Hybrid mesh/particle meshless method for modeling geological flows with discontinuous transport properties

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Abstract: Geodynamic modeling is an important branch of Earth Sciences. Direct observation of geodynamic processes is limited in both time and space, while on the other hand numerical methods are capable of simulating millions of years in a matter of days on a desktop computer. The model equations can be reduced to a set of Partial Differential Equations with possibly discontinuous coefficients, governing mass, momentum and heat transfer over the domain. Some of the major challenges associated with such simulations are (1) geological time scales, which require long (in physical time) simulations using small time steps; (2) the presence of localization zones over which large gradients are present and which are much smaller than the overall physical dimensions of the computational domain and require much more refined discretization than for the rest of the domain, much like in fracture or shear band mechanics. An added difficulty is that such structures in the solution may appear after long periods of stagnant behaviour; (3) the definition of boundary conditions, material parameters and that of a suitable computational domain in terms of size; (4) a posteriori error estimation, sensitivity analysis and discretization adaptivity for the resulting coupled problem, including error propagation between different unknown fields. Consequently, it is arguable that any suitable numerical methods aimed at the solution of such problems on a large scale must be able to (i) provide ease of discretization refinement, including possible partition of unity enrichment; (ii) offer a large stability domain, so that "large" time steps can be chosen; (iii) ease of parallelization and good scalability. Our approach is to rely on "meshless" methods based on a point collocation strategy for the discretization of the set of PDEs. The method is hybrid Eulerian/Lagrangian, which enables to switch easily between stagnant periods and periods of localization. Mass and momentum equations are solved using a meshless point collocation Eulerian method, while energy equation are solved using a set of particles, distributed over the spatial domain, with the solution interpolated back to the Eulerian grid at every time step. This hybrid approach allows for the accurate calculation of fine thermal structures, through the ease of adaptivity offered by the flexibility of the particle method. The approximation space is constructed using the Discretization Correction Particle Strength Exchange (DC PSE) method. The proposed scheme gives the capability of solving flow equations (Stokes flow) in fully irregular geometries while particles, "sprinkled" in the spatial domain, are used to solve convection-diffusion problems avoiding the oscillation produced in the Eulerian approach. The resulting algebraic linear systems were solved using direct solvers. Our hybrid approach can capture sharp variations of stresses and thermal gradients in problems with a strongly variable viscosity and thermal conductivity as demonstrated through various benchmarking test cases such as the development of Rayleigh-Taylor instabilities, viscous heating and flows with non-Newtonian rheology.