Weighted Fair Multicast Multigroup Beamforming under Per-antenna Power Constraints

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Abstract

A multi-antenna transmitter that conveys independent sets of common data to distinct groups of users is considered. This model is known as physical layer multicasting to multiple co-channel groups. In this context, the practical constraint of a maximum permitted power level radiated by each antenna is addressed. The per-antenna power constrained system is optimized in a maximum fairness sense with respect to predetermined quality of service weights. In other words, the worst scaled user is boosted by maximizing its weighted signal-to-interference plus noise ratio. A detailed solution to tackle the weighted max-min fair multigroup multicast problem under per-antenna power constraints is therefore derived. The implications of the novel constraints are investigated via prominent applications and paradigms. What is more, robust per-antenna constrained multigroup multicast beamforming solutions are proposed. Finally, an extensive performance evaluation quantifies the gains of the proposed algorithm over existing solutions and exhibits its accuracy over per-antenna power constrained systems.

Index Terms

Physical layer Multigroup Multicasting; Per-antenna Power Constraints; Weighted Max Min Fair Optimization; Semidefinite Relaxation; Gaussian Randomization;

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I. Introduction & Related Work

The spatial degrees of freedom offered by multiple antenna arrays are a valuable interference mitigation resource. Advanced signal processing techniques are currently employed to boost the performance of the multi-antenna transmitters without compromising the complexity of single antenna receivers. These beamforming (or equivalently precoding) techniques efficiently manage the co-channel interferences to achieve the targeted service requirements (Quality of Service–QoS targets). As a result, the available spectrum can be aggressively reused towards increasing the system throughput.

The optimal downlink transmission strategy in the sense of minimizing the total transmit power whilst guaranteeing specific QoS targets at each user, was given in [1], [2]. Therein, the tool of Semi-Definite Relaxation (SDR) reduced the non-convex quadratically constrained quadratic problem (QCQP) into a relaxed semi-definite programming instance by changing the optimization variables and disregarding the unit-rank constraints over the new variable. The solution of the relaxed problem was proven to be optimal. The multiuser downlink beamforming problem in terms of maximizing the minimum SINR, was optimally solved in [3]. The goal of the later formulation is to increase the fairness of the system by boosting the SINR of the user that is further away from a targeted performance. Hence, the problem is commonly referred to as *max–min fair*. In [3], this problem was solved using the principles of uplink/downlink duality. Therein, *Schubert and Boche* developed a strongly convergent iterative alternating optimization algorithm for the equivalent uplink problem. In the same work, the power minimization problem of [1] was also solved by acknowledging its inherent connection with the max-min fair problem. Consequently, a significantly less complex framework to solve the optimal beamforming problem was established. Extending these works, the practical per-antenna power constraints (PAC) were considered in [4]. Generalized power constraints, including sum power, per-antenna power and per-antenna array power constraints were considered in [5], where the proposed max-min fair solution was derived on an extended duality framework. This framework accounted for both instantaneous and long term channel state information (CSI). PACs are motivated from the practical implementation of systems that rely on precoding. The lack of flexibility in sharing energy resources amongst the antennas of the transmitter is usually the case, since a common practice in multi-antenna systems is the use of individual amplifiers per antenna. Despite the fact that flexible amplifiers could be incorporated in multi-antenna transmitters, specific communication systems cannot afford this design. Typical per antenna power limited systems can be found in multibeam satellite communications [6], where flexible on board payloads are difficult to implement and in cooperative multicell systems (also known as distributed antenna systems, DAS), where the physical co-location of the transmitting elements is not a requisite and hence power sharing might be infeasible.

A fundamental consideration of the aforementioned works is that independent data is addressed to multiple users. However, the new generation of multi-antenna communication standards has to adapt the physical layer design to the needs of the higher network layers. Examples of such cases include highly demanding applications (e.g. video broadcasting) that stretch the throughput limits of multiuser broadband systems. In this direction, physical layer (PHY) multicasting has the potential to efficiently address the nature of future traffic demand and has become part of the new generation of communication standards. PHY multicasting is also relevant for the application of beamforming without changing the framing structure of standards.Such a scenario can be found in satellite communications where the communication standards are optimized to cope with long propagation delays and guarantee scheduling efficiency by framing multiple users per transmission [6], [7].

In [8], the NP-hard multicast problem was accurately approximated by SDR and Gaussian randomization. The natural extension of the multicast concept lies in assuming multiple interfering groups of users. A unified framework for physical layer multicasting to multiple co-channel groups, where independent sets of common data are transmitted to groups of users by the multiple antennas, was given in [9], [10]. Therein, the QoS and the fairness problems were formulated, proven NP-hard and solved for the sum power constrained multicast multigroup case. In parallel to [9], the independent work of [11] involved complex dirty paper coding methods. Also, a convex approximation method was proposed in [12] that exhibits superior performance as the number of users per group grows. Finally, in [13] the multicast multigroup problem under SPC, was solved based on approximations and uplink-downlink duality [3]. In the context of coordinated multicast multicell systems[1], max–min fair beamforming with per base-station (BS) constraints has been considered in [14] where each BS transmits to a single multicast group. Hence, a power constraint over each precoder was imposed while no optimization weights were considered.This formulation still considers power sharing amongst the multiple antennas at each transmitter.

Towards deriving the optimal multigroup multicast precoders when a maximum limit is imposed on the transmitted power of each antenna, a new optimization problem with one constraint per transmit antenna needs to be formulated. Amid the extensive literature on multigroup multicast beamforming,

[1] Coordinated multicell networks consist of connected base stations (BS), with each BS serving a single multicast group, a case tackled in [14]. Extending this, the methods presented herein can be applied in cooperative multicell systems where all BSs will jointly transmit to several multicast groups [15].

the PACs have only been considered in [16], where an equally fair multicast multigroup solution is presented. Extending these considerations, the present work accounts optimization weights. Therefore, a consolidated solution for the weighted max–min fair multigroup multicast beamforming under PACs is hereafter presented. The contributions of the present work are summarized as follows

- The PAC weighted fair multigroup multicast beamforming problem is formulated and solved.
- Practical system design insights are given by examining the implications of the PACs on multigroup multicast distributed antenna systems (DAS), modulation constrained systems and uniform linear array (ULA) transmitters.
- A robust to erroneous CSI multigroup multicast design under PACs is proposed.
- The performance of the solution is evaluated through extensive numerical results under various system setups.

The rest of the paper is structured as follows. The multigroup multicast system model is presented in Sec. II while the weighted fair problem is formulated and solved in Sec. III. In Sec. IV, the performance of the design is evaluated for various system setups along with a robust extension of the derived algorithm and a weighted multigroup multicast application paradigm. Finally, Sec. V concludes the paper.

Notation: In the remainder of this paper, bold face lower case and upper case characters denote column vectors and matrices, respectively. The operators $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\dagger}$, $|\cdot|$, $\mathrm{Tr}\,(\cdot)$ and $||\cdot||_2$, correspond to the transpose, the conjugate transpose, the absolute value, the trace and the Frobenius norm operations, while $[\cdot]_{ij}$ denotes the i,j-th element of a matrix. The principal eigenvalue of a matrix $\mathbf{X}$ are denoted as $\lambda_{max}(\mathbf{X})$. Calligraphic indexed characters denote sets.

II. System Model

Herein, the focus is on a multi-user (MU) multiple input single output (MISO) multicast system. Assuming a single transmitter, let N_t denote the number of transmitting elements and N_u the total number of users served. The input-output analytical expression will read as $y_i = \mathbf{h}_i^{\dagger}\mathbf{x} + n_i$, where $\mathbf{h}_i^{\dagger}$ is a $1 \times N_t$ vector composed of the channel coefficients (i.e. channel gains and phases) between the i-th user and the N_t antennas of the transmitter, $\mathbf{x}$ is the $N_t \times 1$ vector of the transmitted symbols and n_i is the independent complex circular symmetric (c.c.s.) independent identically distributed (i.i.d) zero mean Additive White Gaussian Noise (AWGN) measured at the i-th user's receive antenna. Focusing in a multigroup multicasting scenario, let there be a total of $1 \leq G \leq N_u$ multicast groups with $\mathcal{I} = \{\mathcal{G}_1, \mathcal{G}_2, \dots \mathcal{G}_G\}$ the collection of index sets and $\mathcal{G}_k$ the set of users that belong to the k-th multicast group, $k \in \{1 \dots G\}$. Each user belongs to only one group, thus $\mathcal{G}_i \cap \mathcal{G}_j = \emptyset, \forall i, j \in \{1 \cdots G\}$. Let

$\mathbf{w}_k \in \mathbb{C}^{N_t \times 1}$ denote the precoding weight vector applied to the transmit antennas to beamform towards the k-th group. The assumption of independent data transmitted to different groups renders the symbol streams $\{s_k\}_{k=1}^{G}$ mutually uncorrelated and the total power radiated from the antenna array is

$$P_{tot} = \sum_{k=1}^{G} \mathbf{w}_k{}^{\dagger}\mathbf{w}_k \tag{1}$$

The power radiated by each antenna element is a linear combination of all precoders [4]:

$$P_n = \left[\sum_{k=1}^{G} \mathbf{w}_k \mathbf{w}_k^{\dagger}\right]_{nn} \tag{2}$$

where $n \in \{1 \dots N_t\}$ is the antenna index. The fundamental difference between the SPC of [10] and the proposed PAC is clear in (2), where instead of one, N_t constraints are realized, each one involving all the precoding vectors. A more general constraint formulation to model power flexibility amongst groups of antennas can be found in [17].

III. Multicast Multigroup Beamforming with Per Antenna Power Constraints

A. *Weighted Max-Min Fair Formulation*

The PAC weighted max-min fair problem is defined as

$$\boxed{\begin{aligned} \mathcal{F}: \quad & \max_{t,\ \{\mathbf{w}_k\}_{k=1}^{G}} \ t \\ & \text{subject to } \frac{1}{\gamma_i}\frac{|\mathbf{w}_k^{\dagger}\mathbf{h}_i|^2}{\sum_{l\neq k}^{G}|\mathbf{w}_l^{\dagger}\mathbf{h}_i|^2+\sigma_i^2} \geq t, \qquad (3) \\ & \qquad \forall i \in \mathcal{G}_k, k, l \in \{1 \dots G\}, \\ & \text{and to } \left[\sum_{k=1}^{G}\mathbf{w}_k\mathbf{w}_k^{\dagger}\right]_{nn} \leq P_n, \qquad (4) \\ & \qquad \forall n \in \{1 \dots N_t\}, \end{aligned}}$$

where $\mathbf{w}_k \in \mathbb{C}^{N_t}$ and $t \in \mathbb{R}^{+}$. Different service levels between the users can be acknowledged in this weighted formulation. Problem $\mathcal{F}$ receives as inputs the PACs vector $\mathbf{p} = [P_1, P_2 \dots P_{N_t}]$ and the target SINRs vector $\mathbf{g} = [\gamma_1, \gamma_2, \dots \gamma_{N_u}]$. Its goal is to maximize the slack variable t while keeping all SINRs above this value. Thus, it constitutes a max-min problem that guarantees fairness amongst users. Following the common in the literature notation for ease of reference, the optimal objective value of $\mathcal{F}$ is denoted as $t^* = \mathcal{F}(\mathbf{g}, \mathbf{p})$ and the associated optimal point as $\{\mathbf{w}_k^{\mathcal{F}}\}_{k=1}^{G}$. Of particular interest is the case where the co-group users share the same target i.e. $\gamma_i = \gamma_k, \ \forall i \in \mathcal{G}_k, k \in \{1 \dots G\}$.

Remark 1: The difference of the present formulation with respect to the weighted max-min fair problem with SPC presented in [8], [10] lies in the N_t power constraints over each individual radiating element. Additionally, this formulation differs from the coordinated multicell multicasting Max-Min formulation of [14] since the constraint is imposed on the n-th diagonal element of the summation of the correlation matrices of all precoders, while weights on each users SINR are also inserted. On the contrary, in [14], the imposed per base station constraints are translated to one power constraint per each precoder. In the present work, weights to differentiate the QoS targets between users are also proposed.

B. Per-antenna power minimization

The relation between the fairness and the power minimization problems for the multicast multigroup case was firstly established in [10]. As a result, by bisecting the solution of the QoS optimization, a solution to the weighted fairness problem can be derived. Nevertheless, fundamental differences between the existing formulations and problem $\mathcal{F}$ complicate the solution. In more detail, the per-antenna constraints are not necessarily met with equality (a discussion on this is also given in Sec. IV-B). Therefore, the fairness problem is no longer equivalent to the sum power minimization under QoS constraints problem. Since the absence of a related, solvable problem prohibits the immediate application of bisection, a novel equivalent per-antenna power minimization problem is proposed as

$$\boxed{\begin{aligned}
\mathcal{Q}:\ & \min_{r,\ \{\mathbf{w}_k\}_{k=1}^{G}} r\\
&\text{subject to } \frac{|\mathbf{w}_k^\dagger \mathbf{h}_i|^2}{\sum_{l\neq k}^{G} |\mathbf{w}_l^\dagger \mathbf{h}_i|^2+\sigma_i^2} \geq \gamma_i, \qquad (5)\\
&\qquad \forall i \in \mathcal{G}_k, k,l \in \{1\dots G\},\\
&\text{and to } \quad \frac{1}{P_n}\left[\sum_{k=1}^{G}\mathbf{w}_k\mathbf{w}_k^\dagger\right]_{nn} \leq r, \qquad (6)\\
&\qquad \forall n \in \{1\dots N_t\},
\end{aligned}}$$

with $r \in \mathbb{R}^+$. Problem $\mathcal{Q}$ receives as input SINR constraints for all users, defined before as $\mathbf{g}$, as well as the per antenna power constraint vector $\mathbf{p}$ of (4). The introduction of the slack-variable r, a common practice in convex optimization [18], constraints the power consumption of each and every antenna. Subsequently, at the optimum r^*, the maximum power consumption out of all antennas is minimized and this solution is denoted as $r^* = \mathcal{Q}(\mathbf{g},\mathbf{p})$. The generic difference of the present min-max formulation and the formulation proposed in [14] lies in the per antenna constraint (6). Instead of constraining the power of each antenna, the authors of [14] impose a constraint over each precoder that serves a common multicast group. In the case tackled herein, the number of constraints is increased from one to N_t, while each constraint is a function of all multigroup precoders as the summation in (6) reveals. The following claim reveals the relation between the described problems.

Claim 1: Problems $\mathcal{F}$ and $\mathcal{Q}$ are related as follows

$$1 = \mathcal{Q}\left(\mathcal{F}\left(\mathbf{g}, \mathbf{p}\right) \cdot \mathbf{g}, \mathbf{p}\right) \tag{7}$$

$$t = \mathcal{F}\left(\mathbf{g}, \mathcal{Q}\left(t \cdot \mathbf{g}, \mathbf{p}\right) \cdot \mathbf{p}\right) \tag{8}$$

Proof: Similar to the line of reasoning in [14] the above claims will be proven by contradiction. Starting with (7), let $t^* = \mathcal{F}(\mathbf{g},\ \mathbf{p})$ denote the optimal value of $\mathcal{F}$ with associated variable $\{\mathbf{w}_k^F\}_{k=1}^G$. Also, let $\hat{r} = \mathcal{Q}\left(t^* \cdot \mathbf{g},\ \mathbf{p}\right)$ be the optimal value of $\mathcal{Q}$ at the point $\{\mathbf{w}_k^Q\}_{k=1}^G$. Then, assuming that $\hat{r} > 1$, the vectors $\{\mathbf{w}_k^F\}_{k=1}^G$ satisfy the feasibility criteria of $\mathcal{Q}$ and produce a lower optimal value thus contradicting the optimality of $\{\mathbf{w}_k^Q\}_{k=1}^G$ and opposing the hypothesis. Alternatively, assuming that $\hat{r} < 1$ then the solutions $\{\mathbf{w}_k^Q\}_{k=1}^G$ can be scaled by the non-negative $\hat{r}$. The vectors $\{\hat{r} \cdot \mathbf{w}_k^Q\}_{k=1}^G$ are feasible solutions to $\mathcal{F}$ which provide the same optimal objective value with however some remaining power budget. Therefore, the power could be scaled up until at least one of the PACs is satisfied with equality and a higher objective value would be derived thus again contradicting the hypothesis. Consequently, $\hat{r} = 1$. The same line of reasoning is followed to prove (8). Let $r^* = \mathcal{Q}(t \cdot \mathbf{g},\ \mathbf{p})$ denote the optimal value of $\mathcal{Q}$ with associated solution $\{\mathbf{w}_k^Q\}_{k=1}^G$. Assuming that the optimal value of $\mathcal{F}$ under constraints scaled by the solution of $\mathcal{Q}$ is different, i.e. $\hat{t} = \mathcal{F}\left(\mathbf{g},\ \mathcal{Q}\left(t \cdot \mathbf{g},\ \mathbf{p}\right) \cdot \mathbf{p}\right)$ with $\{\mathbf{w}_k^F\}_{k=1}^G$, the following contradictions arise. In the case where $\hat{t} < t$, then the precoders $\{\mathbf{w}_k^Q\}_{k=1}^G$ are feasible solutions to $\mathcal{F}$ which lead to a higher minimum SINR, thus contradicting the optimality of $\hat{t}$. Alternatively, if $\hat{t} > t$ then the solution set $\{\mathbf{w}_k^F\}_{k=1}^G$ can be scaled by a positive constant $c = t/\hat{t} < 1$. The new solution $\{c\mathbf{w}_k^F\}_{k=1}^G$ respects the feasibility conditions of $\mathcal{Q}$ and provides a lower optimal value, i.e. $c \cdot r^*$, thus again contradicting the hypothesis. As a result, $\hat{t} = t$ $\square$.

C. Semidefinite Relaxation

Problem $\mathcal{Q}$ belongs in the general class of non-convex QCQPs for which the SDR technique is proven to be a powerful and computationally efficient approximation technique [19]. The relaxation is based on the observation that $|\mathbf{w}_k^\dagger \mathbf{h}_i|^2 = \mathbf{w}_k^\dagger \mathbf{h}_i \mathbf{h}_i^\dagger \mathbf{w}_k = \mathrm{Tr}(\mathbf{w}_k^\dagger \mathbf{h}_i \mathbf{h}_i^\dagger \mathbf{w}_k) = \mathrm{Tr}(\mathbf{w}_k \mathbf{w}_k^\dagger \mathbf{h}_i \mathbf{h}_i^\dagger)$. With the change of variables $\mathbf{X}_i = \mathbf{w}_i\mathbf{w}_i^\dagger$, $\mathcal{Q}$ can be relaxed to $\mathcal{Q}_r$

$$\boxed{\begin{aligned}
\mathcal{Q}_r: \quad & \min_{r,\ \{\mathbf{X}_k\}_{k=1}^G} r \\
& \text{subject to} \quad \frac{\mathrm{Tr}\left(\mathbf{Q}_i\mathbf{X}_k\right)}{\sum_{l\neq k}^{G}\mathrm{Tr}\left(\mathbf{Q}_l\mathbf{X}_k\right)+\sigma_i^2} \geq \gamma_i, \qquad (9)\\
& \qquad\qquad \forall i \in \mathcal{G}_k, k, l \in \{1\ldots G\}, \\
& \text{and to} \quad \frac{1}{P_n}\left[\sum_{k=1}^{G}\mathbf{X}_k\right]_{nn} \leq r \qquad (10)\\
& \text{and to} \quad \mathbf{X}_k \succeq 0,\ \forall n \in \{1\ldots N_t\},
\end{aligned}}$$

where $\mathbf{Q}_i = \mathbf{h}_i\mathbf{h}_i^\dagger$, $r \in \mathbb{R}^+$, while the constraint $\mathrm{rank}(\mathbf{X}_i) = 1$ is dropped. Now the relaxed $\mathcal{Q}_r$ is convex, thus solvable to an arbitrary accuracy. This relaxation can be interpreted as a Lagrangian bi-dual of the original problem [18]. The weighted max-min fair optimization is also relaxed as

$$\boxed{\begin{aligned}
\mathcal{F}_r: \quad & \max_{t,\ \{\mathbf{w}_k\}_{k=1}^G} t \\
& \text{subject to} \quad \frac{1}{\gamma_i}\frac{\mathrm{Tr}\left(\mathbf{Q}_i\mathbf{X}_k\right)}{\sum_{l\neq k}^{G}\mathrm{Tr}\left(\mathbf{Q}_l\mathbf{X}_k\right)+\sigma_i^2} \geq t, \qquad (11)\\
& \qquad\qquad \forall i \in \mathcal{G}_k, k, l \in \{1\ldots G\}, \\
& \text{and to} \quad \left[\sum_{k=1}^{G}\mathbf{X}_k\right]_{nn} \leq P_n, \qquad (12)\\
& \qquad\qquad \forall n \in \{1\ldots N_t\}, \\
& \text{and to} \quad \mathbf{X}_k \succeq 0, \qquad (13)
\end{aligned}}$$

which, however, remains non-convex due to (11), as in detail explained in [10]. However, this obstacle can be overcome by the following observation.

Claim 2: Problems $\mathcal{F}_r$ and $\mathcal{Q}_r$ are related as follows

$$1 = \mathcal{Q}_r\left(\mathcal{F}_r\left(\mathbf{g},\mathbf{p}\right)\cdot\mathbf{g},\mathbf{p}\right) \tag{14}$$

$$t = \mathcal{F}_r\left(\mathbf{g},\mathcal{Q}_r\left(t\cdot\mathbf{g},\mathbf{p}\right)\cdot\mathbf{p}\right) \tag{15}$$

Proof: Follows the steps of the proof of *Claim 1* and is therefore omitted. □

D. Gaussian Randomization

Due to the NP-hardness of the multicast problem, the relaxed problems do not necessarily yield unit rank matrices. Consequently, one can apply a rank-1 approximation over $\mathbf{X}^*$. Many types of rank-1 approximations are possible depending on the nature of the original problem. The solution with the highest provable accuracy for the multicast case is given by the Gaussian randomization method [19]. In more detail, let $\mathbf{X}^*$ be a symmetric positive semidefinite solution of the relaxed problem. Then, a candidate solution to the original problem can be generated as a Gaussian random variable with zero mean and covariance equal to $\mathbf{X}^*$, i.e. $\hat{\mathbf{w}} \backsim \mathbb{CN}(0, \mathbf{X}^*)$. After generating a predetermined number of candidate solutions, the one that yields the highest objective value of the original problem can be chosen. The accuracy of this approximate solution is measured by the distance of the approximate objective value and the optimal value of the relaxed problem and it increases with the predetermined number of randomizations [10], [19]. Nonetheless, an intermediate problem dependent step between generating a Gaussian instance with the statistics obtained from the relaxed solution and creating a feasible candidate instance of the original problem still remains, since the feasibility of the original problem is not yet guaranteed.

E. Feasibility Power Control

After generating a random instance of a Gaussian variable with statistics defined by the relaxed problem, an additional step comes in play to guarantee the feasibility of the original problem. In [8], the feasibility of the candidate solutions, as given by the Gaussian randomization, was guaranteed by a simple power rescaling. Nevertheless, since in the multigroup case an interference scenario is dealt with, a simple rescaling does not guarantee feasibility. Therefore, an additional optimization step is proposed in [10] to re-distribute the power amongst the candidate precoders. To account for the inherently different PACs, a novel power control problem with per antenna power constraints is proposed. Given a set of Gaussian instances, $\{\hat{\mathbf{w}}_k\}_{k=1}^G$, the *Multigroup Multicast Per Antenna power Control* (MMPAC) problem reads as

$$\boxed{\begin{aligned} \mathcal{S}^{\mathcal{F}}: \max_{t,\ \{p_k\}_{k=1}^G} \quad & t \\ \text{subject to} \quad & \frac{1}{\gamma_i}\frac{|\hat{\mathbf{w}}_k^\dagger \mathbf{h}_i|^2 p_k}{\sum_{l\neq k}^G |\hat{\mathbf{w}}_l \mathbf{h}_i|^2 p_l + \sigma_i^2} \geq t, && (16) \\ & \forall i \in \mathcal{G}_k, k, l \in \{1 \dots G\} \\ \text{and to} \quad & \left[\sum_{k=1}^G \hat{\mathbf{w}}_k \hat{\mathbf{w}}_k^\dagger p_k\right]_{nn} \leq P_{n,} && (17) \\ & \forall n \in \{1 \dots N_t\}, \end{aligned}}$$

with $\{p_k\}_{k=1}^G \in \mathbb{R}^+$. Problem $\mathcal{S}^{\mathcal{F}}$ receives as input the PACs as well as the SINR targets and returns the maximum scaled worst SINR $t^* = \mathcal{S}(\mathbf{g}, \mathbf{p})$ and is also non-convex like $\mathcal{F}$. The difference of this problem compared to [10] lies in (17).

Remark 2: A very important observation is clear in the formulation of the power control problem. The optimization variable $\mathbf{p}$ is of size G, i.e. equal to the number of groups, while the power constraints are equal to the number of antennas, N_t. In each constraint, all the optimization variables contribute. This fact prohibits the total exploitation of the available power at the transmitter. Once at least one of the N_t constraints is satisfied with equality and remaining power budget, then the rest can not be scaled up since this would lead to at least one constraint exceeding the maximum permitted value.

F. Bisection

The establishment of claims 1 and 2, allows for the application of the bisection method, as developed in [8], [10]. The solution of $r^* = \mathcal{Q}_r\left(\frac{L+U}{2}\mathbf{g}, \mathbf{p}\right)$ is obtained by bisecting the interval $[L, U]$ as defined by the minimum and maximum SINR values. Since $t = (L+U)/2$ represents the SINR, it will always be positive or zero. Thus, $L = 0$. Also, if the system was interference free while all the users had the channel of the best user, then the maximum worst SINR would be attained, thus $U = \max_i\{P_{tot}\mathbf{Q}_i/\sigma_i\}$. If $r^* < 1$, then the lower bound of the interval is updated with this value. Otherwise the value is assigned to the upper bound of the interval. Bisection is iteratively performed until an the interval size is reduced to a pre-specified value ϵ. This value needs to be dependent on the magnitude of L and U so that the accuracy of the solution is maintained regardless of the region of operation. After a finite number of iterations the optimal value of $\mathcal{F}_r$ is given as the resulting value for which L and U become almost identical. This procedure provides an accurate solution to the non-convex $\mathcal{F}_r$. Following this, for each and every solution $\{\hat{\mathbf{w}}_k\}_{k=1}^G$, the power of the precoders needs to be controlled. Consequently, problem $\mathcal{S}^F$ can be solved using the well established framework of bisection [18] over its convex equivalent problem, which reads as

$$\boxed{\begin{aligned}
\mathcal{S}^{\mathcal{Q}}: \min_{r,\ \{p_k\}_{k=1}^G} \quad & r \\
\text{subject to} \quad & \frac{|\hat{\mathbf{w}}_k^\dagger \mathbf{h}_i|^2 p_k}{\sum_{l\neq k}^G |\hat{\mathbf{w}}_l^\dagger \mathbf{h}_i|^2 p_l + \sigma_i^2} \geq \gamma_i, && (18)\\
& \forall i \in \mathcal{G}_k, k, l \in \{1\dots G\}, \\
\text{and to} \quad & \frac{1}{P_n}\left[\sum_{k=1}^G \hat{\mathbf{w}}_k \hat{\mathbf{w}}_k^\dagger p_k\right]_{nn} \leq r, && (19)\\
& \forall n \in \{1\dots N_t\},
\end{aligned}}$$

Problem $\mathcal{S}^{\mathcal{Q}}$ is an instance of a linear programming (LP) problem.

Remark 3: For completeness, the possible reformulation of the non-convex problem $\mathcal{S}^{\mathcal{F}}$ into the following geometric problem (GP) is considered, thus surpassing the need for bisection:

$$\boxed{\begin{aligned} \mathcal{S}^{\mathcal{F}}_{\mathcal{GP}}:& \min_{t,\ \{p_k\}_{k=1}^{G}} \quad t^{-1} \\ \text{s. t.}& \sum_{l\neq k}^{G} |\hat{\mathbf{w}}_l\mathbf{h}_i|^2 p_l + \sigma_i^2 \leq \frac{t^{-1}}{\gamma_i}|\hat{\mathbf{w}}_k^{\dagger}\mathbf{h}_i|^2 p_k, \\ & \forall i \in \mathcal{G}_k, k, l \in \{1\dots G\} \\ \text{and to}& \left[\sum_{k=1}^{G}\hat{\mathbf{w}}_k\hat{\mathbf{w}}_k^{\dagger}p_k\right]_{nn} \leq P_n, \forall n \in \{1\dots N_t\}, \end{aligned}} \tag{20}$$

G. Complexity

An important discussion involves the complexity of the employed techniques to approximate a solution of the highly complex, NP-hard multigroup multicast problem under PACs. Focusing on the proposed algorithm (cf. Alg. 1), the main complexity burden originates from the solution of a SDP. The present work relies on the CVX tool [18] which calls numerical solvers such as SeDuMi to solve semi-definite programs. The complexity of the SDR technique has been exhaustively discussed in [19] and the references therein. To calculate the total worst case complexity of the solution proposed in the present work, the following are considered.

Initially, a bisection search is performed over $\mathcal{Q}_r$ to obtain the relaxed solution. This bisection runs for $N_{iter} = \lceil \log_2 (U_1 - L_1)/\epsilon_1 \rceil$ where ϵ_1 is the desired accuracy of the search. Typically ϵ_1 needs to be at least three orders of magnitude below the magnitudes of U_1, L_1 for sufficient accuracy. In each iteration of the bisection search, problem $\mathcal{Q}_r$ is solved. This SDP has G matrix variables of $N_t \times N_t$ dimensions and $N_u + N_t$ linear constraints. The interior point methods employed to solve this SDP require at most $\mathcal{O}\left(\sqrt{GN_t}\log(1/\epsilon)\right)$ iterations, where ϵ is the desired numerical accuracy of the solver. Moreover, in each iteration not more than $\mathcal{O}(G^3N_t^6 + GN_t^3 + N_uGN_t^2)$ arithmetic operations will be performed. The increase in complexity stems from increasing the number of constraints, i.e. $N_t + N_u$ constraints are considered instead of only N_u as in [10]. However, this increase is not significant, since the order of the polynomial with respect to the number of transmit antennas is not increased. The solver used also exploits the specific structure of matrices hence the actual running time is reduced. Next, a fixed number of Gaussian random instances with covariance given by the previous solution are generated. The complexity burden of this step is given by the following considerations. For each randomization, a second bisection search is performed this time over the LP $\mathcal{S}^{Q}$. An $\epsilon-$optimal solution of this problem can be generated with a worst case complexity of $\mathcal{O}(G^{3.5}\log(1/\epsilon))$ [20] . The second bisection runs for $N_{iter} = \lceil \log_2 (U_2 - L_2) / \epsilon_2 \rceil$ iterations, which are significantly reduced since the upper bound U_2 is now the optimal value of the relaxed problem. Moreover, the Gaussian randomization is executed for a fixed number of iterations. The accuracy of the solution increases with the number of randomizations [8], [10], [19]. Finally, the complexity burden can be further reduced by the reformulation of the non-convex $\mathcal{S}^{\mathcal{F}}$ into the GP, $\mathcal{S}^{\mathcal{F}}_{\mathcal{GP}}$ which is efficiently solved by successive approximations of primal-dual interior point numerical methods [18]. Thus the need for the second bisection can be surpassed.

Input: $N_{rand}, \mathbf{p}, \mathbf{g}, \mathbf{Q}_i, \sigma_i^2 \ \forall i \in \{1 \dots G\}$
Output: $\{\mathbf{w}_k^{opt}\}_{k=1}^G$, t^*_{opt} of $\mathcal{F}$ $\{\mathbf{w}_k^{out}\}_{k=1}^G$ t^*_{out}
begin
 Step 1: Solve $t_{opt} = \mathcal{F}_r(\mathbf{g}, \mathbf{p})$ by bisecting $\mathcal{Q}_r\left(\frac{L+U}{2}\mathbf{g}, \mathbf{p}\right)$, (see Sec. III-F). Let the associated point be $\{\mathbf{w}_k^{opt}\}_{k=1}^G$.
 if $\text{rank}(\mathbf{X}_k^{opt}) = 1, \forall\ k \in \{1 \dots G\}$ **then**
  $\{\mathbf{w}_k^{out}\}_{k=1}^G$ is given by $\lambda_{max}(\mathbf{X}^{opt})$.
 else
  Step 2: : Generate N_{rand} precoding vectors $\{\hat{\mathbf{w}}_k\}_{k=1}^G$, (see Sec. III-D). $t^*_{(0)} = 0$;
  for $i = 1 \dots N_{rand}$ **do**
   Step 3: Solve $\mathcal{S}^{\mathcal{F}}(\mathbf{g}, \mathbf{p})$ by bisecting $\mathcal{S}^{\mathcal{Q}}\left(\frac{L+U}{2}\mathbf{g}, \mathbf{p}\right)$ to get a $\{\mathbf{w}_k^{can}\}_{k=1}^G$ with $t^*_{(i)}$.
   if $t^*_{(i)} > t_{(i-1)}$ **then**
    $t^*_{out} = t^*_{(i)}, \{\mathbf{w}_k^{out}\}_{k=1}^G = \{\mathbf{w}_k^{can}\}_{k=1}^G$
   end
  end
 end
end

Algorithm 1: Fair multigroup multicasting under PACs.

IV. Performance Evaluation & Applications

A. Multigroup multicasting over Rayleigh Channels

The performance of linear multicast multigroup beamforming under per antenna power constraints is examined for a system with $N_t = 5$ transmit antennas, $G = 2$ groups and $N_u = 4$ users. Rayleigh fading is considered, thus the channels are generated as Gaussian complex variable instances with unit variance and zero mean. For every channel instance, the approximate solutions of the max-min fair SPC and the proposed PAC problems are evaluated using $N_{rand} = 100$ Gaussian randomizations [10]. The results are averaged over one hundred channel realizations, while the noise variance is normalized to one for all receivers and all SINR targets are assumed equal to one.

The achievable minimum rate is plotted for the SPC and the PAC optimization in Fig. 1 with respect to the total transmit power in dBWs. Noise is assumed normalised to one. For fair comparison, the total power constraint P_{tot} [Watts] is equally distributed amongst the transmit antennas when PACs are considered, hence each antenna can radiate at most P_{tot}/N_t [Watts]. The accuracy of the approximate solutions for both problems, given by comparing the actual solution to the relaxed upper bound [8], [10], is clear across a wide range of SNR. Nevertheless, the accuracy due to the PACs is slightly reduced. This accuracy degradation is intuitively justified. A Gaussian randomization instance is less likely to approach the optimal point when the number of constraints is increased while the same number of Gaussian randomizations are performed ($N_{\mathrm{rand}} = 100$). Towards quantifying the gains of the proposed solution, the performance of the SPC solution re-scaled to respect the PACs is also included in Fig. 1. Re-scaling is achieved by multiplying each line of the precoding matrix with the square root of the inverse level of power over satisfaction of the corresponding antenna. In Fig. 1 it is clear that more than 1 dB of gain can be obtained by the proposed method over the suboptimal re-scaling approach.

A significant issue for the SDR techniques in multicast applications is the tightness of the approximate solution versus an increasing number of receivers per multicast. In the extreme case of one user per group, it was proven in [1] that the relaxation provides an optimal solution. Thus the solution is no longer approximate but exact. However, the increasing number of users per group degrades the solution, as depicted in Fig. 2 for both problems. It is especially noticed that the PAC system suffers more than the SPC of [10] as the number of users per multicast group increases. An attempt to solve this inaccuracy, but only under sum power constraints, is presented in [12].

B. Power Consumption in DAS

The main difference between the SPC and the PAC optimization problems is the utilization of the available on board power in each system architecture. In [10], the sum power constraint is always satisfied with equality, since any remaining power budget can be equally distributed to the precoding vectors and the solution is further maximized. On the contrary, the PAC system includes N_t constraints which are coupled via the precoders. According to the relation between $\mathcal{F}$ and $\mathcal{Q}$, i.e. (7), the ratio of transmitted power over the power constraint (i.e. r) is one. Since this ratio applies for at least one of the N_t power

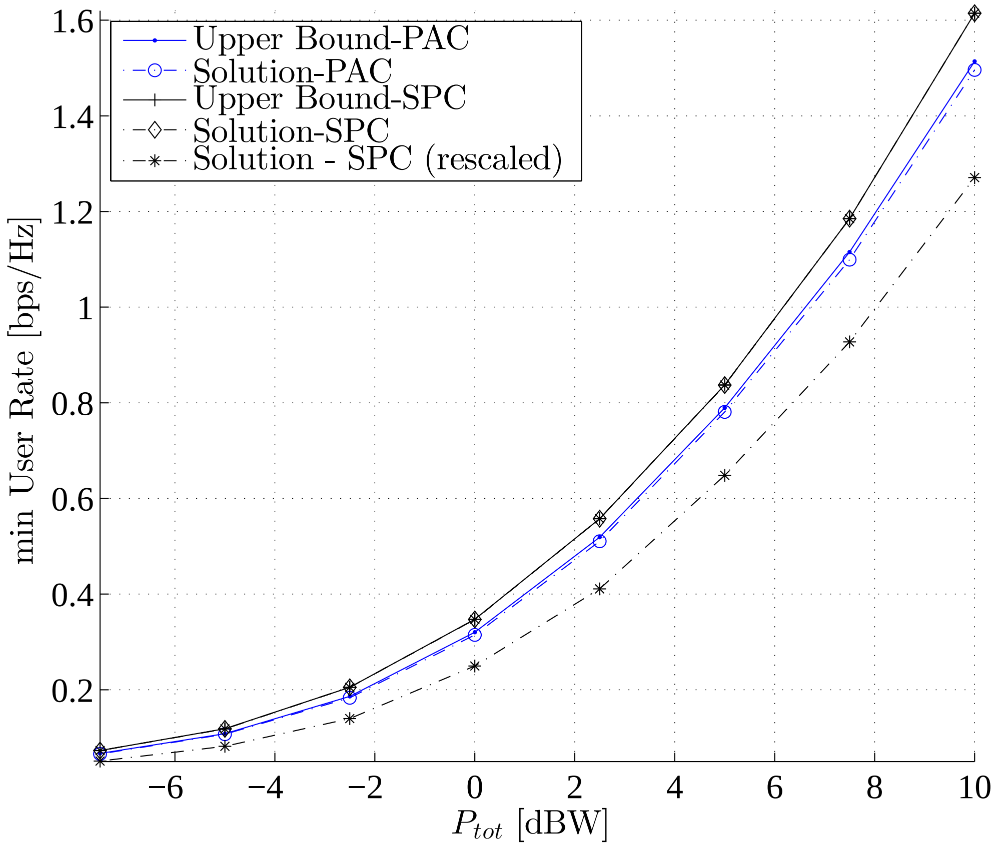

Fig. 1. Minimum user rate with SPC and PAC. Results for $N_u = 4$ users, $N_t = 5$ antennas, $L = 2$ groups and $\rho = 2$ users per group.

constraints, if one is met with equality and the remaining $N_t - 1$ are not, then no more power can be allocated to the precoders. Let us assume a channel matrix with one compromised transmit antenna, i.e. $\mathbf{H} =$

$$\begin{bmatrix} 2.94\angle 41^\circ & 11\angle -25^\circ & 4.4\angle 50^\circ & 6.6\angle -4^\circ \\ 13.2\angle -150^\circ & 4.8\angle 14^\circ & 15.2\angle -7^\circ & 4.8\angle -37^\circ \\ 12\angle -155^\circ & 1.5\angle 163^\circ & 13.5\angle -105^\circ & 3.9\angle -46^\circ \\ 0.02\angle -53^\circ & 0.03\angle -66^\circ & 0.03\angle 120^\circ & 0.03\angle -129^\circ \\ 5.66\angle 137^\circ & 9.2\angle 49^\circ & 13\angle -175^\circ & 2.45\angle 126^\circ \end{bmatrix}^{\mathrm{T}},$$

where 4 users, divided into 2 groups, are served by 5 antennas. One of the antennas (the 4-th antenna) has severely degraded gains towards all users. This practical case can appear in a DAS where the physical separation of the transmit antennas not only imposes per antenna constraints but can also justify highly

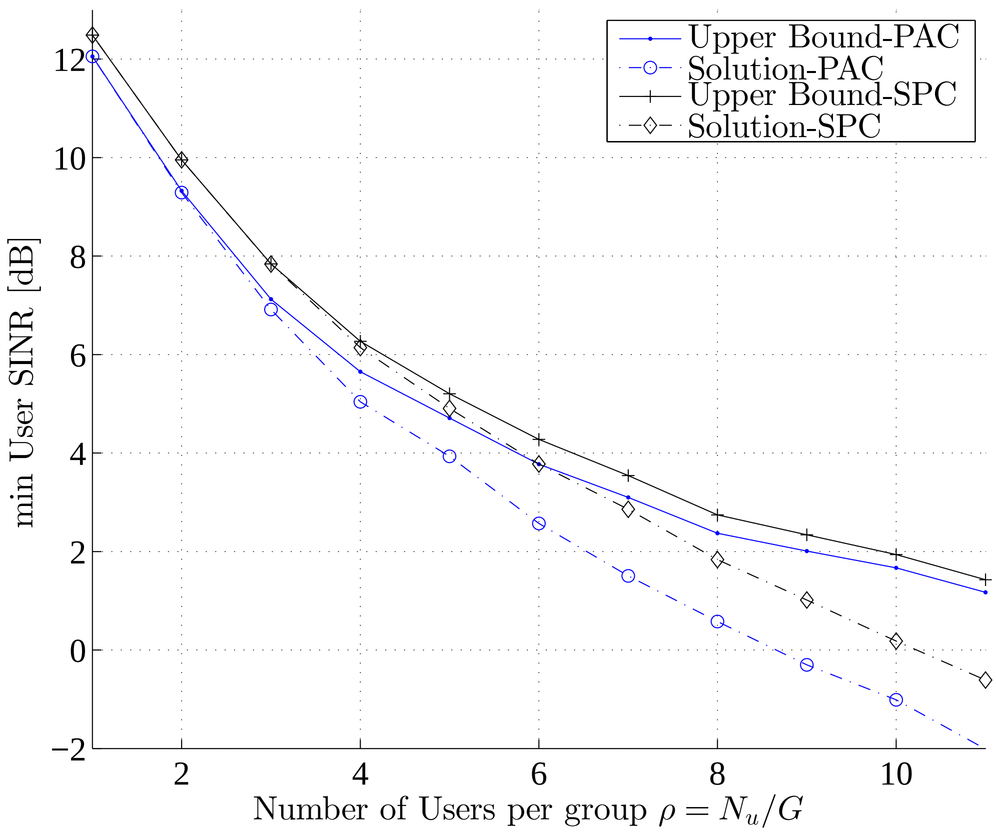

Fig. 2. Minimum SINR with SPC and PAC versus an increasing ratio of users per group $\rho = N_u/G$, for $P_{tot} = 10$ dBW.

unbalanced channel conditions around the environment the antennas. The power utilization of the solution of the optimization for each of the two problems is defined as the total transmitted power over the total available power P_{tot}, that is $P_u = \left(\sum_{k=1}^{G} \mathbf{w}_k{}^{\dagger}\mathbf{w}_k\right)/P_{tot}$, and is plotted versus an increasing power budget in Fig. 3. It is clear that in the low power regime the available power is not fully utilized. As the available power increases, however, the power consumption of the PAC increases. This result is in accordance with the optimality of equal power allocation in the high power regime and renders the PAC formulation relevant for power limited systems. Further insights for this PAC system are given in Fig. 4, where the power utilization of each antenna is shown, for different total power budgets. Interpreting these results, it can be concluded that the PAC problem is highly relevant for power-over-noise limited systems. Otherwise, in the high power regime, the solution of the SPC problem with less constraints could be also used as an accurate approximation.

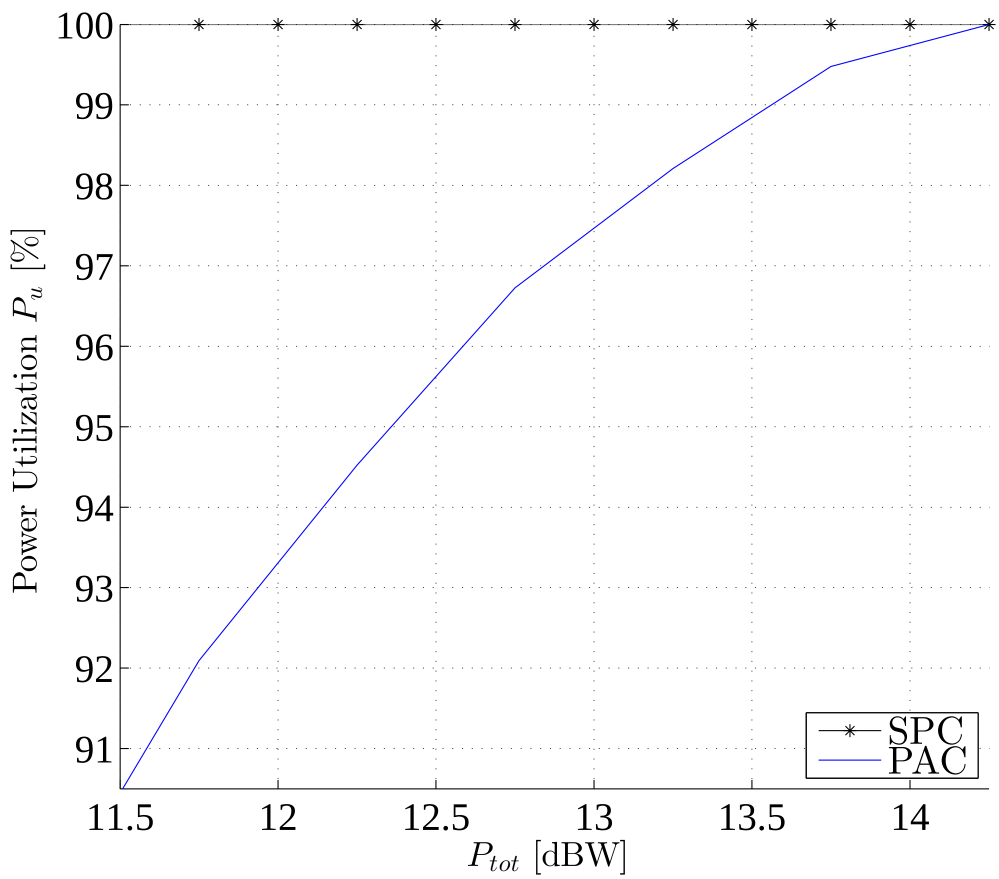

Fig. 3. Total power consumption of a PAC system versus available power.

C. *Weighted Fairness Paradigm*

To the end of establishing the importance of the weighted optimization, a simple paradigm is elaborated herein. Under the practical assumption of a modulation constrained system, the weighted fair design can be exploited for rate allocation towards increasing the total system throughput. More specifically, the considered system employs adaptive modulation and allocates binary phase shift keying (BPSK) modulation if the minimum $\mathrm{SINR_i}$ in the k-th group is less than the ratio for which the maximum modulation constrained spectral efficiency is achieved. This ratio is simply given by $\log_2 M$, where M is the modulation order. Hence for BPSK, $\gamma_2 = 0$ dB, and so forth. If for some group k, $\min_i \mathrm{SINR_i} \geq \gamma_2, \ \forall i \in \mathcal{G}_k$, then quaternary phase shift keying (QPSK) is used for all users in the group. Forward error correction is not assumed. Let there be a two antenna transmitter that serves four users grouped into two

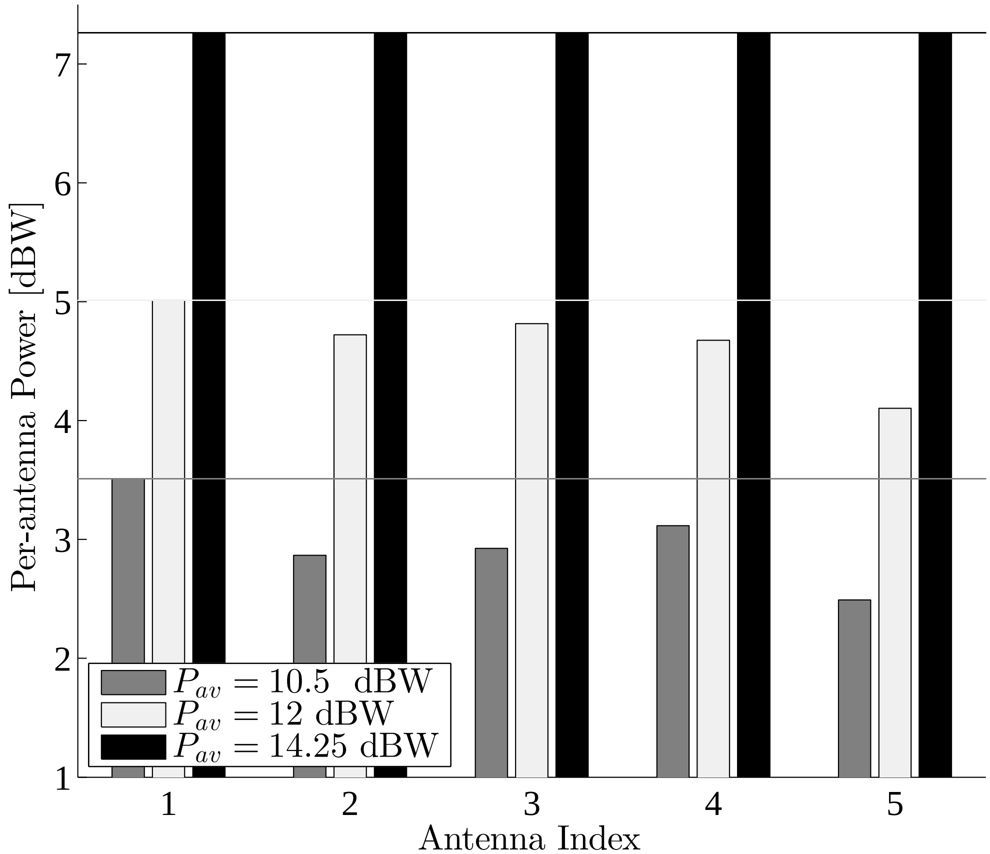

Fig. 4. Per-antenna consumption in a PAC system versus transmit power.

groups. The considered channel matrix reads as

$$\mathbf{H} = \begin{bmatrix} 0.2\angle 106^\circ & 90\angle{-69^\circ} & 0.5\angle{-99^\circ} & 0.5\angle 61^\circ \\ 0.8\angle 111^\circ & 120\angle{-112^\circ} & 1\angle 127^\circ & 1.5\angle 49^\circ \end{bmatrix}^{\mathrm{T}}.$$

The attributes of the specific channel matrix depict one possible instance of the system where one user with a good channel state (i.e. user two) is in the same group with a jeopardized user, namely user one. On the other hand, the second group contains relatively balanced users in terms of channel conditions. For an un-weighted optimization (i.e. $\mathbf{g} = [1\ 1\ 1\ 1]$) the spectral efficiency of each user is shown in Fig. 5. Baring in mind that each user is constrained by the minimum group rate, the actual rate at which all users will receive data is 0.52 [bps/Hz]. Both groups achieve the same spectral efficiency since the minimum SINRs and hence the minimum rates are balanced between the groups. Subsequently, a modulation constrained multicast transmitter will employ BPSK for all users. By heuristically choosing the constraint vector to be $\mathbf{g} = [1\ 1\ 5.3\ 5.3]$ each user rate is modified. As depicted in Fig. 5 both users in the second group are achieving adequate SINR to support a higher order modulation. This gain is achieved at the expense of the rates of the users of the first group. Following this paradigm, the weight optimization can lead to an improved modulation assignment and thus higher throughput in practical systems. Hence, the weighted formulation offers the substantial degrees of freedom to maximize the total throughput of a modulation constrained multicast system by properly allocating the rates amongst the groups.

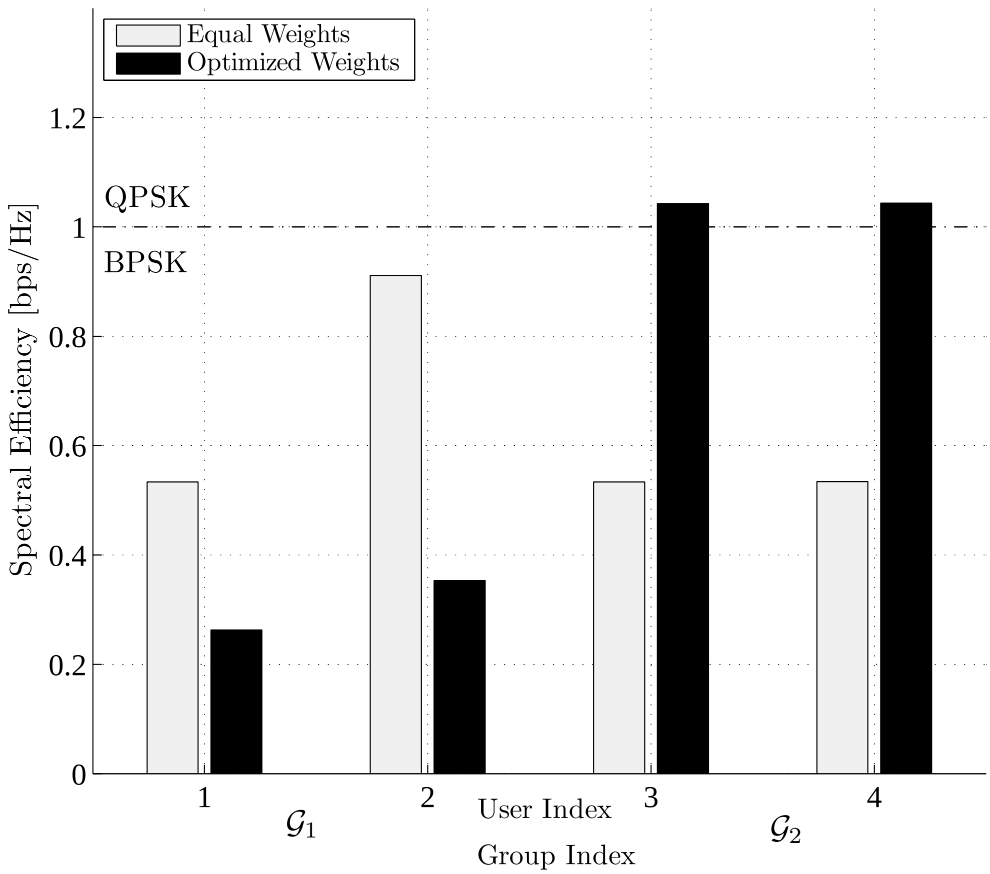

Fig. 5. Modulation constrained paradigm.

D. Uniform Linear Arrays

To the end of investigating the sensitivity of the proposed algorithm with respect to the angular separation of co-group users, a uniform linear array (ULA) transmitter is considered. Assuming far-field, line-of-sight conditions, the user channels can be modeled using Vandermonde matrices. For this important special case, the SPC multicast multigroup problem was reformulated into a convex optimization problem and solved in [21], [22]. These results where motivated by the observation that in ULA scenarios, the relaxation consistently yields rank one solutions. Thus, for such cases, the SDR is essentially optimal [8]. The fact that the SDR of the sum power minimization problem is tight for Vandermonde channels was established in [22]. Let us consider a ULA serving 4 users allocated to 2 distinct groups. In Fig. 6, its radiation pattern for co-group angular separation $\theta_a = 35^\circ$ is plotted. The symmetricity due to the inherent ambiguity of the ULA is apparent. Clearly, the multigroup multicast beamforming optimizes the lobes to reduce interferences between the two groups. The SPC solution, re-scaled to respect the PACs are also included in Fig. 6. The superiority of the proposed solution is apparent.

In Fig. 7, the performance in terms of minimum user rate over the area with respect to an increasing angular separation is investigated. When co-group users are collocated, i.e. $\theta_a = 0^\circ$, the highest performance is attained. As the separation increases, the performance is reduced reaching the minimum when users from different groups are placed in the same position, i.e. $\theta_a = 45^\circ$. In Fig. 7, the tightness of the relaxation for the SPC problem [22] is clear. However, the same does not apply for the proposed PAC. As co-group channels tend to become orthogonal, the approximation becomes less tight. Nevertheless, $N_{\mathrm{rand}} = 200$ randomizations are sufficient to maintain the solution above the re-scaled SPC, as shown in Fig. 7. Consequently, the proposed solution outperforms a re-scaled to respect the per-antenna constraints, SPC solution, over the span of the angular separations.

Remark 4: The semidefinite relaxation of the per-antenna power minimization problem in ULA transmitters is not always tight.

For every optimum high rank set of matrices $\{\mathbf{X}_k^{opt}\}_{k=1}^G$, there exists a set of rank one positive semidefinite matrices $\{\bar{\mathbf{X}}_k^{opt}\}_{k=1}^G$, i.e. $\mathrm{rank}(\bar{\mathbf{X}}_k^{opt}) = 1, \forall k \in \{1 \dots G\}$, which is equivalent with respect to the power received at each user, i.e $\mathrm{Tr}(\mathbf{X}_k^{opt}\mathbf{Q}_i) = \mathrm{Tr}(\bar{\mathbf{X}}_k^{opt}\mathbf{Q}_i), \forall i \in \mathcal{G}_k, k, l \in \{1 \dots G\}$. This result is based on the Riesz-Féjer theorem on real valued complex trigonometric polynomials [22]. Therefore, the Vandermonde channels impose a specific structure to the SPC solution that allows for a convex reformulation. The difference in the case tackle herein lies in the N_t PACs, i.e. $\left[\sum_{k=1}^G \mathbf{X}_k\right]_{nn} \leq P_n, \forall n \in \{1 \dots N_t\}$, in which the channel structure is not involved. Thus, a rank-1 matrix is equivalent in terms of per user received power [22] but not necessarily in terms of per-antenna consumed power, as shown herein.

E. Robust Design under PAC*s*

When beamforming under uncertainty is considered, three different designs can be realized [23]. Namely, the probabilistic design, where acceptable performance is guaranteed for some percentage of

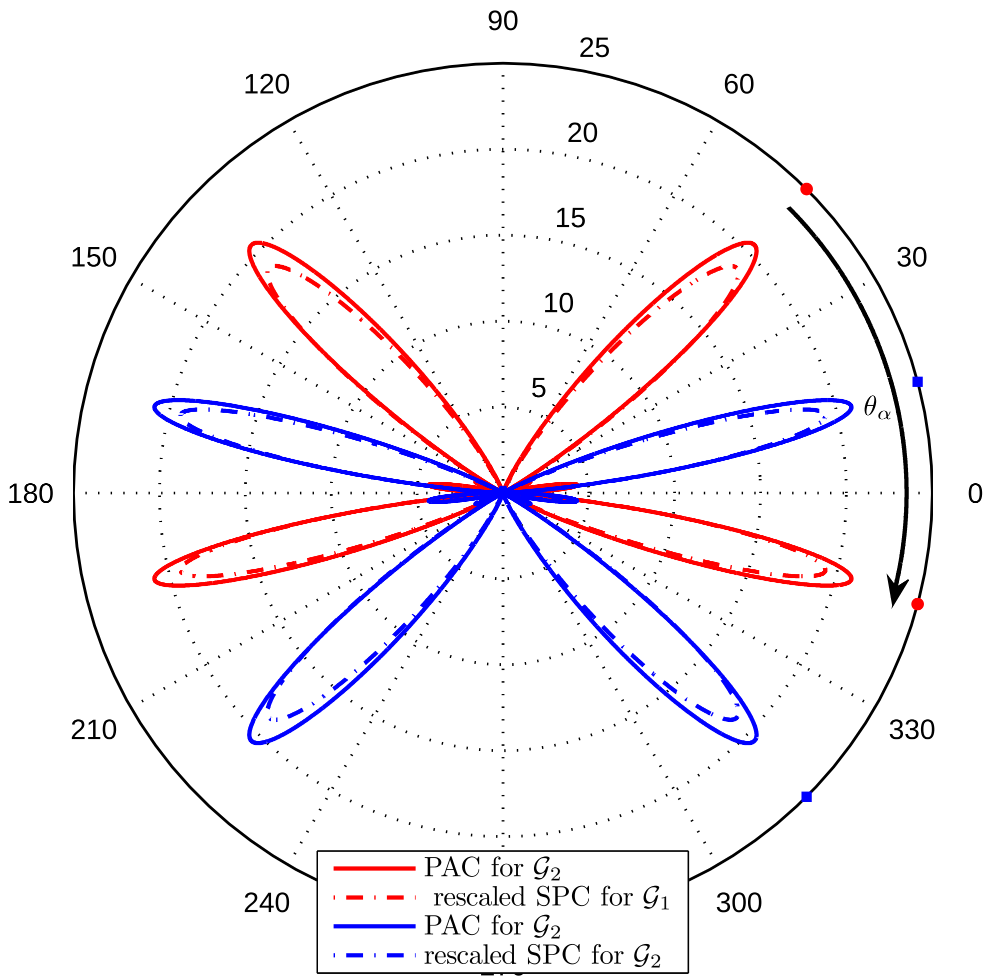

Fig. 6. ULA beampattern for PAC and re-scaled SPC solutions.

time, the expectation based design that requires knowledge of the second order channel statistics but cannot guarantee any outage performance and the worst-case design. The latter approach guarantees a minimum QoS requirement for any error realization.

Focusing on a worst-case design, let us assume an elliptically bounded error vector. In this context, the actual channel is given as $\mathbf{h}_i = \bar{\mathbf{h}}_i + \mathbf{e}_i$ where $\bar{\mathbf{h}}_i$ is the channel available at the transmitter and $\mathbf{e}_i$ is an error vector bounded by $\mathbf{e}_i^\dagger \mathbf{C}_i \mathbf{e}_i \leq 1$. The hermitian positive definite matrix $\mathbf{C}_i$ defines the shape and size of the ellipsoidal bound. For $\mathbf{C}_i = 1/\sigma_\epsilon^2 \mathbf{I}_{N_t}$, then $||\mathbf{e}_i||_2^2 \leq \sigma_\epsilon^2$ and the error remains in a spherical region of radius σ_ϵ [24]. This spherical error model is mostly relevant when the feedback quantization

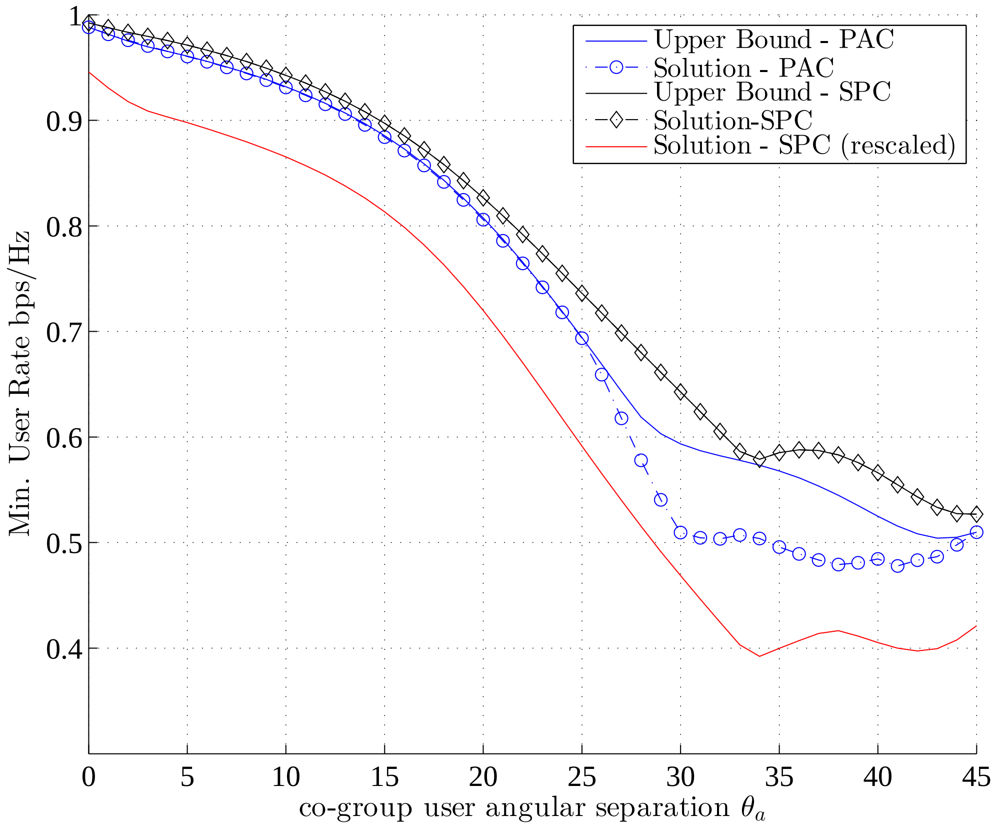

Fig. 7. ULA performance for increasing co-group user angular separation.

error of a uniform quantizer at the receiver is considered [25]. The proposed design is formulated as

$$\boxed{\begin{aligned} \mathcal{F}_{\mathcal{RB}}: \quad & \max_{t,\ \{\mathbf{w}_k\}_{k=1}^{G}} \ t \\ \text{s. t.}\ & \frac{1}{\gamma_i} \frac{|\mathbf{w}_k^\dagger \left(\bar{\mathbf{h}}_i + \mathbf{e}_i\right)|^2}{\sum_{l\neq k}^{G} |\mathbf{w}_l^\dagger \left(\bar{\mathbf{h}}_i + \mathbf{e}_i\right)|^2 + \sigma_i^2} \geq t, \qquad (21) \\ & \forall i \in \mathcal{G}_k, k, l \in \{1 \ldots G\}, \\ \text{and to}\ & \left[\sum_{k=1}^{G} \mathbf{w}_k \mathbf{w}_k^\dagger\right]_{nn} \leq P_n, \forall n \in \{1 \ldots N_t\}, \qquad (22) \end{aligned}}$$

and involves the channel imperfections only in the SINR constraints. The novelty of $\mathcal{F}_{\mathcal{RB}}$ over existing robust multicast formulations lies in (22). The SINR constraints of $\mathcal{F}_{\mathcal{RB}}$, i.e. (21), are over all possible error realizations and cannot be handled. However, by applying the S-lemma [18], the error vector in (21) can be eliminated. This procedure is analytically described in [26]. Thus, $\mathcal{F}_{\mathcal{RB}}$ can be converted to a SDP and solved efficiently using the methods described in Sec. III. The performance gain of the proposed robust design for a ULA with $N_t = 2$ transmit antennas, serving $N_u = 6$ users is given in Fig. 8, versus an increasing error radius σ_ϵ, for different user per group configurations, ρ. These results exhibit the significant gains of the proposed technique as the error and the group sizes increase.

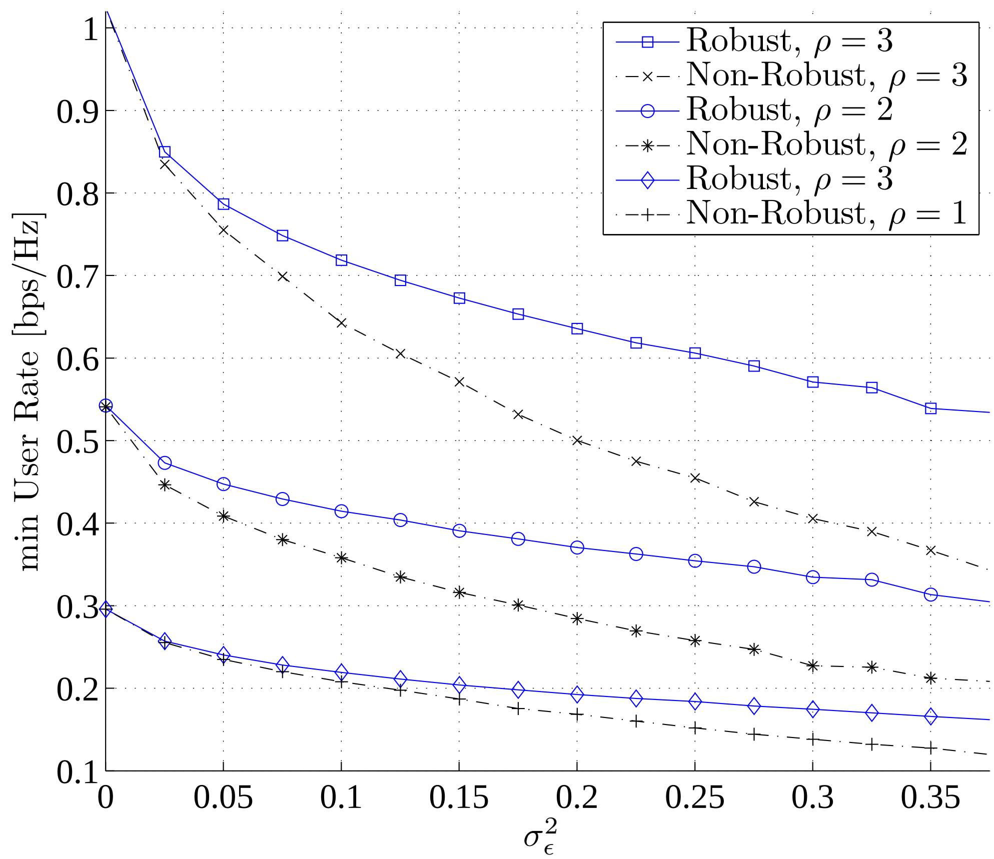

Fig. 8. Robust performance for various user per group configurations.

To establish the importance of the novel formulation, the performance in terms of minimum user rate over 1000 error realizations is given in Fig. 9, versus a wide range of the error radius σ_ϵ for the proposed $\mathcal{F}_{\mathcal{RB}}$ as well as the existing SPC solutions re-scaled to respect the per-antenna constrains. For this figure, a ULA with $N_t = 3$ transmit antennas is considered, serving $N_u = 6$ users partitioned into $L = 2$ multicast groups. The co-group angular separation is $\theta_a = 10°$ and the number of Gaussian randomizations chosen is $N_{\text{rand}} = 200$ and $N_{\text{rand}} = 1000$ for the high and low precision curves respectively. According to Fig. 9, the proposed robust PAC formulation (i.e. $\mathcal{F}_{\mathcal{RB}}$) outperforms existing solutions, in a per-antenna power constrained setting, for a wide range of channel error radius. However, as the error radius increases, a slight performance degradation is noted, especially for the low precision results. To further investigate on this result, the following remark is given.

Remark 5: The semidefinite relaxation of robust multigroup multicasting under PACs yields non rank-1 solutions with higher probability as the channel errors increase.

The accuracy of the minimum rate results of Fig. 9, is presented in Fig. 10. The accuracy is measured by the distance of the randomized solution from the upper bound given by the relaxation, following the standards of Sec. IV-A and [8], [10]. In Fig. 10, the results are also normalized by the value of the upper bound. According to these results, the probability for the SDR to yield rank-1 solutions is reduced as the error radius increases, for all problems. The accuracy reduction of the SDR technique as the channel errors increase was also reported via simulations in [27], but for unicast scenarios. What is more, $\mathcal{F}_{\mathcal{RB}}$ yields non rank-1 solutions as the errors increase, with higher probability than the SPC problem. However, 1000 randomizations are sufficient to reduce the inaccuracy of all solutions to less than 7%, as illustrated in Fig. 10. It is therefore concluded that although the relaxation of the robust formulations does not consistently yield rank-1 solutions, especially for higher values of error radius, the Gaussian randomization can provide solutions with adequate accuracy. Finally, the proposed solutions surpass the performance of existing approaches, in practical per-antenna power constrained settings.

V. Conclusions

In the present work, optimum linear precoding vectors are derived under per antenna power constraints, when independent sets of common information are transmitted by an antenna array to distinct co-channel sets of users. The novel weighted max–min fair multigroup multicast problem under PACs is formulated. An approximate solution for this NP-hard problem is presented based on the well established methods of semidefinite relaxation. The performance of the weighted max–min fair multigroup multicast optimization is examined under various system parameters and important insights on the system design are gained. Moreover, an application paradigm of the new system design is described while robust to imperfect CSI extensions are given. Consequently, an important practical constraint towards the implementation of physical layer multigroup multicasting is alleviated.

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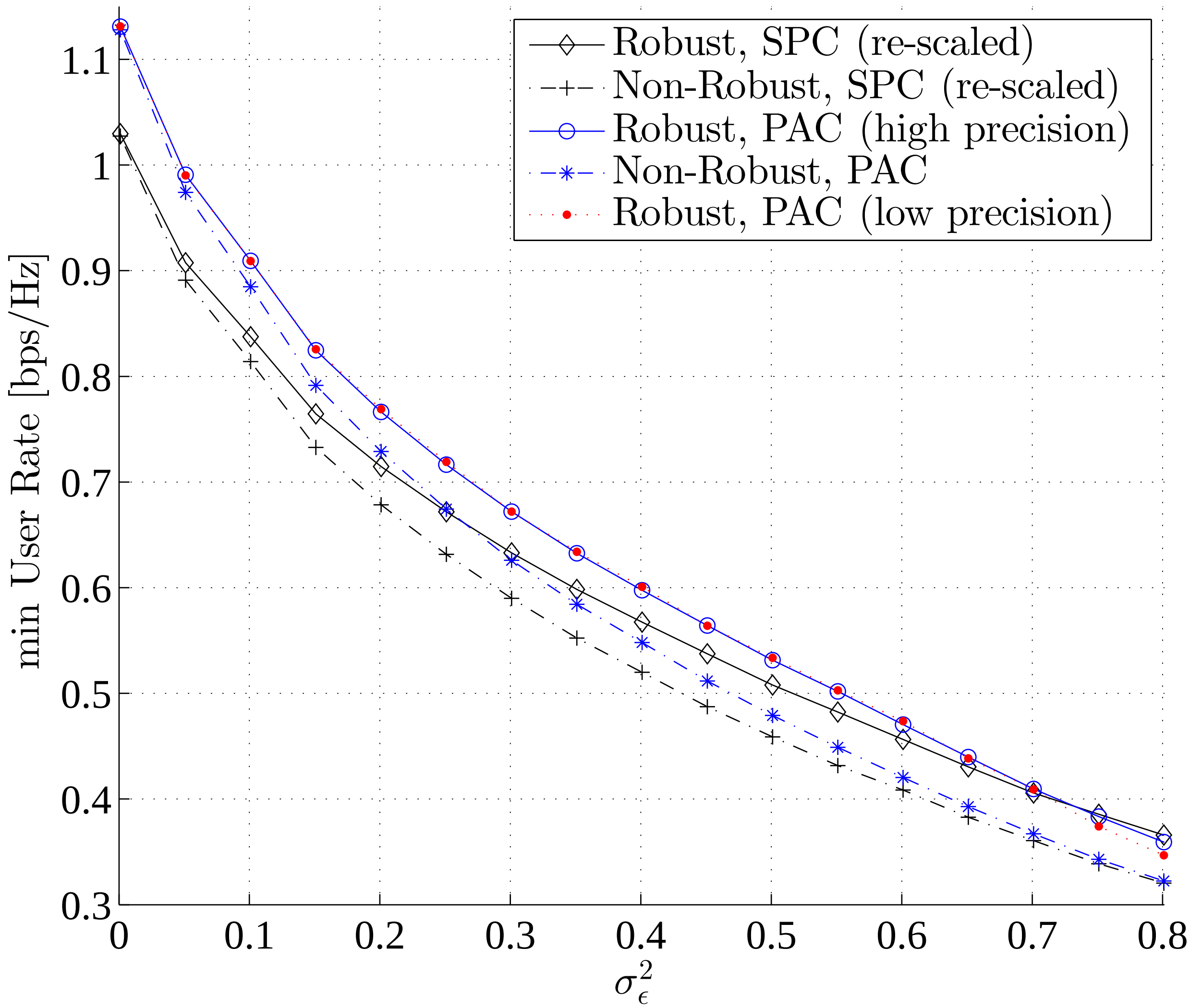

Fig. 9. Minimum user rate versus increasing CSI error.

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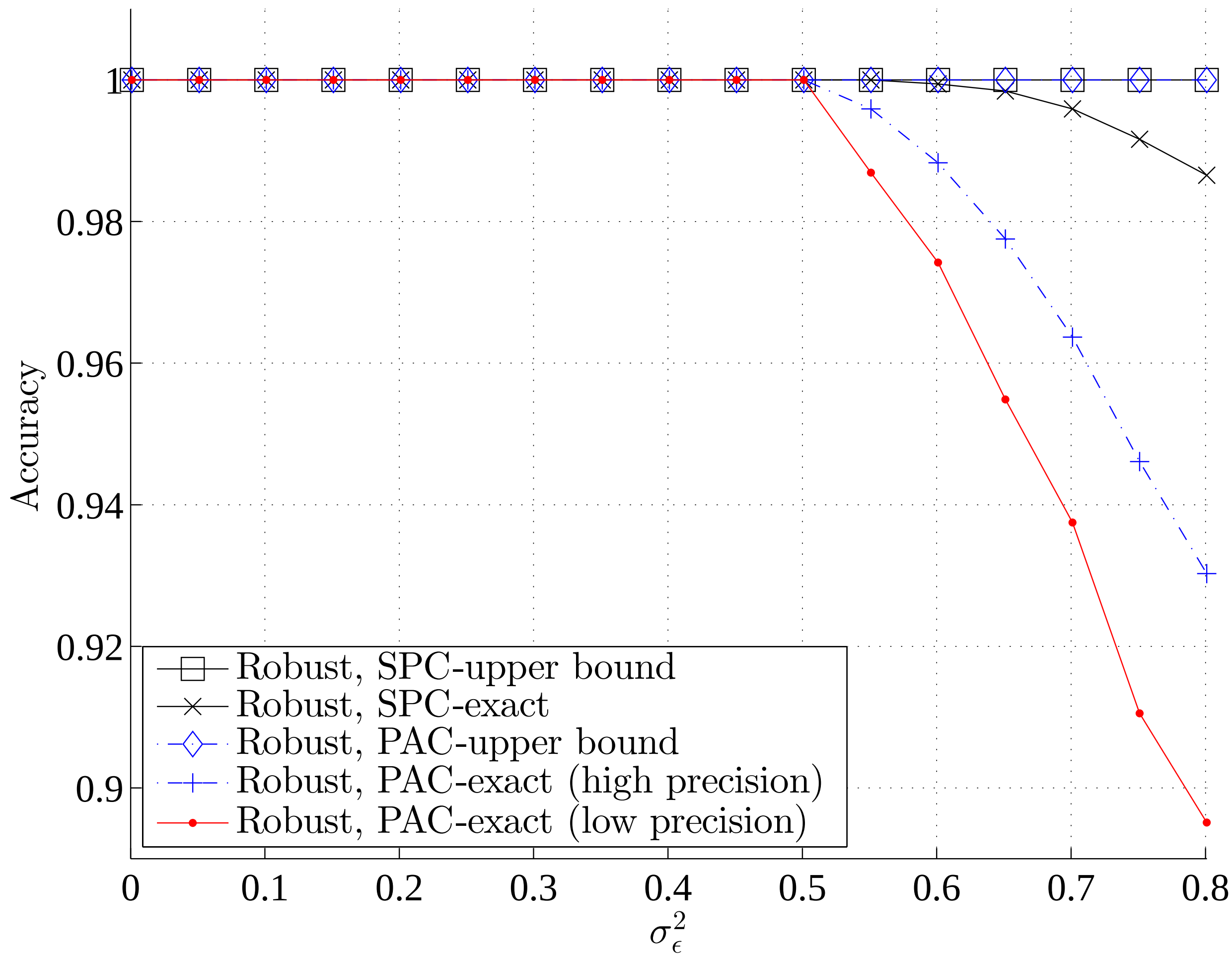

Fig. 10. Accuracy of the semidefinite relaxation versus an increasing CSI error.

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