied to a wide variety of market situations and corresponding payoffs' structures. Nevertheless, the definition of stability assumes that each member of the agreement receives the same share of profits, $\Pi^c(K)$, and, similarly, that each outsider obtains an equal amount of profits, $\Pi^f(K)$. One way to rationalize this assumption consists in supposing that all firms in the industry are identical, with the sole exception that either they participate to the collusive agreement, or are outsiders. This assumption then allows to share equally the profits inside the entity among its members through an argument of equal treatment. Furthermore, supposing an identical strategic behavior for the outsiders allows to state by an argument of symmetry that each one of them also should obtain an equal amount of profits at the solution. With these assumptions, given an entity of size K, profit sharing among firms is fully described by two numbers: the share of profits received by each participant, $\Pi^c(K)$, and the profits received by each outsider, $\Pi^f(K)$.

A first example of market structure and associated payoffs for which stability has been analyzed can be found in the paper referred to above (d'Aspremont *et al.*, 1983). The solution at which the profits are evaluated are those corresponding to the *price leadership* model introduced by Markham (1951). In this version, the cartel (collusive agreement) is assumed to be the price-leader (*the dominant firm*) maximizing its profits on the "residual demand function", while the outsiders are behaving competitively, taking the price set by the cartel as given. It is easily seen that, for this specific market situation and payoff's structure, any cartel is always internally unstable simply because, according to the argument put forward by Stigler (1950), the per-firm profits of a cartel member are smaller than the profits obtained by each outsider. But this remark does not prevent the existence of at least one stable cartel, as demonstrated in d'Aspremont *et al.* (1983).

A second example of market structure in which a collusive agreement is contemplated corresponds to the version proposed by Salant, Schwitzer and Reynolds (1983) of the Cournot model. Compared with the price-leadership model, this approach consists in assuming that a collusive agreement among K firms takes place, according to which these firms maximize per firm profits against the output choice of the outsiders. This situation represents the Cournot equilibrium of the game consisting of the entity and $n - K$ outsiders. In this setup, stability has been analyzed by Shaffer (1995), Bloch (1997) and Belleflamme and Peitz (2010). Assuming linear inverse demand and constant marginal cost, Shaffer (1995) shows that, whenever $n \geq 3$, there exists no stable horizontal cartel in the Cournot game. In a recent paper, Zu *et al,* (2012), borrowing from Konishi and Lin (1999), study the size of horizontal stable cartels using general specifications for the output demand and cost function. These authors confirm that, even with a more general specification, the size of stable horizontal cartels remains quite reduced. The main reason why cartel stability fails in Cournot competition refers to the Stigler statement reminded in the beginning of this paper: outsiders are always better off than insiders, which destroys internal stability in the Cournot game. Accordingly, contrary to the price leadership model in which there always exists a stable cartel, the Cournot game with linear output demand and constant marginal cost has never a stable cartel when the number of firms exceeds three.

The above studies all refer to collusive agreements embodying downstream firms only, excluding thereby more general forms of collusive agreements, like those arising when downstream and upstream firms are allowed to combine together. Extending stability analysis to such collusive agreements is undoubtedly interesting and important. Real-life collusive entities often share the property that, among the participating firms, some of them operate in the upstream market(s) and produce the input(s) used by the downstream firms in the production of the final good. Such agreements have been studied by Salinger (1988), Gaudet and van Long (1996) and, more recently, by Gabszewicz and Zanaj (2011). However, to the best of our knowledge, the analysis of collusive stability in this context has not yet been pursued. Probably this is so because some assumptions used in the traditional approach seem inappropriate in this new set-up. Among these assumptions is the one stating that each member of the agreement must receive the same share of the entity's profits. From the very nature of the problem, the firms participating the entity belong to different types, some producing the final good (downstream firms) while the others produce the input (upstream firms). When downstream and upstream firms are allowed to combine together, with both upstream and downstream firms in the collusive agreement, there is no longer any reason to assume that both types of firms should get the same share of the entity's profits. Thus a conceptual problem arises: how profits should be shared among the members of the collusive entity, knowing that these members are not all identical, but belong to two different types? We meet this difficulty in the present paper by assuming that participants share equally the marginal variation of profits of the entity caused by the entry (exit) of a new participant. For instance, consider an entity composed by one downstream and one upstream firm. If a new upstream (downstream) firm enters the entity, the three participants share equally only the marginal profit variation created by the entry of the new firm in the agreement.

We give a definition of stability that is a direct extension of the definition of stability in d'Aspremont *et al* (1983). It requires that no upstream firm in the entity would get more when leaving the entity than when staying inside (internal stability), taking into account the change in profits resulting from its move. Furthermore it requires that no firm outside the identity would obtain more when entering the entity than staying outside, again taking into account the change in profits resulting from its move (external stability). Thus this definition requires not only that the cartel should be internally (resp. externally) stable with respect to moves of firms of the same type, but also with respect to moves of firms of the alternative type. It is interesting to notice that a cartel embodying K firms in d'Aspremont *et al.* (1983) is stable if, and only if, it corresponds to a Nash equilibrium of the game with n players, two (pure) strategies: "enter the entity-remain outside the entity" and payoffs evaluated at the price leadership solution with $n - K$ firms in the fringe.

A vertical agreement has three effects: *(i)* it softens double marginalization for the entity boosting its profit; *(ii)* it reduces the number of active upstream firms in the upstream market leading to a shift upwards of the input supply and it reduces the number of downstream firms that buy the input in the input market causing a shift downwards of the input demand schedule. The balance of these two shifts can lead to an increase of a decrease of the equilibrium input price and, finally, *(iii)* it creates asymmetries in the production costs of the downstream firms: the entity produces at the marginal cost while the downstream firms at the market price. The effect on the equilibrium output price is ambiguous. The equilibrium output price increases if the output quantity increases due to the presence of the entity, or it may decrease, otherwise. The stability of the vertical entity depends on the balance of these three effects. We show that the presence of upstream firms in the agreement allows stable cartels even for n exceeding three. This is the case because Stigler's statement according to which the only ones who benefit from a collusive agreement are the outsiders need not be valid in vertical agreements.

2 Stability of vertical agreements and successive oligopolies

In the following we call *vertical* agreement any collusive entity involving simultaneously both downstream and upstream firm(s). In order to examine the question of stability in the case of vertical agreements, we need a framework in which these are analyzed and firms' payoffs defined. This framework is provided in Salinger (1988) and, more recently, by the authors in Gabszewicz and Zanaj (2011), in which they propose a definition of *successive oligopolies* allowing a precise concept of vertical agreement. The nature of the agreement concerns *(i)* the payoff division among downstream and upstream participants, *(ii)* the price of the input for the insider downstream firms and *(iii)* the behavior of the insider upstream firms with respect to the input market. In the present paper, we assume that participants share equally the marginal variation of profits of the entity caused by the entry (exit) of a new participant.

To define precisely the notion of stability of vertical agreements, consider two successive markets embodying n identical downstream firms and m identical upstream firms, $m \geq 2$. Assume that $K, K < n$, downstream firms and $H, H < m$, upstream firms decide to collude. Notice this entity now involves *two* types of agents, all identical in each type. An entity of size $K + H$ is stable when it is both internally and externally stable, for each type of agents. Define formally $\Pi^f(K, H)$ (resp. $\Pi^c(K, H)$) the payoff received by each downstream outsider (resp. by each participant), and $\Gamma^f(K, H)$ (resp. $\Gamma^c(K, H)$) the payoff received by each upstream outsider (resp. by participant). Then,

Definition *A vertical entity of size $K + H$ is stable if both the sets of inequalities*

$$\Pi^c(K,H) \geq \Pi^f(K-1,H) \text{ and } \Pi^c(K,H) \geq \Pi^f(K,H-1) \quad (1)$$
$$\Gamma^c(K,H) \geq \Gamma^f(K-1,H) \text{ and } \Gamma^c(K,H) \geq \Gamma^f(K,H-1) \quad (2)$$

(internal stability) and

$$\Pi^f(K+1,H) \geq \Pi^c(K,H) \text{ and } \Pi^f(K,H+1) \geq \Pi^c(K,H) \quad (3)$$
$$\Gamma^f(K+1,H) \geq \Gamma^c(K,H) \text{ and } \Gamma^f(K,H+1) \geq \Gamma^c(K,H) \quad (4)$$

(external stability) hold simultaneously.

This definition directly extends the definition of stability provided by d'Aspremont *et al* (1983) to agreements that include two types of firms. More precisely, a vertical collusive agreement embodying $K+H$ is said *internally stable* when the profits realized by each type of firm, downstream and upstream, member of the entity, exceeds the profits obtained when being outside of it, taking into account the change in the profits of an outsider, of any type, resulting from this exit. Similarly, a vertical entity with $K+H$ members is externally stable when the profits of a firm, upstream or downstream, member of a entity of size $K+1$ or/and $H+1$ are smaller than the profits realized by an outsider, downstream and upstream, when the cartel is of size $K+H$. This definition of stability translates into demanding four conditions for internal stability and other four for the external stability of the collusive entity.

Now let us apply the definition of stability to the well-known case of linear output demand and constant returns to scale in successive Cournot oligopolies. Let the demand function for some output in the downstream market be given by $p(Q) = 1 - Q$, where Q denotes aggregate supply. Consider n *downstream* firms producing the output *via* a constant returns technology $f(z) = \alpha z, \alpha > 0$, as well as m *upstream firms* initially supplying the market for the input z at a constant marginal cost equal to $\beta, \beta > 0$. Now assume that H upstream firms, $h = 1, 2...H$, form a vertical collusive agreement with K downstream firms $k = 1, 2, ...K$, and maximize joint profits together. Also assume that $K < n$ and $H < m$.[1] After this agreement, the downstream and upstream markets move from an initial situation with n active downstream firms and m active upstream firms, to a market structure with $n - K + 1$ active firms in the downstream market and $m - H$ in the upstream one. An example of this type of vertical collusive agreement is the agreement taking place in a market among one or several wholesalers with one or several retailers that fixes the price at which the market product is sold to the final consumers.[2]

Consider first how the profit functions write in the downstream market after the collusive agreement. To this end denote by I the entity resulting from the agreement. The profits Π_I of the entity I to which K downstream firms participate is given by

$$\Pi_I(q_I,q_{-I}) = (1 - q_I - \sum_{i \neq I} q_i)q_I - \beta \frac{q_I}{\alpha} \tag{5}$$

where q_I (resp. $\sum_{i\neq I} q_i$) denotes the supply of the entity (resp. firms not in the agreement) in the downstream market and $\frac{\beta}{\alpha}$ is its unit production cost. As for the downstream firms that do not participate in the agreement, each of them obtains a payoff Π_i defined by

[1] This assumption guarantees that there always exists at least one firm on each side of the upstream market so that the cartel cannot exclude the outsider downstream firms to have access to the input. A similar asssumption in another approach to collusion has been used by Gabszewicz and Hansen (1971).

[2] For example, the French Competition Council in 2008 sanctioned five toys manufacturers and three distributors on grounds of collusion during the Christmas period between 2001 and 2004. (La Revue, 2008)

$$\Pi_i(q_i, q_k) = (1 - q_I - q_i - \sum_{k \neq i} q_k)q_i - \omega(\frac{q_i}{\alpha}), \tag{6}$$

with $i, i \neq I$, and ω denoting the unit price in the input market.[3] Notice that from the comparison between (5) and (6), it appears immediately that, while the collusive members in the downstream market pay their input at marginal cost β, the rivals pay the input price ω. Since Π_I is concave in q_I, we may use the first order condition to get the best reply function q_I of the entity in the downstream market game as

$$q_I(q_{i \neq I}) = \frac{1 - \frac{\beta}{\alpha} - \sum_{i \neq I} q_i}{2}.$$

As for an outsider downstream firm i, its best reply q_i in the downstream market is conditional on the input price ω realized in the upstream market, namely

$$q_i(q_I, q_k, \omega) = \frac{1 - \frac{\omega}{\alpha} - \left(q_I + \sum_{k \neq i, k \neq I} q_k\right)}{2}.$$

Assuming a symmetric equilibrium *among the colluded downstream firms*, we get the resulting Cournot equilibrium in the downstream market, namely, the optimal supply coming from the entity q_I^* and from each of the rivals q_i^* which do not belong to the cartel, namely

$$q_I^*(K, \omega) = \frac{\alpha - \beta + (n - K)(\omega - \beta)}{\alpha(n - K + 2)}, \tag{7}$$

and

$$q_i^*(K, \omega) = \frac{\alpha + \beta - 2\omega}{\alpha\,(n - K + 2)}. \tag{8}$$

It is worth noting that the equilibrium in the downstream market depends on the input price obtained in the upstream market as an immediate consequence of supply and demand for the input. Taking into account (8) and the fact that $q = f(z) = \alpha z$, it is easy to derive the input demand resulting from the $n-K$ outsider firms in the downstream market, i.e. $\sum_{i \neq I} z_i(\omega) = (n - K)(\frac{\beta + \alpha - 2\omega}{\alpha^2(N-K+2)})$. As for the input supply, it comes from the strategies s_j, $j \neq I$, selected by the outsider upstream firms in the input market. Consider the j_{th} upstream firm not participating in the entity. Its profits Γ_j at the vector of strategies (s_j, s_{-j}) write as

$$\Gamma_j(s_j, S_{-j}) = \omega(s_j, S_{-j})s_j - \beta s_j, \tag{9}$$

with $S_{-j} = \sum_{-j \neq I} s_{-j}$. Taking into account that $\omega(s_j, s_{-j})$ has to make demand equal to supply in the upstream market, namely, $\sum_{j \neq I} s_j = \sum_{i \neq I} z_i(\omega)$, we obtain

[3] Notice that the set $\{k : k \neq i\}$ includes the index I.

$$\omega(s_j, s_{-j}) = \frac{(\alpha+\beta)(n-K) - \alpha^2(n-K+2)\sum_{j\neq I} s_j}{2\,(n-K)} \tag{10}$$

where $\sum_{j\neq I} s_j = S_{-j} + s_j$. Accordingly, the payoff of the j-th upstream firm writes as

$$\Gamma_j(s_j, s_{-j}) = \left(\frac{(\alpha+\beta)(n-K)-\alpha^2(n-K+2)\sum_{j\neq I} s_j}{2(n-K)}\right) s_j - \beta s_j.$$

It is immediate to derive from the above the best reply function $s_j = s_j(S_{-j})$. Using the symmetry condition $S_{-j} = (m-H-1)s_{-j}$, we derive the optimal input supply s_j^* coming from the j-th outsider firm, namely

$$s_j^*(K,H) = \frac{(\alpha-\beta)\,(n-K)}{\alpha^2(n-K+2)(m-H+1)}.$$

Substituting the expression of s_j^* in (10) we get the equilibrium input price

$$\omega^*(H) = \frac{\alpha+\beta+2\beta(m-H)}{2\,(m-H+1)}. \tag{11}$$

Substituting (11) in (8) and (7) we get the output supply of each outsider downstream firm q_i^* and that of the cartel q_I^*, respectively,

$$q_i^*(K,H) = \frac{(m-H)\,(\alpha-\beta)}{\alpha\,(n-K+2)\,(m-H+1)}, \tag{12}$$

and

$$q_I^*(K,H) = \frac{(\alpha-\beta)(n-K+2(m-H+1)}{2\alpha(n-K+2)(m-H+1)}.$$

It follows immediately that profits at equilibrium of the entity Π_I^*, and of the outsider firms $\Pi_i^*(K,H)$ in the downstream market, write as

$$\Pi_I^*(K,H) = \frac{(\alpha-\beta)^2}{4\alpha^2}\frac{(2(m-H)+n-K+2)^2}{(n-K+2)^2(m-H+1)^2}, \tag{13}$$

and

$$\Pi_i^*(K,H) = \frac{(\alpha-\beta)^2}{\alpha^2}\frac{(m-H)^2}{(n-K+2)^2(m-H+1)^2}, \tag{14}$$

respectively. The profit of an outsider upstream firm is

$$\Gamma_j^*(K,H) = \frac{(\alpha-\beta)^2}{2\alpha^2}\frac{(n-K)}{(n-K+2)(m-H+1)^2} \tag{15}$$

Now we are in a position to examine whether, as in the case of pure horizontal collusive agreements, no stable equilibrium exists (for $n \geq 3$) when the entity involves simultaneously downstream and upstream firms.

Recall that the profit sharing rule is as follows. Participants share equally the marginal variation of profits of the entity caused by the entry (exit) of a new participant. For instance, consider an entity composed by one downstream and one upstream firm. When no entity exists the profit for a downstream firm is $\Pi^*(0,0)$.[4] Hence as a participant of the entity, the dowstream firm obtains $\Pi^*(0,0)$ plus half of the marginal variation in payoffs, namely $\left[\Pi_I^*(1,1) - \Pi_i^*(0,0)\right]/2$. Whereas the upstream participant obtains $\left[\Pi_I^*(1,1) - \Pi_i^*(0,0)\right]/2$.

Using the profit functions for the entity and for the outsider firms, we can state the following

Proposition 1 *Assume that participating firms in a vertical collusive entity share equally the marginal profit obtained by the entity due to the entry of a new participant. Then, there exist at least a stable vertical collusive entity, including one downstream firm and one upstream firm, if the number of downstream firms n, satisfy the following condition:*

$$\bar{n}(m) < n < \tilde{n}(m).$$

where $\bar{n}(m)$ and $\tilde{n}(m)$ defined in the proof.

Proof. Consider the entity composed by one downstream and one upstream

firm, i.e., $H = 1$ and $K = 1$. The conditions for external stability are:

(i) for the upstream participant with respect to the entry of a downstream firm

$$\frac{\Pi_I^*(2,1) - \Pi_I^*(1,1)}{3} \leq \Gamma_j^*(1,1). \tag{16}$$

Substituting (13) and (15), the above condition is satisfied if and only if the inequality

$$-3n^4 + (2m+1)\,n^2 + \left(4m^2 - 6m + 2\right)n + \left(2m^2 - 4m + 2\right) \leq 0$$

is true. This is a polynomial of 4th degree of n, whose coefficients change their sign only once. Therefore this polynomial accepts only one positive root called $\hat{n}(m)$. Then, the inequality is true if $n > \hat{n}(m)$.

(ii) for the upstream participant with respect to the entry of a new upstream firm:

$$\frac{\Pi_I^*(1,2) - \Pi_I^*(1,1)}{3} - \Gamma_j^*(1,1) \leq 0. \tag{17}$$

Substituting (13) and (15), we find that the above condition is satisfied if the inequality $(-6n-2)\,m^2 + (14n+6)\,m - (7n+5) \leq 0$ is true. This is always the case because the polynomial $(-6n-2)\,m^2 + (14n+6)\,m - (7n+5) = 0$ has two two roots which are both inferior to 2. Hence for $m > 2$, the inequality always holds.

[4] Recall that if no agreement is established between the downstream and upstream firm then no entity exist.

(iii) The same statement can be made for the condition of external stability of a downstream firm with respect to entry of a downstream firm.

(iv) downstream firm with respect to entry of an upstream firm:

$$\frac{\Pi_I^*(1,2) - \Pi_I^*(1,1)}{3} \leq \Pi_i^*(1,1). \tag{18}$$

This condition is satisfied if and only if the inequality $(4-3m)\,n^2 + (2m-1)\,n + (m-1) \leq 0$ is true, namely if and only if $n \leq \tilde{n}(m) \equiv -\frac{-4m+2m^2+1-2(m-1)\sqrt{12m-14m^2+6m^3-3}}{2m-1}$.

The condition for internal stability are

$$\begin{aligned} \Pi_i^*(0,0) + \frac{\Pi_I^*(1,1) - \Pi_0^*(0,0)}{2} &\geq \Pi_i^*(0,0) \\ \frac{\Pi_I^*(1,1) - \Pi_0^*(0,0)}{2} &\geq \Gamma_j^*(0,0). \end{aligned}$$

for the downstream and the upstream firm, respectively. Clearly, the condition for the downstream is always satisfied being $\Pi_I^*(1,1) - \Pi_0^*(0,0) > 0$. Whereas, for the second condition, substituting the expressions of payoffs, we find that this is satisfied if and only if $n > \check{n}(m) \equiv \frac{2m\sqrt{-4m-7m^2+6m^3+m^4+4}-m^2+2m^3-1}{-2m+7m^2-1}$. Denote by $\bar{n} = max\left\{\check{n}(m), \hat{n}(m)\right\}$. Then, the entity composed of one downstream and one upstream firm is stable if

$$\bar{n}(m) < n < \tilde{n}(m) \tag{19}$$

It remains to be shown that the set defined by (19) is not empty. To show this, we first check that the set is not empty for the smallest admissible value of m, namely $m = 2$. Since $\bar{n}(m)$ and $\tilde{n}(m)$ are both monotonically increasing in m, with $\tilde{n}(m)$ increasing faster than $\bar{n}(m)$, if the set is not empty for $m = 2$, then it is not empty for any m such that $m > 2.QED.$ ■

An immediate consequence of the above proposition is that the entity composed of one downstream and upstream firm is stable for $n = 3$ and $m = 3$. Hence, the above proposition should be contrasted with the main result of Shaffer (1995) stating that no stable agreement can exist for horizontal mergers when $n \geq 3$. In the framework of successive oligopolies, the marginal cost of downstream firms is no longer exogenous, as in the case of a single market where horizontal mergers are analyzed. Consequently, a vertical entity comprising upstream firms diminishes the number of input suppliers in the input market, restricting accordingly competition in this market and leading ultimately to an increase of the input price. In turn, this increase in the input price increases the production costs of outsider downstream firms. Therefore, downstream outsiders in the case of vertical collusive agreements have now an incentive to be inside the entity, differently from what is argued by Stigler (1950). This is why we have just found at least one case of a weakly stable vertical agreement. Nonetheless, the classical effect documented by Stigler is still present. The output price may increase due to existence of the vertical entity, increasing the incentive for downstream outsiders to stay outside the cartel. In sum, a vertical agreement has three effects: *(i)* it softens double marginalization for the entity boosting its profit; *(ii)* it reduces the number of active upstream firms in the upstream market leading to a shift upwards of the input supply and it reduces the number of downstream firms that buy the input in the input market causing a shift downwards of the input demand schedule. The balance of these two shifts can lead to an increase of a decrease of the equilibrium input price and, finally, *(iii)* it creates asymmetries in the production costs of the downstream firms: the entity produces at the marginal cost while the downstream firms at the market price. The effect on the equilibrium output price is ambiguous. The equilibrium output price increases if the output quantity increases due to the presence of the entity, or it may decrease, otherwise.

The stability of the vertical entity depends on the balance of these three effects which all depend on the parameters n, m, H and K. Internal stability of the entity depends on the extent of the effect in *(i)*. More specifically, internal stability depends on the size of double marginalization. The higher the difference between the input price and the marginal cost to produce the input, the higher the resulting double marginalization and thus the incentive to create the entity. As far as upstream firms are concerned, their incentive to stay out of the entity depends on the extent of the effect in *(ii)*. More precisely, they wish to stay out of the entity if the increase in the input price due to the existence of the entity is more profitable than the profit obtained being in the entity due to the effect in *(i)*. As for the downstream firms, their incentive to stay out depend on the extent of the effect in *(ii)* as well as in *(iii)*. Namely, they have an interest to be outside the entity if buying the input in the market allows a residual output demand (captured in *(iii)*) that yields profits higher than their share of profits in the entity.

Two last remarks are in order. First, in the case of the entity composed of one downstream and one upstream firm both insiders receive a payoff that exceeds the payoff of the outsiders:

$$\begin{aligned} \Pi_i^*(0,0) + \frac{\Pi_I^* - \Pi_i^*(0,0)}{2} &\geq \Pi_i^*(1,1), \\ \frac{\Pi_I^* - \Pi_i^*(0,0)}{2} &\geq \Gamma_j^*(1,1). \end{aligned}$$

Hence, Stigler's statement according to which the only ones who benefit from a collusive agreement are the outsiders need not be true in vertical agreements.

Second, the analysis of stable vertical agreements developed above required a precise profit sharing rule to specify the payoff of participants in the entity. The rule we put forward is not the only rule that can be used in vertical agreements. For instance, we could imagine that downstream firms share equally the profit of the entity net of the profit attributed to upstream firm who receive the same level of profits as the outsider upstream firms. It turns out that using this sharing rule, no stable cartel exists. The condition that guaranties the external stability of the upstream firms with respect to entry (or exit) of a downstream firm fails to hold. In fact, the profit of outsider upstream firms (which is also the payoff that they receive in the entity) is a decreasing function of K. Hence, the condition $\Gamma^f(K+1,H) = \Gamma^c(K+1,H) > \Gamma^c(K,H)$ is always violated.

This feature of the analysis shows that the assumption on profit sharing is crucial for the analysis of stability, but it also reveals that our definition of stability is quite strong. Should a vertical agreement be stable it implies in particular that, given the H upstream firms, the vertical agreement of the K downstream participants satisfy the inequalities $\Pi^c(K,H) \geq \Pi^f(K-1,H)$ and $\Pi^f(K+1,H) \geq \Pi^c(K,H)$, and similarly, given the K downstream firms, the vertical agreement of the H upstream participants satisfy the inequalities $\Gamma^c(K,H) \geq \Gamma^f(K-1,H)$ and $\Gamma^f(K,H+1) \geq \Gamma^c(K,H)$, only. Therefore, a weakly less demanding notion of stability would require only these inequalities to hold. Nonetheless, this notion of stability does not correspond to a Nash equilibrium of the game with $n+m$ players and 2 (pure) strategies: "enter the entity-remain outside the entity".

3 Conclusion

In this paper, we tackle the stability problem of collusive agreements not only involving some downstream firms, but also embodying some upstream firms, providing the final market with a specific input. In other words, we extend the stability analysis from pure horizontal collusive agreements to entities involving some degree of vertical agreements. This endeavour is made possible due to the flexibility of the stability concept introduced above, but also to the framework of *successive oligopolies* introduced elsewhere by the authors (see Gabszewicz and Zanaj (2011)). This extension is crucial because many real-life collusive agreements embody both upstream and downstream firms, influencing thereby the outcomes obtained both in the upstream and downstream market.

Our analysis reveals that stable entities exist for market structure in which horizontal cartels would be unstable. It also reveals that the introduction of some degree of vertical agreements weakens the Stigler statement according to which "the major difficulty in forming a cartel is that it is more profitable to be outside a cartel than to be a participant". In the framework of successive oligopolies, the marginal cost of downstream firms is no longer exogenous, as in the case of horizontal agreements. When the entity also comprises upstream firms, it reduces the number of input suppliers in the upstream market, restricting thereby competition in this market, leading in turn to an increase in the input price. In turn, this increase enlarges the production costs of the outsider downstream firms. Therefore, these downstream may have now an incentive to be inside the agreement, differently from what is argued by Stigler (1950).

Our paper has only scratched the surface of what looks a promising territory for further research. Many questions are still remaining open after our analysis. First, how robust are the conclusions of the paper ? It is clear that, like most of the previous research in this field, its conclusions hold in the framework of examples. Second, in reality, the institutional forms of collusive agreements are by far more complex than those evoked in this paper where the agreement reduces simply to the acceptance to belong to the entity or not. In particular, merging existing firms often takes the form of acquisition of one firm by another one. Such acquisitions reveal the existence of a market where firms are exchanged among firms, in order to identify the optimal structure of a specific industry. Finally, it would be natural to examine the effects of entry in the upstream and downstream markets, affecting thereby the number n and m, viewed here as parameters of the stability problem.

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