A signal processing method to remove environmental effects for damage detection in bridge structures

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Abstract— **This paper consists in damage diagnosis for several real bridges in Luxembourg. Before, different analysis methods were applied to the data measured from these structures showing interesting results. However, some difficulties are faced, especially due to environmental influences (temperature and soil-behaviour variations) which overlaid the structural changes caused by damage or confuse damage levels. These environmental effects are investigated in detail and removed in this work through Principal Component Analysis. Damage index is based on outlier analysis.**

Keywords— Civil engineering structures; Damage detection; Principal component analysis; Statistics; Eigenfrequency

I. INTRODUCTION

As for mechanical systems, the condition of civil engineering structures such as bridges may be monitored through vibration features identified regularly during their life. Damage detection is often performed by comparison of modal characteristics between current states and an earlier healthy state considered as "reference". However, detection based on the comparison of modal parameters like natural frequencies, mode shapes etc. is not always obvious, because damage is not the only source that disturbs those parameters. Indeed, environmental factors, i.e. temperature, temperature gradients, soil-behavior variations, traffic etc., show an important influence on modal parameters as well. It was shown through a bridge's monitoring in HongKong [1] that the normal environmental changes can bring variance error from 0.2% to 1.52% for the first ten eigenfrequencies. Such a high variation may mask the frequency change due to structural damage, which begs the question: how to remove the environmental effects in the damage detection problem.

In face of the last intricate question, several investigations have been carried out on civil engineering structures. Data reduction is often performed from time sequences into modal features. In [2-4], damage detection was achieved by some derivatives of Principal Component Analysis and factor analysis where numerical data were collected according to a quite large range of temperature. Furthermore experiments in laboratory [4-7] and on real structures have been examined. In the last decade, the real bridge Z24 in Switzerland was studied in several works [2, 8, 9] with various methods. Damage detection and localization in the I-40 bridge in New Mexico (USA) was also studied in [6, 10]; however, temperature effect was modeled numerically in [6].

The examples considered in this paper are three bridges located in Luxembourg. The first is in Useldange, over the river Attert; the second is named "Deutsche Bank" and the third is the Champangshiehl bridge. The bridge in Useldange is in good condition and has been monitored since 2006. By contrast, the last two bridges were destroyed by urban planning reason. Before their demolitions, artificial damages were introduced gradually and the bridges were monitored for a short period. Damage detection performed on these bridges has been addressed in previous works [11-15]. In the "Deutsche Bank" bridge, 31% of prestressed tendons were cut but no crack was observed and the eigenfrequency shift from the intact state to increasing damage levels does not show any tendency. In the case of the Champangshiehl bridge, damages were well detected, but close examination of eigenfrequency shifts and damage indexes did not allow to clearly identify the levels of damages. It was suspected that this could be due to the variation of environmental conditions. In this context, the present work seeks to remove environmental effects, namely temperature or soil-abutment from the bridge diagnosis. The method relies on Principal Component Analysis (PCA) of the identified features, which allows to separate the changes due to environmental variations from the changes due to damage sources. The examination of the three different bridges demonstrates the efficiency of the method toward real complicated civil engineering structures.

II. METHODOLOGY

Regarding to damage detection, many authors consider eigenfrequencies as good features, although they may be very sensitive to temperature variation. Mode-shapes are less influenced by temperature. In [3], it was shown through a numerical example that damage detection including temperature effect is better when high mode-shapes (modes 6-10) rather than mode-shapes at low frequency are considered as features. However, in real-time monitoring of bridges, modes at high frequency are more difficult to identify from vibration measurements. Therefore the features considered in this paper are eigenfrequencies of the bridge. In a first step, eigenfrequencies are identified from the recorded signals. Next, they are analyzed using PCA in order to differentiate the temperature effect from the damage effect in the feature variations.

The present study exploits the technique and Novelty Index proposed in [2], which does not require any measurement of environmental parameters because they are considered as embedded variables. Their effect can be simply observed from the variation of the identified features. For the sake of conciseness, the method is briefly recalled here.

If a vector of vibration features $\mathbf{x}_k \in \mathbf{R}^m$ is identified for an instant k, let us collect the features in a matrix $\mathbf{X} \in \mathbf{R}^{m \times N}$:

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_k & \dots & \mathbf{x}_N \end{bmatrix}, \quad k = 1, \dots, N \tag{1}$$

where m is the number of features and N is the number of samplings. If eigenfrequencies are considered as the features, m is the number of identified modes. PCA provides a linear mapping of data from the original dimension m to a lower dimension p:

$$\mathbf{S} = \mathbf{L}\mathbf{X} \tag{2}$$

where $\mathbf{S} \in \mathbf{R}^{p \times N}$ is the score matrix which characterizes the environmental-factor space and $\mathbf{L} \in \mathbf{R}^{p \times m}$ is the loading matrix. The dimension p presents the number of combined environmental factors disturbing vibration features.

In practice, PCA is often performed by singular value decomposition (SVD) of matrix $\mathbf{X}$, i.e.

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{T}} \tag{3}$$

where $\mathbf{U}$ and $\mathbf{V}$ are orthonormal matrices, the columns of $\mathbf{U}$ defining the principal components (PCs). The number p of the most important components is determined by selecting the first p non-zero singular values in $\mathbf{\Sigma}$ which have a significant magnitude ("energy"). If noise is negligible, environmental factors often show strong influence. Practically for civil engineering structures, temperature reveals itself as the only important environmental factor; in that case p is limited to 1.

The loading matrix $\mathbf{L}$ may be constructed by the first p columns $\mathbf{U}_1$ of matrix $\mathbf{U} \in \mathbf{R}^{m \times m}$ that $\mathbf{L} = \mathbf{U}_1^{\mathrm{T}}$. A residual error matrix $\mathbf{E}$ is assessed by comparing the original data and the loss of information in the re-mapping of score data $\mathbf{S}$ back to the original space:

$$\mathbf{E} = \mathbf{X} - \hat{\mathbf{X}}; \quad \hat{\mathbf{X}} = \mathbf{L}^{\mathrm{T}}\mathbf{L}\mathbf{X} \tag{4}$$

For an instant k, the Novelty Index (NI) is defined following the Mahalanobis norm:

$$NI_k = \sqrt{E_k^{\mathrm{T}} \Delta^{-1} E_k} \tag{5}$$

where $\mathbf{\Delta} = \left(\mathbf{X}\mathbf{X}^{\mathrm{T}}\right)/N$ is the covariance matrix of the features and residual error $\mathbf{E}_k$ computed for instant k is the kth column of matrix $\mathbf{E}$. Let us note $\overline{NI}^r$ and σ respectively the mean and standard deviation values of NI in the reference state, an outlier limit may be estimated by the value: OL = $\overline{NI}^r + 3\sigma$. A state may be identified as a damage state when a considerable percentage of samples exceed the outlier limit and when the ratio $\overline{NI}^d / \overline{NI}^r$ is high where $\overline{NI}^d$ is the mean value for the current state.

III. Applications

A. Fault-positive in a bridge in Useldange

The bridge is a composite two-span structure with a total length of 37.3m divided into two fields of 23.9m and 13.4m as sketched in Fig. 1. The time data were recorded using 8 accelerometers and 7 temperature sensors installed on the bridge as shown in Fig. 1. The analysis of the data and the identification of the modal parameters are done by the Stochastic Subspace Identification (SSI) [16] method for several years, from 2007 to 2010 and presented in Fig. 2. There is no structural damage, however, a strong variation of eigenfrequencies is observed following the temperature alteration during every year.

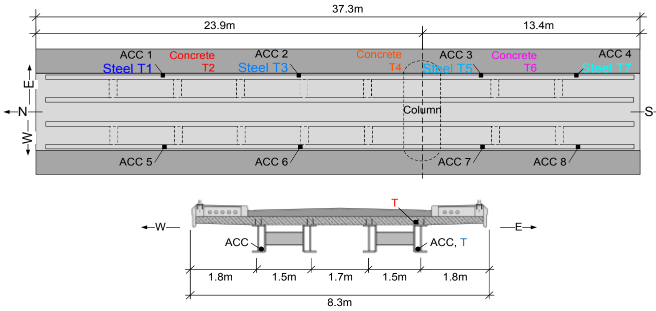

Fig. 1: Positions of the temperature transducers and the accelerometers

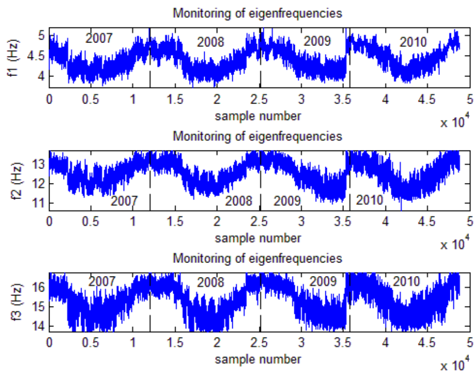

Fig. 2. Eigenfrequencies identified for 4 years 2007-2010

To perform the monitoring by PCA, the eigenfrequencies are now collected in the feature matrix $\mathbf{X}$ according to (1). For maximizing useful information for the PCA procedure, all the first three structural modes are considered (Fig. 2). The SVD of $\mathbf{X}$ (3) for the intact state reveals that the first singular value concentrates about 99% of the energy, which means that only one environmental factor has a significant influence on the three eigenfrequencies. It means that one principal component is enough to characterize the system dynamics. The other singular values are small and may be attributed to the effect of noise; their influence is so small that they do not affect the diagnostics.

Novelty Index by PCA is presented in Fig. 3 for a period from 2007 to 2010 that is divided in a number of data sets that each set consists of 3000 samples. Dotted bold horizontal lines give NI mean values of all the sets of data. Dashed horizontal lines show standard deviation σ. The first healthy set is chosen as reference and its mean value is indicated by $\overline{NI}^{r}$. In each set, the percentage of samples exceeding the outlier limit and the ratio $\overline{NI}^{d} / \overline{NI}^{r}$ are also shown in the top of the figure. Any alarm is given as the exceeding percentages are small and the ratios $\overline{NI}^{d} / \overline{NI}^{r}$ get along unity. All the upper lines of standard deviation do not overpass the outlier limit. So it can be stated that no damage occurs in the structure.

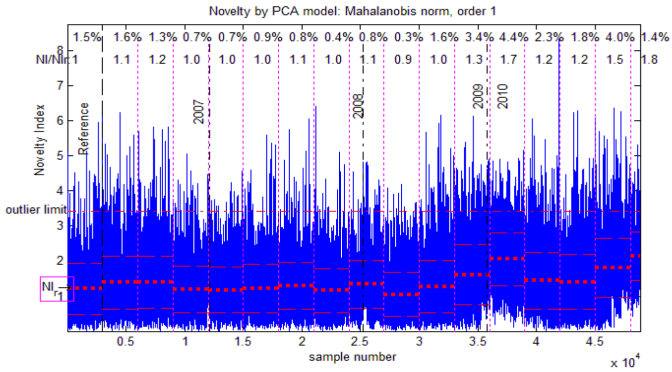

Fig. 3. Monitoring of Novelty Index (NI) from 2007 to 2010

B. The "Deutsche Bank" bridge

It is a three-span concrete bridge with a total length of 51m (Fig. 4), post-tensioned by 29 tendons with subsequent grouting. In order to simulate damage due to corrosion, several prestressed tendons were cut locally at different positions as pointed out in Table 1. The data considered in this work were achieved after the removal of asphalt layer (170t) that reduced mass rather than stiffness of the structure. Under the excitations of an electric shaker, vibration responses of the bridge were captured by 12 sensors allocated on two sides of the bridge deck.

Because of the safety regulations and time restrictions, no further damage was recorded before the demolition.

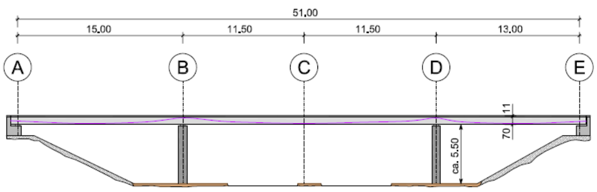

Fig. 4. The elevation and the cross-section (axis B/D) of the "Deutsche Bank" bridge

TABLE 1. Eigenfrequencies identified by MEscope

State	# 0	# 1	# 2	# 3	# 4
Number of cut tendons	0	1 tendon axis C	5 tendons axis C	9 tendons axis C	9 tendons, axis B ,C, D
f_1	4.15	4.17	4.13	4.12	4.20
f_2	5.08	5.08	5.06	5.11	5.13
f_3	10.20	10.20	10.20	10.20	10.30
f_4	11.90	12.00	12.00	12.00	12.00

As reported in [14], the measurements at the center of the mid-span showed an increase of about 15% in the static vertical displacement and of about 35% in the longitudinal strain of the passive reinforcement all along the damage scenarios. Modal analysis was carried out by means of the Global Polynomial method available in the MEscope software [17] and shown in Table 1 [18]. However, by observing dynamic responses such as eigenfrequencies and damping ratios, no damage could be discovered [14, 18]. It concerns with the cutting of the third of tendons (9 out of 29) that may be inadequate to provoke visible crack. (Due to the removal of the asphalt layer, stresses did not exceed tensile strength of concrete). As for ambient condition, the bridge was tested during 10 days in autumn with no significant weather alterations. However, the environmental variation may mask changes in modal parameters and so obstruct the detection of the produced damages. But unfortunately, no complete monitoring of temperature was achieved.

In the present work, to exploit statistical Novelty Index cited above, modal parameters are identified here using the Wavelet Transform (WT)[19] and eigenfrequencies are chosen as system features. The reason is that the data obtained by WT responses are abundant, so that each eigenfrenquency may be periodically picked up to 600 times for each state. In total there are 3000 sets of results for all the states. For the sake of conciseness, only the first frequency is presented in Fig. 5.

Similarly to the results obtained by the Global Polynomial method from Frequency Response Functions in Table 1, the eigenfrequencies identified by the Wavelet Transform (WT) do not reveal any tendency corresponding to the increasing damages. WT frequencies are presented by spectrum and the examined states show different spectrum dispersions. With these dispersions, the observation of eigenfrequencies in Fig. 5 does not give any direct detection of damage.

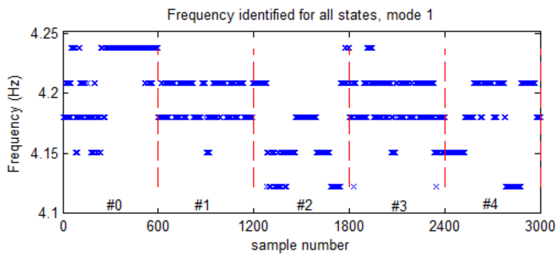

Fig. 5. The first eigenfrequencies identified for #0 ÷ #4 by Wavelet Transform

All the first four eigenfrequencies are then gathered for the PCA detection. Only one principal component keeping near 100% energy of the system is retained. Fig. 6 presents the results of the PCA-based detection using the first principal component. In this figure, three data sets are considered for each state. Each data set contains 200 samples. Dotted bold horizontal lines give mean values of the Novelty Index (*NI*) of all the 15 sets of data. The first undamaged set is chosen as reference. The percentage of samples exceeding the outlier limit is given in Fig. 7 and the ratio $\overline{NI}^{d} / \overline{NI}^{r}$ in Fig. 8. Among the first three set of data including 600 no-damage samples, the third set shows the highest rate of overshoot the outlier limit (27.5%). However in the whole intact state, this drop is only minority that keeps the mean of *NI* always under the outlier limit and the maximum ratio $\overline{NI}^{d} / \overline{NI}^{r}$ is only 1.5. For the first two damaged states #1 and #2, Novelty Index does not show any alarm, except the last set of #2. However, a clear distinction is given from damage #3 when *NI* overshoot reaches 50-80%; all $\overline{NI}^{d} / \overline{NI}^{r}$ ratios overpass the outlier limit with values from 2.1 to 3.5. Thus due to small cutting proportion, damages are only detected from state #3, however that is an interesting improvement in relation to former studies in this structure.

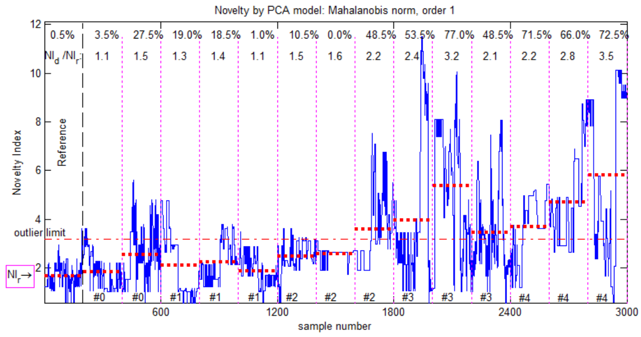

Fig. 6. Monitoring of Novelty Index (*NI*)

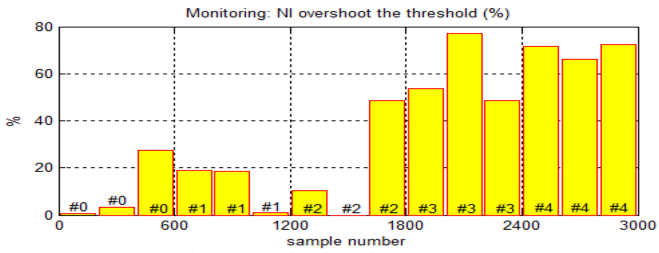

Fig. 7. Overshoot rate by *NI*

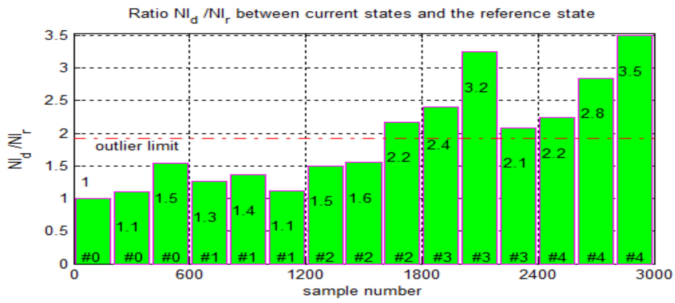

Fig. 8. Ratio $\overline{NI}^{d} / \overline{NI}^{r}$

C. *The Champangshiehl bridge*

With a total length of 102 m, the bridge is divided into two spans of 37 m and 65 m (henceforth noted *L*) respectively (Fig. 9). It was pre-stressed by 112 steel wires as illustrated in Fig. 10. Before its complete destruction, the bridge was monitored and a series of damages were artificially introduced as summarized in Table 2.

Vibration monitoring under swept sine excitation force and impact excitation were performed on the healthy structure and at each damage state during June 2011.Ten sensors were located on each side of the deck (the distance between each sensor is about 10 m). More detailed descriptions of the bridge can be found in [13]. Processing of the data was performed in [11, 13] using the Stochastic Subspace Identification (SSI) method and the Global Polynomial Method. Those earlier analysis can be used for the comparison with the present study combining the Wavelet Transform and the Principal Component Analysis.

Moreover, in order to broaden false-positive tests that avoid false alarms for undamaged states, data in the healthy condition are enriched and gathered from different days and different excitations. As presented in Figs. 11 and 12, two sections #0 correspond to the impact and then swept sine excitations, respectively, which were performed in two separate days. The damaged states are examined under the swept sine excitations.

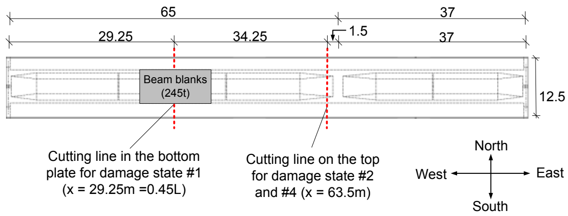

Fig. 9. Longitudinal section of the Champangshiehl bridge

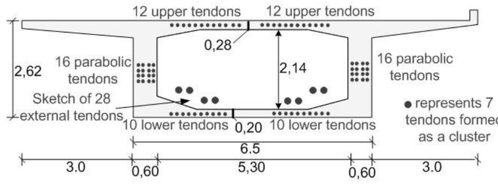

Fig. 10. Schematic cross section of the box girder with location of the tendons

TABLE 2. Description of the damage scenarios according to the cutting sections shown in Fig. 10

State		# 0	# 1	# 2	# 3	# 4
Damage		Undamaged state	Cutting straight lined tendons in the lower part, at 0.45L (20 tendons)	#1 + Cutting 8 straight lined tendons in the upper part, over the pylon	#2 + Cutting external tendons (56 wires)	#3 + Cutting 16 straight lined tendons in the upper part and 8 parabolic tendons
Percentage cutting (100% equals all tendons in the defined section	0.45L		33.7%	33.7%	46.1%	46.1%
	Over the pylon		0%	12.6%	24.2%	62.12%

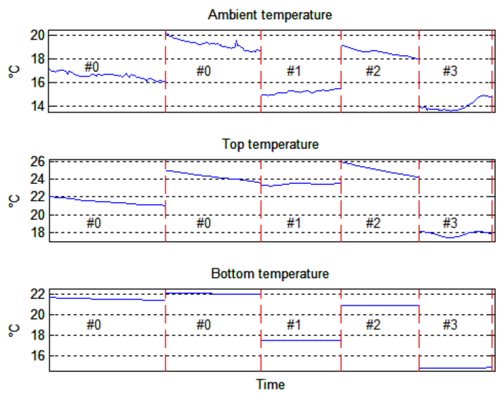

Fig. 11. Evolution of the temperature during states #0 ÷ #3 (no data recorded for state #4)

During the monitoring of the bridge, temperature was also measured outside and inside of the bridge. As all the states were carried out one after the other and so recorded at different days in June, the environmental conditions changed from one measurement set to the other. Fig. 11 displays the ambient temperature (under the bottom plate of the superstructure) as well as temperatures measured at the top and at the bottom of the bridge. For security reason, the temperature monitoring system was removed before the most significant damage #4 so that the temperature was not recorded for the last state. Modal identification and damage detection were performed in [11-13] without taking into account temperature variations. Table 3 reports frequency identification in these previous works.

TABLE 3. Change in the eigenfrequencies (identified by the SSI method)

State	#0 (H)	#1 (D1)	#2 (D2)	#3 (D3)	#4 (D4)
f_1 (Hz)	1.92	1.87	1.95	1.82	1.75
Δf_1 (%)		-2.6	**+1.6**	-5.21	-8.85
f_2 (Hz)	5.54	5.45	5.24	5.39	5.3
Δf_2 (%)		-1.62	**-5.42**	-2.71	-4.33

It can be asserted from Table 3 that all the damage states are well detected. However, according to these results, state #2 shows a particular behavior: the frequency of mode 1 increases slightly with respect to the healthier states while the frequency of mode 2 exhibits the most important drop. The variation of temperature shown in Fig. 11 may be suspected as responsible for this particular behavior. For damaged state #2, the corresponding ambient temperature does not look unusual as it is in the same range of the ambient temperature recorded for the healthy state #0; however the temperature at the top of the bridge during state #2 is the highest. This observation is probably the reason why state #2 has a non-conventional behavior compared to the other states. In the following, it will be shown that the proposed method allows to answer this problem.

As said before, eigenfrequencies are identified here using the Wavelet Transform (WT) and are chosen as system features. Each eigenfrenquency is here periodically picked up to 300 times for each damaged state, and 2×300 times for the reference state. In total there are 1800 sets of results for all the states. They are plotted in Fig. 12 for the first two modes. As revealed in the figure, the first eigenfrequency provides a quite clear distinction between different states as the decrease in frequency is monotonous from one state to the others. However, the identification results confirm that the second eigenfrequency do not allow to detect damage except in states #3 and #4.

With the first singular value occupying near 100% of energy, the SVD of vibration feature matrix **X** shows also that there is only one environmental factor that influence the most on the features. In this case, the environmental factor is the temperature (it is the only one that is noticeable). Thus 1 principal component is kept to define the loading matrix to characterize the environmental-factor space. The PCA detection is shown in Figs. 13-15 that for each case of damage, three data sets are considered and the reference state has six data sets. Each data set contains 100 samples.

An overall look at Fig. 13 reveals an interesting result: despite the variation of the *NI* for the 6 undamaged states #0 (which results from the variation of the eigenfrequencies with the temperature), most of the *NI* values lies below the outlier limit line. The few samples crossing this line are influenced by other factors (presence of nonlinear effects, noise). Fig. 13 also reveals that all the damage states are detected and well classified in accordance with their levels. Despite the fact that all healthy states are not measured continuously and that their temperature range does not cover every states, consistent results are obtained.

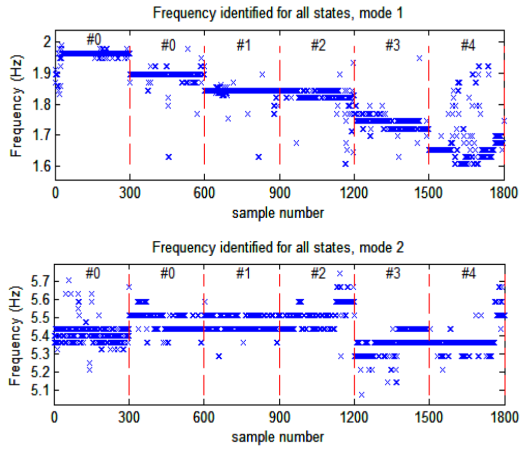

Fig. 12. Eigenfrequencies identified by WT for #0 ÷ #4

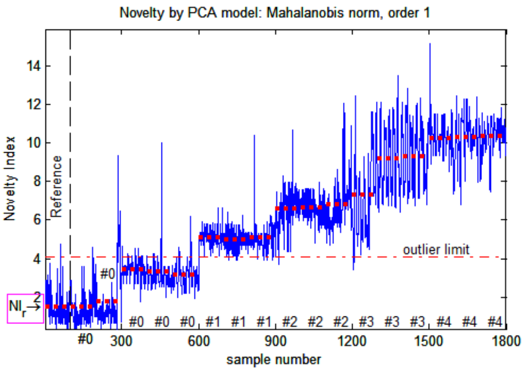

Fig. 13. Monitoring of Novelty Index (*NI*)

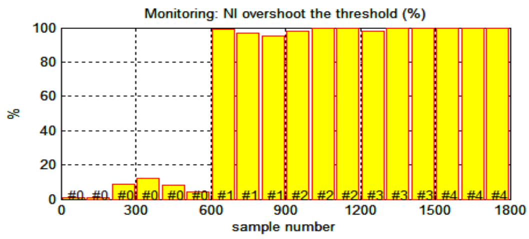

Fig. 14. Overshoot rate by *NI*

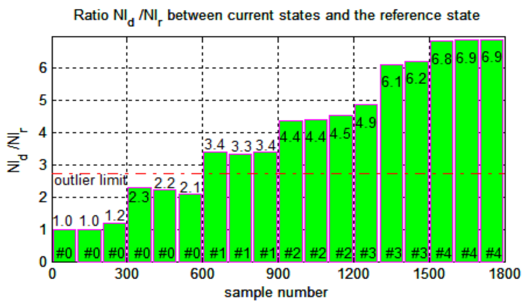

Fig. 15. Ratio $\overline{NI}^{\,d} / \overline{NI}^{\,r}$

In Fig. 14, the percentage of NI overpassing the limit is close to 100% for all the damaged states showing that even the smallest damage is clearly detected. In Fig. 15, the ratio $\overline{NI}^d / \overline{NI}^r$ is use to exhibit the progression of damage. By the distinct levels of damage, each damaged state is clearly classified from undamaged state with homogenous ratios.

IV. CONCLUSION

Data recorded from three real concrete bridges are processed in this paper. Sometimes an increase of eigenfrequencies follows a damage of increasing level, which is not theoretically expected. The same phenomenon was observed in other structure like in the bridge I-40 in New Mexico (cited in [6, 10]). Environmental effects that are assumed responsible of this phenomenon could be removed using PCA of the identified features (eigenfrequencies in this work) and statistics as damage indexes. The present work shows meaningful improvements with respect to earlier analyses [11-14, 18]. In previous studies on the "Deutsche bank" bridge, the damages were not directly detected by modal features. As discussed in [14], it may be a consequence of prestressed structures: visible cracks appear very late, shortly before collapse and only with important damage. Non-sizable damage is not revealed directly from the monitoring of modal parameters. In the particular case of the Champangshiehl bridge, damage #2 showed uncommon behaviors (apparent by eigenfrequency, damage index [11]). This uncommon behavior is cancelled out through damage indexes used in this paper. It shows that the detection in the Champangshiehl bridge is clearly easier than in the "Deutsche bank" because the cutting ratio is considerably higher in the Champangshiehl bridge.

As for damage indices, the $\overline{NI}^d / \overline{NI}^r$ ratio is not in the same range for every structure in healthy state. So looking in the overshoot rate is also an inherent task which may give more accurate information.

The advantage of the detection here is its simplicity. No-environmental measurement is needed. The feature collection is achieved by SSI or Wavelet Transform, then PCA is used for analysis, they are all very practical and convenient for automation. It is shown that even if reference data is not collected according to a full range of temperature covering other states, the detection is still faithful.

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REFERENCES

[1] J. M. Ko , K. K. Chak , J. Y. Wang, Y. Q. Ni, T. H. T. Chan , "Formulation of an uncertainty model relating modal parameters and environmental factors by using long-term monitoring data", Smart Structures and Materials 2003: Smart Systems and Nondestructive Evaluation for Civil Infrastructures, 298, 2003.

[2] A.-M. Yan, G. Kerschen, P. De Boe, J.-C. Golinval, "Structural damage diagnosis under varying environmental conditions- Part I, II", Mechanical Systems and Signal Processing 19, pp. 847-880, 2005.

[3] Deraemaeker, Reynders, De Roeck, Kullaa, "Vibration-based structural health monitoring using ouput-only measurements under changing environment", Mechanical Systems and Signal Processing 22, pp. 34-56, 2008.

[4] A. Bellino, A. Fasana, L. Garibaldi, S. Marchesiello, "PCA-based detection of damage in time-varying systems", Mechanical Systems and Signal Processing 24, 2010, pp. 2250-2260.

[5] J. Kullaa, "Elimination of environmental influences from damage-sensitive features in a structural health monitoring system", Structural Health Monitoring – the Demands and Challenges, CRC Press, in Fu-Kuo Chang (Ed.), Boca Raton, FL, pp. 742-749, 2001.

[6] M.P. Limongelli, "Frequency response function interpolation for damage detection under changing environment", Mechanical Systems and Signal Processing 24, pp. 2898-2913, 2010.

[7] Y. Sun, "Combined Neural Network and PCA for Complicated Damage Detection of Bridge," Natural Computation, Fifth international conference, 2009, pp. 524 – 528.

[8] J. Kullaa, "Damage detection of the Z24 bridge using control charts", Mechanical Systems and Signal Processing 17(1), pp. 163-170, 2003.

[9] E. Reyders, G. Wursten, G. De Roeck, "Output-only structural health monitoring by vibration measurements under changing weather conditions", Proceedings of the third International Symposium on Life-Cycle Civil Engineering IALCCE, 3-6 October 2012, Vienna, Austria, 2012.

[10] V. H. Nguyen, J.-C. Golinval, "Damage localization in Linear-Form Structures Based on Sensitivity Investigation for Principal Component Analysis", Journal of Sound and Vibration 329, pp. 4550-4566, 2010.

[11] V.H. Nguyen, C. Rutten, J.-C. Golinval, J. Mahowald, S. Maas & D. Waldmann, "Damage Detection on the Champangshiehl Bridge using Blind Source Separation", Proceedings of the Third International Symposium on Life-Cycle Civil Engineering, IALCCE'12, pp. 172-176, 2012.

[12] J. Mahowald, S. Maas, F. Scherbaum, D. Waldmann, A. Zuerbes, "Dynamic damage identification using linear and nonlinear testing methods on a two-span prestressed concrete bridge", Proceedings of the Third International Symposium on Life-Cycle Civil Engineering, IALCCE'12, pp. 157-164, 2012.

[13] J. Mahowald, S. Maas, D. Waldmann, A. Zürbes, F. Scherbaum, "Damage Identification and Localisation Using Changes in Modal Parameters for Civil Engineering Structures", Proceedings of the International Conference on Noise and Vibration Engineering, Leuven, Belgium, pp. 1103–1117, 2012.

[14] S. Maas, A. Zürbes, D. Waldmann, M. Waltering, V. Bungard, G. De Roeck, "Damage assessment of concrete structures through dynamic testing methods.Part 2: Bridge tests", Engineering Structures 34, pp. 483–494, 2012.

[15] V. H. Nguyen, M. Peeters, J.-C. Golinval, "Detection of nonlinearity in a dynamic system using deformation modes obtained from the wavelet transform of measured responses", Shock and Vibration 17, 2010, pp. 491-506.

[16] Overschee P. V., De Moor B., "Subspace identification for linear systems-Theory-Implementation-Applications", Kluwer Academic Publishers, 1997.

[17] ME'scope VES5.0, Tutorial Manual, August 2008, Vibrant Technology.

[18] M. Waltering, V. Bungard, A. Zuerbes, S. Maas, D. Waldmann, "Non-destructive Damage Assessment Using Non-linear Vibration", International Modal Analysis Conference, IMAC-XXVI, 2008.

[19] V. H. Nguyen, J. Mahowald, S. Maas, J.-C. Golinval, "Damage Detection in Civil Engineering Structure Considering Temperature Effect", Proceedings of IMAC XXXII Dynamics of Coupled Structures, February 2014, Orlando, USA.