Reasoning About Coalitional Effectivity in Modal Extension of Łukasiewicz Logic

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Strategic game forms

$$G = \langle N, \{\Sigma_i \mid i \in N\}, S, o \rangle$$

$N = \{1, \ldots, n\}$ is a set of *players*

Σ_i is the *strategy set* of player $i \in N$

S is a set of *outcome states*

$o : \prod_{i \in N} \Sigma_i \to S : \sigma \mapsto o(\sigma)$ is an *outcome function*

Measuring the power of coalitions

$$G = \langle N, \{\Sigma_i \mid i \in N\}, S, o \rangle$$

For a *coalition* $C \in \mathcal{P}N$:

$\overline{C}$ stands for $N \setminus C$

σ_C stands for a tuple of strategies for players in C

Definition. The *effectivity function* $E^\flat_G : \mathcal{P}N \to \mathcal{P}\mathcal{P}S$ *associated with* G is defined by

$$X \in E^\flat_G(C) \qquad \text{if} \qquad \exists \sigma_C \forall \sigma_{\overline{C}} \;\; o(\sigma_C \sigma_{\overline{C}}) \in X$$

Towards a quantitative notion of effectivity

$$G = \langle N, \{\Sigma_i \mid i \in N\}, S, o \rangle$$

$N =$ set of countries in EU

$\Sigma_i =$ set of economic policies that country i can adopt

$o(\sigma) = (r_i)_{i \in N}$ where $r_i =$ rate of employment of country i if strategy vector σ is applied

The limitation of $E^{\flat}_G$

An outcome sate $(r_i)_{i \in N}$ is composed of quantitative information

while

$E^{\flat}_G$ qualifies the power of a coalition in a $\{0, 1\}$-way

Towards a quantitative notion of effectivity

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Effectivity of C on...

$$f_C : \quad (r_i)_{i \in N} \mapsto \min_{i \in C} r_i$$

$$\text{mean} : \quad (r_i)_{i \in N} \mapsto \sum_{i \in N} \frac{1}{|N|} r_i$$

We define generalized effectivity functions

$$G = \langle N, \{\Sigma_i \mid i \in N\}, S, o \rangle$$

$Ł_k := \{0, \frac{1}{k}, \ldots, \frac{k-1}{k}, 1\}$ is the scale

Definition. The *$Ł_k$-valued effectivity function of G*

$$E_G : \mathcal{P}N \times Ł_k^S \to Ł_k$$

is defined by

$$E_G(C, f) \;=\; \max_{\sigma_C} \min_{\sigma_{\overline{C}}} \; f(o(\sigma_C \sigma_{\overline{C}}))$$

Ł$_k$-valued effectivity function in the EU example

$$E_G(C, f) = \max_{\sigma_C} \min_{\sigma_{\overline{C}}} f(o(\sigma_C \sigma_{\overline{C}}))$$

$f\left((r_i)_{i \in N}\right)$	$E_G(C, f)$
$\min_{i \in C} r_i$	the minimum rate of employment for countries in C that coalition C can enforce 'employment sovereignty of C'
$\text{mean}_{i \in N} r_i$	the mean country level rate of employment in EU that coalition C can enforce 'global employment policy efficiency of C'

The poster session

1. Examples illustrating the relevance of $Ł_k$-valued effectivity function
2. Axiomatization of functions $E : \mathcal{P}N \times Ł_k^S \to Ł_k$ that are *playable*, *i.e.*, for which there is a G such that $E = E_G$

★ $Ł_k$-valued version of superadditivity ★

3. A modal language to talk about $Ł_k$-valued effectivity functions (based on Łukasiewicz logic)
4. A $Ł_n$-valued neighborhood semantic and a completeness result for the smallest $Ł_k$-valued coalitional logic