A JOINT DAY-TO-DAY MODE AND WITHIN-DAY DEPARTURE TIME CHOICE MODEL FOR THE ANALYSIS OF DYNAMIC RIDESHARING

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Abstract

Sharing travels is an effective way to increase car occupancy rates and to reduce the total number of cars for the same distances travelled. A potentially attractive service based on the sharing concept is dynamic ridesharing (DRS), where a service provider matches up driver and passenger with similar itineraries and time schedules with an automated program.

To analyse how modal shift from single passenger use of the car to high occupancy one can be realized and what are the demand management solutions that can effectively be recommended, we deal with a theoretically interesting basic model structure of a single-link case, in which we study the complex interaction of multiple factors involved in the dynamic ridesharing problem, and the joint sensitivity to the most relevant parameters involved in this behavioural process.

The main contribution of this study is therefore a unified framework that is able to model the dynamics of the departure time, characterized as a within-day process, while keeping the mode choice as a day-to-day process, and characterizing explicitly the matching process of DRS.

Introduction

The freedom offered by cars is an extremely valued asset for the drivers. However, excessive car usage leads to congestion and related emission problems. Car occupancy rates in Western Europe are nowadays stabilizing around 1.54 persons per vehicle, and these values become 1.1 if only commuting travels are considered [1]. Sharing concepts (carpools, carsharing, etc.) have been often presented to the travellers as an effective way for solving energy and emission problems already from the energy and fuel crises after World War II. Sharing travels and traveling modes (cars, bikes, etc.) are an effective way to reduce the number of vehicles on the road by increasing occupancy rates, contributing to relieve networks from congestion, and in turn saving fuel and reducing pollution. This is one of the reasons why more and more attention is given to travel sharing services in the last years, both from research and from the public and private service providers.

Promoting shared mobility is a goal of critical importance for both the society and for the individual traveller. Societal benefits are expected in terms of total cost savings and better utilization of the road capacity through higher vehicle occupancies. With the current world's economic crisis, fuel costs and other car-related taxes have increased dramatically, and owning a car is considered by a great portion of the travellers a luxury. With travel sharing options, car users have the possibility to share car expenses such as fuel costs and tolls, with great reductions of individuals' and total system costs. However, these systems will effectively attract car users only when they will be able to substitute the flexibility and comfort of the car in their weekly activity-travel agenda.

A potentially attractive service, which is currently receiving a growing commercial interest, is dynamic ridesharing (DRS), where a service provider matches up driver and passenger with similar itineraries and time schedules with an automated program. The high penetration of Information and Communication Technologies (ICT) through personal devices such as smartphones has been an important driver for the establishment of these services in many countries, and in different forms. Despite much interest, very few systems have been tried out and even less have shown significant appeal to users. Despite many incentives and some theoretical work, DRS is still far from reaching the modal share sought. Many review papers addressing these issues have been published recently, including Agatz et al. [2] and Furuhata et al. [3].

When modelling ridesharing, one must take into account that a user can decide whether to participate or not to the sharing service, and if he/she participates, whether to offer his/her car or just offering to share expenses as passenger. Each of these decisions determines a different (expected) cost, depending on a variety of factors (matching availability and at what costs in terms of rerouting or rescheduling, what is the policy for sharing expenses, how people value handing over privacy, etc.). The different groups cannot be modelled separately since the proportion between ridesharing participants and not (and between ridesharing drivers and passengers) and the costs for each user category depend on the final equilibrium state. This makes the problem different from more conventional equilibrium models with pre-defined user classes. Another difference with respect to conventional equilibrium models is that the system performance normally does not degrade with increased number of users, but it should actually become more efficient and more attractive for a range of the demand. This is because more matches will be possible and at smaller rerouting and rescheduling costs, and because more matches leads to less low-occupancy cars and therefore less congestion for the same total demand.

Despite the apparent simplicity of this system, modelling its service performance and functionality is not an easy task. Unlike other collective transport services, DRS performance strongly depends on the number of service participants and their distribution among the two possible states, i.e. being the driver and offering the ride, or being a passenger and looking for a ride. This distribution may depend on travellers' attitudes (value of time, value of flexibility, etc.), on the costs associated to the trip (duration of the trip, detour needed to match rides, etc.) and on the interaction between travellers, mainly the rules adopted to share the travel expenses. Two perspectives have to be considered together, namely the matching problem, typical of operations research (how to best match travels in time and space, what level of similarity for the routes is acceptable,…); and many behavioural challenges (on which conditions users choose to share travels, which type of users is willing to share the travel, what is the accepted detour and rescheduling elasticity, how to incentivize the participation to DRS services,…). To the best of our knowledge, a model incorporating all these factors at once is missing. Our goal is to advance on the latter modelling aspects by introducing a behavioural model that can be easily plugged in the ride-matching problem.

In recent research, we have developed a basic link model where ridesharing has been considered as an alternative mode [4]. Equilibrium has been analysed in mode choice, but without considering the time-dependent nature of scheduling the ride matches. In this paper we extend the previous link model to consider a simple scheduling strategy, where DRS participants, after choosing which role to play (driver, passenger) look for the best time window where to look for a ride match. This time window depends, among the other time-independent factors used in the previous model form proposed in [4], on the chance to find a DRS partner at that time, on the queue of DRS users looking for a matching partner, and on the costs incurred in arriving early and in late departures. In this respect, this model can be seen also as a rather straightforward extension of Vickrey's departure time choice model that includes the scheduling of dynamic ridesharing systems.

Literature review

Different types of sharing solutions are offered in the form of private and public services, and involve different modalities (cars, vans, bikes). We can broadly categorize these services into travel-sharing and mode-sharing systems. While in travel-sharing (e.g., carpool) the main requirement is the joint use of a travel mode, e.g., a car, so traveling options such as routes and departure times must be somewhat in common, in mode-sharing the mode (car, bike, etc.) has a concurrent use, i.e. the requirement/target is to intensify the car usage. These systems essentially cover different trip frequencies and purposes (i.e. the travel-sharing option is more suited for recurrent trips, while mode-sharing solutions for more occasional trips). In this study we focus on the travel sharing options.

Travel-sharing services can be offered on a large scale and for obtaining high occupancy rates, for instance by organizing mobility services for companies in the form of, e.g., vanpools, or as demand-responsive solutions such as dial-a-ride, and fill the grey area between the private need and the benefits of a public transport service.

Among the different travel sharing options, carpooling is a well-established mode of transport in US since decades, thanks also to the different travel demand management solutions dedicated to HOVs (e.g. high occupancy lanes, dedicated parking lots, etc.). In Europe, these options are only recently being considered, especially with the new EU targets of energy efficiency and sustainability. However, these systems will effectively attract a significant share of the users only when they will be able to compensate the flexibility and comfort of the privately owned car with greater advantages, in terms of cost savings, increased accessibility, high availability and flexibility, etc.

The idea of dynamic ride-sharing has generated much interest in the last years, and several services have been established both in US (e.g., Carticipate, EnergeticX, Lyft, SideCar) and in Europe (e.g., Avego/CarMa, Flinc, Cocar), which provide applications that run on Internet-enabled mobile phones that match car-sharers almost in real time (30 to 90 seconds updates). A more detailed description of these systems is given in [2]. Being recently offered in the market, long run assessments are not yet reported. Nevertheless the general opinion is that dynamic ride-sharing apps will soon become competitive alternatives to traditional car use, and will have an impact on car-ownership [5].

Despite the existence and fast development of ride-sharing services and the development of optimization algorithms capable of handling the complexity of matching rides (see e.g., [6]), the impact on travellers' behaviour and on transport system costs is relatively unexplored. Exceptions include studies that evaluated travel-sharing matches of individuals using different preferred departure times and ride-matching strategies in real time ([7-8]), or that quantified the reduction in travel costs when sharing rides [9-11]. Additional to time and space constraints, other studies focused on the influence of gender [12], users' groups [13-14], sharing expenses policies [15-16] and different tolling schemes [17]. As concerns the behavioural modelling of DRS, several papers consider an agent-based system where autonomous passenger and driver agents locally establish ride-shares (e.g., [18-19]), while others modify or extends other existing collective transport models (e.g., dial-a-ride, pick-up and delivery problems, carpooling and car-sharing; see again [2] for an overview).

Equilibrium of ridesharing networks has been so far explored in the context of deterministic and stochastic user equilibrium (Huang et al., [20]), or using a traditional bottleneck model (Qian and Zhang, [21]). A factor still overlooked is the departure time choice as function of the matching rate, which has a direct impact on the travel time experienced by people and thus their decisions.

The research area of ridesharing has also great analogy with studies on taxi sharing, characterized by a similar problem (users accepting to share part of their rides with other customers) but with fundamentally different issues, e.g. trade-offs have to be considered between tax drivers and customers, and not between road users only (see e.g., [20]). Nevertheless, we may think of extending the models developed in this paper to also address the taxi-sharing problems, but we leave this exercise to future papers.

Methodology

In this section we briefly summarize the main elements of the day-to-day mode choice model, and the main findings of the equilibrium formulation without within-day scheduling choice. Later we introduce the departure time and arrival costs and extend the day-to-day formulation to include the within-day dynamics.

The concept of dynamic ridesharing (DRS) is characterized by the following features:

- The match can be established on short-notice (a few minutes to a few hours from departure); travellers indicate the relevant characteristics of their trip (place of origin, destination, number of seats offered/requested, desired time of departure/arrival,…);
- The drivers are independent private entities, normally offering their own private vehicle;
- The trip-related costs (fuel, tolls,…) are shared among the participants; the fraction paid by each partner depends on the DRS regulation;
- A system matches up DRS drivers and DRS passengers and communicates the matches to the users. Normally in this done in the form of First-In-First-Out service.

Modelling assumptions

In the model presented in this section, it is assumed that all users in the transportation system have access to the dynamic ridesharing service and are capable of offering a ride, i.e. they all own a car. This makes the assignment of the trips easier and also leaves the focus on the mode choice or modal split model. It will be interesting in future extensions to add a decisional layer where travellers who systematically find a match as passengers may decide to sell or not replace their car, but due to the significant increase of decisional variables in this context, involving the whole activity-travel behaviour seems at the moment not sound.

Three different modes will be possible (without loss of generality, as other modes such as Public Transport can always be added to the different mode alternatives):

- Travelling with own car, without participating to the DRS system (*SOLO-driver*);
- Travelling with own car, and offering a trip in the DRS system (*DRS-Driver*);
- Travelling as a passenger within the DRS system (*DRS-Passenger*).

DRS-Driver and Passenger are in the following generically referred to also as *DRS participant or DRS partner*. In the first case we indicate a service user without essentially specifying his/her role, while in the second we refer to a matched driver (passenger) with a specific passenger (driver).

Each of the three modes has its own cost function expressing the general cost of making a trip. We also assume, for simplicity’s sake, that every supplier offers only one spot in his car. In theory it would be possible (and interesting to analyse) to offer more seats, but it is highly unlikely that a supplier would be willing to make multiple detour trips to pick up passengers from different locations, while it would be more likely that a DRS participant would accept more partners from the same location (household). In principle this can be modelled by considering different fractions paid by each DRS participant.

Regarding the mode choice model, we assume that choices are made such that each traveller chooses the alternative that maximizes his/her utility. In the following we consider only the costs associated to travelling, so travellers aim to minimize the costs. Both deterministic user equilibrium and stochastic user equilibrium can be adopted. In the latter case, we have analysed the behaviour of a Nested Logit structure for the choice process, to take into account the higher degree of similarity between DRS participants’ alternatives.

Finally, it is assumed that the demand for trips is constant and all the users will make their trip, no matter how high the cost is. This assumption can be changed by altering the model, for example by taking public transportation into account by adding a fourth mode, or by changing the demand from constant to elastic.

Regarding the cost components, we used very simple forms for each component. The travel time cost is represented by the traditional BPR-function [21], which represents congestion as non-linearly increasing with the flow-to-capacity rate; distance-dependent costs (fuel consumptions, tolls, etc.), together with distance-independent costs (cost for loss of flexibility, for accepting detours, etc.) are highly simplified as well. This allows us to analyse the modelling behaviour using very simple cost and choice structures.

We are aware that the above assumptions make the model not ready for the practice. They nevertheless are necessary at this initial stage, as extra complexity would not allow us to acquire insight into the macroscopic relations arising in this problem. It is our intention in the future to relax these assumptions towards a more realistic and ready-to-use model.

Cost functions

Notation

All used variables in this section are explained in the following list (in parenthesis the default values used later in the numerical examples):

c_0^i	(time-independent) Cost of mode *i*
q_i	Traffic load on mode *i*
a, b	BPR-curve parameters [0.15; 4]
T	Total demand of trips [10000 trips]
S	Capacity of the link [=T for the whole evaluation interval]
T_0	Travel time on the link in free flow conditions [20 mins]
L	Length of the link [20 km]
C_{km}	Cost per kilometre driven (e.g. fuel, toll, etc.) [0.2 EUR/km]
f	Fraction of C_{km} the DRS-driver pays if he has a passenger [0.5]
Dt	Cost for making a detour to pick up a passenger [1 EUR]
Cf	Cost for the lost comfort due to sharing a trip. This can however be a positive utility in some conditions. [1 EUR]
Fl	Valued cost by the passengers for the loss of flexibility (e.g. rescheduling activities, walking to a pickup point, etc.) [1 EUR]
VoT	Value of time [10 EUR/hour]

Cost functions per mode For each link, the following costs are associated to each mode:

- SOLO-driver (solo):
  $$c_0^{solo} = T_0\left(1 + a\left(\frac{q_{\text{solo}} + q_{\text{drsd}}}{S}\right)^b\right) \cdot VoT + L \cdot C_{km}$$
- DRS driver (drsd):

$$c_0^{drsd} = T_0\left(1 + a\left(\frac{q_{\text{solo}} + q_{\text{drsd}}}{S}\right)^b\right) \cdot VoT + \ f \cdot L \cdot C_{km} + Dt + Cf_{solo}$$

- DRS passenger (pax):

$$c_0^{pax} = T_0 \cdot \left(1 + a\left(\frac{q_{\text{solo}} + q_{\text{drsd}}}{S}\right)^b\right) \cdot VoT + (1 - f) \cdot L \cdot C_{km} + Fl + Cf_{pax}$$

The cost function curves take into account that the cost of a trip will increase if the congestion along the link increases. The volume/capacity ratio takes both the flows of solo-drivers and DRS drivers into account since both these modes generate a car on the link. The cost for car ownership and maintenance is not included in the cost functions as it just complicates the model unnecessarily at this stage. Additional to the congestion costs, people who drive alone experience a cost for having to pay their travel fully by themselves. This cost consists of the fuel they have to pay per kilometer, and the eventual extra costs (tolls, parking fees, etc.). We assume this cost to be dependent on the distance travelled, only.

The additional costs for DRS participants are slightly more complex. The cost for using a car may decrease when an increasing number of passengers may become available. When a supplier finds a passenger, travel expenses may be shared; but when the supplier does not find a match, he still has to pay 100% of the usage costs. Additional terms are added, taking into account that a supplier might have to make a detour for picking up a passenger and loses comfort when having a passenger driving along. In general, factors affecting the DRS participants will have the same effect of (and thus represent) subsidies offered by the government. So any eventual fee to be paid to participate to the service, or in an opposite case a monetary incentive to participate, would be equivalent to the above fixed costs.

DRS passengers have also decreasing expected total costs with an increasing number of DRS Drivers as more cars become available. Additionally, they also may experience some extra cost due to loss of flexibility in choosing the preferred departure or arrival time (we assume here that drivers are offering their car at their own scheduled departure time, or that anyway the most significant rescheduling costs are associated to the passengers), and they also experience the (dis-)comfort of sharing a car. This component can be different for drivers or passengers.

Other possible costs to be considered in the problem may be a possible fare the users have to pay for maintaining the dynamic ridesharing system (server costs, employees, etc.), and incentive schemes (subsidies for participants) can be added in an analogous way to increase the DRS participation rate.

Equilibrium analysis

At equilibrium, DRS drivers and passengers must equally match. In a very basic attempt to guarantee this in an iterative scheme, in Viti and Corman [4] the balance between DRS drivers and passengers changed by having people changing their stated decision (for instance, a would-be DRS passenger who does not find a match become a DRS driver and then pick up some other DRS passenger). In the previous approach we assumed that a user chooses the option that gives the maximum expected utility of the alternative two options. This means that, if more convenient, a non-matched participant may decide to drive as SOLO driver. This simple rule is modified when within-day dynamics are explicitly considered, as in the next section.

At equilibrium, the costs of all three mode alternatives must be equal. In [4] we used a basic Method of Successive Averages (flow averages) to calculate the equilibrium states for different combinations of the above cost components. One of the most interesting findings is that equilibrium states can shift rather significantly if the cost parameters are opportunely set. For example, it would seem logical that a 50-50% separation of the costs would be the fairest solution among DRS participants, but the equilibrium behaviour as modelled by our equilibrium model is not as straightforward (Fig. 1). In fact, with small fractions paid by drivers (i.e. passengers pay the majority of the travel expenses) not many passengers become available, forcing the drivers mainly to become solo drivers. When the fraction paid by the driver increases, the interest in the DRS system also grows as the number of solo drivers decreases. It is therefore interesting to observe an interval where the DRS solution becomes more attractive than solo driving (in between 45% and 75% of costs paid by the driver). This change in behaviour can be linked to a change in the second-best alternative for non-matched participants. As for small fractions the best option for unmatched participants is to drive alone, in the above interval they will try to find a match by changing role within the DRS service. The change in modal shift rates around the values 45% and 75% indicate that gradually more and more DRS participants decide to remain DRS participant and simply change role. However, once drivers have to pay too high shares, then the interest in offering a ride decreases and the number of solo drivers increases again.

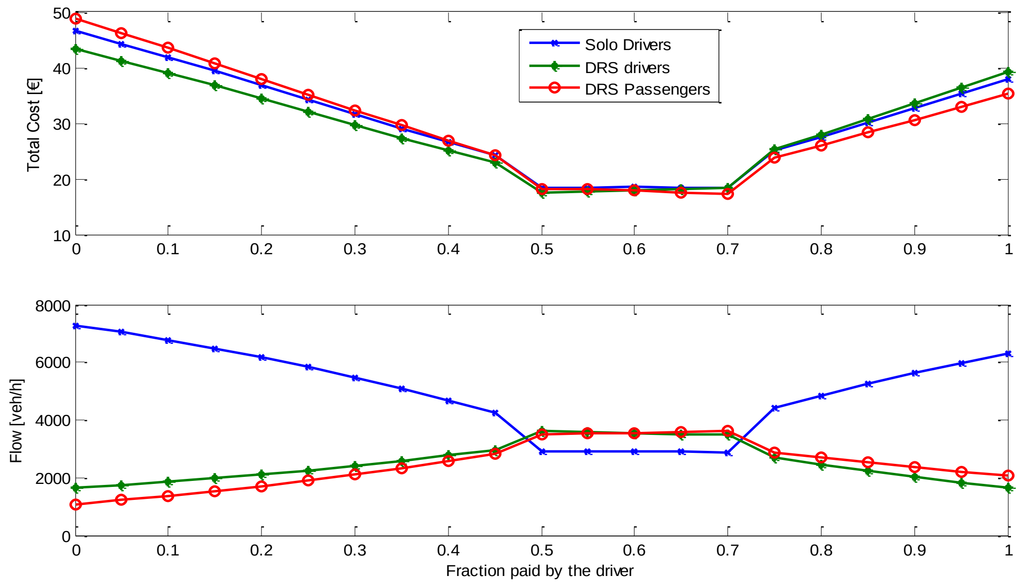

Fig. 1: Equilibrium for different cost sharing policies

By performing sensitivity analysis of the equilibrium wrt the different combination of the above fractions with the other cost parameters (distance-related costs, flexibility, detour and comfort costs) we concluded that through an opportune selection of these cost parameters we can determine a stable region where DRS systems can be self-supporting and create a "critical mass" effect, i.e. ridesharing becomes a competitive mode for the single car mode [4].

The equilibrium formulation described and solved in [4] is not limited to specific iterative solution schemes such as MSA. A more detailed description of the behaviour of this system, including the analysis of existence, convergence, uniqueness and stability of the solution is beyond the scope of this paper, which instead focuses on studying these properties in the time-dependent case, described in the following section.

Joint day-to-day mode and within-day departure time model

We study now the theoretical equilibrium possible in such systems considering explicitly the departure time choice problem in a doubly-dynamic approach. Keeping the mode choice described in [4] as a day-to-day process, we aim in this study to make a step forward by modelling the dynamic of the departure time, characterized as a within-day process. This means

- for those being solo drivers choosing the most convenient departure time, i.e. a single element in time. This can be dealt with as a classical Vickrey model [22]
- for DRS participants, the choice of the time interval where to participate in the DRS service, as a DRS driver or as passenger, i.e. deciding at what earliest time start being available for matches with to a DRS partner, and for how long to wait (until a latest time), after which the travel must be done as solo driver to avoid experiencing higher costs.

We assume again that the population is fixed (i.e. no elastic demand), and the distribution of participants on the different modes is known and it can be determined at the beginning of each day, based e.g. on the day-to-day mode choice module described in Viti and Corman [4]; we further consider the within day dynamics as a time-discretized setting.

Cost functions including departure time costs

Based on the above described process, we can express the costs in a very general way as

- Solo drivers

$$c_{solo}(t) = c_0^{solo}(N^{solo}(t,k) + \ N^{drsd}(t,k)) \ + \qquad \textit{[travelling cost]}$$

$$+\alpha(\max\{0, PAT - (t + tt_d)\}) \qquad \textit{[early arrival cost]}$$

$$-\beta(\min\{0, PAT - (t + tt_d)\}) \qquad \textit{[schedule delay]}$$

- Drivers(passengers) in case of match

$$c_{drsd(pax)}(t) = c_0^{drsd(pax)} \left(N^{solo}(t,k) + \ N^{drsd}(t,k)\right) + \qquad \textit{[travelling cost]}$$

$$+\alpha\left(\max\left\{0, PAT - \left(T_{EDT}(k) + tw_{drsd(pax)} + tt_k\right)\right\}\right) \qquad \textit{[early arrival cost]}$$

$$-\beta(\min\left\{0, PAT - \left(T_{EDT}(k) + tw_{drsd(pax)} + tt_k\right)\right\}) \qquad \textit{[schedule delay]}$$

$$+\gamma \cdot tw_{drsd(pax)} \qquad \textit{[waiting costs]}$$

The above expressions indicate that solo-drivers can freely choose their departure time according to their preferred time of arrival, and to minimise the journey costs, which depend on the travel time on the route and on the early or late arrival at destination. For DRS participants, extra costs may incur because of a mismatch between DRS-drivers and DRS-passengers, which result in a queue of participants waiting for a match.

The symbols indicate:

$c_i\ (t)$	(time-dependent) Cost of mode *i*
$N^i(t,k)$	Cumulative number of travellers for mode *i* at time *t* and day *k*
α, β	Departure time parameters [0.5; 2 EUR/hour]
γ	Value of waiting time in the DRS system [0.2 EUR/hour]
PAT	Preferred arrival time [3 hours after beginning of evaluation period of 5 hours]
$T_{EDT}(k)$	Earliest preferred departure time
tw_i	Waiting time in the DRS system due to queue of requests
tt_k	Travel time experienced during day *k*, considering a waiting time tw_i

In case of no match we assume that the DRS participant will switch to the SOLO-driver mode as soon as the expected costs of waiting for a matched ride becomes higher than driving alone. We assume in this case that the DRS service user become aware (is informed) of the queue of requests and has some indication of the expected waiting time to obtain a match, and that earlier participants in queue do not decide to leave the DRS service unless the solo-driving costs are lower.

Within-day model

Demand can be modelled by considering the amount of SOLO drivers, DRS-Drivers, DRS-Passengers that entered the system at time period t and for day k. In fact, the concept of departure time is to be extended to the earliest departure time at which users might look for a matched ride. The whole procedure can be explained along the graphical description of Figure 2.

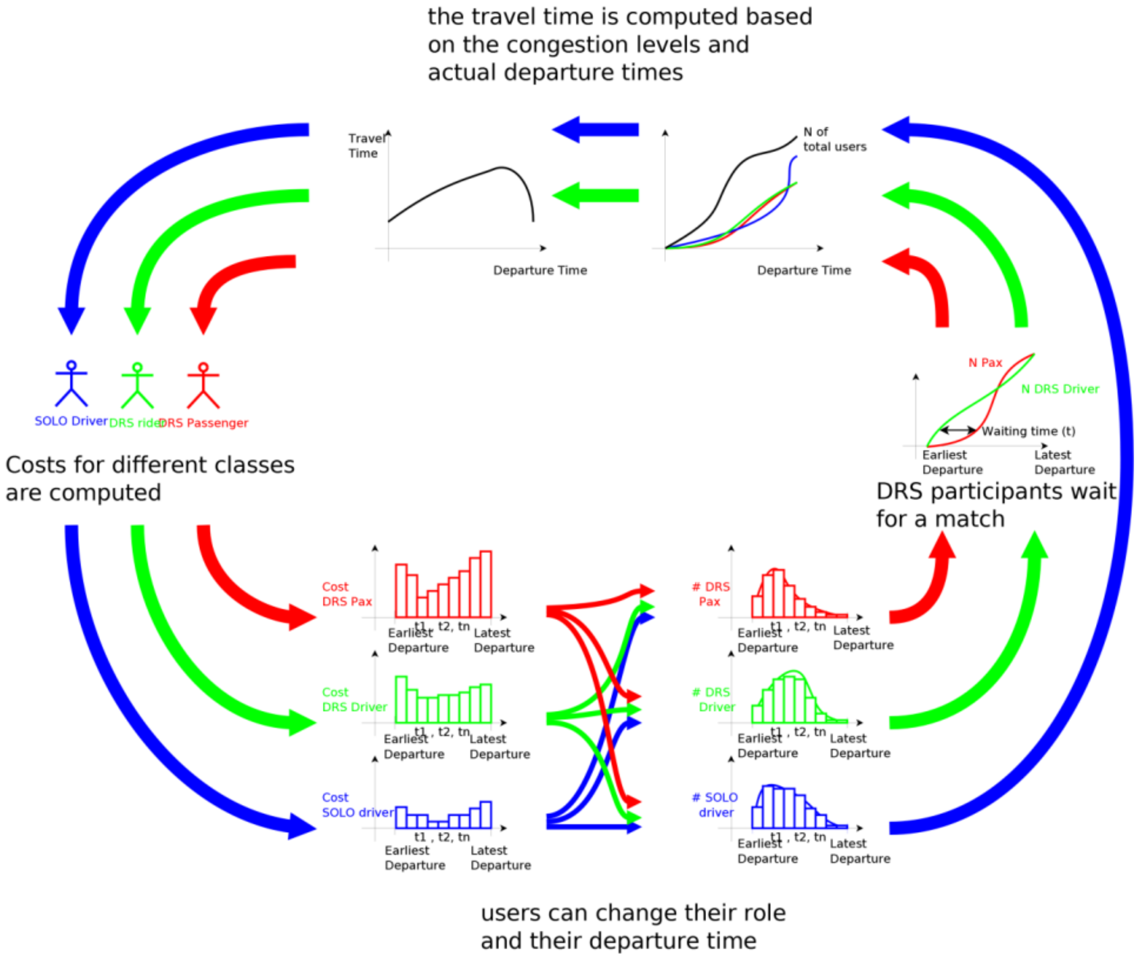

Figure 2. Graphical flow chart of the algorithm analysing the ridesharing system. The within-day dynamic is what happens within a cycle depicted; the day-to-day dynamic happens between different cycles

We can start by the item at the bottom of the Figure 2, i.e. given a subdivision in classes, and departure times, we determine first the time DRS participants have to spend waiting for a match, $tw_{drsd(pax)}$. To do so, we consider a cumulative representation, depicted on the right item in Fig. 5 and separately on Fig. 6. The elements $N^{solo}(t,k)$, $N^{drsd}(t,k)$, $N^{pax}(t,k)$ represent the amount of participants that have entered the system until time t and for day k; clearly, all solo drivers can depart as soon as they enter the system. Instead, $N^{drsd}(t,k)$, $N^{pax}(t,k)$ include the amount of participants starting offering/requesting a ride at the beginning of time period t, plus some DRS participant from previous time periods that are still in the process of finding a match (queues of requests). As for the matching, a FIFO discipline is used for determining the matches between DRS users. This means that a user (say DRS Driver) identified by $N^{drsd}(t,k) = M$ entering the system at time t, will have to wait till the passenger identified by $N^{pax}(\tau,k) = M$ will enter the system at time τ, for a total waiting time of $t - \tau$.

We graphically report the matching process in Figure 3. In the picture, the x-axis represents times, starting from an earliest possible departure time $t_{ed}(k)$ (i.e. nobody perceives departing earlier attractive, for a given Preferred Arrival Time) and going to a latest possible departure time $t_{ld}(k)$

(departing after which will result in excessive scheduling delays even in free flow conditions). The three curves $N^{drsd}(t,k)$, $N^{pax}(t,k)$, $N^{solo}(t,k)$ represent the cumulative amount of participants that have entered the system (i.e. for DRS drivers and passengers, they are either waiting for a match, or driving; for solo-driver, they are driving towards the destination). The lower envelop of the curves $N^{drsd}(t,k)$, $N^{pax}(t,k)$ is the amount of matches in time, which, together with the rate of departures of the solo-drivers, represents the flow rate at time t during day k. The number of DRS participants left at the latest acceptable departure time $t_{ld}(k)$, i.e. the difference $N^{drsd}(t_{ld}(k),k) - N^{pax}(t_{ld}(k),k)$ in the picture, is then departing as solo drivers.

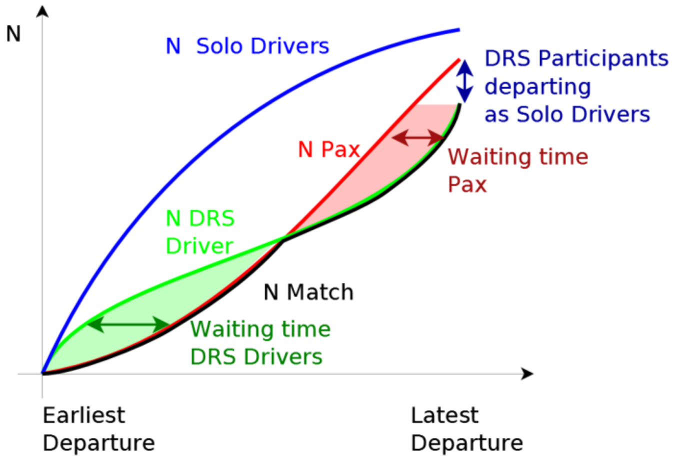

Figure 3. Graphical representation of how a matching algorithm determines waiting time for the DRS participants. Cumulative curves are used.

The within-day model proposed has therefore a few key aspects. Every day, the users evaluate whether to change mode of travelling, and at what time this decision has to be taken. The departure time for the solo-drivers and the latest departure times for the DRS participants should coincide and depend on the tradeoff between congestion costs and schedule delay costs. The earliest departure times of the DRS participants is instead a tradeoff between the early arrival costs, the queue of requests and therefore the waiting time to find a match. So the interesting result of the above model is to find both distribution and rate of mode choices, as well as to find the time window where DRS participants look for a DRS partner at equilibrium.

Algorithm

The main challenge and contribution of this study is therefore an *assignment algorithm* that allows to assign users to classes and departure times, i.e. the bottom step in Figure 2. The underlying mathematical problem is challenging in many aspects, for instance existence, convergence and uniqueness of an equilibrium in assignment is far from trivial, given the fact that travellers characteristics change when they switch group. Great attention is also given to the calculation of the externalities, i.e. changing group for instance from car-sharer to solo-driver increases his/her own costs in terms of fuel consumption for the sake of higher flexibility but also may reduce the opportunities for passengers to find a car-sharer and then force them to take their own car, thus increasing car usage and eventually congestion, which then will affect the same solo-driver who will experience extra travel time.

A few algorithms, illustrated in the following, have been defined and tested, able to solve the assignment problem (and re-assignment problem, in the day to day perspective) with increasing degrees of freedom. The goal of the assignment problem is to find flows that result in an equilibrium point. In this study we use a simple Method of Successive Averages (MSA) to calculate the assignment, as well as a simple Mode-Time-Switching (MTS) where the rate of flow passing to less costly routes is proportional to the gain in terms of cost savings.

Method of successive averages

In the first case each traveller, after experiencing a certain travel cost at the end of day $k-1$, decides which combination of (mode, time t) is giving the least expected costs for the next day k. Then the new fraction of flows from each time period to the least costly alternative is calculated using a flow averaging (FA) criterion. To take into account that the elasticity of adapting the scheduling times may be different than switching mode, we consider different flow averaging parameters for the two choice levels. The procedure goes along the following pseudo-code:

Given the cost vector for day $k-1$
Calculate mode choice probabilities
Calculate total flow amount per mode
Calculate earliest departure time choice probabilities
Calculate new flow amount for time t by taking total flow per mode
Perform an MSA-FA and update flow observed at $k-1$ to flow at day k

Different probability functions can be considered for the mode choice and the departure time choice, taking into account the different elasticity of the travellers to switch mode or transport or adjust departure time from one day to the other.

Mode switching

In the mode-time-switching algorithm, each cost difference between pairs of travel costs is calculated and the flow swapped from travel alternative i to alternative j is then proportional to the cost difference c_{ij} and with the flow amount to be swapped. Also in this case we consider the mode and time choice decision process as two-stage, and consider different elasticity for the swapping rates. The procedure goes along the following pseudo-code:

Given the cost vector for day $k-1$
Calculate cost differences for each pair of modes
Calculate total swapping rate per mode
Calculate cost difference for each pair of earliest departure times
Calculate new flow amount for time t by multiplying cost difference by flow at time $k-1$

The two assignment algorithms have been embedded in the following main pseudo-code addressing the overall procedure, also addressing convergence to a fixed point:

Initialize

$k = 0$
Flow matrix (dimension: number of mode alternatives and departure time intervals)
Cost matrix (same dimension as above)

Calculate

for each day, until convergence or a maximum number k is reached:
for each mode and for each time interval calculate
The cumulative number of travellers per mode and per time
The cumulative number of departing DRS travellers
The queue of DRS participants not finding a match
The waiting time incurred in the DRS system
The flow rate by summing up SOLO-Drivers and DRS-Drivers
The travel time considering the above flow rates
The costs incurred by each traveller
The number of DRS participants needing to switch to solo driving as expected waiting time exceeds the solo driving costs

Perform MSA or MTS assignment algorithm to determine the flows and costs at $k + 1$

Update $k = k + 1$
Determine whether convergence has been reached based on a criterion of choice

end

end

Like for the MSA algorithm, the mode switching and the departure time adjustments have been coded using different sensitivity parameters. In particular, a pure switching algorithm has been evaluated where the users jointly switch mode and time every day, and a mixed mode choice – time switching where mode choice is calculated using a choice probability function and the departure time choice is calculated using a time switching algorithm.

Numerical examples

The main goal of this paper is to systematically identify the conditions for which ridesharing can become a competitive mode, and which policies can be proposed to facilitate the penetration of ridesharing in a transportation system.

The joint day-to-day mode and within-day scheduling model is used to study the interplay between DRS costs (dependent on the role chosen), departure time (i.e. possible time spent waiting for a match, plus experienced travel time) and congestion levels caused by DRS drivers and solo drivers. Thus users have multiple choices: changing role; changing departure time, changing latest time of participating in DRS, after which they will become SOLO driver in order to reach their destination on time. Different elasticity can be assumed to these choices. The generic framework allows also for more sophisticated matching rules, as far as they can be modelled based on cumulative counts, and they would give the time to wait for the match, in real value, expected value, or probability distribution.

A main subject is the evaluation on a simulated network of the final assignment equilibrium, as dependent of departure time choice. This analysis is done evaluating different combination of the cost parameters, and by testing the above-described assignment algorithms. For sake of illustration we limit our presentation of results to only a few of the combinations that can be done with the developed algorithms.

Impact of initial conditions
Figures 4 and 5 shows the result of the model by running the code with the default parameters specified in the previous sections, using the joint mode-time MSA-FA, using different initial conditions for the mode shares.

The default parameters have been chosen opportunely, in order to obtain at equilibrium an exact amount of users distributing on the three different modes. This makes easier to show how the model behaves in terms of departure time choices. Figure 4 shows in fact that if conditions are equally fair for the participants, i.e. the fixed costs incurred by DRS drivers and passengers equal the distance-based costs, which in turn are shared equally between them, and initial conditions are set such that mode spits are also equally set, the mode shares are equal in all time periods. Because no real advantage is considered whether being DRS driver or passenger here, perfect matches are realised and the travellers will simply try and distribute themselves in a period around the preferred departure time, so that costs will be equal. This is realised in the figure for a period of about 2 hours. The sharper costs incurred with the scheduling delays are reflected in the sharp reduction decrease of flows in around 4h. The results using the mode-time switching assignment procedures are fully analogous.

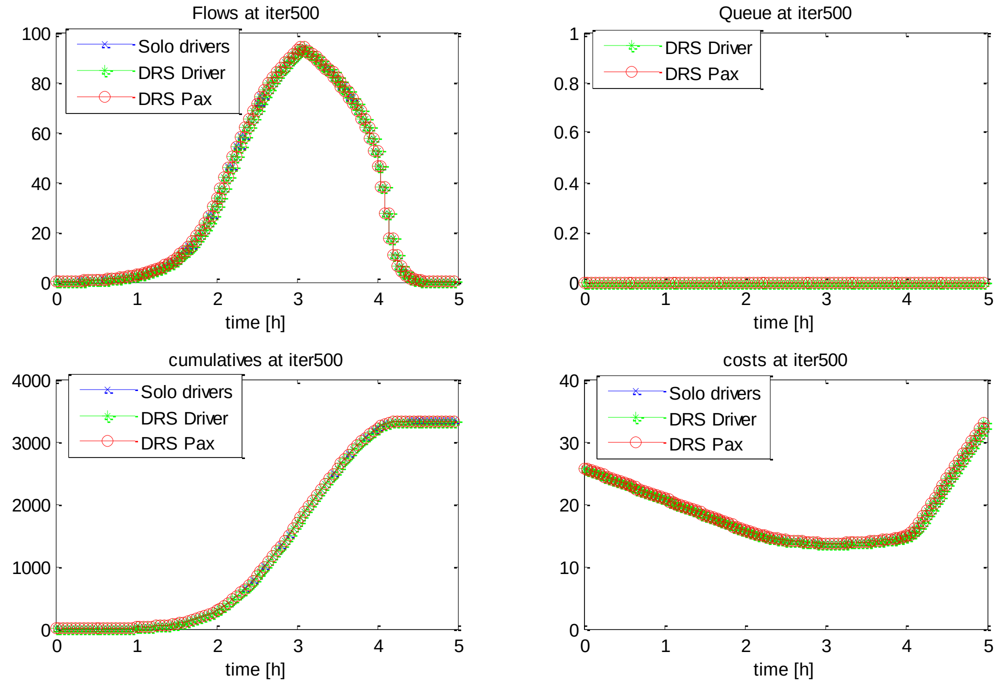

Figure 4. Equilibrium results of the link model using the default parameters in terms of (a) flows at each time period, (b) costs per mode, (c) cumulative travellers and (4) departures

However, the results differ significantly if the initial conditions are set by considering simply more DRS drivers than passengers (or vice-versa), as one can see from Figure 5. In this case, because a queue of passengers (respectively, drivers) is present at each iteration, DRS users in queue choose to enter the DRS system earlier as the cost of waiting is in this case equivalent to the travel time savings.

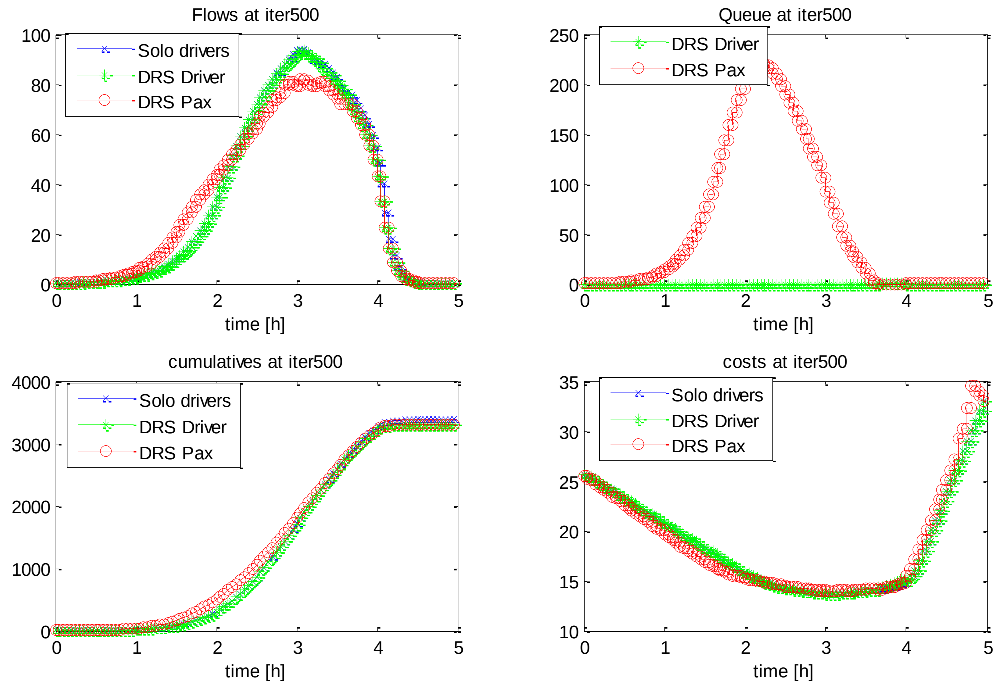

Figure 5. Equilibrium results of the link model in terms of (a) flows and (b) cumulative flows using the default parameters and using initial conditions [0.5, 0.3, 0.2].

Therefore the initial condition with the same amount of initial DRS users represents an instable condition for the results in terms of flows, which determines a bifurcation of the solution set into two regions where either a queue of passengers or of drivers is observed. No significant changes in the equilibrium costs were calculated with respect to this instable point for the solo driving costs.

The results of the mode-time switching are again analogous, except that the different updating rates and the sensitivity of the switching rates are reflected into a stronger shift from DRS service to solo-driving and a larger time period is used by the travellers**.** Because the general trends do not differ significantly, and the model seems to stabilise with both assignment procedures, we will from now on only present the results of the MSA-FA algorithm.

Sensitivity analysis

We now analyse the behaviour of the system under different congestion levels, and the relationship between the travel cost functions and the aforementioned time intervals is evaluated. The results of such a study will describe the properties, stability and extent of the equilibrium point. An important policy implication is the impact of the waiting time during the matching process to the attractiveness of the whole DRS systems (complementary to the research in Agatz et al 2012), that is possibly one main reason for the limited success of existing DRS systems.

Impact of congestion

In the first analysis we observe the behaviour under different congestion effects. Different total demand-capacity values (total number of travellers vs. link capacity over the whole evaluation period) were evaluated. An example of such evaluations is represented in Figure 6, where a more limiting bottleneck is simulated for the demand.

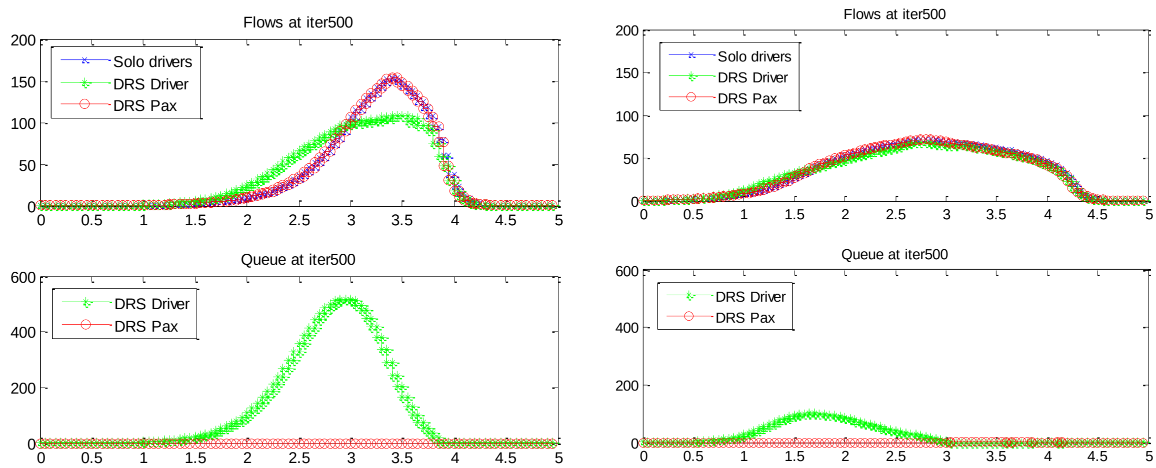

Figure 6. Comparison between (a) moderate congestion and highly congested conditions (b) using default parameters and MSA-FA.

In Figure 6, left pictures, enough capacity is still available such that more solo-drivers switch to DRS services and drivers use a larger departure interval. In Figure 6, right pictures, all travellers are not capable of finding a departure time within the evaluation period without experiencing significant delays. In this case, a higher match of DRS participants is observed on all time periods, and queues are significantly lower.

Impact of different fractions paid by DRS participants

Let us also consider different fractions paid by the DRS drivers. Due to the choice of using default values identical for the DRS drivers and the passengers, the results are fully analogous if the same fractions indicated in the numerical experiment were assigned to the passengers. Figure 7 shows the computation of flows in case of 20% (Figure 7, left pictures) and 40% (Figure 7, right pictures) of distance costs paid by the drivers.

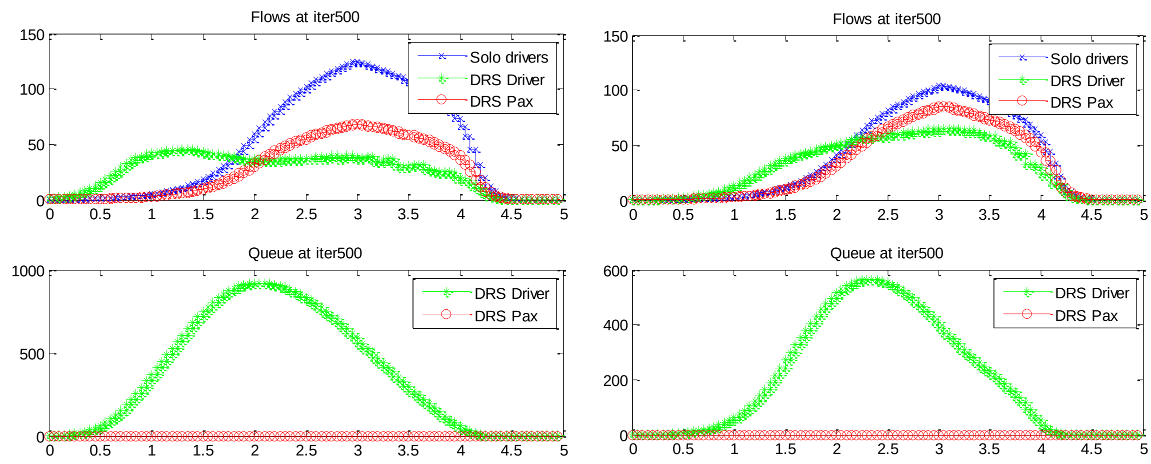

Figure 7. Results using fractions paid by the drivers respectively 20% (a) and 40% (b).

As one can observe, modifying the fraction paid by the drivers changes the mode shares quite significantly. The distance cost savings are then traded off with higher waiting costs for a match.

Impact of asymmetric costs between DRS participants

If we consider different cost functions for the DRS participants the system becomes evidently less attractive, due to a systematic imbalance and the emergence of queues in the service.

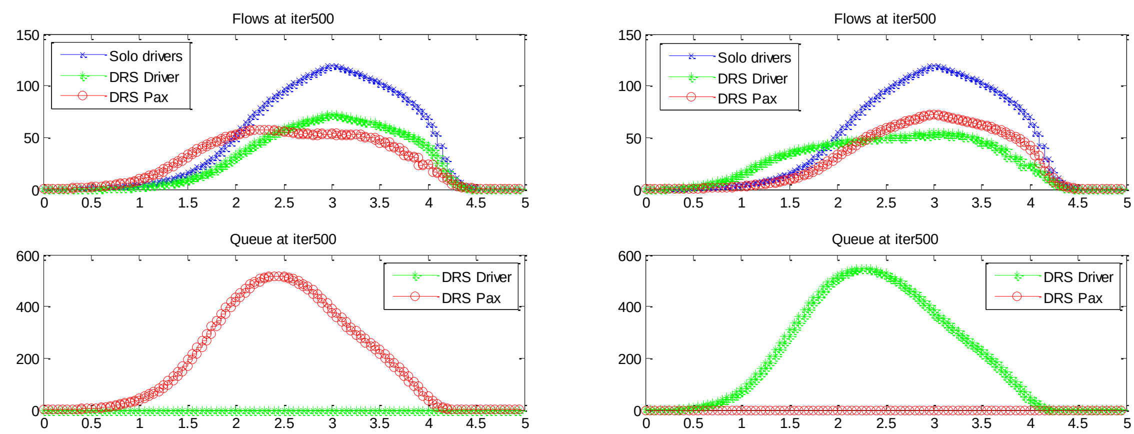

Figure 8. Results using flexibility costs for the DRS passengers as 2 EUR in terms of flows (a) and in terms of queue length (b).

As one can see from Figure 8, if one sets a higher cost for the passenger (the flexibility cost was set to 2 EUR), a clear advantage for the DRS drivers was observed, especially in terms of participant queuing for a match. This makes a significant reduction of DRS matches, due to a significantly lower number of DRS passengers, but also that DRS drivers find advantageous to concentrate nearer to the latest acceptable departure time, and if a match was not found they quickly switch to solo-driving. This stresses even further the need for a quick and predictable matching process. The queue size observed until the end of the evaluation period is clearly dependent on the solo-driving flow.

Impact of increasing distance-dependent costs (e.g. fuel)

A natural question one may be interested in answering is what are the effects of the increasing fuel prices, or in general distance-dependent costs (tolls, car maintenance, etc.), to the utilisation of DRS services. Using the numerical example with 0.3 Eur/km and 0.4 Eur/km (Figure 10), which are seemingly far from unrealistic results in the near future, we can see how the desirable “critical mass” effect observed already in our previous static equilibrium approach [4] are confirmed. Moreover, under those conditions solo-driving becomes very quickly a dominated mode in favour of shared mobility.

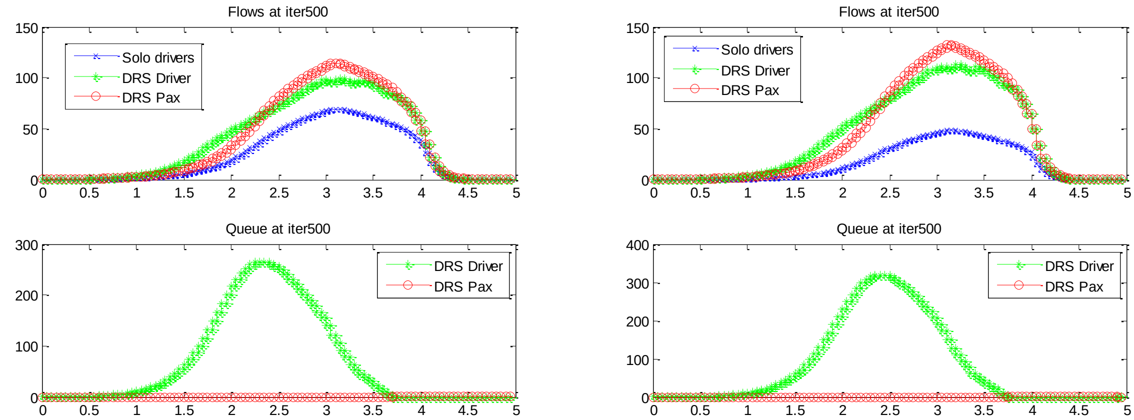

Figure 9. Results using distance-dependent costs as 0.3 EUR/km (a) and 0.4 EUR/km (b).

Summary of findings and conclusions

Dynamic ridesharing has evident benefits from the point of view of the society (less cars running, less emissions) and users (lower traveling costs due to sharing expenses and reduced externalities), balanced by some behavioural obstacles; that overall made the system not successful as in the intention of the policy makers.

We developed a model that analyses dynamic ridesharing services from a transport equilibrium point of view, aiming at analysing the most influencing factors that contribute to the success – or the failure - of such a solution. Based on our studies on a simple one-link model, and looking at a limited but relevant set of parameters, we conclude that many factor are heavily interrelated, and contribute to determine a large stable region where DRS systems can be self-supporting and create a "critical mass" effect. This reinforces and extends recent findings of the authors using a static equilibrium model.

Sensitivity of modal shares and departure time rates has been performed with respect to the main influencing factors incorporated in the cost functions. With respect to the static equilibrium model, we could numerically show in this study that at equilibrium, if total capacity is sufficient, an early acceptable departure time is clearly observed, as well as a late acceptable departure time, after which DRS participants find more reasonable to switch to solo-driving in order to avoid excessive schedule delays, which then would be summed to waiting time costs in the system.

The next steps in this research will be to (1) test the impact of different Travel Demand Management solutions (e.g. parking management schemes, congestion pricing, carpool lanes, etc.), (2) evaluate the impact of past positive or negative experiences in a more realistic day-to-day model (e.g. considering positive feedback in matching DRS partners, which very likely may prefer to be re-matched), (3) design and evaluate different bidding schemes to optimise the way drivers trade off service queues with fractions paid in order to reduce their waiting times and last but not least (4) evaluate and test the model on more realistic network sizes.

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