# GROTHENDIECK-TEICHMÜLLER AND BATALIN-VILKOVISKY

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Abstract. It is proven that, for any affine supermanifold $M$ equipped with a constant odd symplectic structure, there is a universal action (up to homotopy) of the Grothendieck-Teichmüller Lie algebra $\mathfrak{grt}_1$ on the set of quantum BV structures (i. e. solutions of the quantum master equation) on $M$.

## 1. Introduction

The Grothendieck-Teichmüller group $GRT_1$ is a pro-unipotent group introduced by Drinfeld in [Dr]; we denote its Lie algebra by $\mathfrak{grt}_1$. It is shown in this paper that, for any affine formal $\mathbb{Z}$-graded manifold $M$ over a field $\mathbb{K}$ equipped with a constant degree 1 symplectic structure $\omega$, there is a universal action (up to homotopy) of the group $GRT_1$ on the set of quantum BV structures on $M$, that is, on the set of solutions, $S$,

$$\hbar\Delta S + \frac{1}{2}\{S, S\} = 0,$$

of the quantum master equation on $M$ (see [Sc] for an introduction into the geometry of the BV formalism). This action is induced by, in general, homotopy non-trivial $L_\infty$ automorphisms of the corresponding dg Lie algebra $(\mathcal{O}_M[[\hbar]], \hbar\Delta, \{\ ,\ \})$, where $\mathcal{O}_M$ is the ring of (formal) smooth functions $M$, and $\mathcal{O}_M[[\hbar]] := \mathcal{O}_M \otimes_{\mathbb{K}} \mathbb{K}[[\hbar]]$.

Our main technical tool is a version of the Kontsevich graph complex, $(\mathsf{GC}_2[[\hbar]], d_\hbar)$ which controls universal deformations of $(\mathcal{O}_M[[\hbar]], \hbar\Delta, \{\ ,\ \})$ in the category of $L_\infty$ algebras. Using the main result of [Wi] we show in Sect. 2 that

$$H^0(\mathsf{GC}_2[[\hbar]], d_\hbar) \simeq \mathfrak{grt}_1.$$

In Sect. 3 we explain how to use this isomorphism of Lie algebras to define a universal homotopy action of $\mathfrak{grt}_1$ on the set of quantum BV structures on any affine odd symplectic manifold $M$.

## 2. A variant of the Kontsevich graph complex

**2.1. From operads to Lie algebras.** Let $\mathcal{P} = \{\mathcal{P}(n)\}_{n\geq 1}$ be an operad in the category of dg vector spaces with the partial compositions $\circ_i : \mathcal{P}(n) \otimes \mathcal{P}(m) \to \mathcal{P}(m+n-1)$, $1 \leq i \leq n$. Then the map

$$\begin{array}{rccc} [\ ,\ ] : & \mathsf{P} \otimes \mathsf{P} & \longrightarrow & \mathsf{P} \\ & (a \in \mathcal{P}(n), b \in \mathcal{P}(m)) & \longrightarrow & [a,b] := \sum_{i=1}^n a \circ_i b - (-1)^{|a||b|} \sum_{i=1}^m b \circ_i a \end{array}$$

makes the vector space $\mathsf{P} := \bigoplus_{n\geq 1} \mathcal{P}(n)$ into a dg Lie algebra [KM]; moreover, the bracket restricts to the subspace of invariants $\mathsf{P}^{\mathbb{S}} := \bigoplus_{n\geq 1} \mathcal{P}(n)^{\mathbb{S}_n}$ making it into a dg Lie algebras as well.

**2.2. An operad of graphs and the Kontsevich graph complex.** For any integers $n \geq 1$ and $l \geq 0$ we denote by $\mathsf{G}_{n,l}$ a set of graphs[1], $\{\Gamma\}$, with $n$ vertices and $l$ edges such that (i) the vertices of $\Gamma$ are labelled by elements of $[n] := \{1, \ldots, n\}$, (ii) the set of edges, $E(\Gamma)$, is totally ordered up to an even permutations. For example, $\overset{1}{\bullet}\!\!-\!\!\!-\!\!\overset{2}{\bullet} \in \mathsf{G}_{2,1}$. The group $\mathbb{Z}_2$ acts freely on $\mathsf{G}_{n,l}$ by changes of the total ordering; its orbit is

[1] A *graph* $\Gamma$ is, by definition, a 1-dimensional $CW$-complex whose 0-cells are called *vertices* and 1-dimensional cells are called *edges*. The set of vertices of $\Gamma$ is denoted by $V(\Gamma)$ and the set of edges by $E(\Gamma)$.

denoted by $\{\Gamma, \Gamma_{opp}\}$. Let $\mathbb{K}\langle \mathsf{G}_{n,l}\rangle$ be the vector space over a field $\mathbb{K}$ spanned by isomorphism classes, $[\Gamma]$, of elements of $\mathsf{G}_{n,l}$ modulo the relation[2] $\Gamma_{opp} = -\Gamma$, and consider a $\mathbb{Z}$-graded $\mathbb{S}_n$-module,

$$\mathsf{Gra}(n) := \bigoplus_{l=0}^{\infty} \mathbb{K}\langle \mathsf{G}_{n,l}\rangle[l].$$

Note that graphs with two or more edges between any fixed pair of vertices do not contribute to $\mathsf{Gra}(n)$ so that we could have assumed right from the beginning that the sets $\mathsf{G}_{n,l}$ do not contain graphs with multiple edges. The $\mathbb{S}$-module, $\mathsf{Gra} := \{\mathsf{Gra}(n)\}_{n\geq 1}$, is naturally an operad with the operadic compositions given by

$$\begin{array}{rccc} \circ_i: & \mathsf{Gra}(n)\otimes \mathsf{Gra}(m) & \longrightarrow & \mathsf{Gra}(m+n-1) \\ & \Gamma_1 \otimes \Gamma_2 & \longrightarrow & \sum_{\Gamma\in \mathsf{G}^i_{\Gamma_1,\Gamma_2}} (-1)^{\sigma_\Gamma}\Gamma \end{array}$$

where $\mathsf{G}^i_{\Gamma_1,\Gamma_2}$ is the subset of $\mathsf{G}_{n+m-1,\#E(\Gamma_1)+\#E(\Gamma_2)}$ consisting of graphs, $\Gamma$, satisfying the condition: the full subgraph of $\Gamma$ spanned by the vertices labeled by the set $\{i, i+1, \ldots, i+m-1\}$ is isomorphic to $\Gamma_2$ and the quotient graph, $\Gamma/\Gamma_2$, obtained by contracting that subgraph to a single vertex, is isomorphic to $\Gamma_1$. The sign $(-1)^{\sigma_\Gamma}$ is determined by the equality

$$\bigwedge_{e\in E(\Gamma)} e = (-1)^{\sigma_\Gamma} \bigwedge_{e'\in E(\Gamma_1)} e' \wedge \bigwedge_{e''\in E(\Gamma_2)} e''.$$

The unique element in $\mathsf{G}_{1,0}$ serves as the unit element in the operad $\mathsf{Gra}$. The associated Lie algebra of $\mathbb{S}$-invariants, $((\mathsf{Gra}\{-2\})^{\mathbb{S}}, [\ ,\ ])$ is denoted, following notations of [Wi], by $\mathsf{fGC}_2$. Its elements can be understood as graphs from $\mathsf{G}_{n,l}$ but with labeling of vertices forgotten, e.g.

$$\bullet\!\!-\!\!\!-\!\!\bullet = \frac{1}{2}\left(\overset{1}{\bullet}\!\!-\!\!\!-\!\!\overset{2}{\bullet} + \overset{2}{\bullet}\!\!-\!\!\!-\!\!\overset{1}{\bullet}\right) \in \mathsf{fGC}_2.$$

It is easy to check that $\bullet\!\!-\!\!\!-\!\!\bullet$ is a Maurer-Cartan element in the Lie algebra $\mathsf{fGC}_2$. Hence we get a dg Lie algebra,

$$(\mathsf{fGC}_2, [\ ,\ ], d := [\bullet\!\!-\!\!\!-\!\!\bullet,\ ])$$

whose dg subalgebra, $\mathsf{GC}_2$, spanned by connected graphs with at least trivalent vertices is called the *Kontsevich graph complex* [Ko1]. We refer to [Wi] for a detailed explanation of why studying the dg Lie subalgebra $\mathsf{GC}_2$ rather than full Lie algebra $\mathsf{fGC}_2$ should be enough for most purposes. The cohomology of $\mathsf{GC}_2$ was partially computed in [Wi].

**2.2.1. Theorem** [Wi]**.** (i) $H^0(\mathsf{GC}_2, d) \simeq \mathfrak{grt}_1$. (ii) *For any negative integer $i$, $H^i(\mathsf{GC}_2, d) = 0$.*

This result implies that the Grothendieck-Teichmüller group $GRT_1$ acts on the set of Poisson structures on an arbitrary manifold.

We shall introduce next a new graph complex which is responsible for the action of $GRT_1$ on the set of quantum master functions on an arbitrary odd symplectic supermanifold.

### 2.3. A variant of the Kontsevich graph complex.

The graph $\circlearrowleft\!\!\bullet$ $\in \mathsf{fGC}_2$ has degree $-1$ and satisfies

$$[\circlearrowleft\!\!\bullet, \circlearrowleft\!\!\bullet] = [\circlearrowleft\!\!\bullet, \bullet\!\!-\!\!\!-\!\!\bullet] = 0.$$

Let $\hbar$ be a formal variable of degree 2 and consider the graph complex $\mathsf{fGC}_2[[\hbar]] := \mathsf{fGC}_2 \otimes \mathbb{K}[[\hbar]]$ with the differential

$$d_\hbar := d + \hbar\Delta, \qquad \text{where} \quad \Delta := [\circlearrowleft\!\!\bullet,\ \ ].$$

The subspace $\mathsf{GC}_2[[\hbar]] \subset \mathsf{fGC}_2[[\hbar]]$ is a subcomplex of $(\mathsf{fGC}_2[[\hbar]], d_\hbar)$.

**2.3.1. Proposition.** $H^0\left(\mathsf{GC}_2[[\hbar]], d_\hbar\right) \simeq \mathfrak{grt}_1$.

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[2] Abusing notations we identify from now an equivalence class $[\Gamma]$ with any of its representative $\Gamma$.

*Proof.* Consider a decreasing filtration of $\mathsf{GC}_2[[\hbar]]$ by the powers in $\hbar$. The first term of the associated spectral sequence is

$$\mathcal{E}_1 = \bigoplus_{i\in\mathbb{Z}} \mathcal{E}_1^i, \qquad \mathcal{E}_1^i = \bigoplus_{p\geq 0} H^{i-2p}(\mathsf{GC}_2, d)\hbar^p$$

with the differential equal to $\hbar\Delta$. As $H^0(\mathsf{GC}_2, d) \simeq \mathfrak{grt}_1$ and $H^{\leq -1}(\mathsf{GC}_2, d) = 0$, we get the desired result. □

**2.4. Remark.** Let $\sigma$ be an element of $\mathfrak{grt}_1$ and let $\Gamma_\sigma^{(0)}$ be any cycle representing the cohomology class $\sigma$ in the graph complex $(\mathsf{GC}_2, d)$. Then one can construct a cycle,

$$\Gamma_\sigma^\hbar = \Gamma_\sigma^{(0)} + \Gamma_\sigma^{(1)}\hbar + \Gamma_\sigma^{(2)}\hbar^2 + \Gamma_\sigma^{(3)}\hbar^3 + \dots, \tag{1}$$

representing the cohomology class $\sigma \in \mathfrak{grt}_1$ in the complex $(\mathsf{GC}_2[[\hbar]], d_\hbar)$ by the following induction:

*1st step*: As $d\Gamma_\sigma^{(0)} = 0$, we have $d(\Delta\Gamma_\sigma^{(0)}) = 0$. As $H^{-1}(\mathsf{GC}_2, d) = 0$, there exists $\Gamma_\sigma^{(1)}$ of degree $-2$ such that $\Delta\Gamma_\sigma^{(0)} = -d\Gamma_\sigma^{(1)}$ and hence

$$(d + \hbar\Delta)\left(\Gamma_\sigma^{(0)} + \Gamma_\sigma^{(1)}\hbar\right) = 0 \bmod O(\hbar^2).$$

*n-th step*: Assume we have constructed a polynomial $\sum_{i=1}^n \Gamma_\sigma^{(i)}\hbar^i$ such that

$$(d + \hbar\Delta)\sum_{i=1}^n \Gamma_\sigma^{(i)}\hbar^i = 0 \bmod O(\hbar^{n+1}).$$

Then $d(\Delta\Gamma_\sigma^{(n)}) = 0$, and, as $H^{-2n-1}(\mathsf{GC}_2, d) = 0$, there exists a graph $\Gamma_\sigma^{(n+1)}$ in $\mathsf{GC}_2$ of degree $-2n-2$ such that $\Delta\Gamma_\sigma^{(n)}) = -d\Gamma_\sigma^{(n+1)}$. Hence $(d + \hbar\Delta)\sum_{i=1}^{n+1} \Gamma_\sigma^{(i)}\hbar^i = 0 \bmod O(\hbar^{n+2})$.

It is these $\hbar$-dependent additions to $\Gamma_\sigma^{(0)}$ in $\Gamma_\sigma^\hbar$ which make the action of $GRT_1$ on quantum master functions different from its action on Poisson structures.

## 3. Quantum BV structures on odd symplectic manifolds

**3.1. On $L_\infty$ automorphisms of $L_\infty$ algebras.** Let $\mathfrak{g} = \oplus_{\imath\in\mathbb{Z}}\mathfrak{g}^i$ be a $\mathbb{Z}$-graded Lie algebra admitting a bounded above complete Hausdorff filtration, $\mathfrak{g}^i = \mathfrak{g}^i_{(0)} \supset \mathfrak{g}^i_{(1)} \supset \mathfrak{g}^i_{(2)} \supset \dots$, and let $\mathcal{MC}(\mathfrak{g}) := \left\{\gamma \in \mathfrak{g}^1 \,|[\gamma,\gamma] = 0\right\}$ be the associated set of Maurer-Cartan elements. The group $G_{(1)} := \exp \mathfrak{g}^0_{(1)}$ acts naturally on $\mathcal{MC}(\mathfrak{g})$,

$$\begin{array}{ccc} G^0_{(1)} \times \mathcal{MC}(\mathfrak{g}) & \longrightarrow & \mathcal{MC}(\mathfrak{g}) \\ (e^g, \gamma) & \longrightarrow & e^{-g}\gamma e^g. \end{array}$$

We are interested in the particular case when $\mathfrak{g}$ is the Lie algebra, $CE^\bullet(V,V)$, of coderivations

$$CE^\bullet(V,V) = \left(\mathrm{Coder}(\odot^{\bullet\geq 1}(V[2])), [\ ,\ ]\right), \quad CE^\bullet(V,V)_{(m)} := \mathrm{Hom}(\odot^{\bullet\geq m+1}(V[2]), V[2]),$$

of the standard graded co-commutative coalgebra, $\odot^{\bullet\geq 1}(V[2])$, co-generated by a vector space $V$. Then the set $\mathcal{MC}(CE^\bullet(V,V))$ can be identified with the set of $L_\infty$ structures[3] on the space $V$. Any element $\gamma \in CE^\bullet(V,V)$ defines a differential, $d_\gamma := [\gamma,\ ]$ in $CE^\bullet(V,V)$, and any element $g \in \mathsf{Ker}\, d_\gamma \cap CE^0(V,V)_{(1)}$ defines an automorphism of $\mathcal{MC}(CE^\bullet(V,V))$ which leaves the point $\gamma$ invariant. Such an element $e^g$ can be interpreted as a $L_\infty$ automorphism of the $L_\infty$ algebra $(V,\gamma)$. Moreover, if the cohomology class of $g$ in $H(CE^\bullet(V,V), d_\gamma)$ is non-trivial, then this $L_\infty$ automorphism is obviously homotopy non-trivial.

[3]In our grading conventions the degree of $n$-th $L_\infty$ operation on $V$ is equal to $3-2n$.

**3.2. Quantum BV manifolds.** Let $M$ be a (formal) $\mathbb{Z}$-graded manifold equipped with an odd symplectic structure $\omega$ (of degree 1). There always exist so called Darboux coordinates, $(x^a, \psi_a)_{1\leq a\leq n}$, on $M$ such that $|\psi_a| = -|x^a| + 1$ and $\omega = \sum_a dx^a \wedge d\psi_a$. The odd symplectic structure makes, in the obvious way, the structure sheaf $\mathcal{O}_M$ into a Lie algebra with brackets, $\{\ ,\ \}$, of degree $-1$. A less obvious fact is that $\omega$ induces a degree $-1$ differential operator, $\Delta_\omega$, on the invertable sheaf of semidensities, $Ber(M)^{\frac{1}{2}}$ [Kh]. Any choice of a Darboux coordinate system on $M$ defines an associated trivialization of the sheaf $Ber(M)^{\frac{1}{2}}$; if one denotes the associated basis section of $Ber(M)^{\frac{1}{2}}$ by $D_{x,\psi}$, then any semidensity $D$ is of the form $f(x,\psi)D_{x,\psi}$ for some smooth function $f(x,\psi)$, and the operator $\Delta_\omega$ is given by

$$\Delta_\omega\left(f(x,\psi)D_{x,\psi}\right) = \sum_{a=1}^{n} \frac{\partial^2 f}{\partial x^a \partial \psi_a} D_{x,\psi}.$$

Let $\hbar$ be a formal parameter of degree 2. A *quantum master function* on $M$ is an $\hbar$-dependent semidensity $D$ which satisfies the equation

$$\Delta_\omega D = 0$$

and which admits, in some Darboux coordinate system, a form

$$D = e^{\frac{S}{\hbar}} D_{x,\psi},$$

for some $S \in \mathcal{O}_M[[\hbar]]$ of total degree 2. In the literature it is this formal power series in $\hbar$ which is often called a quantum master function. Let us denote the set of all quantum master functions on $M$ by $\mathcal{QM}(M)$. It is easy to check that the equation $\Delta_\omega D = 0$ is equivalent to the following one,

$$\hbar\Delta S + \frac{1}{2}\{S,S\} = 0, \tag{2}$$

where $\Delta := \sum_{a=1}^n \frac{\partial^2}{\partial x^a \partial \psi_a}$. This equation is often called the *quantum master equation*, while a triple $(M,\omega,S\in\mathcal{QM}(M))$ a *quantum BV manifold*.

Let us assume from now on that a particular Darboux coordinate system is fixed on $M$ up to affine transformations[4].

**3.3. An action of $GRT_1$ on quantum master functions.** The vector space $V := \mathcal{O}_M[[\hbar]]$ is a dg Lie algebra with the differential $\hbar\Delta$ and the Lie brackets $\{\ ,\ \}$. These data define a Maurer-Cartan element, $\gamma_{\mathcal{QM}} := \hbar\Delta \oplus \{\ ,\ \}$ in the Lie algebra $CE^\bullet(V,V)$.

The constant odd symplectic structure on $M$ makes $V$ into a representation,

$$\begin{array}{rccc} \rho: & \mathsf{Gra}(n) & \longrightarrow & \mathsf{End}_V(n) = \mathrm{Hom}(V^{\otimes n}, V) \\ & \Gamma & \longrightarrow & \Phi_\Gamma \end{array}$$

of the operad $\mathsf{Gra} := \{\mathsf{Gra}(n)\}_{n\geq 1}$ as follows:

$$\Phi_\Gamma(S_1,\ldots,S_n) := \pi\left(\prod_{e\in E(\Gamma)} \Delta_e\left(S_1(x_{(1)},\psi_{(1)},\hbar)\otimes S_2(x_{(2)},\psi_{(2)},\hbar)\otimes\ldots\otimes S_n(x_{(n)},\psi_{(n)},\hbar)\right)\right)$$

where, for an edge $e$ connecting vertices labeled by integers $i$ and $j$,

$$\Delta_e = \sum_{a=1}^{n} \frac{\partial}{\partial x^a_{(i)}} \otimes \frac{\partial}{\partial \psi_{(j)a}} + \frac{\partial}{\psi_{(i)a}} \otimes \frac{\partial}{\partial x^a_{(j)}}$$

and $\pi$ is the multiplication map,

$$\begin{array}{rccc} \pi: & V^{\otimes n} & \longrightarrow & V \\ & S_1\otimes S_2\otimes\ldots\otimes S_n & \longrightarrow & S_1 S_2\cdots S_n. \end{array}$$

[4]This is not a serious loss of generality as any quantum master equation can be represented in the form (2). Our action of $GRT_1$ on $\mathcal{QM}(M)$ depends on the choice of an affine structure on $M$ in exactly the same way as the classical Kontsevich's formula for a universal formality map [Ko2] depends on such a choice. A choice of an appropriate affine connection on $M$ and methods of the paper [Do] can make our formulae for the $GRT_1$ action invariant under the group of symplectomorphsims of $(M,\omega)$; we do not address this *globalization* issue in the present note.

This representation induces in turn a morphism of dg Lie algebras,

$$(\mathsf{GC}_2[[\hbar]],[\ ,\ ],d_h) \longrightarrow (CE^\bullet(V,V),[\ ,\ ],\delta:=[\gamma_{\mathcal{QM}},\ ])\,,$$

and hence a morphism of their cohomology groups,

$$\mathfrak{grt}_1 \simeq H^0\left(\mathsf{GC}_2[[\hbar]],d_\hbar\right) \longrightarrow H^0\left(CE^\bullet(V,V),\delta\right).$$

Let $\sigma$ be an arbitrary element in $\mathfrak{grt}_1$ and let $\Gamma^\hbar_\sigma$ be a cycle representing $\sigma$ in the graph complex $(\mathsf{GC}_2[[\hbar]],d_\hbar)$. For any $u\in\mathbb{R}$ the adjoint action of $e^{u\Phi_{\Gamma^\hbar_\sigma}}\in G_{(1)}$ on $CE^\bullet(V,V)$ (see §**3.1**) can be interpreted as a $L_\infty$ automorphism,

$$F^\sigma=\{F^\sigma_n:\odot^n V\longrightarrow V[2-2n]\}_{n\geq 1}\,,$$

of the dg Lie algebra $(V,\hbar\Delta,\{\ ,\ \})$ with $F^\sigma_1=\mathrm{Id}$. Hence, for any real analytic quantum master function $S\in\mathcal{QM}(M)$ and sufficiently small $u\in\mathbb{R}^+$, the series

$$S^\sigma := S+\sum_{n\geq 2}\frac{1}{n!}F^\sigma_n(S,\ldots,S)$$

if convergent, gives again a quantum master function. This is the acclaimed homotopy action of $GRT_1$ on $\mathcal{QM}(M)$ for any affine odd symplectic manifold $M$.

**3.4. Remark.** In QFT one often works with quantum master functions $S$ which are formal power series in the Darboux coordinates rather than real analytic functions. In that case one should view $u$ as a degree 0 formal parameter so that the adjoint action of $e^{u\Phi_{\Gamma^\hbar_\sigma}}\in G_{(1)}$ on $C^\bullet(V,V)[[u]]$ gives a continuous (in the adic topology) $L_\infty$ automorphism of of the dg Lie algebra $(V[[u]],\hbar\Delta,\{\ ,\ \})$, and hence induces a transformation of master functions from $\mathcal{O}_M[[u,\hbar]]$.

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