

NUMERICAL MODELING OF PIEZOELECTRIC ENERGY HARVESTING DEVICES DRIVEN BY FLOW INDUCED VIBRATIONS

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The focus of this research is to scavenge energy from the environment to power small scale electrical devices. Research interest in harvesting energy from ambient vibrations has seen a big surge in recent years owing to the rapid technological progress in micro-electro mechanical systems (MEMS) and wireless systems. Piezoelectric materials possess the ability to instantaneously convert mechanical energy to electrical energy and vice-versa by the virtue of their crystalline structure and are widely used in energy scavenging applications due to their high coupling coefficients, high energy density and small form factor.

The research aims to utilize flow induced structural vibrations to harvest useful electrical energy with the help of piezoelectric material and develop a numerical model to solve the discretized equations in a monolithic framework. In other words, a holistic approach that provides simultaneous solution to the coupled fields which involve volume-coupled piezoelectric mechanics, surface-coupled fluid structure interaction and a controlling harvesting circuit. The weak form of the governing equations is discretized by space-time finite element method based on mixed velocity-stress/ rate of potential-dielectric displacement setting.

An exemplary harvester model consists of a cantilever plate structure, with thin piezoelectric patches, placed in fluid flow. The piezoelectric patches are connected to a suitable harvesting circuit which stores or transforms the energy generated by the harvester. The behavior of the structure in the time interval $I = (t_a, t_b)$ and occupying the space-time domain $Q_0 = \Omega_0 \times I$ is described in the reference configuration using a *Lagrangian* description to account for large deformations. The fluid flow within the same time interval and occupying a space-time domain $Q = \Omega \times I$ is described in the current configuration. The harvesting circuit, during the initial analysis, consists of a resistor element

connected either in series or parallel with the piezoelectric patches. The system is modeled in a 3-dimensional framework to enable straightforward application of constitutive models without reducing them based on assumptions.

The method of weighted residuals is utilized to arrive at the weak form of the governing equations of the multi-physics problem and the resulting equations are discretized both in space and time using space-time finite element method in a mixed-hybrid framework. Time integration is performed with the discontinuous Galerkin method. A simultaneous solution strategy is carried out wherein the strongly coupled system is described as a single algebraic system and the various unknowns are solved simultaneously. The temporal axis is included in the finite element discretization wherein the space-time domain Q^n is divided into N time slabs as $Q^n = \Omega^n \times [t_n, t_n + 1]$ which are solved successively. Additional jump terms are added in the weak form to account for the time-discontinuous approximation of the unknowns since the energy of the discretized system at time slab t_n^- has to be equal to the energy at the beginning of the next time slab t_n^+ . The non-linear equations are linearized and solved using a Newton-Raphson scheme.

The research contributes to the mathematical modeling and numerical discretization of complex multi-physics system in an efficient way by simultaneously solving the various fields involved which facilitates an ideal basis for precise and transient coupling. This may lead to improved convergence and numerical efficiency in comparison with partitioned approaches. This methodology also provides new insights and in-depth understanding on design requirements on such energy harvesting devices in terms of their robustness and efficiency.