Shape Optimization Directly from CAD: an Isogeometric Boundary Element Approach

Haojie Lian

Supervisors: Professor S.P.A. Bordas
Dr. P. Kerfriden
Dr. R.N. Simpson

Advanced Materials and Computational Mechanics Group
The School of Engineering, Cardiff University

Outline

1. Motivation

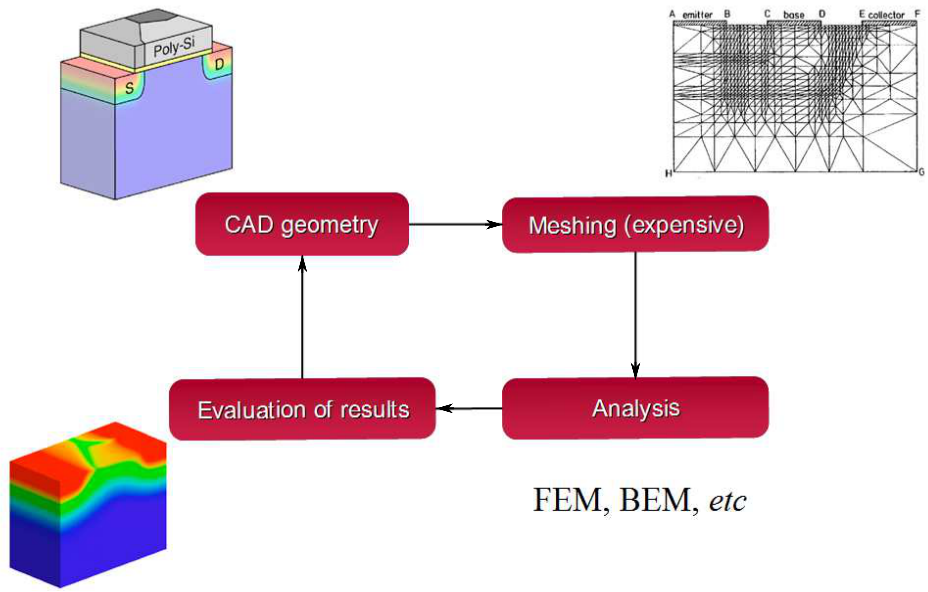
2. CAD techniques (B-splines, NURBS, T-splines)

3. Isogeometric boundary element methods in shape optimization

4. Conclusions

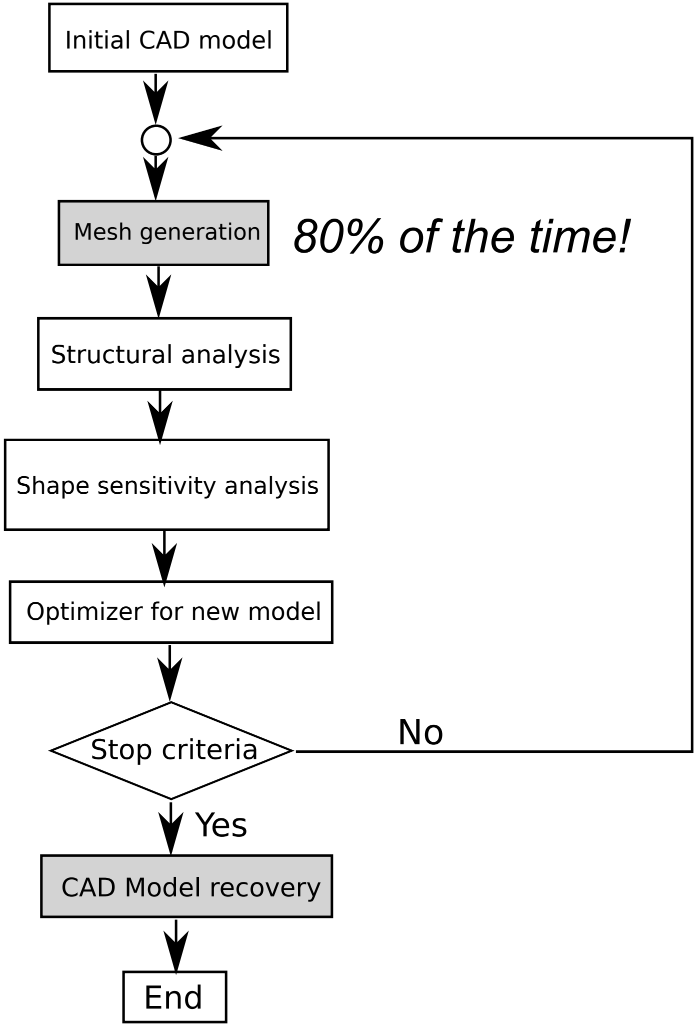
Mesh Burden in Shape Optimization

Current Engineering Analysis/Design Process



Meshing:

1. Time consuming.
2. Human intervention.
3. CAD recovery from mesh.

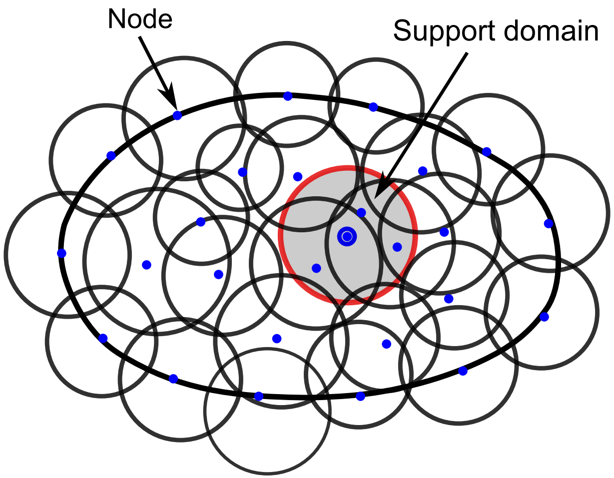


FEM optimization process

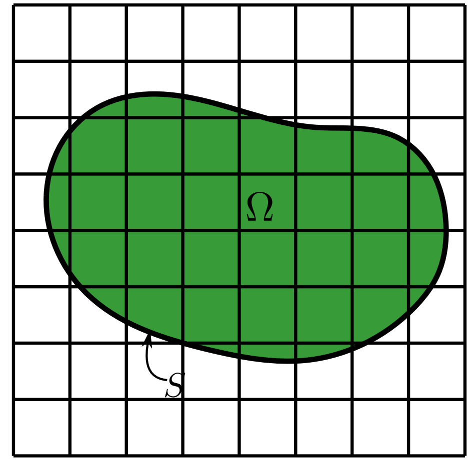
CARDIFF UNIVERSITY PRIFYSGOL CAERDYDD

Numerical Methods for Alleviating Mesh Burden

1. Separate the CAD from the analysis

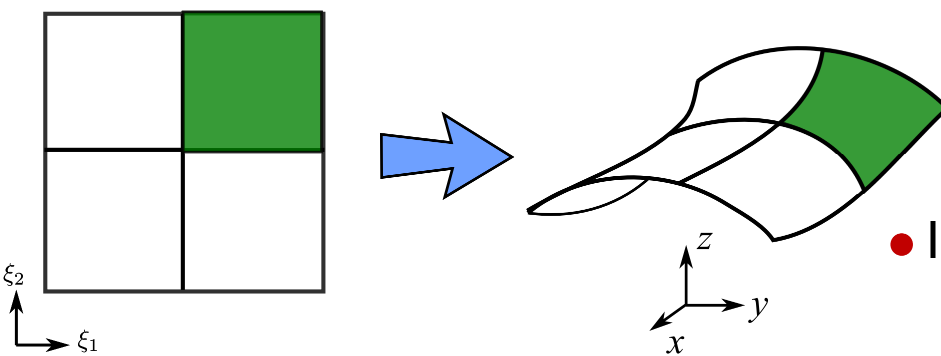


- Meshfree Methods (Belytschko et al. 1994)



- XFEM (Belytschko, et al., 1999)

2. Integrate the CAD and the analysis



- Isogeometric Analysis (Hughes et al., 2005)

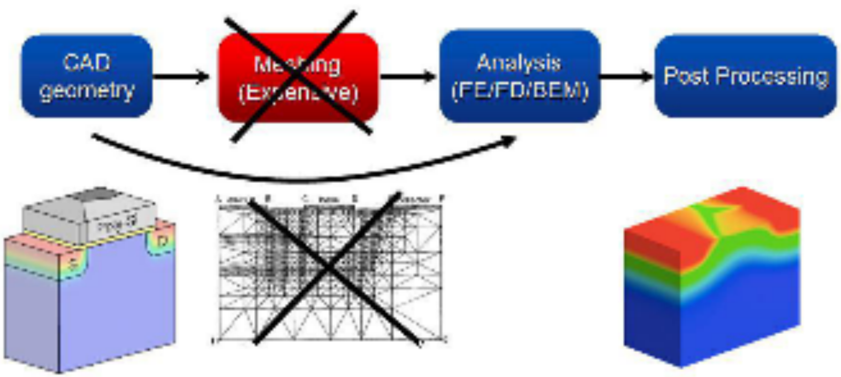
Isogeometric Boundary Element Methods (IGABEM)

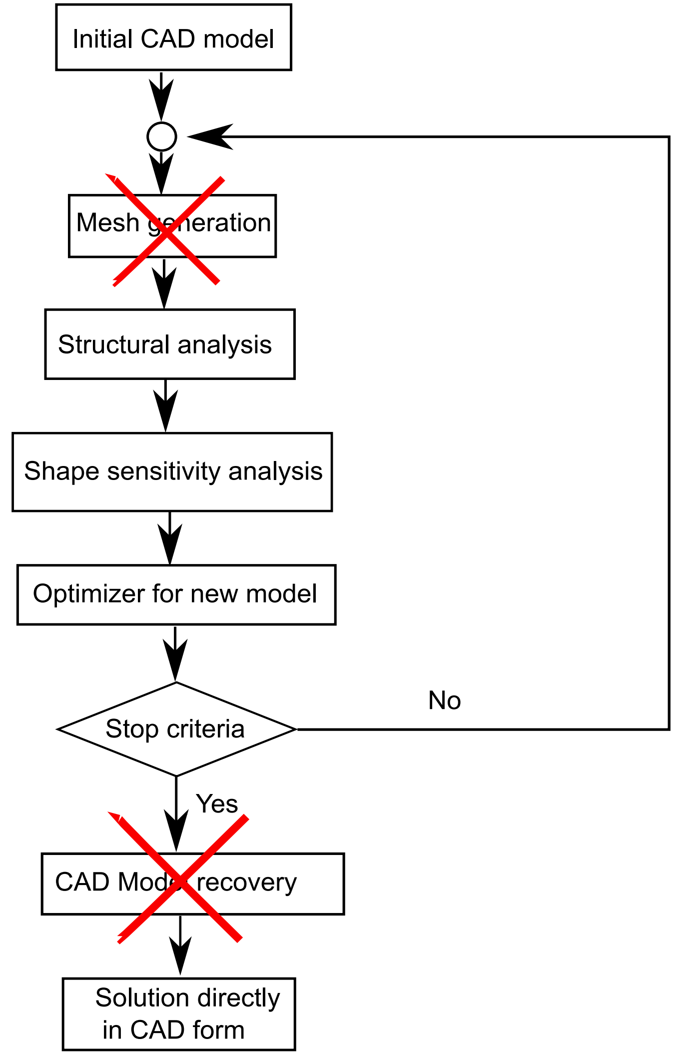
Use the same basis functions in CAD to discretize Boundary Integral Equations (BIE). (Simpson et al., 2010)

1. Seamlessly compatible with CAD due to the boundary representation.

2. No meshing and no CAD model recovery step throughout the optimization.

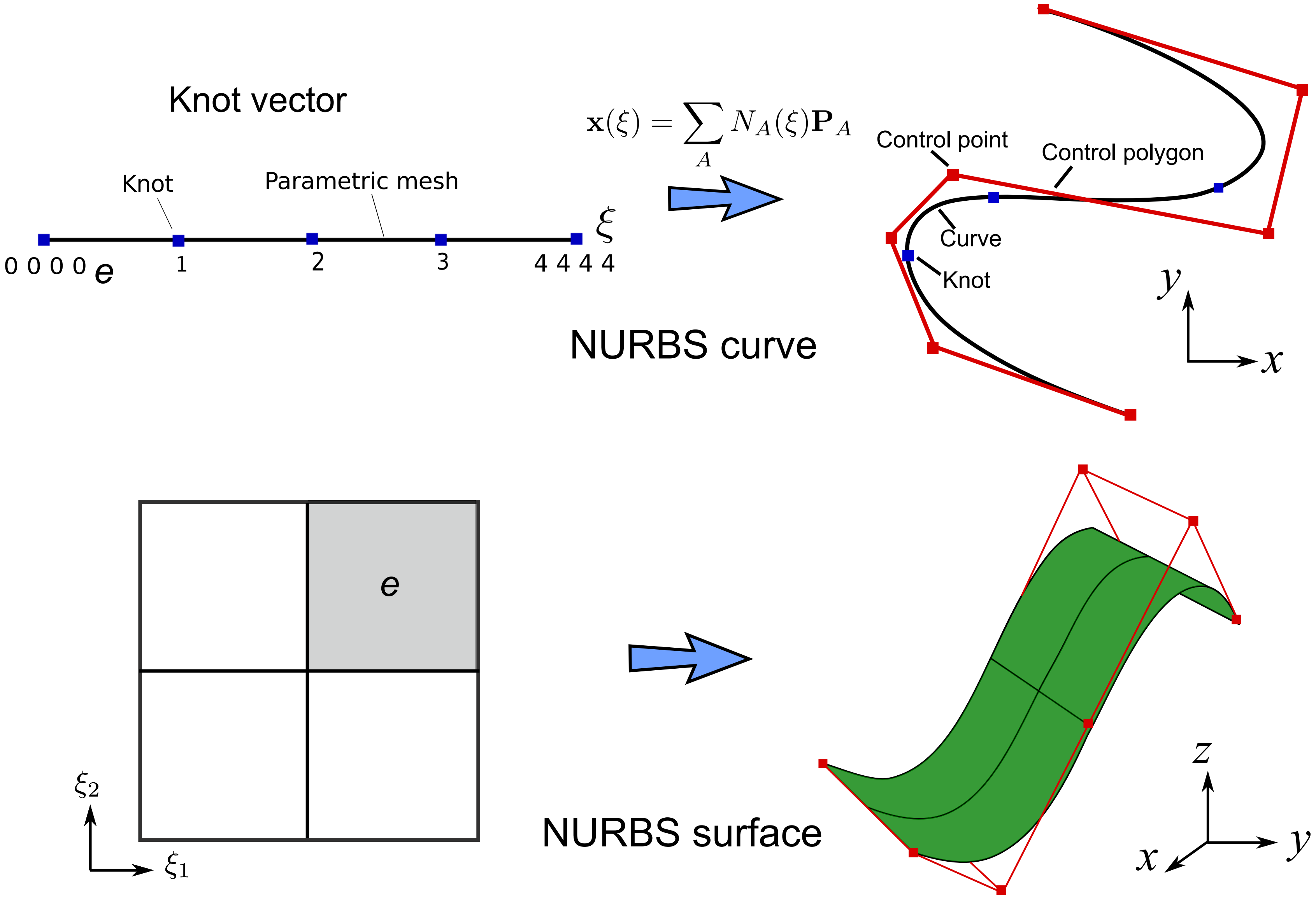
3. The control points can be naturally chosen as design parameters.





IGABEM shape optimization flowchart

B-splines

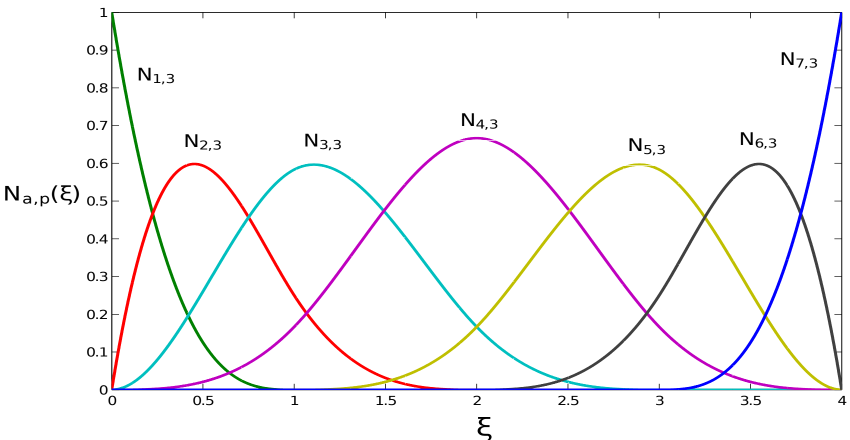

Knot vector
$\mathbf{x}(\xi) = \sum_A N_A(\xi)\mathbf{P}_A$
Knot
Parametric mesh
ξ
0 0 0 0 e
1
2
3
4 4 4 4
Control point
Control polygon
Curve
Knot
y
x
NURBS curve
e
ξ_2
ξ_1
NURBS surface
z
x
y

B-spline Basis

B-spline basis

$$N_{A,0}(\xi) = \begin{cases} 1, & \text{if } \xi_A \leqslant \xi < \xi_{A+1}, \\ 0, & \text{otherwise,} \end{cases}$$

$$N_{A,p}(\xi) = \frac{\xi - \xi_A}{\xi_{A+p} - \xi_A} N_{A,p-1}(\xi) + \frac{\xi_{A+p+1} - \xi}{\xi_{A+p+1} - \xi_{A+1}} N_{A+1,p-1}(\xi).$$



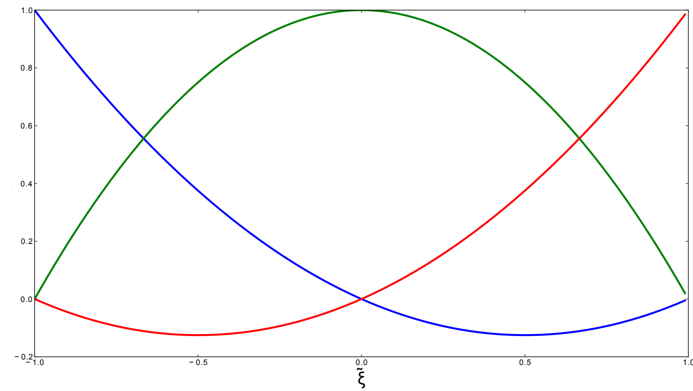


Fig 1. B-spline basis (left) vs quadratic polynomials (right)

Linear independence.
The partition of unity.
Locally supported.
No *Kronecker delta* property.

NURBS

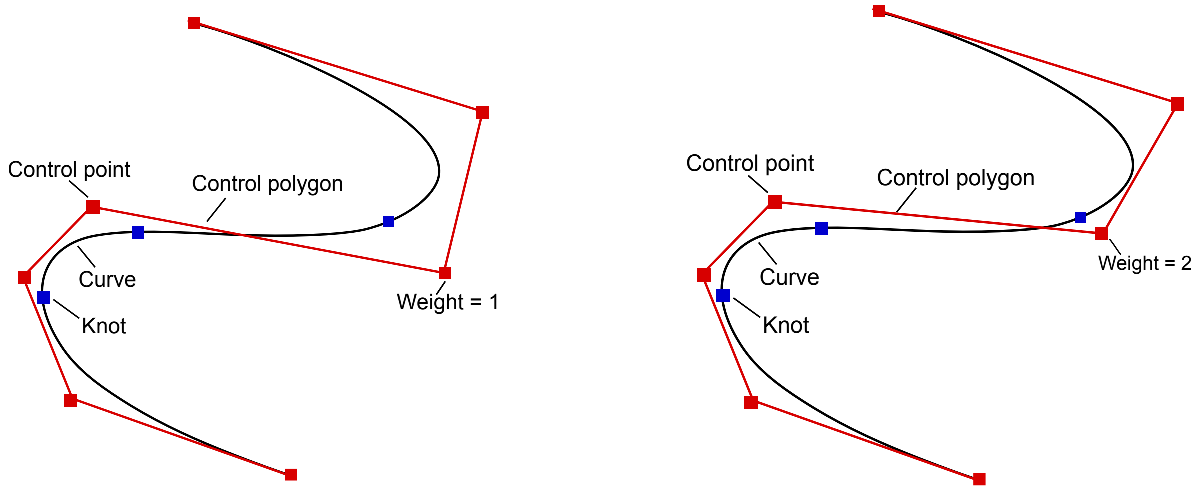
Non-Uniform B-splines (NURBS) is obtained by

$$x(\xi) = \sum_{A=1}^{n} R_{A,p}(\xi)\mathbf{P}_A$$

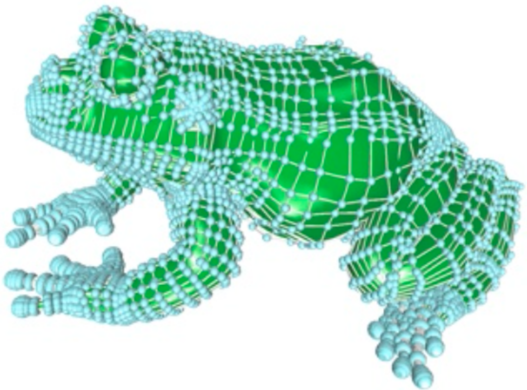
and

$$R_{A,p}(\xi) = \frac{N_{A,p}(\xi)w_A}{\sum_{A=1}^{n} w_A N_{A,p}(\xi)} \quad \text{and} \quad \mathbf{P} = \{x, y, z, w\}^{\mathrm{T}}$$

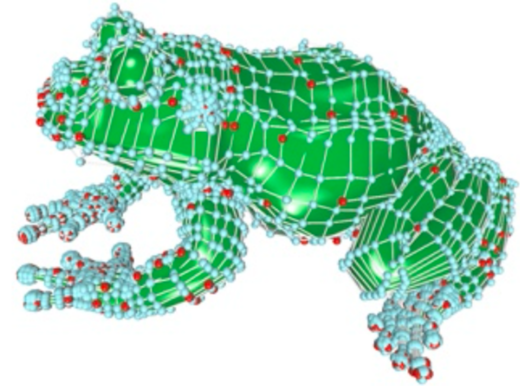
where R is the NURBS basis function, N is the B-spline basis function, and w is the weight.



T-splines



NURBS (source: Rhino3D website)



T-spline (source: Rhino3D website)

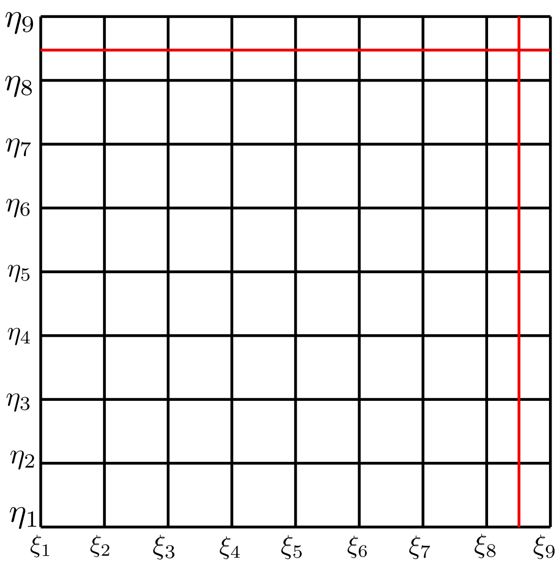


Fig. 1: NURBS mesh topology

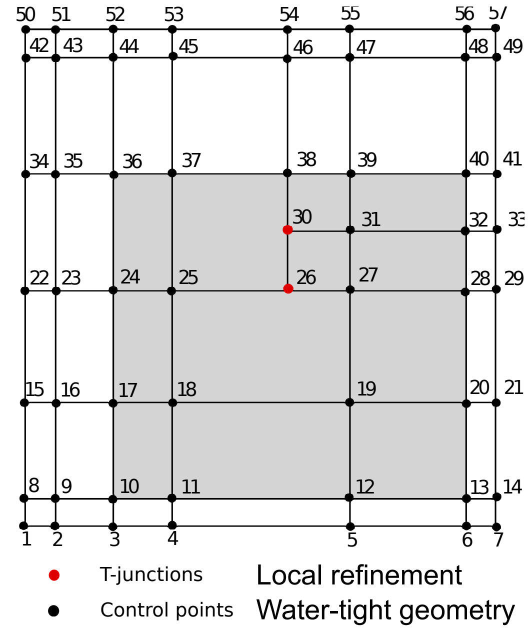


Fig. 2: T-spline mesh topology

Bézier Extraction

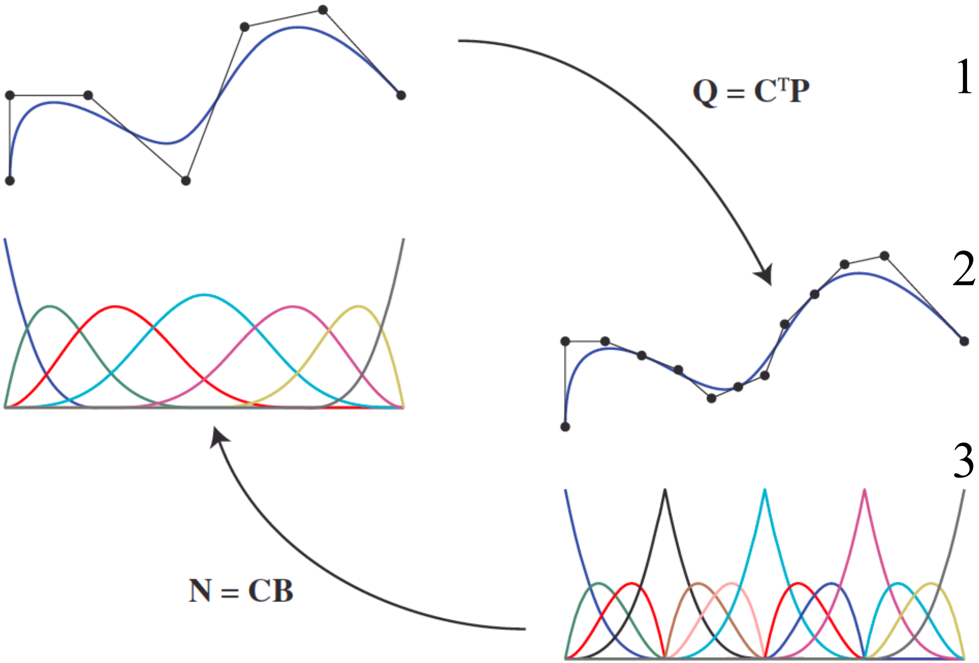
Bézier extraction formulation (Borden, 2011 and Scott, 2012):

N=CB

N: NURBS basis functions.

C: Bézier extraction coefficient matrix (vary from element to element).

B: Bézier basis functions (the same for all elements).



1. Easy to be incorporated into the existing FEM/BEM code.
2. Accelerate basis function evaluation time.
3. The extraction coefficients do NOT depend on the control points.

Fig. 1: Bézier extraction (source: Scott, 2011, IJNME)

IGABEM Formulation

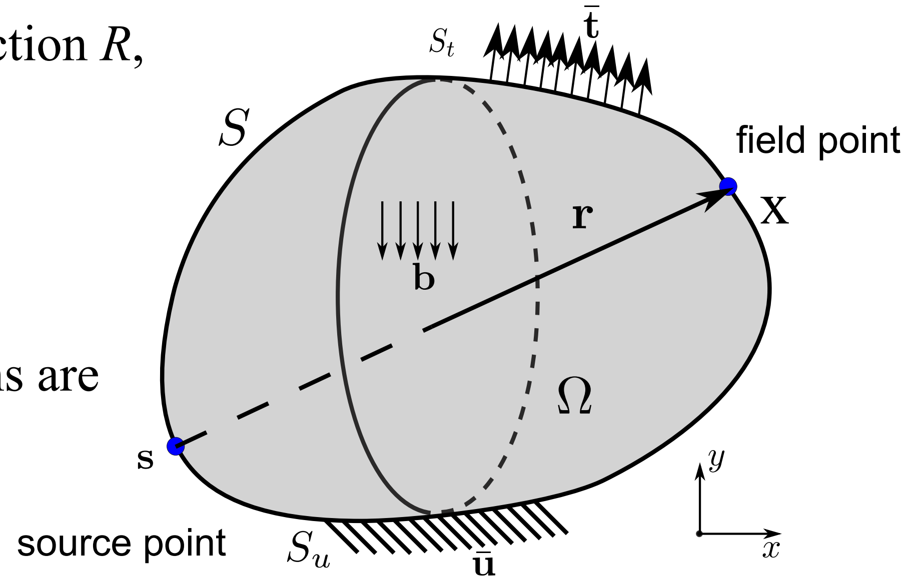
Boundary Integral Equation (BIE)

$$C_{ij}(\mathbf{s})u_j(\mathbf{s}) + \int_S T_{ij}(\mathbf{s},\mathbf{x})u_j(\mathbf{x})\mathrm{d}S(\mathbf{x}) = \int_S U_{ij}(\mathbf{s},\mathbf{x})t_j(\mathbf{x})\mathrm{d}S(\mathbf{x})$$

Discretize BIE with CAD basis function R,

$$u_j(\xi) = R_A(\xi)\tilde{u}_j^A$$

$$t_j(\xi) = R_A(\xi)\tilde{t}_j^A$$



After discretization, matrix equations are

$$\mathbf{Hu} = \mathbf{Gt}$$

The main implementations challenges:

1. Singular Integrals.
2. Jump terms.

Regularized IGABEM Formulation

1. Regularized Boundary Integral Equation

$$\int_S T_{ij}(\mathbf{s},\ \mathbf{x})\left[u_j(\mathbf{x}) - u_j(\mathbf{s})\right]\mathrm{d}S(\mathbf{x}) = \int_S U_{ij}(\mathbf{s},\ \mathbf{x})t_j(\mathbf{x})\,\mathrm{d}S(\mathbf{x})$$

No jump terms.
No strongly singular integrals.
Also available for sensitivity analysis.
Reduce to rigid body motion method.

2. Boundary condition enforcement:

$$\int_{S_u} \mathbf{R}^{\mathrm{T}}\mathbf{u}\mathrm{d}S = \int_{S_u} \mathbf{R}^{\mathrm{T}}\bar{\mathbf{u}}\mathrm{d}S \quad \text{on } S_u,$$

$$\int_{S_t} \mathbf{R}^{\mathrm{T}}\mathbf{t}\mathrm{d}S = \int_{S_t} \mathbf{R}^{\mathrm{T}}\bar{\mathbf{t}}\mathrm{d}S \quad \text{on } S_t,$$

Impose boundary conditions in "weak" sense.

IGABEM Sensitivity Formulation

Recall the regularized BIE:

$$\int_S T_{ij}(\mathbf{s},\ \mathbf{x})\left[u_j(\mathbf{x}) - u_j(\mathbf{s})\right]\mathrm{d}S(\mathbf{x}) = \int_S U_{ij}(\mathbf{s},\ \mathbf{x})t_j(\mathbf{x})\,\mathrm{d}S(\mathbf{x})$$

Direct differentiate the above equation:

$$\begin{aligned}
&\int_S \left\{\dot{T}_{ij}(\mathbf{s},\mathbf{x})\left[u_j(\mathbf{x}) - u_j(\mathbf{s})\right] + T_{ij}(\mathbf{s},\mathbf{x})\left[\dot{u}_j(\mathbf{x}) - \dot{u}_j(\mathbf{s})\right]\right\}\mathrm{d}S(\mathbf{x}) \\
&+ \int_S T_{ij}(\mathbf{s},\mathbf{x})\left[u_j(\mathbf{x}) - u_j(\mathbf{s})\right]\left[\mathrm{d}\dot{S}(\mathbf{x})\right] \\
= \ &\int_S \left[\dot{U}_{ij}(\mathbf{s},\mathbf{x})\,t_j(\mathbf{x}) + U_{ij}(\mathbf{s},\mathbf{x})\,\dot{t}_j(\mathbf{x})\right]\mathrm{d}S(\mathbf{x}) \\
&+ \int_S U_{ij}(\mathbf{s},\mathbf{x})\,t_j(\mathbf{x})\left[\mathrm{d}\dot{S}(\mathbf{x})\right].
\end{aligned}$$

$$\dot{\mathbf{H}}\mathbf{u} + \mathbf{H}\dot{\mathbf{u}} = \dot{\mathbf{G}}\mathbf{t} + \mathbf{G}\dot{\mathbf{t}}$$

Critical points: analytical sensitivities of Green's functions $\dot{T}_{ij}, \dot{U}_{ij}$.

IGABEM Analysis (NURBS)

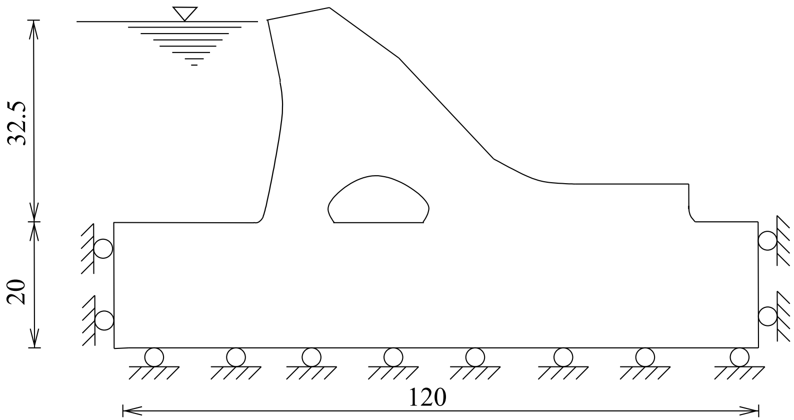


Fig. 1: Problem definition

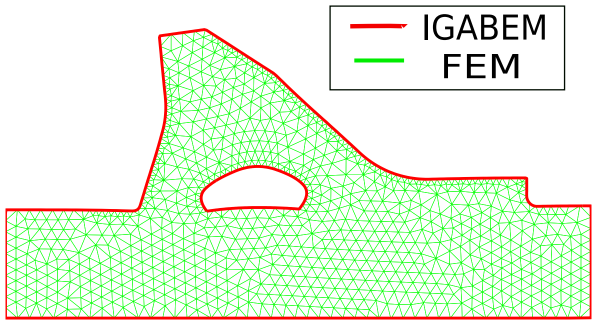


Fig. 2: Deformation

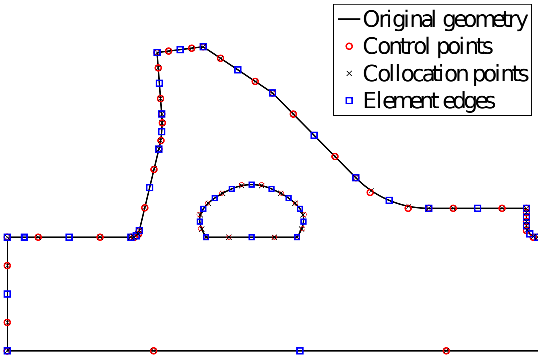


Fig. 3: Geometry description

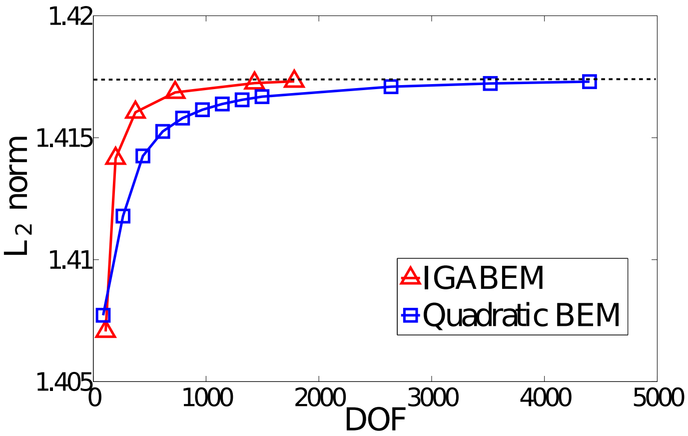


Fig. 4: Convergence study

IGABEM Analysis (T-splines)

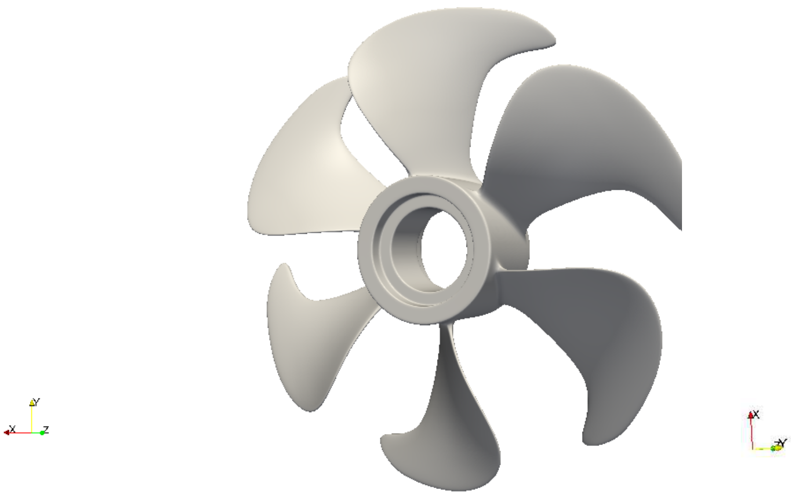


Fig. 1: Propeller CAD model

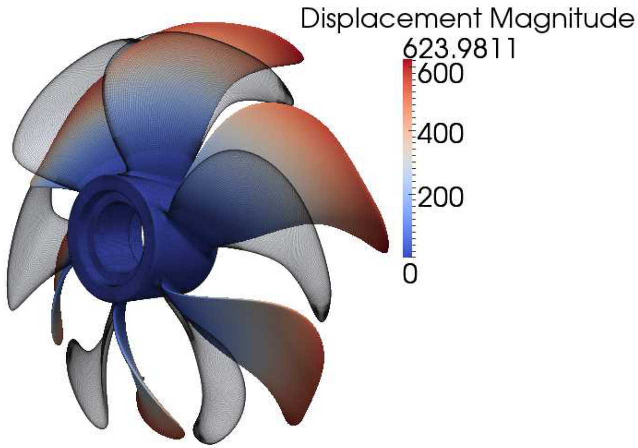


Fig. 2: Displacement

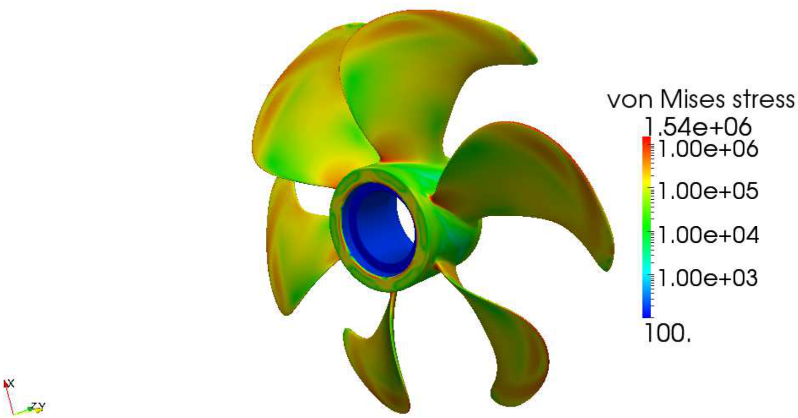


Fig. 3: Stress

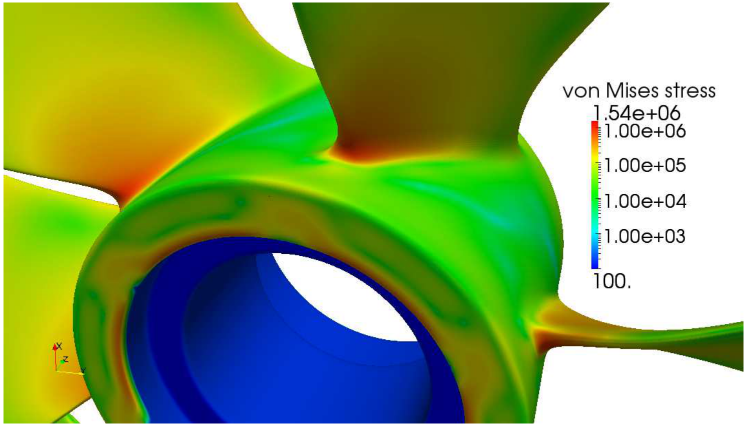
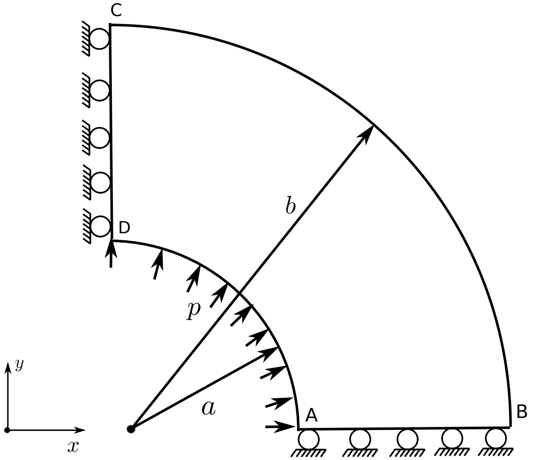
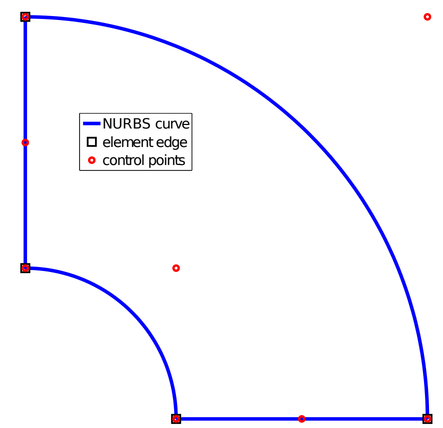


Fig. 4: Stress close-up

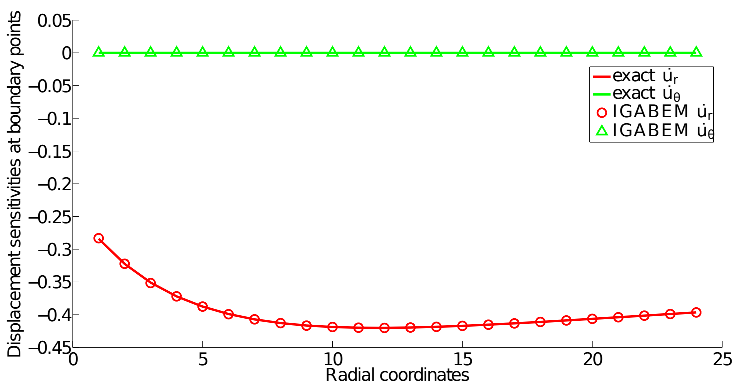
Sensitivity Analysis of Lamé problem



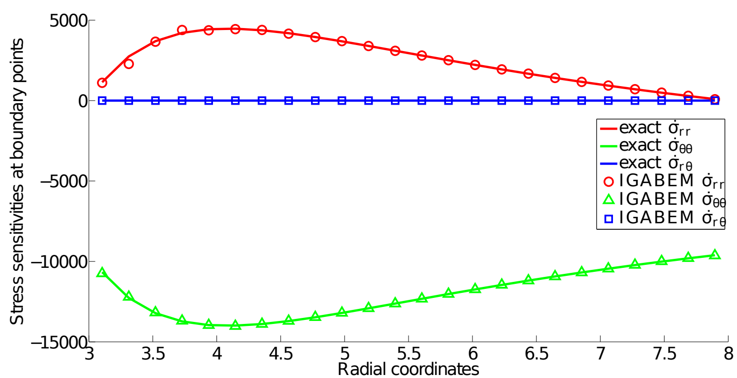
Problem definition



Design model

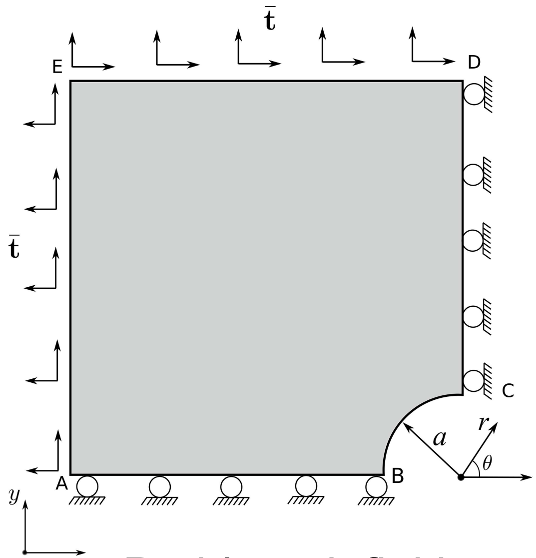


Displacement sensitivities on AB

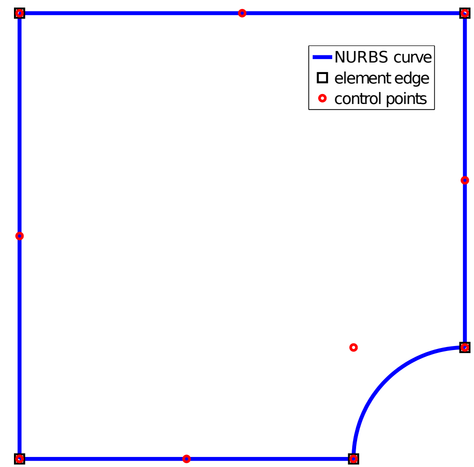


Stress sensitivities on AB

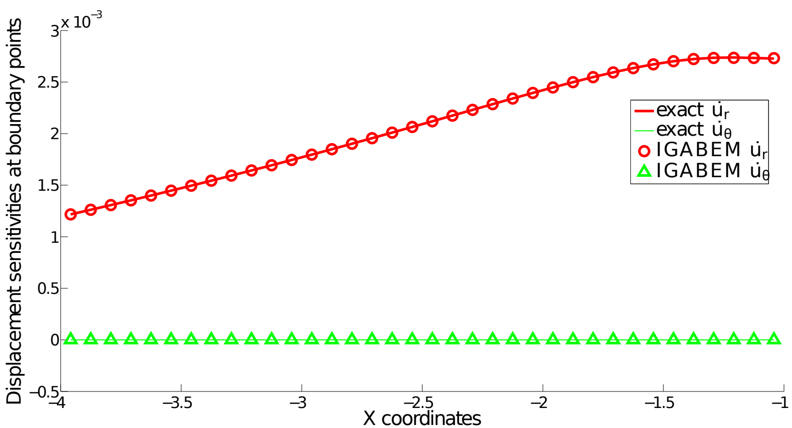
Sensitivity Analysis of Kirsch problem



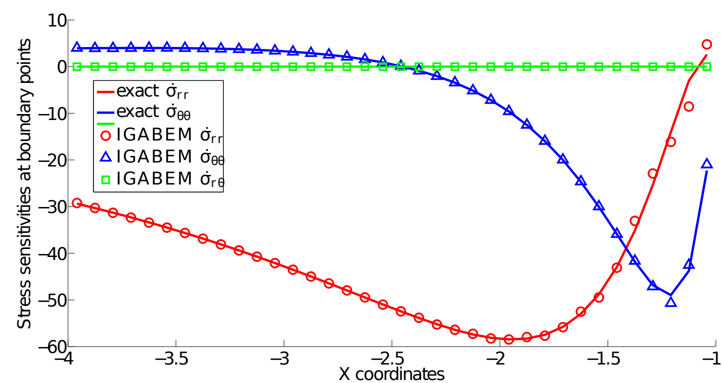
Problem definition



Design model



Displacement sensitivities on AB



Stress sensitivities on AB

Shape Optimization of a 2D Cantilever

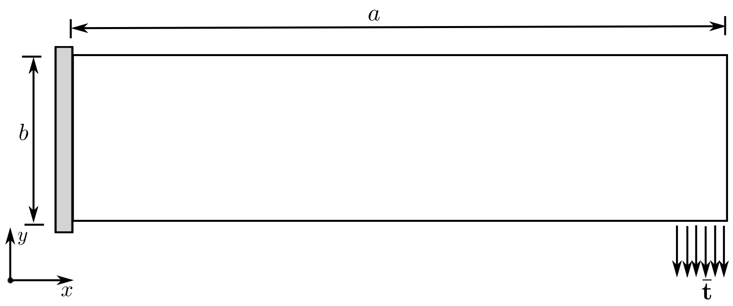


Fig. 1: Problem definition

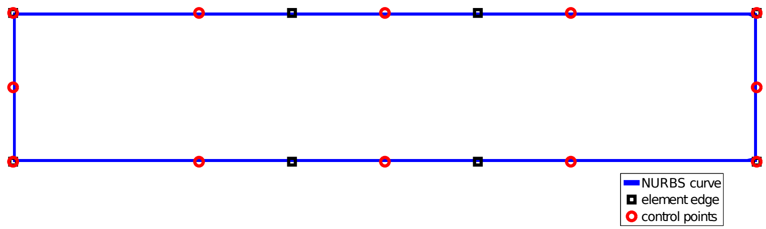


Fig. 2: Design model

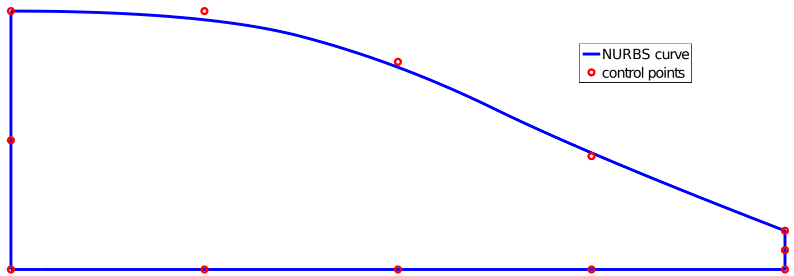


Fig. 3: Optimal model

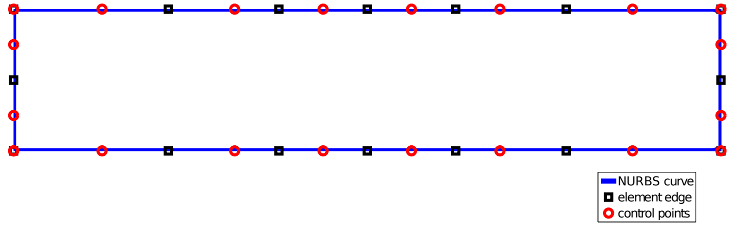


Fig. 4: Analysis model

Shape Optimization of a Fillet

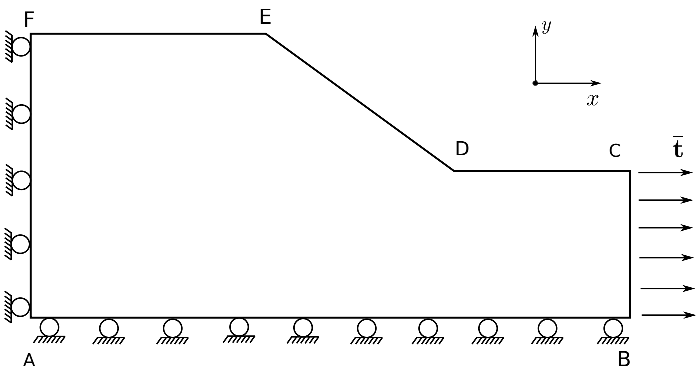


Fig. 1: Problem definition

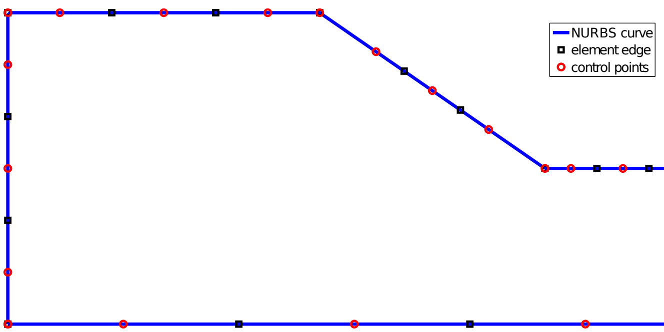


Fig. 2: Design model

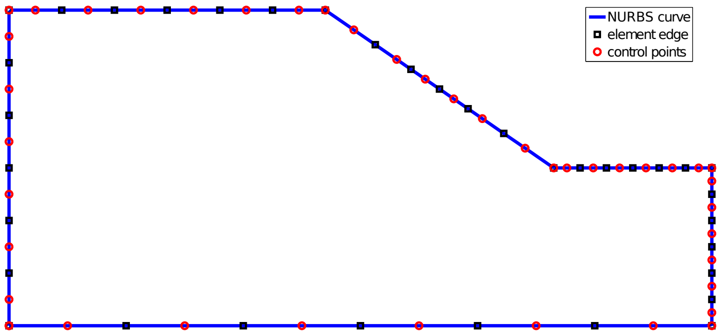


Fig. 3: Analysis model

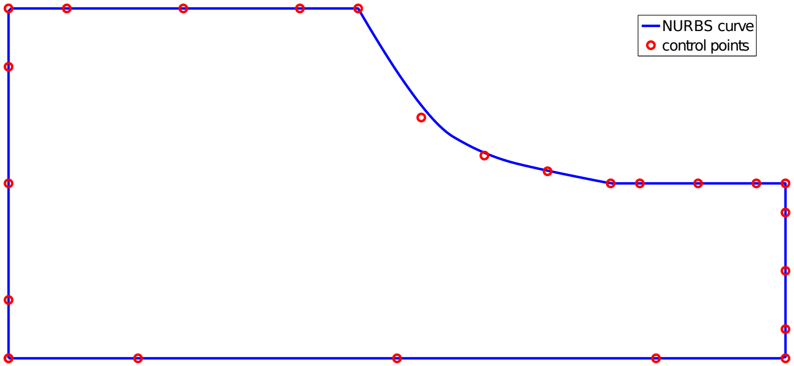


Fig. 4: Optimal models

Shape Optimization of a Connecting Rod

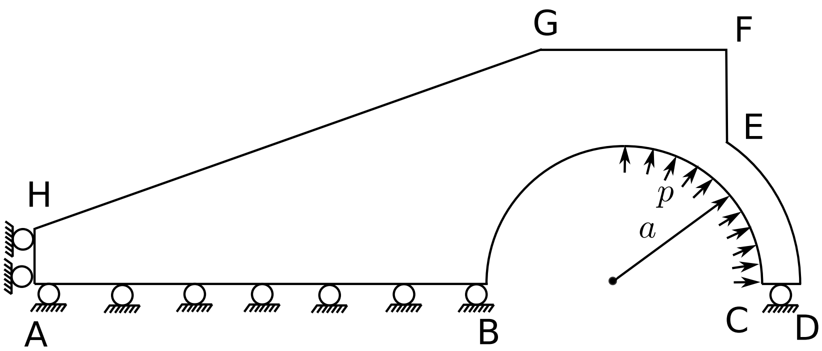


Fig. 1: Problem definition

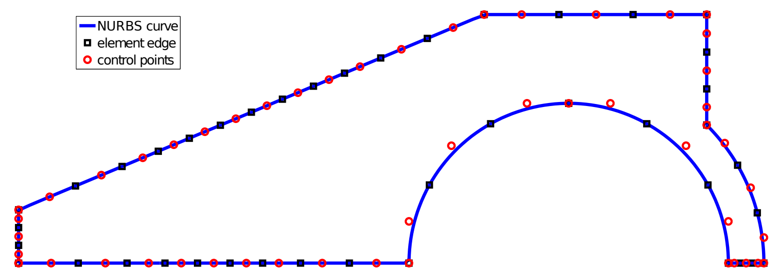


Fig. 2: Analysis model

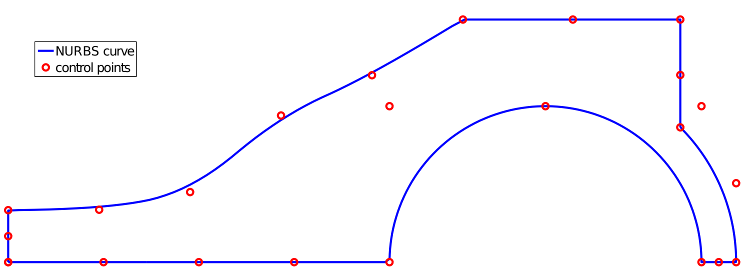


Fig. 3: Optimal shape

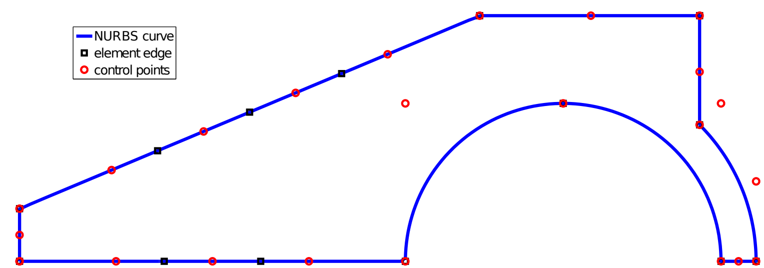
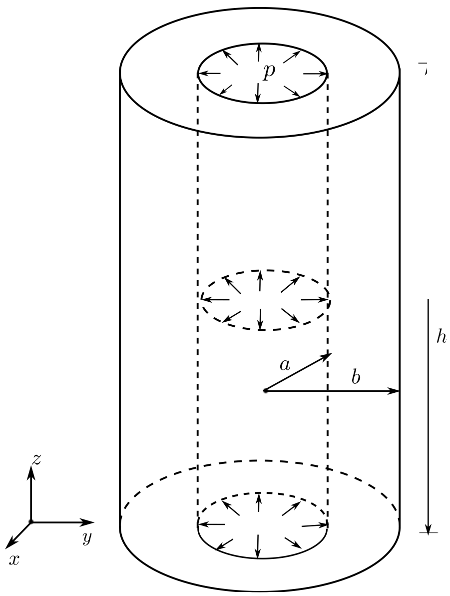
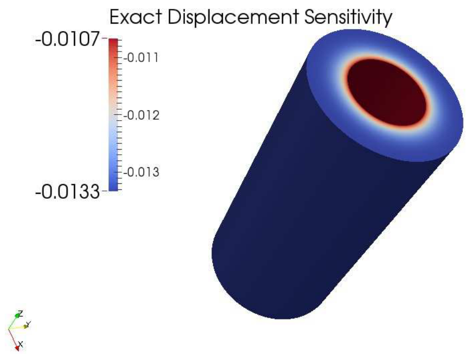


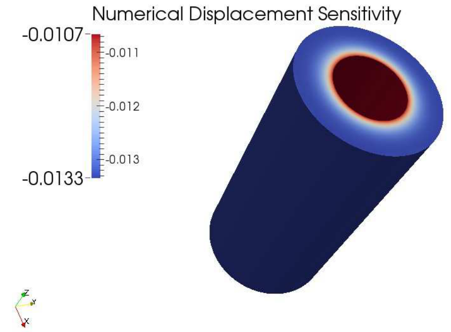
Fig. 4: Design model

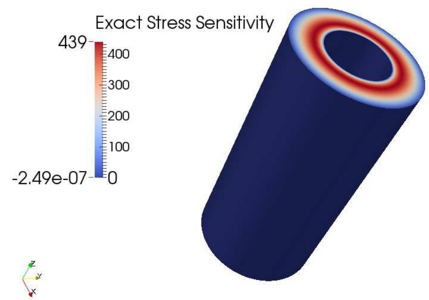
Senstivity Analysis of a Cylinder

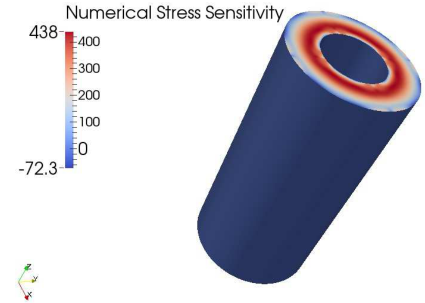


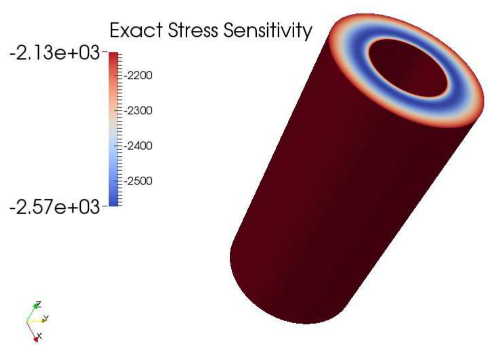
Problem definition

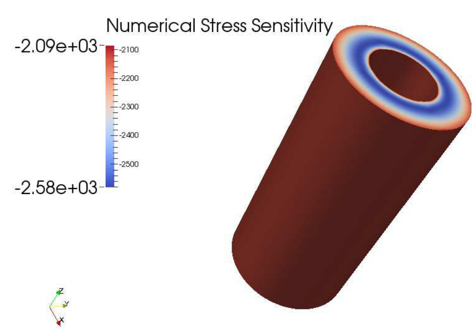




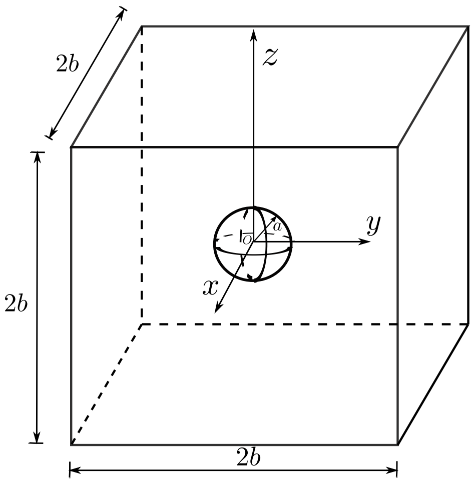




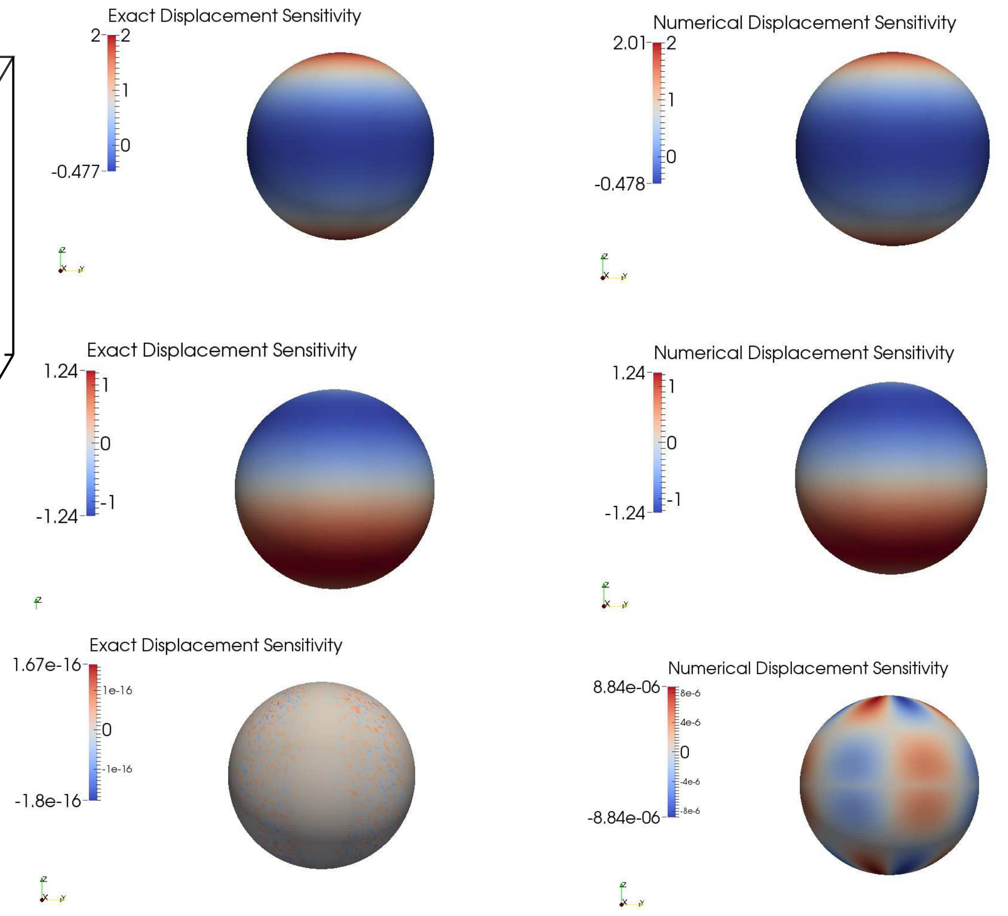




Sensitivity Analysis of Cavity Problem

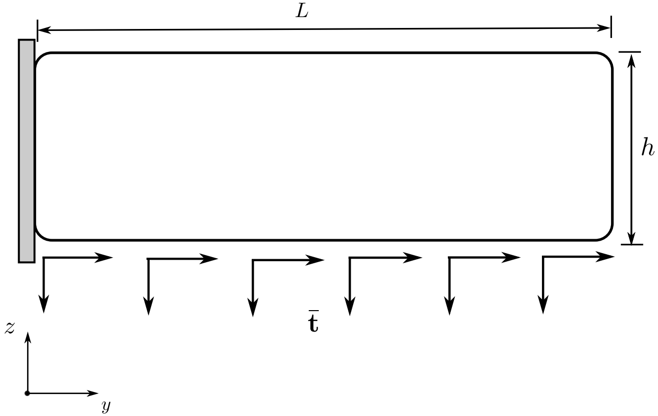


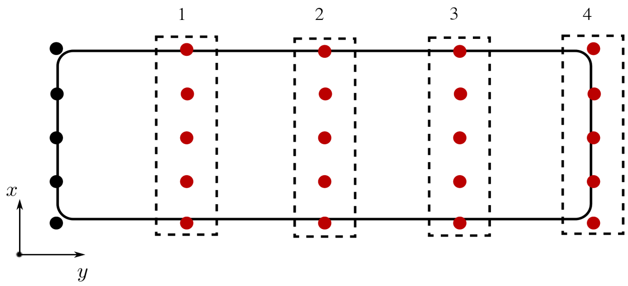
Cavity problem

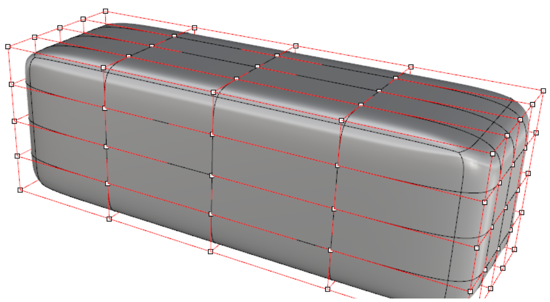


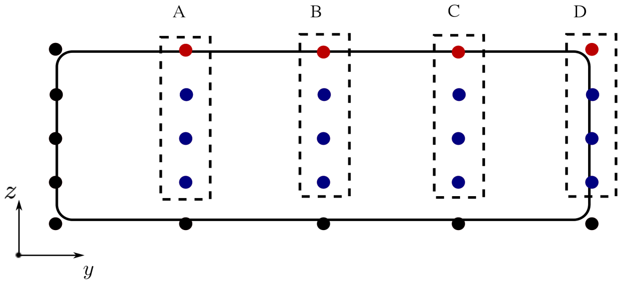
Displacement sensitivity comparison

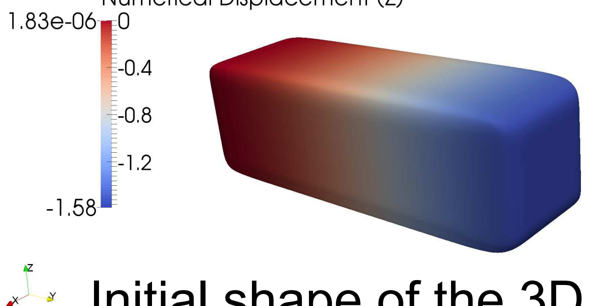
Shape Optimization of a 3D Beam



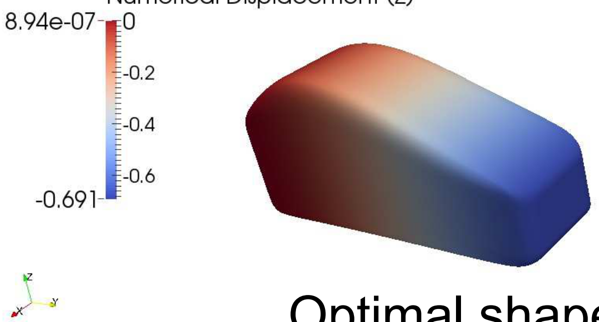








Initial shape of the 3D beam



Optimal shape

Shape Optimization of a Hammer

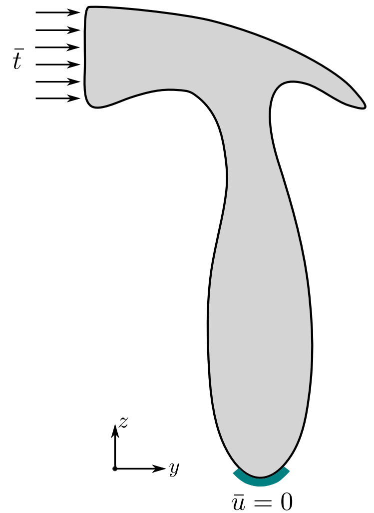


Fig. 1: Problem definition

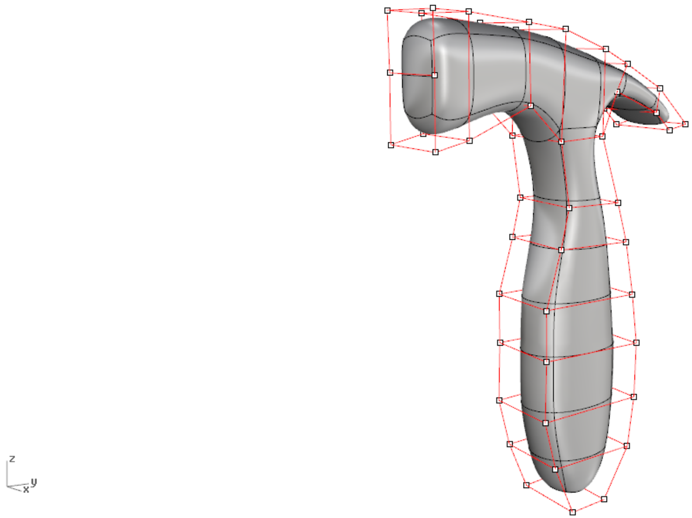


Fig. 2: CAD model

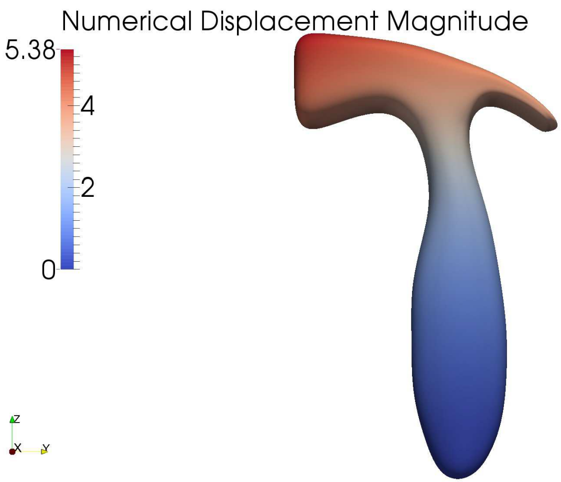


Fig. 3: Initial geometry

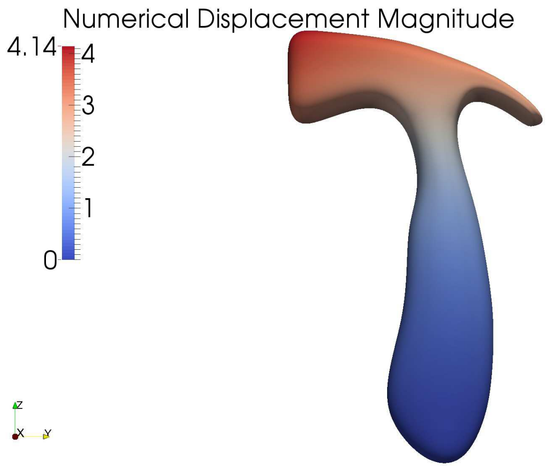


Fig. 4: Optimal geometry

CARDIFF UNIVERSITY PRIFYSGOL CAERDYDD

Shape Optimization of a T-shape component

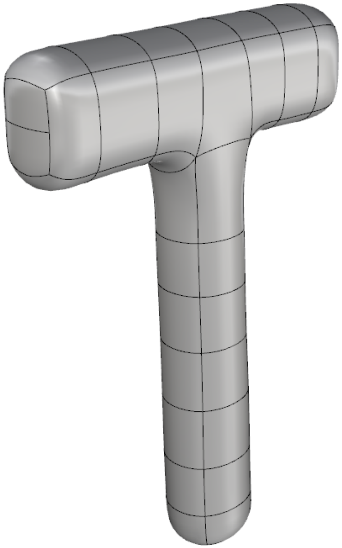
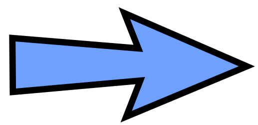


Fig. 1: CAD model



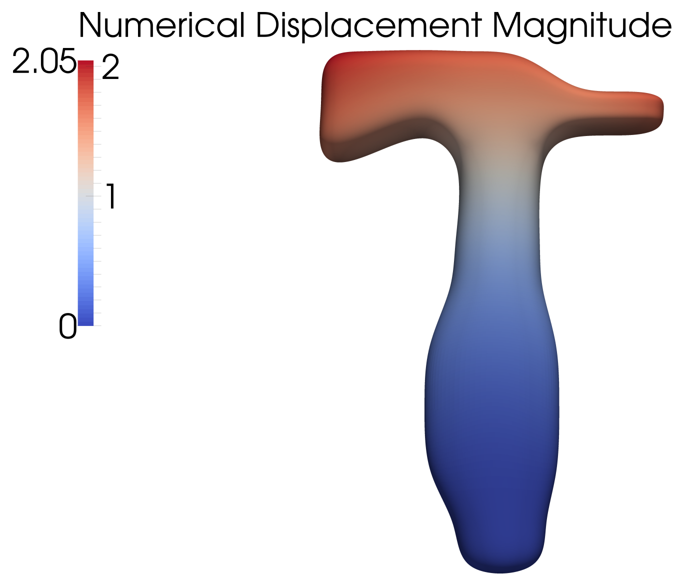


Fig. 3: Optimal geometry

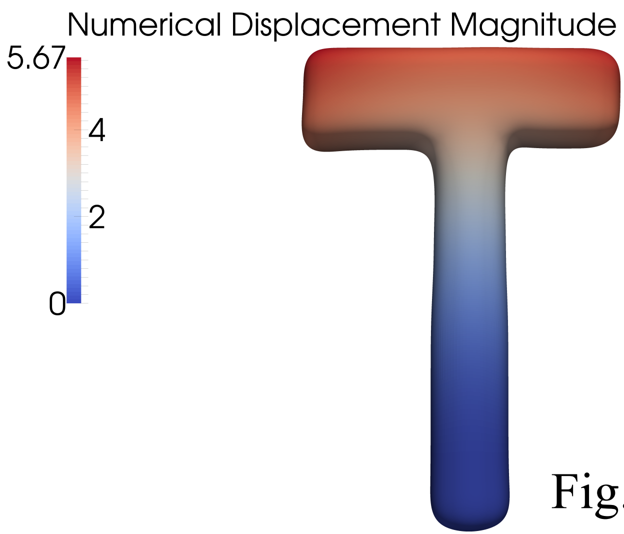


Fig. 2: Initial geometry

Shape Optimization of a Chair

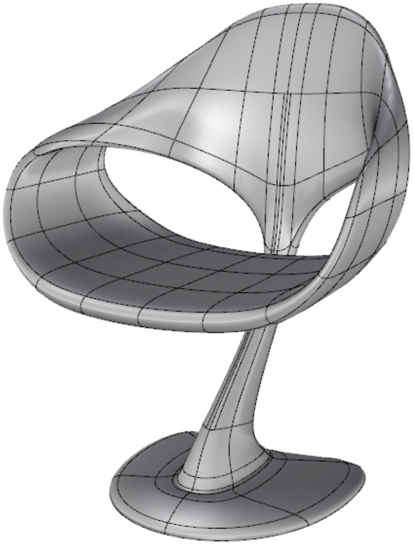


Fig. 1: CAD model

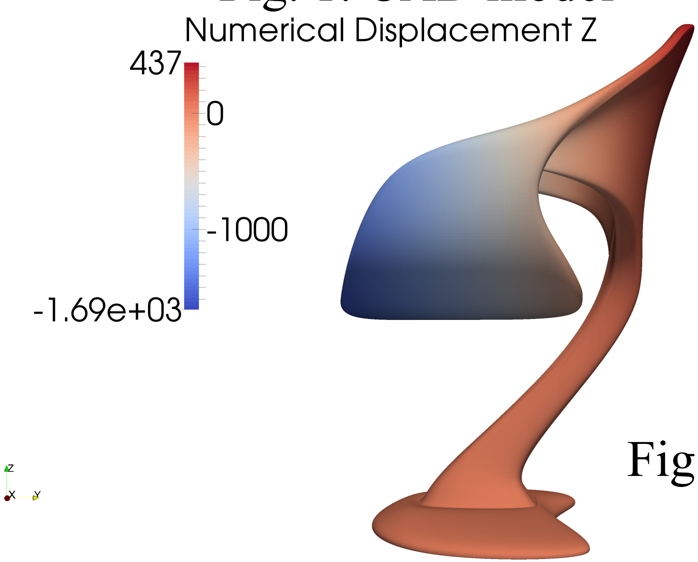


Fig. 2: Inital geometry

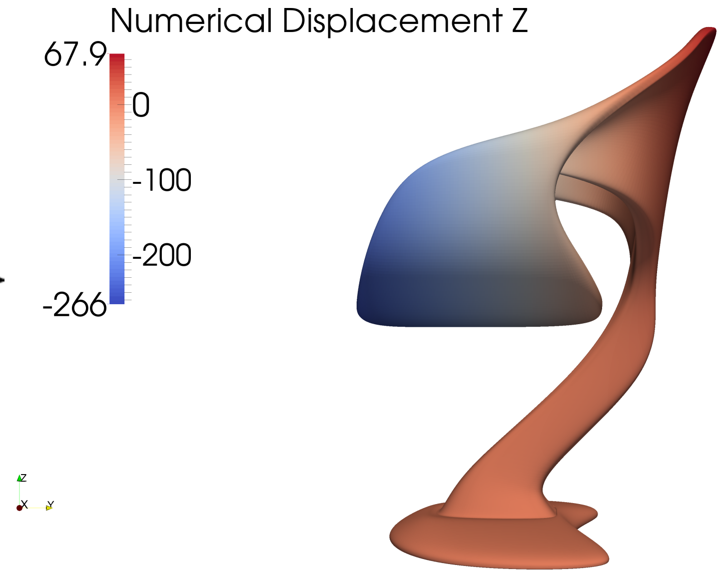


Fig. 3: Optimal geometry

Conclusions and Future Work

A shape optimization scheme using Isogeometric Boundary Element Methods (IGABEM) was presented, which possesses the following advantages:

1. Seamless integration with CAD.

2. No meshing during the steps throughout the iterative steps.

3. The CAD provides a natural parametrization for optimization.

4. NURBS for 2D and T-splines for 3D, so a water-tight geometry and local refinement is guaranteed.

Future work:

1. Acceleration algorithm, to save the memory and time.

2. Application in open domain problem optimization, such as acoustic and electromagnetics shape optimization.


Thank you!