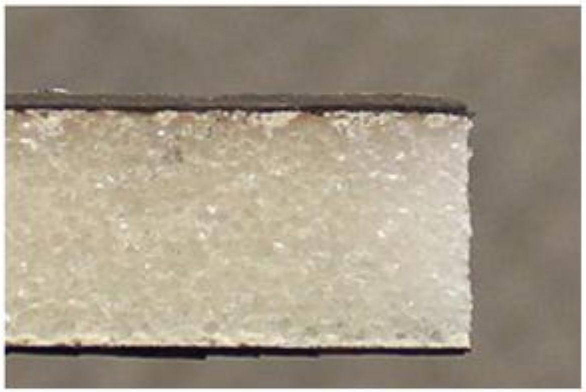
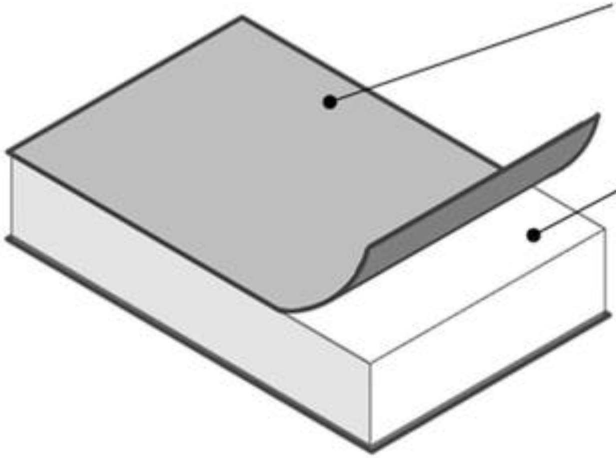
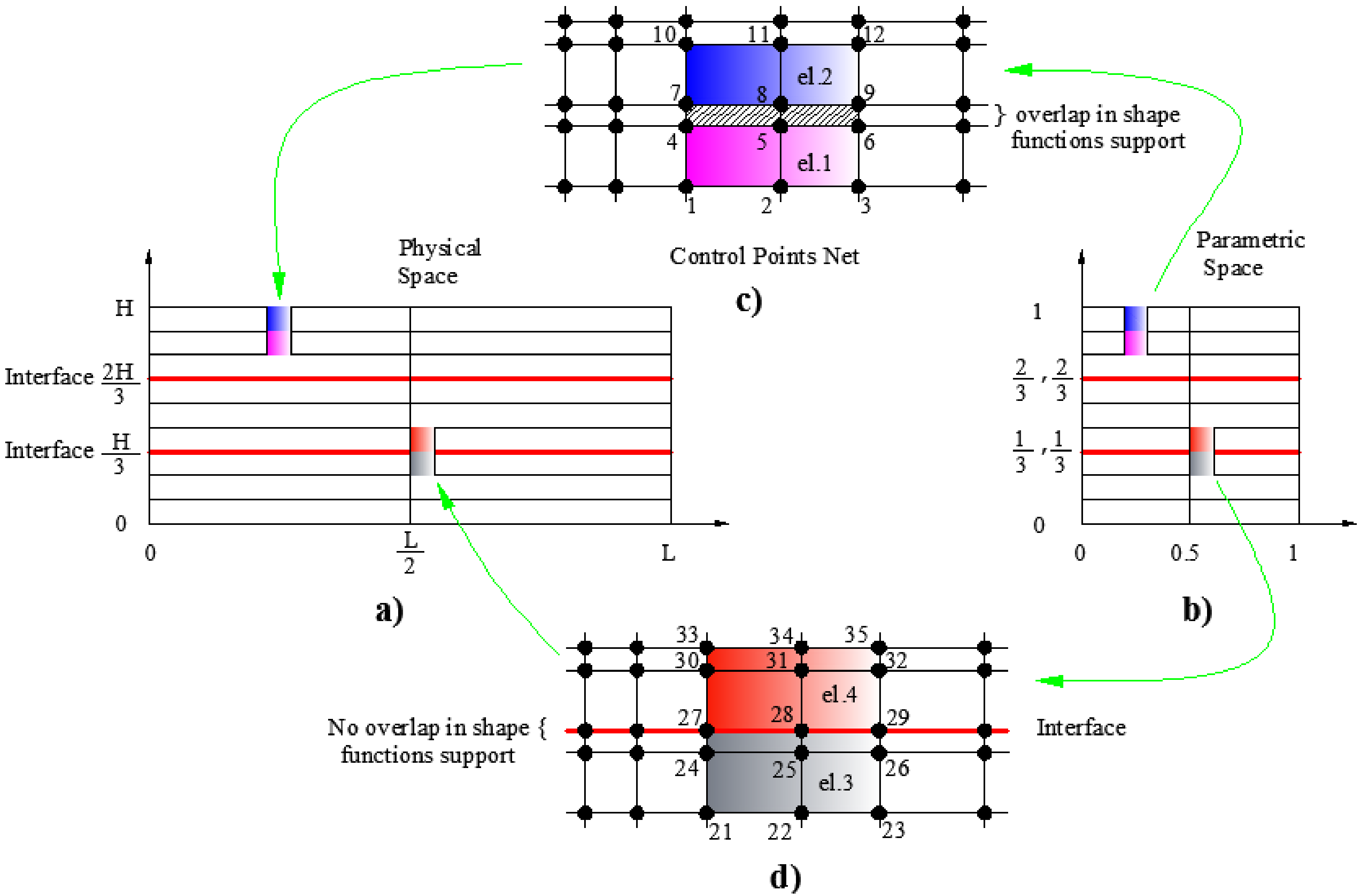
Interfacial shear stress optimization in a sandwich beam

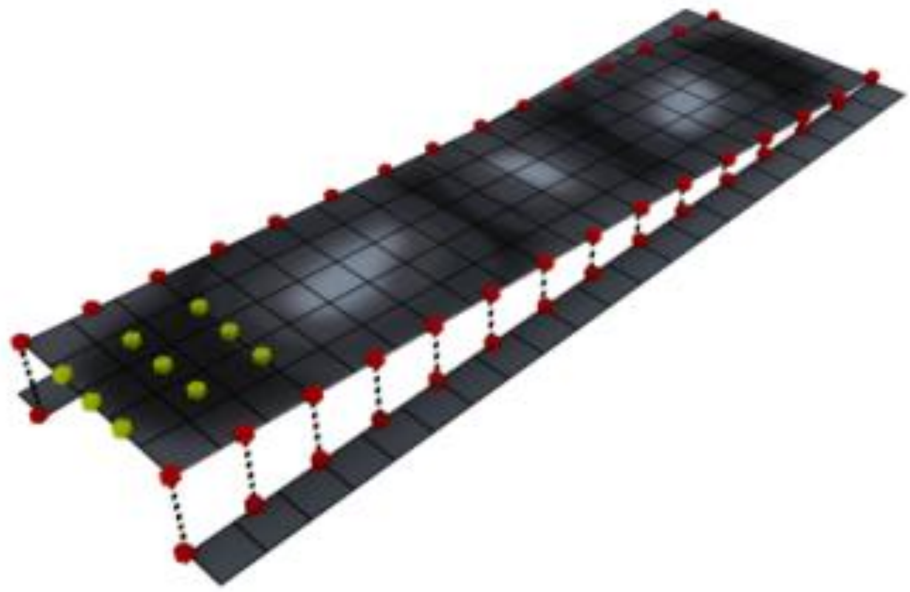




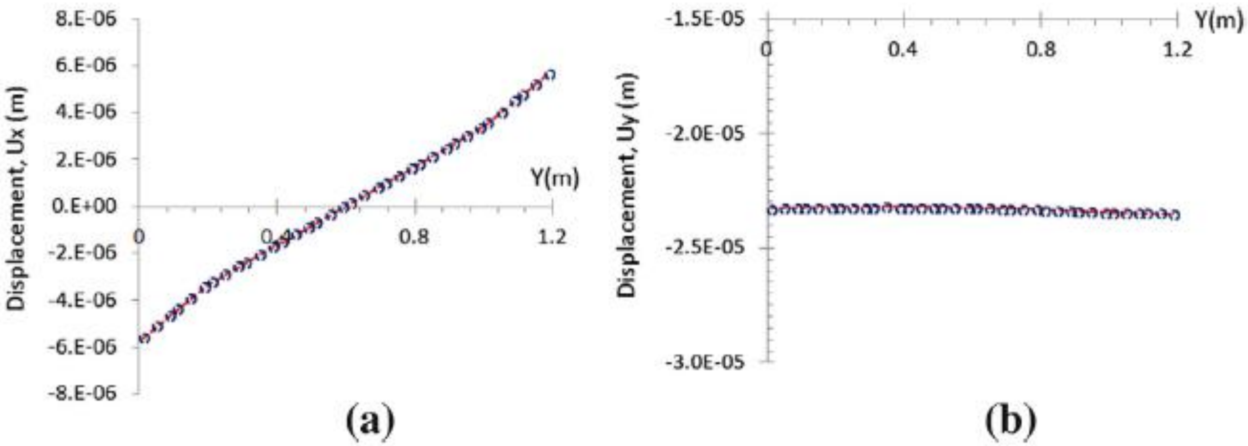
Material discontinuity



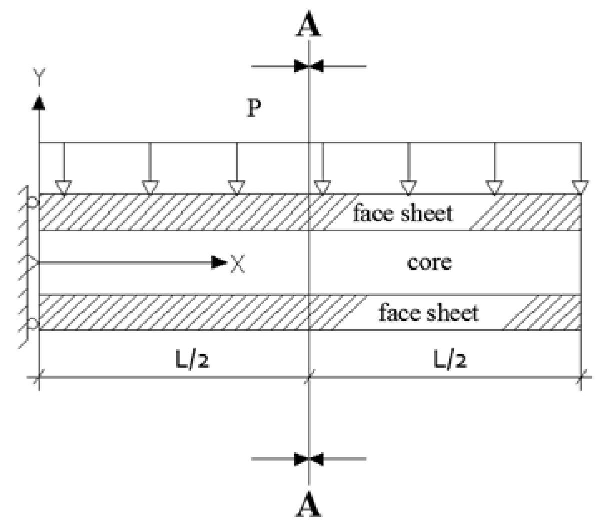
Material discontinuity

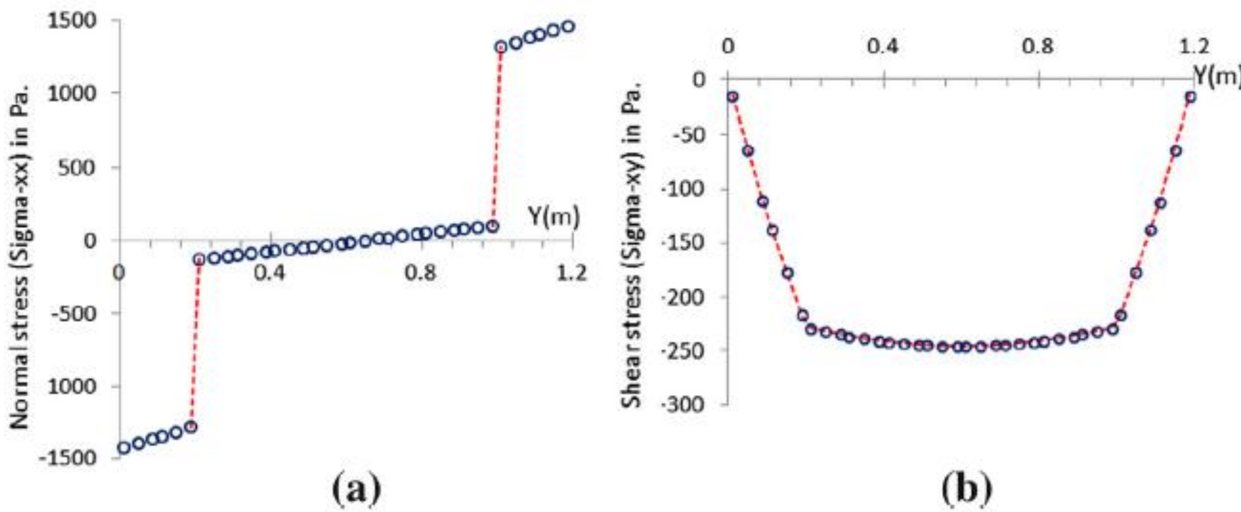


Verification of the analysis model



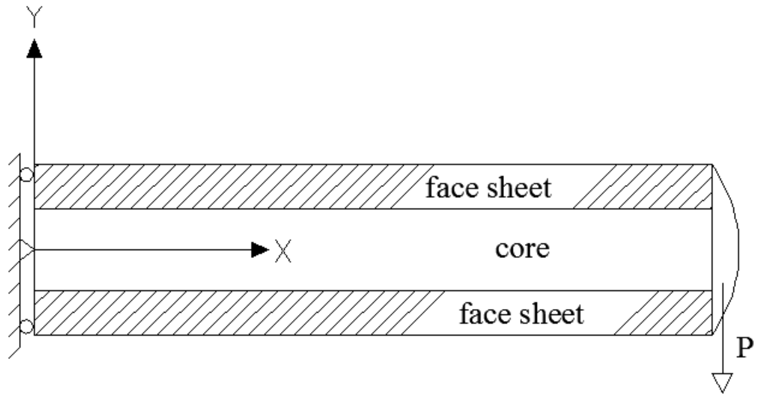
Displacements in the x (a) and y direction (b) of a sandwich cantilever beam along cut $A-A(x=\frac{L}{2})$

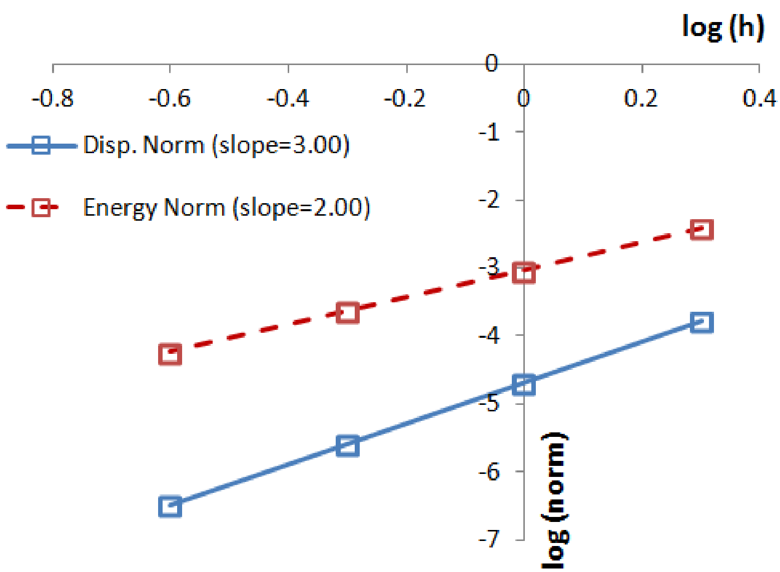




Normal stress (a) and shear stress (b) in a sandwich cantilever beam along cut $A-A(x=\frac{L}{2})$

Verification of the analysis model

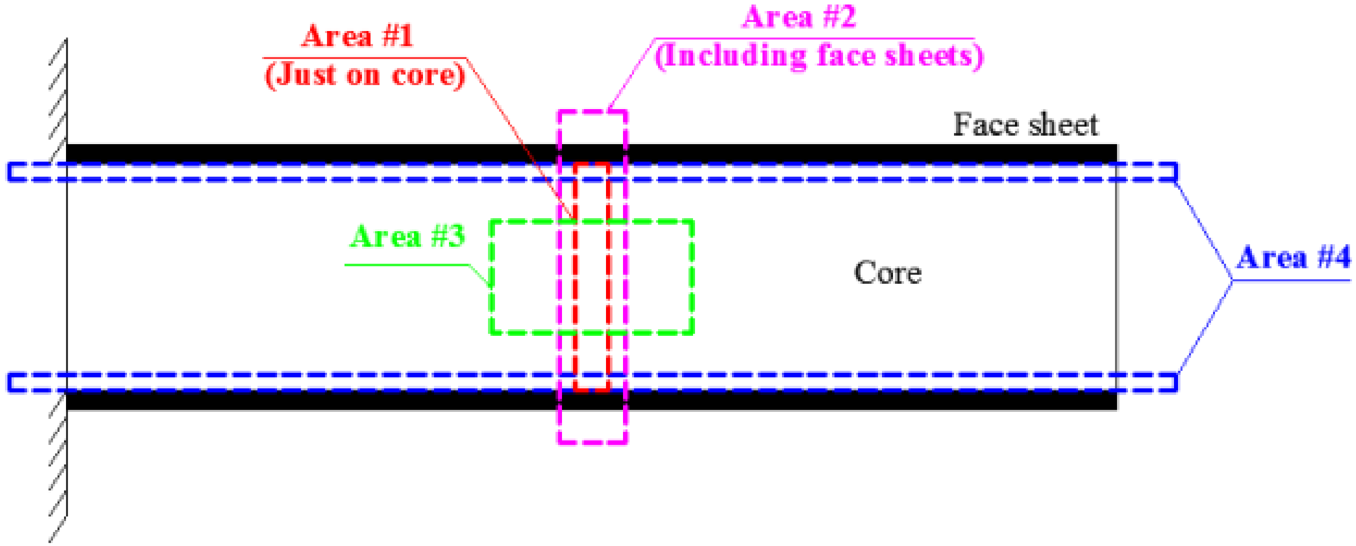




$$e_{energy} = \left[\frac{1}{2}\int_{\Omega}(\epsilon_{num} - \varepsilon_{exact}) \cdot \boldsymbol{D} \cdot (\epsilon_{num} - \varepsilon_{exact})d\Omega\right]^{\frac{1}{2}}$$

$$e_{displacement} = \left\{\frac{\int_{\Omega}[(\boldsymbol{u}_{num} - \boldsymbol{u}_{exact}) \cdot (\boldsymbol{u}_{num} - \boldsymbol{u}_{exact})]d\Omega}{\int_{\Omega}[\boldsymbol{u}_{exact} \cdot \boldsymbol{u}_{exact}]d\Omega}\right\}^{\frac{1}{2}}$$

Optimization problem and results



$$\text{Minimize:} \quad J(\boldsymbol{u}(\boldsymbol{\varphi}), \boldsymbol{\varphi}) = \frac{1}{|\Omega_1|} \int_{\Omega_1} \sigma_{xy} d\Omega_1$$

$$\text{Subjected to:} \; V_f = \int_{\Omega} \eta_p d\Omega \leqslant V_{f0}$$

$$\boldsymbol{Ku} = \boldsymbol{f}$$

$$\varphi_{i,j} - 1 \leqslant 0$$

$$-\varphi_{i,j} \leqslant 0$$

Adjoint sensitivity technique

$$l = J - (V_f - V_{f0}) - \sum_{i,j=1}^{ncp} \psi_1(\varphi_{i,j} - 1) - \sum_{i,j=1}^{ncp} \psi_2(-\varphi_{i,j})$$

$$\frac{dl}{d\varphi} = \frac{dJ}{d\varphi} - \frac{dV_f}{d\varphi} - \psi_1 + \psi_2 = 0$$

$$\frac{dJ}{d\varphi} = \frac{\partial J}{\partial \boldsymbol{u}} \frac{\partial \boldsymbol{u}}{\partial \varphi} + \frac{\partial J}{\partial \varphi}$$

$$\left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}\right)^T \frac{\partial \boldsymbol{u}}{\partial \varphi} + \frac{\partial \boldsymbol{f}}{\partial \varphi} = 0$$

$$\frac{dJ}{d\varphi} = -\frac{\partial J}{\partial \boldsymbol{u}} \left[\left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}\right)^{-T} \frac{\partial \boldsymbol{f}}{\partial \varphi} \right] + \frac{\partial J}{\partial \varphi}$$

$$\frac{dV_f}{d\varphi} = \frac{\partial V_f}{\partial \varphi} = \int_\Omega \frac{\partial \eta_p}{\partial \varphi} \, d\Omega$$

Optimization results (#1)

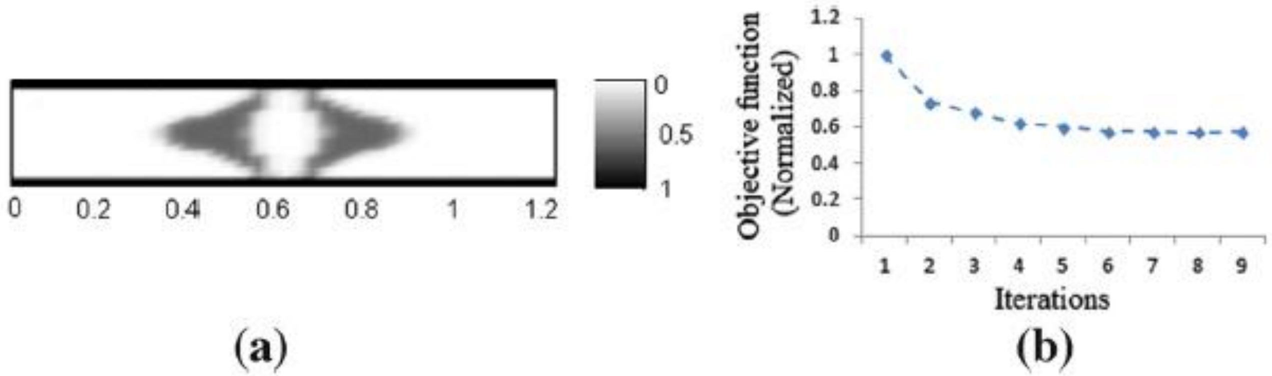


Fig. 10. Optimal distribution of reinforcing ingredients considering area of interest #1 (a), objective function versus iterations (b).

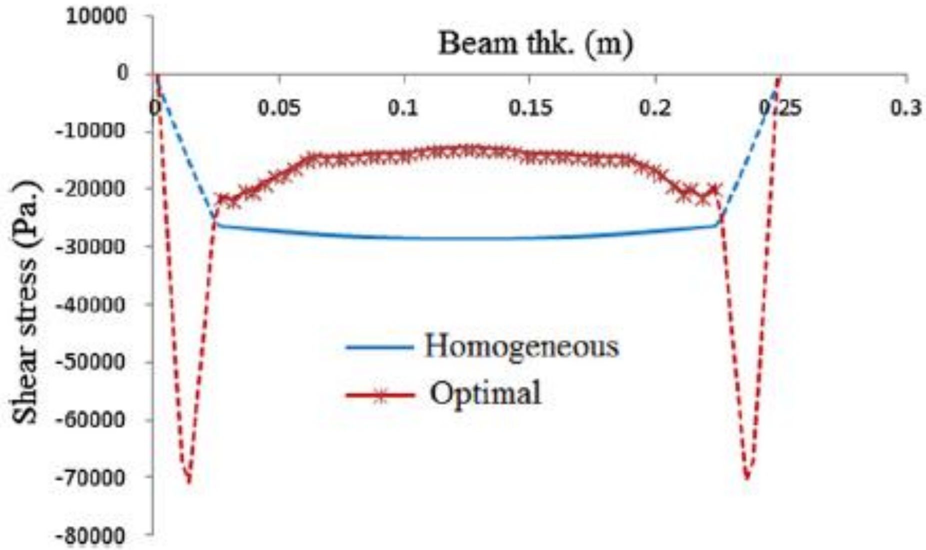


Fig. 11. Shear stress profile for area #1 considering homogeneous and optimal distribution of reinforcements, dash lines stand for face sheets which are outside of area #1.

Optimization results (#2)

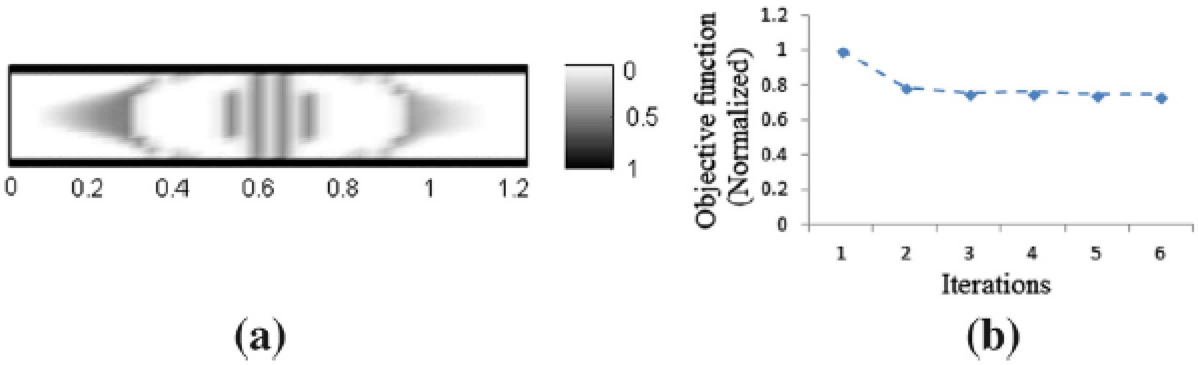


Fig. 12. Optimal distribution of reinforcing ingredients considering area of interest #2 (a), objective function versus iterations (b).

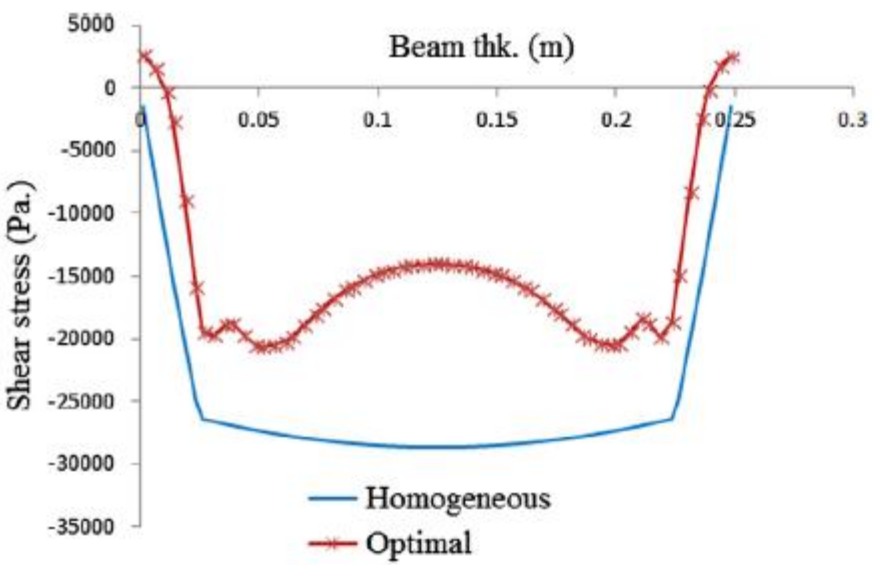
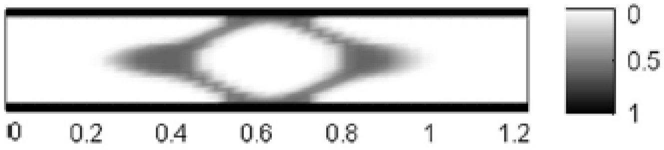
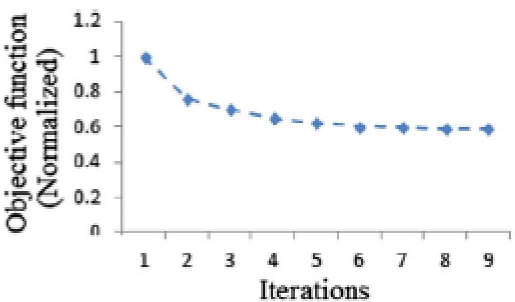


Fig. 13. Shear stress profile for area #2 considering homogeneous and optimal distribution of reinforcements.

Optimization results (#3)





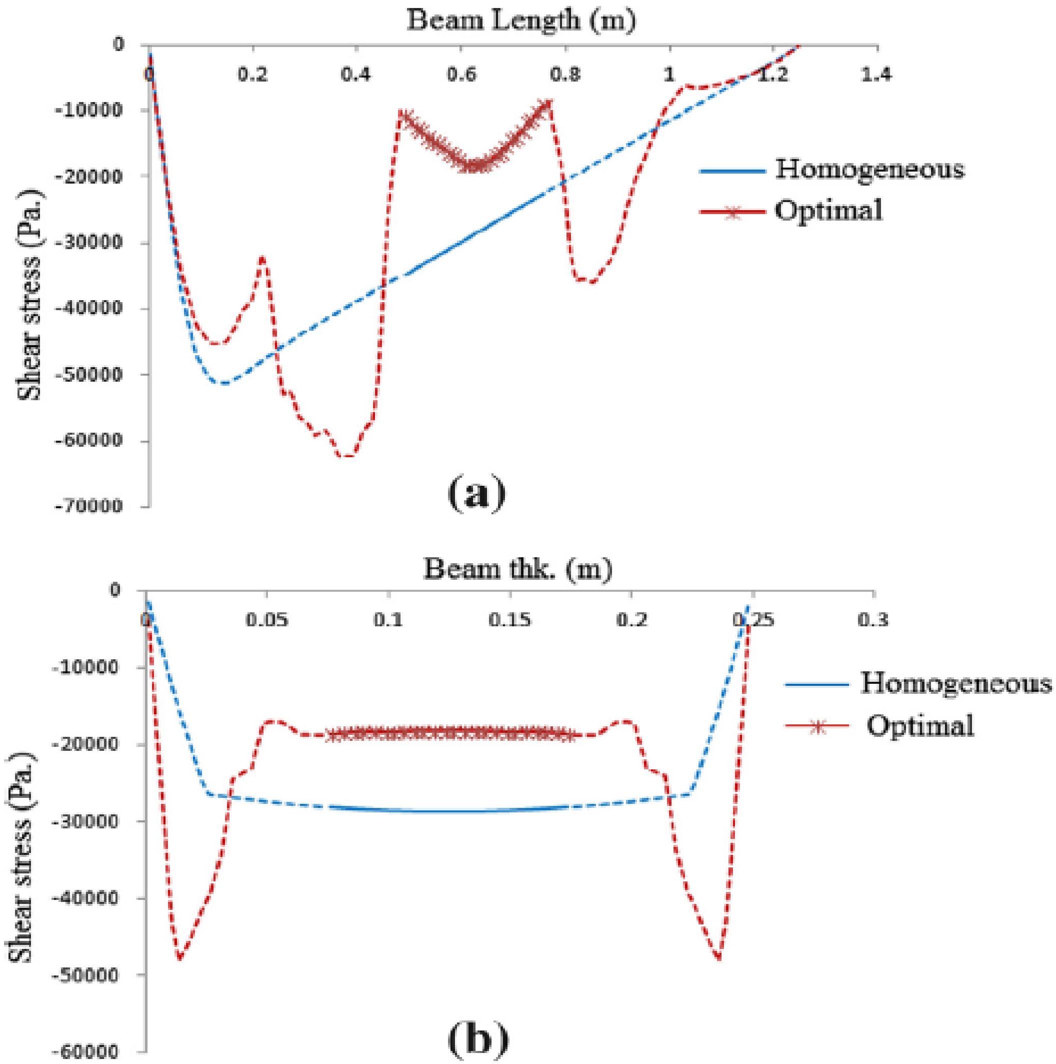
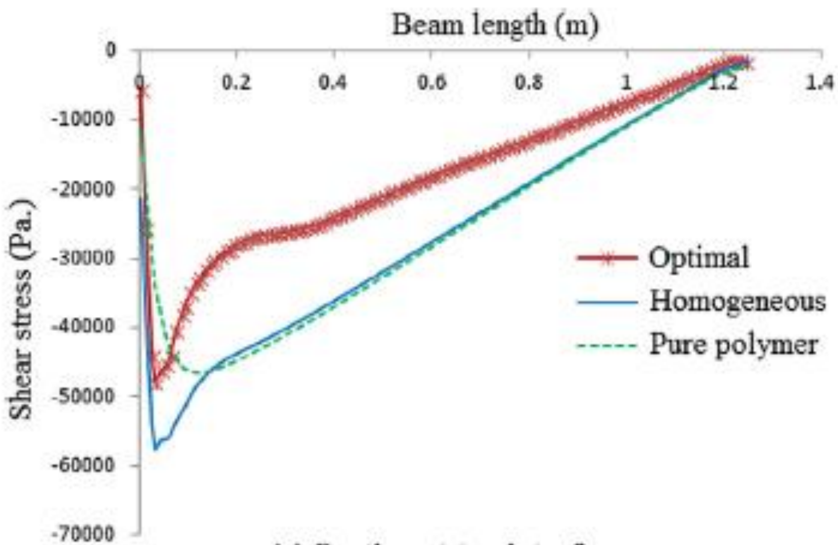
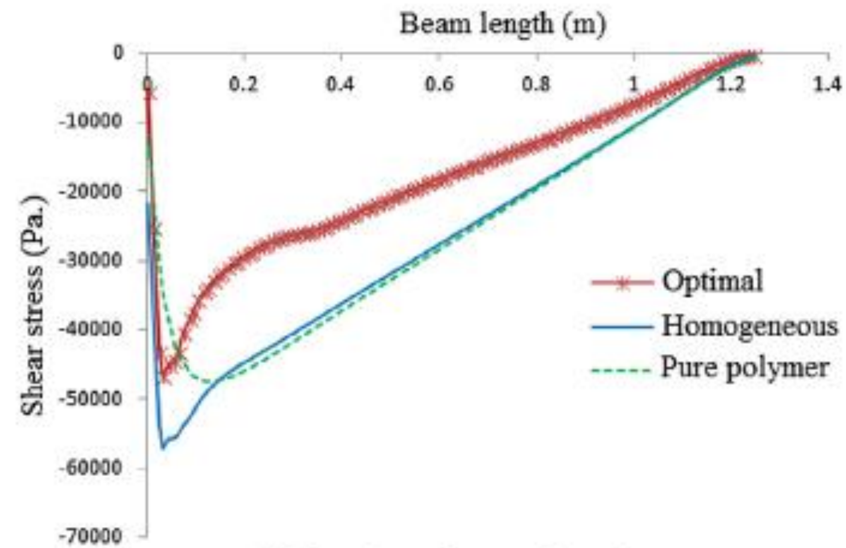


Fig. 15. Shear stress profile for area #3 along a section at mid width (a) and mid length (b) of the beam considering homogeneous and optimal distribution of reinforcements. In both figures, dash lines refer to zones which are outside the interested area #3.

Optimization results (#4)



(a) Section at top interface



(b) Section at bottom interface

Fig. 18. Nodal values of shear stress along a longitudinal sections at top interface (a) and bottom interface (b) in area of interest #4 comparing characteristics obtained by pure polymer, homogeneous FRP and FRP with optimal distribution of reinforcements.