Compressive Sparsity Order Estimation for Wideband Cognitive Radio Receiver

Shree Krishna Sharma, Symeon Chatzinotas, and Björn Ottersten
SnT - securityandtrust.lu, University of Luxembourg
Email:{shree.sharma, symeon.chatzinotas, bjorn.ottersten}@uni.lu

Abstract— **Compressive Sensing (CS) has been widely investigated in the Cognitive Radio (CR) literature in order to reduce the hardware cost of sensing wideband signals assuming prior knowledge of the sparsity pattern. However, the sparsity order of the channel occupancy is time-varying and the sampling rate of the CS receiver needs to be adjusted based on its value in order to fully exploit the potential of CS-based techniques. In this context, investigating blind Sparsity Order Estimation (SOE) techniques is an open research issue. To address this, we study an eigenvalue-based compressive SOE technique using asymptotic Random Matrix Theory. We carry out detailed theoretical analysis for the signal plus noise case to derive the asymptotic eigenvalue probability distribution function (aepdf) of the measured signal's covariance matrix for sparse signals. Subsequently, based on the derived aepdf expression, we present a technique to estimate the sparsity order of the wideband spectrum with compressive measurements using the maximum eigenvalue of the measured signal's covariance matrix. The performance of the proposed technique is evaluated in terms of normalized SOE Error (SOEE). It is shown that the sparsity order of the wideband spectrum can be reliably estimated using the proposed technique.**

I. Introduction

Cognitive Radio (CR), which can exploit the unused licensed spectrum opportunistically, is considered a promising candidate to enhance the spectral efficiency in future wireless systems [1, 2]. In this direction, Spectrum Sensing (SS) is an important spectrum awareness mechanism required by the CRs. Several SS techniques such as matched filter, cyclostationary feature detection, energy detection, autocorrelation and eigenvalue-based detection have been proposed in the literature for sensing the presence of a Primary User (PU) [3–6]. Most of the existing SS techniques focus on the detection of narrowband signals considering a single radio channel. However, in practical scenarios, the CRs need to detect and acquire information about a wide spectrum band in order to utilize the spectrum efficiently. Furthermore, CRs do not have prior knowledge about the PU's signal and channel. In this aspect, investigating efficient blind wideband spectrum awareness techniques is an important and relevant research challenge. One of the main challenges in implementing a wideband CR is the design of the Radio Frequency (RF) front-end. The sensing RF chain of a CR receiver should be able to receive a wideband signal, sample it using a high speed Analog to Digital Converter (ADC) and perform measurements for the detection of PU signals. Furthermore, the main limitation in an RF front-end's ability to detect weak signals is its dynamic range i.e., the requirement for a large number of bits in the ADC. For this purpose, the wideband sensing requires multi-GHz speed ADCs, which together with high resolution might be infeasible with current technology [7].

In the above context, Compressive Sensing (CS) has emerged as an important technique which can significantly reduce the acquisition cost at the CR node [8, 9]. According to CS theory, certain signals can be recovered from far fewer samples or measurements than those required by conventional methods [8]. In sparse signals, most of the signal energy is concentrated in a few non-zero spectral coefficients. Furthermore, it's not necessary for the signal itself to be sparse but it can be compressible within sparse representations of the signal in some known transform domain, which depends on the nature of the waveform [10, 11]. For example, smooth signals are sparse in the Fourier basis, whereas piecewise smooth signals are sparse in a wavelet basis. Most of the CS literature has focused on improving the speed and accuracy of recovering the original sparse signal from compressive measurements [12–14]. In the context of CR networks, CS techniques are suitable for acquiring the spectrum usage information in a wide spectrum band as in many cases the spectrum occupancy is sparse in the frequency and time domains [15].

A. Contributions

Most of the CS contributions in the context of a wideband CR assume that the wideband signal is sparse in some domain and the number of measurements is kept fixed based on the assumed sparsity order. In addition, in most of the contributions, it is assumed that the sparsity order of the signal is known beforehand. However, in the context of CR networks, this prior information is not available at the CR sensor and has to be estimated. Furthermore, in compressive wideband systems, the required number of measurements to achieve a successful recovery rate proportionally varies with the sparsity order of wideband signals [16]. In this context, Sparsity Order Estimation (SOE) is crucial in order to choose the appropriate number of measurements based on the estimated sparsity order.

In the above context, a two-step CS algorithm has been recently proposed in [16]. In the proposed algorithm, the sparsity order of the wideband signals is estimated in the first step and the total number of collected samples are adjusted in the second step based on the estimated sparsity order. However, the considered estimation approach is based on Monte Carlo simulations and the proposed simulation-based approach requires the reconstruction of the original sparse signal. In this aspect, we consider an eigenvalue-based approach using the eigenvalues of the CS measurement vector in order to estimate the sparsity order of the wideband spectrum using Random Matrix Theory (RMT). In our previous works [17–19], the eigenvalue-based approach has been used for Signal to Noise Ratio (SNR) estimation in various noise/channel correlated scenarios. Our proposed method requires no prior information about the PU signals neither the knowledge of channel nor the noise covariance. Another main advantage of the proposed method is that compressive measurements can be used for acquiring the sparsity order information with a trade-off between estimation performance (expressed in terms of SOE Error (SOEE)) and hardware cost (number of measurements). It should be noted that this work focuses on the SOE and the support recovery of the sparsity pattern is not addressed here. After estimating the sparsity order, already existing support recovery algorithms [20, 21] can be straightforwardly applied. The signal model used in this paper has been inspired from the model used in [20] which focuses on the input-output mutual information and the support recovery rate in the asymptotic limit. Our theoretical analysis differs from [20] as we derive the aepdf expression based on the Multiple Measurement Vector (MMV) model instead of the Single Measurement Vector (SMV) model considered in [20], which is the main contribution of this paper. Based on the derived aepdf expressions, we propose a technique in order to estimate the sparsity order of the channel occupancy within the considered wideband spectrum.

B. Structure and Notation

The remainder of this paper is structured as follows: Section II describes the system and signal models. Section III presents theoretical analysis using the RMT approach. Section IV proposes an eigenvalue-based SOE technique based on the derived aepdf expression. Section V evaluates the performance of the proposed SOE technique with numerical simulations. Section VI presents conclusions. The appendix includes some preliminaries on random matrix transforms.

Throughout this paper, boldface upper and lower case letters are used to denote matrices and vectors respectively, $\mathbb{E}[\cdot]$ denotes expectation, $\mathbb{C}$ denotes complex numbers, $(\cdot)^T$ and $(\cdot)^\dagger$ denote the transpose and the conjugate transpose respectively, $\mathcal{P}[\cdot]$ denotes the probability, $\mathbf{X}_{i,j}$ denotes the (i,j)th element of $\mathbf{X}$, $f_{\mathbf{X}}(\cdot)$ denotes the eigenvalue distribution function of $\mathbf{X}$, $\mathbf{R_X}$ represents the covariance matrix of $\mathbf{X}$, $\hat{\mathbf{R}}_{\mathbf{X}}$ represents the sample covariance of $\mathbf{X}$, $\mathcal{S}_{\mathbf{X}}$ represents the Stieltjes transform of $\mathbf{X}$, $\mathcal{R}_{\mathbf{X}}$ represents the R transform, $\Sigma_{\mathbf{X}}$ represents the Σ transform and $\eta_{\mathbf{X}}$ represents the η transform [22].

II. System and Signal Model

Let us consider a total bandwidth of W Hz with N number of carriers each having W/N channel bandwidth. Since all the carriers may not be occupied all the time, we assume sparse channel occupancy in the considered wideband spectrum. In this context, we consider multiple subbands within the considered wideband spectrum and represent each subband with a center carrier frequency. Let σ be the sparsity order, which is defined as the ratio of the number of the occupied carriers to the total number of carriers over the considered wideband spectrum. Let ρ be the compression ratio, which indicates the number measurements as defined later. As in [20], we formulate the SMV problem with the following complex-valued observation model

$$\mathbf{y} = \mathbf{AUXb} + \mathbf{z}, \tag{1}$$

where $\mathbf{A}$ is an $N \times N$ diagonal matrix with independent and identically distributed (i.i.d.) diagonal Bernoulli distributed elements i.e., $\mathcal{P}[\mathbf{A}_{i,i} = 1] = \rho = 1 - \mathcal{P}[\mathbf{A}_{i,i} = 0]$, $\mathbf{U}$ is an $N \times N$ random matrix with i.i.d. elements, $\mathbf{X}$ is an $N \times N$ random matrix with i.i.d. elements, $\mathbf{b}$ is an $N \times 1$ vector with i.i.d. complex components b_i distributed with Bernoulli distribution i.e., $\mathcal{P}[b_i = 1] = \sigma = 1 - \mathcal{P}[b_i = 0]$, and $\mathbf{z}$ is an i.i.d. complex Gaussian $N \times 1$ vector with components $z_i \sim \mathcal{CN}(0, 1)$. It should be noted that the $N \times N$ matrix $\mathbf{AU}$ denotes the compressed sensing matrix while the $N \times 1$ vector $\mathbf{Xb}$ denotes the sparse vector representing the sparseness of the carrier occupancy. It should be noted that the parameter ρ is equivalent to the ratio of the dimensions of the measurement matrix considered in the literature [23].

In this paper, we are interested in analyzing the MMV scenario, where the combination of concatenated multiple measurement vectors has been represented in the form of a matrix $\mathbf{Y}$. We consider $N \times N$ sensing matrix $\mathbf{AU}$ consisting of ρN number of non-zero rows. This is equivalent to the scenario with a CR node equipped with $M = \rho N$ number of frequency selective filters considered in [15], where M filters are used to measure M different linear combinations of the received signals of all N carriers. Let N also be the number of samples collected by a sensor during the measurement process and therefore each measurement vector of $\mathbf{Y}$ contains N number of samples. We assume that the PU channel occupancy status remains constant during the period of measurement. In this context, we extend the above SMV problem (1) into the following MMV model:

$$\mathbf{Y} = \mathbf{AUBX} + \mathbf{Z} = \mathbf{AUS} + \mathbf{Z}, \tag{2}$$

where $\mathbf{B} = \mathrm{diag}(\mathbf{b})$ is an $N \times N$ diagonal matrix with the diagonal having i.i.d. Bernoulli distributed elements i.e., $\mathcal{P}[\mathbf{B}_{i,i} = 1] = \sigma = 1 - \mathcal{P}[\mathbf{B}_{i,i} = 0]$. The $N \times N$ matrix $\mathbf{S} = \mathbf{BX}$ is a sparse signal matrix with uniform sparsity (sparsity order σ) across all the columns. It can be noted that $\mathbf{Y}$ contains ρN number of non-zero rows and each non-zero row contains N number of samples. We assume that the matrices $\mathbf{A}$, $\mathbf{U}$, $\mathbf{B}$, $\mathbf{X}$ and $\mathbf{Z}$ are mutually independent. Furthermore, we consider $\mathbf{U}$ and $\mathbf{X}$ to be $N \times N$ random matrices having i.i.d. entries with zero mean and variance $1/N$. The sensing matrix $\mathbf{AU}$ is assumed to be known by the receiver.

Assuming that the source signal is independent from the noise, the covariance matrix of the measured signal, denoted by $\mathbf{R_Y}$, can be calculated as [5]:

$$\mathbf{R_Y} = \mathbb{E}[\mathbf{YY}^\dagger] = \mathbb{E}\left[(\mathbf{AUBX})(\mathbf{AUBX})^\dagger\right] + \mathbb{E}[\mathbf{ZZ}^\dagger]. \tag{3}$$

In this paper, we are interested in finding out the eigenvalue distribution $f(\lambda)$ of $\mathbf{R_Y}$. Since all the matrices $\mathbf{A}, \mathbf{U}, \mathbf{B}, \mathbf{X}$ and $\mathbf{Z}$ are square, $f_{\mathbf{R_Y}}(\lambda) = f_{\bar{\mathbf{R}}_\mathbf{Y}}(\lambda)$, where $\bar{\mathbf{R}}_\mathbf{Y} = \mathbb{E}\left[\mathbf{U}^\dagger\mathbf{A}^\dagger\mathbf{AUBXX}^\dagger\mathbf{B}^\dagger\right] + \mathbb{E}[\mathbf{ZZ}^\dagger] = \mathbf{RR}_1 + \mathbf{R_Z}$ with $\mathbf{R} = \mathbb{E}\left[\mathbf{U}^\dagger\mathbf{A}^\dagger\mathbf{AU}\right]$, $\mathbf{R}_1 = \mathbb{E}\left[\mathbf{BXX}^\dagger\mathbf{B}^\dagger\right]$ and $\mathbf{R_Z} = \mathbb{E}[\mathbf{ZZ}^\dagger]$. In practice, the covariance matrix $\mathbf{R_Y}$ is not available and we have to rely on the sample covariance matrix. Let us define the sample covariance matrices of the measured signal and noise as: $\hat{\mathbf{R}}_\mathbf{Y}(N) = \frac{1}{N}\mathbf{YY}^\dagger$ and $\hat{\mathbf{R}}_\mathbf{Z}(N) = \frac{1}{N}\mathbf{ZZ}^\dagger$. Similarly, let $\hat{\mathbf{R}}$ and $\hat{\mathbf{R}}_1$ be the sample covariance matrices corresponding to the covariance matrices $\mathbf{R}$ and $\mathbf{R}_1$ respectively. It can be noted that $\hat{\mathbf{R}}$ and $\hat{\mathbf{R}}_1$ are asymptotically free from any deterministic matrix for the considered $\mathbf{X}$ and $\mathbf{U}$ [20].

In this study, we are interested in studying the constant power case considering equal received power across all the carriers. The detailed analysis for time varying power and correlated scenarios has been carried out in [24]. The signal model can be written as:

$$\mathbf{Y} = \mathbf{AU}\sqrt{p}\mathbf{BX} + \mathbf{Z}, \tag{4}$$

where p denotes the constant power across all the carriers. Since we assume normalized noise variance, SNR $\equiv p$. The value of SNR is assumed to be known and it can be acquired through SNR estimation techniques like in [17, 18].

III. Analysis

Assuming that signal and noise are uncorrelated with each other, for large values of N, the measured signal's sample covariance matrix can be written using the following asymptotic approximation [5]:

$$\lim_{N\to\infty} \hat{\mathbf{R}}_\mathbf{Y}(N) \approx p\hat{\mathbf{R}}\hat{\mathbf{R}}_1 + \hat{\mathbf{R}}_\mathbf{Z}. \tag{5}$$

The aepdf of the measured signal's sample covariance matrix given by (5) can be used to estimate the sparsity order for the considered case. Due to noncommutative nature of random matrices, it's not straightforward to calculate the eigenvalue distribution of $\hat{\mathbf{R}}_\mathbf{Y}(N)$ by knowing the eigenvalue distributions of $\hat{\mathbf{R}}$, $\hat{\mathbf{R}}_1$ and $\hat{\mathbf{R}}_\mathbf{Z}$. Using free probability analysis, the asymptotic spectrum of the sum or product can be obtained from the individual asymptotic spectra without involving the structure of the eigenvectors of the matrices [22]. The asymptotic eigenvalue distribution of $\mathbf{Y}$ in this context can be obtained by applying Σ and R transforms [22]. To recover the aepdf of $\hat{\mathbf{R}}_\mathbf{Y}(N)$, we need to know the Stieltjes transform of its asymptotic density function (see Theorem 6.7). In this section, we use free probability theory to derive the Stieltjes transform of $\hat{\mathbf{R}}_\mathbf{Y}(N)$, which is then subsequently used to find the aepdf and to estimate the sparsity order in our considered problem.

From [22, Theorem 2.39], the η transform of $\hat{\mathbf{R}}$ satisfies the following relation:

$$1 = \frac{1-\eta_{\hat{\mathbf{R}}}(z)}{1-\eta_\mathbf{F}(z\eta_{\hat{\mathbf{R}}}(z))} \tag{6}$$

with $\mathbf{F} = \mathbf{A}^\dagger\mathbf{A}$. Since $\mathbf{A}$ is diagonal with Bernoulli i.i.d. diagonal elements, its η transform can be written as [20]:

$$\eta_\mathbf{F}(z) = \eta_\mathbf{A}(z) = 1 - \rho + \frac{\rho}{1+z}. \tag{7}$$

Using (7) in (6), the η transform of $\hat{\mathbf{R}}$ is given by the positive solution of the following polymonial:

$$z\eta^2_{\hat{\mathbf{R}}}(z) - ((1-\rho)z - 1)\eta_{\hat{\mathbf{R}}}(z) - 1 = 0. \tag{8}$$

Using the similar procedure, the η transform of $\hat{\mathbf{R}}_1$ is given by the positive solution of the following polymonial:

$$z\eta^2_{\hat{\mathbf{R}}_1}(z) - ((1-\sigma)z - 1)\eta_{\hat{\mathbf{R}}_1}(z) - 1 = 0. \tag{9}$$

Equation (8) corresponds to the η transform of the $\mathbf{HH}^\dagger$ with $N \times \rho N$ random matrix $\mathbf{H}$ with i.i.d. elements having zero mean and variance $1/N$. Similarly, (9) corresponds to the η transform of the $\mathbf{HH}^\dagger$ with $\mathbf{H}$ of dimension $N \times \sigma N$. Since the $\mathbf{HH}^\dagger$ follows the Marchenko-Pastur (MP) law, the Σ transforms of $\hat{\mathbf{R}}$ and $\hat{\mathbf{R}}_1$ can be written as [22]:

$$\Sigma_{\hat{\mathbf{R}}}(z) = \frac{1}{\rho+z}, \Sigma_{\hat{\mathbf{R}}_1}(z) = \frac{1}{\sigma+z}. \tag{10}$$

Theorem 3.1: The Stieltjes transform $\mathcal{S}_{\hat{\mathbf{R}}_\mathbf{Y}}(z)$ of the asymptotic distribution of eigenvalues of $\frac{1}{N}\mathbf{YY}^\dagger$, where $\mathbf{Y} = \mathbf{AU}p^{1/2}\mathbf{BX} + \mathbf{Z}$ for arbitrary value of p can be obtained for any $z \in \mathbb{C}$ by solving a polymonial with the following coefficients

$$\begin{aligned}
&c_0 = -p^2,\\
&c_2 = p^2(\rho\sigma - 1 - z) - p^3(\rho+\sigma) + p^4,\\
&c_3 = -p^3(\rho+\sigma)(z+1) + 2p^2(\rho\sigma - z + zp^2),\\
&c_4 = zp^4(2-z) - 2zp^3(\rho+\sigma) - p^2(z - \rho\sigma),\\
&c_5 = 2z^2p^4 - zp^3(\rho+\sigma),\\
&c_6 = p^4z^2,
\end{aligned} \tag{11}$$

where c_n is the nth order coefficient of the polymonial, ρ and σ denote the compression ratio and sparsity order respectively, and p is the common receive SNR of all the PU signals.

Proof: Since $\hat{\mathbf{R}}$ and $\hat{\mathbf{R}}_1$ are independent Wishart matrices, they are asymptotically free [22]. As a result, the combined aepdf of the term $\hat{\mathbf{R}}\hat{\mathbf{R}}_1$ in (5) can be obtained by applying multiplicative free convolution property of Σ transform in the following way

$$\Sigma_{\hat{\mathbf{R}}\hat{\mathbf{R}}_1}(z) = \Sigma_{\hat{\mathbf{R}}}(z) \cdot \Sigma_{\hat{\mathbf{R}}_1}(z). \tag{12}$$

The η transform corresponding to $\Sigma_{\hat{\mathbf{R}}\hat{\mathbf{R}}_1}(z)$ in (12) can be obtained using (22) and its polymonial can be written as:

$$z(\eta(z)+1)(\eta(z)+\rho)(\eta(z)+\sigma) - \eta(z). \tag{13}$$

Then using the relation between η and Stieltjes transform given by (23) in Appendix, the polymonial for Stieltjes transform of the asymptotic distribution of the eigenvalues of the product of $\hat{\mathbf{R}}$ and $\hat{\mathbf{R}}_1$ can be written as:

$$z^2\mathcal{S}^3(z) + z(2-\rho-\sigma)\mathcal{S}^2(z) + (-z + (\rho-1)(\sigma-1))\mathcal{S}(z) - 1 = 0. \tag{14}$$

Let $\mathcal{R}_{\hat{\mathbf{R}}_c}$ be the R transform of the product term $\hat{\mathbf{R}}\hat{\mathbf{R}}_1$ and is calculated using (14) and (19), which is given by

$$\mathcal{R}_{\hat{\mathbf{R}}_c}(z) = \frac{-(z\rho + z\sigma - 1)}{2z^2} - \frac{\sqrt{(z^2\rho^2 - 2z^2\rho\sigma - 2z\rho + z^2\sigma^2 - 2z\sigma + 1)}}{2z^2}. \quad (15)$$

Then the R transform of $p\hat{\mathbf{R}}\hat{\mathbf{R}}_1$ in the second term of (5) becomes $p\mathcal{R}_{\hat{\mathbf{R}}_c}(pz)$. Since the term $\hat{\mathbf{R}}_{\mathbf{Z}}$ in (5) follows the MP distribution, the R transform of $\hat{\mathbf{R}}_{\mathbf{Y}}$ using additive free convolution property can be written as:

$$\mathcal{R}_{\hat{\mathbf{R}}_{\mathbf{Y}}}(z) = p\mathcal{R}_{\hat{\mathbf{R}}_c}(pz) + \mathcal{R}_{\hat{\mathbf{R}}_{\mathbf{Z}}}(z). \quad (16)$$

Then the polymonial for the Stieltjes transform of the density of $\hat{\mathbf{R}}_{\mathbf{Y}}$ in (11) is obtained using (19). ■

The aepdf of $\hat{\mathbf{R}}_{\mathbf{Y}}$ can be obtained using Stieltjes inversion formula (24).

IV. Proposed Compressive Sparsity Order Estimation Method

The SOE is the process of identifying the number of non-zero elements of a sparse vector and does not need to have the exact knowledge of their amplitudes or positions. In this section, we propose a SOE method based on the maximum eigenvalue of the measured signal's covariance matrix. Based on the polymonial of the Stieltjes transform specified in the above section, the support range of the corresponding aepdf is obtained using (24). For convenience, a lookup table (Table I) is provided in order to illustrate the SOE method in the considered scenario (see Section V). In the lookup table, we present the maximum eigenvalues of $\hat{\mathbf{R}}_{\mathbf{Y}}$ for the considered case and the corresponding values of σ. For any instantaneous maximum eigenvalue ($\lambda_{\max}$) of $\hat{\mathbf{R}}_{\mathbf{Y}}$, its value is compared with the values of $\lambda_{\max}$ stored in the lookup table and the corresponding value of σ is obtained. Furthermore, for any intermediate values of $\lambda_{\max}$, suitable interpolation method can be applied for estimating the corresponding value of σ.

The Stieltjes transform of $\hat{\mathbf{R}}_{\mathbf{Y}}$ for the considered constant power case is calculated using polymonial (11). The value of p is assumed to be known and in practice, its value can be obtained by using SNR estimation techniques like in [17]. Furthermore, the parameter ρ is assumed as an operating parameter of the CR sensing module and its value depends on how much compressed measurements we want to carry out in order to reduce the hardware costs at the expense of some estimation error. Since we know the value of ρ and p in (11), we can estimate the value of σ by sensing the maximum eigenvalue of $\hat{\mathbf{R}}_{\mathbf{Y}}$, where $\mathbf{Y} = \mathbf{A}\mathbf{U}p^{1/2}\mathbf{B}\mathbf{X} + \mathbf{Z}$, obtained using (11) and (24). To clarify the above process, we include procedures for lookup table formation and sparsity order estimation below.

Procedure for lookup table formation

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1) Select the operating parameter ρ.
2) Select the value of p based on SNR estimation techniques like in [17, 18].
3) Evaluate $\mathcal{S}_{\hat{\mathbf{R}}_{\mathbf{Y}}}(z)$ using (11).
4) Find $\lambda_{\max}(\hat{\mathbf{R}}_{\mathbf{Y}})$ using (24).
5) For each value of σ in $(0, 1)$, repeat steps 3 and 4.
6) Store all $\lambda_{\max}(\hat{\mathbf{R}}_{\mathbf{Y}})$ and corresponding σ e.g., Table I.

Procedure for sparsity order estimation

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1) Calculate instantaneous $\hat{\mathbf{R}}_{\mathbf{Y}}(N) = \frac{1}{N}\mathbf{Y}\mathbf{Y}^{\dagger}$.
2) Calculate $\lambda_{\max}$ $(\hat{\mathbf{R}}_{\mathbf{Y}}(N))$.
3) Find σ corresponding to $\lambda_{\max}$ from the lookup table.
4) Use suitable interpolation for any intermediate value of $\lambda_{\max}$.

V. Numerical Results

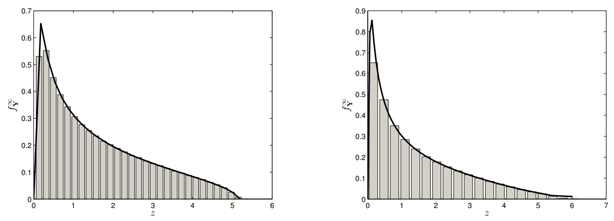
To evaluate the performance of the proposed SOE method for the considered scenario, the normalized SOE Error (SOEE) is used and defined as:

$$\text{SOEE} = \frac{\mathbb{E}[(\hat{\sigma} - \sigma)^2]}{\sigma^2}, \quad (17)$$

where $\hat{\sigma}$ is the estimated sparsity order with the proposed method and σ is the actual sparsity order. In (17), the expectation is taken over the square of the difference between the instantaneous value of the estimated sparsity order and the actual sparsity order considering 10^3 number of realizations of the considered MMV observation model. In the following subsections, we present numerical results for SOE for the considered scenario. In the numerical results, we compare the performance of the proposed SOEE technique in compressive and full measurement cases. The compressive measurement case corresponds to the signal model given by (2) while the full measurement case corresponds to the signal model given by $\mathbf{Y} = \sqrt{p}\mathbf{B}\mathbf{X} + \mathbf{Z}$. It should be noted that the theoretical analysis for the full measurement case has already been carried out in [17]. From practical perspectives, the difference between full measurement and compressive measurement cases is that the former considers all the measurements across the carriers whereas the later case considers the sparse linear combinations of carrier measurements.

To validate our theoretical analysis presented in Section III, we plot the theoretical and simulated eigenvalue distributions of $\hat{\mathbf{R}}_{\mathbf{Y}}(N)$ in Fig. 1 with parameters $\rho = 0.8, \sigma = 0.6, N = 100, \text{SNR} = 0\text{dB}$. The theoretical aepdf in this case was obtained by solving the polymonial given by (11) and using the Stieltjes inversion formula in (24). From the figure (Fig. 1), it can be noted that the theoretical curve perfectly matches with the simulated one.

For sparsity order estimation purpose, we provide a lookup table (Table I), where the maximum eigenvalues of $p\hat{\mathbf{R}}\hat{\mathbf{R}}_1 + \hat{\mathbf{R}}_{\mathbf{Z}}$ are provided for different values of σ for compressive and full measurement cases. The value of σ can be estimated using this table based on the SOE method described in Section IV. For example, if the maximum eigenvalue of an instantaneous $\hat{\mathbf{R}}_{\mathbf{Y}}$ is 6.05 for the compressive measurement case, it can be estimated that the sparsity order of the occupancy of the provided set of the values of σ and $\lambda_{\max}$ for the considered value of ρ. It can be noted that $\rho = 1$ corresponds to no-compression and the estimation error in terms of normalized SOEE increases as ρ decreases i.e., with the increase in the compression.



(a) (b)

Fig. 1: Theoretical and simulated eigenvalue distributions of $\hat{\mathbf{R}}_{\mathbf{Y}}(N)$ $(\rho = 0.8, \sigma = 0.6, N = 100, \mathrm{SNR} = 0\mathrm{dB})$ (a) $\mathbf{Y} = \sqrt{p}\mathbf{BX} + \mathbf{Z}$ (b) $\mathbf{Y} = \mathbf{AU}\sqrt{p}\mathbf{BX} + \mathbf{Z}$

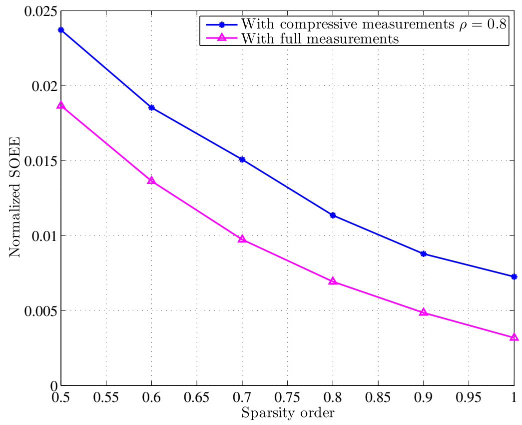


Fig. 2: Normalized SOEE versus sparsity order with compressive and full measurements $(\mathrm{SNR} = 2dB, N = 100)$

considered wideband spectrum is 0.4. Figure 2 presents the normalized SOEE versus sparsity order for compressive and full measurement cases for SNR $= 2$ dB. From this figure, it can be noted that the normalized SOEE is higher for the compressive case than for the full measurement case. For SNR $= 2$ dB as shown in Fig. 2, the normalized SOEE for the compressive case is almost 2.4 % and for the full measurement case is nearly about 1.8 % at the sparsity order of 0.5. On the other hand, the advantage is that we have used 80 % compression i.e., 20 % saving can be achieved in terms of hardware resources, which is a considerable gain. Furthermore, Fig. 3 presents the normalized SOEE versus SNR considering a fixed sparsity order of 0.6. Our simulation results show that at lower values of SNR, the compressive case with $\rho = 0.8$ performs better than the full measurement case in terms of the normalized SOEE (below SNR value of -0.5 dB in Fig. 3). An intuitive explanation is that in the full measurement case, the contribution of the noise in the aepdf becomes dominant with a faster rate compared to the compressive measurement case.

Figure 4 depicts the estimation error in terms of normalized SOEE versus compression ratio ρ for SNR value of 2 dB. In this simulation settings, the value of σ was considered as 0.6 and the estimation error for each ρ was calculated by interpolating the instantaneous maximum eigenvalue with the

TABLE I: Lookup table for sparsity order estimation for the considered scenario ($\rho = 0.8$, SNR $= 2$ dB)

Sparsity Level (σ)	**Compressive**	**Full**
1	9.63	7.44
0.9	9.03	7.14
0.8	8.41	6.83
0.7	7.85	6.53
0.6	7.26	6.21
0.5	6.67	5.88
0.4	6.05	5.52
0.3	5.44	5.15
0.2	4.85	4.76
0.1	4.25	4.31
0	3.79	3.79

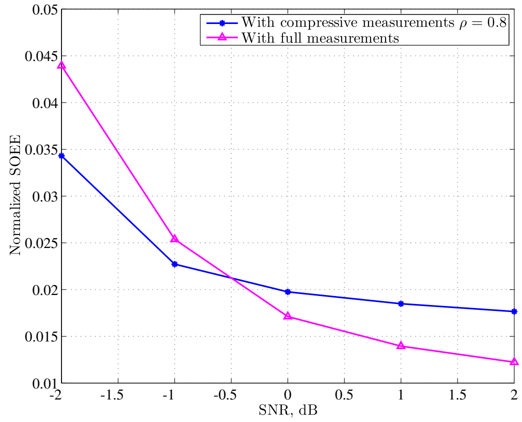


Fig. 3: Normalized SOEE versus SNR with compressive and full measurements $(\sigma = 0.6, N = 100)$

VI. Conclusion

In this paper, firstly, the theoretical expression for aepdf of the measured signal's covariance matrix has been derived for the constant received power scenario using an RMT-based approach. Then a technique has been proposed for estimating the sparsity order of spectrum occupancy within a wideband spectrum in the context of a wideband CR. The performance of the proposed method has been evaluated in terms of the normalized SOEE. It can be concluded that the proposed technique can reliably estimate the sparsity order for the considered scenario even with compressive measurements. Furthermore, it has been noted that there exists a trade-off between the hardware sensing cost and the estimation error while using compressive measurements. In our future work, we plan to apply the proposed compressive SOE technique for implementing an adaptive CS at the CR receiver.

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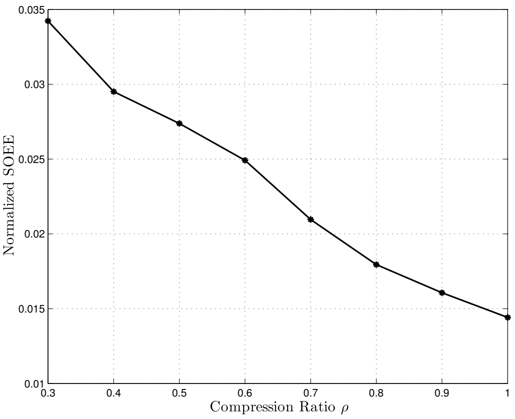

Fig. 4: Normalized SOEE versus compression ratio ρ (SNR $= 2$ dB, $\sigma = 0.6, N = 100$)

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APPENDIX
Random Matrix Theory Preliminaries

Let $F_{\mathbf{X}}(x)$ be the eigenvalue probability density function of a matrix $\mathbf{X}$.

Theorem 6.1: The Stieltjes transform $\mathcal{S}_{\mathbf{X}}(z)$ of a positive semidefinite matrix $\mathbf{X}$ is defined by [22]

$$\mathcal{S}_{\mathbf{X}}(z) = \mathbb{E}\left[\frac{1}{\mathbf{X} - z}\right] = \int_{-\infty}^{\infty} \frac{1}{\lambda - z} dF_{\mathbf{X}}(\lambda). \quad (18)$$

Theorem 6.2: The R transform is related to the inverse of Stieltjes transform as [22]:

$$\mathcal{R}_{\mathbf{X}}(z) = \mathcal{S}_{\mathbf{X}}^{-1}(-z) - \frac{1}{z}. \quad (19)$$

Theorem 6.3: For a Wishart random matrix $\mathbf{X}$, the R transform of the density of eigenvalues of $\mathbf{X}$ is defined as [22]:

$$\mathcal{R}_{\mathbf{X}}(z) = \frac{\beta}{1 - z}. \quad (20)$$

For any $a > 0$, $\mathcal{R}_{a\mathbf{X}}(z) = a\mathcal{R}_{\mathbf{X}}(az)$.

Theorem 6.4: For a Wishart random matrix $\mathbf{X}$, the Σ transform of the density of eigenvalues of $\mathbf{X}$ is defined as [22]:

$$\Sigma_{\mathbf{X}}(z) = \frac{1}{z + \beta}. \quad (21)$$

Theorem 6.5: The Σ transform of the density of eigenvalues of $\mathbf{X}$ is related to the η transform by the following relation [22]:

$$\Sigma_{\mathbf{X}}(z) = -\frac{1 + z}{z}\eta_{\mathbf{X}}^{-1}(1 + z). \quad (22)$$

Theorem 6.6: The η transform of the density of eigenvalues of $\mathbf{X}$ is related to the Stieltjes transform by the following relation [22]:

$$\eta_{\mathbf{X}}(z) = \frac{\mathcal{S}_{\mathbf{X}}(-\frac{1}{z})}{z}. \quad (23)$$

Theorem 6.7: The aepdf of $\mathbf{X}$ is obtained by determining the imaginary part of the Stieltjes transform $\mathcal{S}_{\mathbf{X}}$ for real arguments in the following way.

$$f_{\mathbf{X}}(x) = \lim_{y \to 0^+} \frac{1}{\pi} \mathrm{Im}\{\mathcal{S}_{\mathbf{X}}(x + jy)\}. \quad (24)$$

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