Robust Direct Adaptive Fuzzy Control of Flexible Joints Robots with Time-Varying Stiffness/Damping Parameters

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Summary/Abstract

In this article, we address the problem of controlling unknown flexible-joint robots with unknown time-varying stiffness and damping parameters. We propose a Robust Direct Adaptive Fuzzy Control (RDAFC) strategy that accommodates the dynamics anonymity and joints stiffness/damping variations. The RDAFC strategy relies on the synergy of the concepts of fuzzy logic approximation and the Sliding Mode Control (SMC). The fuzzy logic approximation relaxes the need for knowing the robot dynamics and the SMC accommodates the parameters variations. We also modify the RDAFC strategy to be suited to the KUKA Lightweight Robot (LWR) and propose a control strategy that can accommodates dynamics anonymity, uncertainty and joints elasticity variations. Experimental results are performed on a KUKA LWR moving in free space with its joints stiffness and damping vary with time in sine and cosine waveforms respectively. From the experiments, we can see that excellent tracking performance is obtained when using the RDAFC strategy despite the joints elasticity parameters time-variance and the robot dynamics unavailability.

1 Introduction

Flexible Joints Robots (FJR) are applied nowadays in many vital applications like industry, medicine, space, ...etc. Joints flexibility adds safer operation of robots for different applications. However, such joints flexility results in more complex control situations. Despite their complexity, the derivation of the control strategies for the FJR attracted interests of many researchers from all over the world.

In the framework of singular perturbation, Spong *et. al.* proposed a control strategy that relies on the concept of integral manifold in realizing the strategy and promising results were obtained [1]. In the FJR, the number of variables is more than the control actions and this urged researchers of using model reduction along with the singular perturbation in enhancing the control performance of the FJR [2]. Simplified PD controller was efficiently used in controlling the FJR with compensating the joints friction torques [3]. In [4, 5, 6], adaptive control was employed in accommodating possible parameters uncertainty in the control problem of the FJR. H_∞ control design was suggested for the FJR and the effect of the external disturbance was attenuated below a certain level [7]. In [8, 9, 10], universal approximators like fuzzy and neural systems were used in building control strategies for such robot systems and good performance was obtained despite the dynamics uncertainties. Online gravity compensation is proposed along with a PD controller and only the motor side position and velocity are needed [11]. Passivity-based control strategy is suggested that can guarantee stable performance for the FJR using the motor side position and stiffness torque feedback [12].

One of the most important aim of almost all researchers is to enable the robot mimicking the human in its behavior since such resemblance would enable the robot of doing more complex tasks. If we consider the assembly tasks as an application example, we can see that human operator can perform such tasks simply because of the human high capability of the environment recognition and excellent arm muscles control. If we focus on the arm muscles control, we can notice that a human operator changes his arm joints elasticity with time so that smoother and safer tasks are performed and possible environment elasticity are accommodated [13, 14]. Of course such a change in joints elasticity are done unintentionally and acquired with human experience. Furthermore, if the human performs a certain task and shifts to another one, then his arm joints elasticity may also be varied (unintentionally) to accommodate such a change (if required). Stemming from this motivation one may need the variation of the joints elasticity parameters in a robot according to the task requirements. A FJR is a nonlinear system and when we have the elasticity terms are time-varying, then we would have a time-varying nonlinear system.

In this article we suggest a Robust Direct Adaptive Fuzzy Control (RDAFC) strategy for unknown FJR with time-varying stiffness/damping parameters. The fuzzy logic approximation and the Sliding Mode Control (SMC) are used in accommodating the dynamics anonymity and the time-variance of the joints elasticity. Then we modify the RDAFC strategy to be suited to the KUKA Lightweight Robot (LWR) with unknown time varying joints elasticity parameters. We will consider the current joint specific controller as a diffeomorphism that relates the the joints state variables (joints position and velocity) to the input added torques which results in an unknown mapping with time-varying parameters and the RDAFC strategy would be used in controlling such a robot system.

The rest of the paper is organized as follows. In section 2, we describe the control problem in hand and section 3 will lodge preliminary concepts and definitions. In section 4, the RDAFC strategy will be presented and section 5 will contain the RDAFC strategy for the KUKA LWR. Experimental validation will be shown in section 6 and section 7 will conclude the article with pinpointing the recommendations for future works.

2 Problem Statement

A Flexible Joint Robot (FJR) can be described by the following dynamics [15]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = K(q_m - q) + D(\dot{q}_m - \dot{q}) \quad (1)$$

$$B\ddot{q}_m + K(q_m - q) + D(\dot{q}_m - \dot{q}) = \tau_m \quad (2)$$

Where $q \in R^n$ is the links position vector, $M(q) \in R^{n\times n}$ is the inertia matrix, $C(q,\dot{q})\dot{q} \in R^n$ is the centripetal and Coriolis vector, $G(q) \in R^{n\times n}$ is the gravity vector, $\tau_m \in R^n$ is the torque vector produced by the actuator, $K \in R^{n\times n}$ is a diagonal matrix whose main diagonal elements k_i are the joints stiffness, $q_m \in R^n$ is the actuator side position, $D \in R^{n\times n}$ is a diagonal matrix whose main diagonal elements d_i are the joints damping. Suppose that $x_1 = q$, $x_2 = \dot{q}$, $x_3 = \theta$, and $x_4 = \dot{\theta}$. One can describe (1) as:

$$\begin{aligned}
\dot{x}_1 &= x_2\\
\dot{x}_2 &= f_1(X)\\
\dot{x}_3 &= x_4\\
\dot{x}_4 &= f_2(X) + g\tau_m + K(x_1 - x_3) + D(\dot{x}_1 - \dot{x}_3)
\end{aligned} \quad (3)$$

where $X \in R^{4\times n} : X = (x_1, x_2, x_3, x_4)^T$, $f_1(X) = M^{-1}(x_1)(K(x_3 - x_1) + D(x_4 - x_2) - C(x_1, x_2)x_2 - G(x_1))$, $f_2(X) = B^{-1}(K(x_1 - x_3) + D(x_2 - x_4))$, and $g = B^{-1}$. (3) can be written in the following compact form:

$$\dot{X} = F(X) + G(\tau_m + K(x_1 - x_3) + D(x_2 - x_4)) \quad (4)$$

with $F(X) \in R^{4\times n} : F(X) = (x_2, f_1(X), x_4, f_2(X))$ and $G = (0, 0, 0, g)^T$. For time-varying damping/stiffness parameters, (4) would be:

$$\dot{X} = F(X) + G(\tau_m + K(t)(x_1 - x_3) + D(t)(x_2 - x_4)) \quad (5)$$

For unknown robot dynamics, (5) results in unknown time-varying nonlinear system and the objective of this article is to propose a robust adaptive fuzzy controller for such kind of robot systems. We will consider the case of a KUKA Lightweight Robot (LWR) as a case study and modify the suggested strategy to be applicable to such a robot system with time-varying joints stiffness and damping parameters.

3 Preliminaries

Before exhibiting the main control strategy suggested in this paper, we will explain the concept of fuzzy logic approximators along with other preliminary concepts, properties and assumptions.

3.1 Fuzzy Logic Approximators

One of the vital applications of the fuzzy set theory is the functions approximation. It gives a feasible way of approximating unknown smooth functions through the use of T-S fuzzy models. Suppose that we desire to approximate the control action of (5), and consider that $(q_1, \dot{q}_1, ..., q_n, \dot{q}_n) = (u_1, u_2, ..., u_{2n})$. Let's assume that the output of each mapping, that will be approximated, is y_f. Such approximation would be feasible in the context of fuzzy *If-Then* rules as:
Controller Rule i:

If u_1 is A_{i1} and u_2 is A_{i2} and... and u_{2n} is A_{i2n}
Then $y_f = y_f^i$ (6)

with $i = 1, 2, ..., L$; L is the total number of the *If-Then* rules; $A_{ij}(i = 1, 2, ..., L; j = 1, 2, ..., m)$ are the premise fuzzy sets; and y_f^i is crisp output of the k^{th} rule. Through using a singleton fuzzifier along with the product inference, the overall output for the fuzzy system above can be computed as [16, 17]:

$$y_f = \theta^T h(u) \quad (7)$$

with:

$$\mu_i(u) = \prod_{j=1}^{2n} A_{ij}(u_j)$$

$$h(u) = \left(\frac{\mu_1(u)}{\sum_{i=1}^{L}\mu_i(u)}, \frac{\mu_2(u)}{\sum_{i=1}^{L}\mu_i(u)}, ..., \frac{\mu_L(u)}{\sum_{i=1}^{L}\mu_i(u)}\right)$$

$$\mu_i(u) \geq 0$$

$$\sum_{i=1}^{L}\mu_i(u) > 0$$

and:

$$\theta = (y_f^1, y_f^2, ..., y_f^L)$$

Hence, the control action τ_m of (5) can be approximated through a fuzzy logic controller $\tau_f = (y_{f1}, ..., y_{fn})$ and this is called direct fuzzy control [17]. That is:

$$\tau_f(q, \dot{q}|\theta) = \theta^T h(q, \dot{q}) \quad (8)$$

3.2 Properties and Assumptions

Below properties are common between robot manipulators [15]:
P1. For all robot manipulators, $M(q)$ is a positive definite and symmetric matrix.
P2. For all robot manipulators, the matrix $\dot{M}(q) - 2C(q,\dot{q})$ is a skew symmetric matrix, that is for all $\acute{x}$: $\acute{x} \in R^n$ and $\acute{x} \neq 0$, we have $\acute{x}^T(\dot{M}(q) - 2C(q,\dot{q}))\acute{x} = 0$.
Define the error vector to be:

$$\tilde{x}_{1,3} = x_{1,3} - x_{d1,d3} \tag{9}$$

and consider the filtered error vector to be described as:

$$s = \dot{\tilde{x}}_{1,3} + \gamma\tilde{x}_{1,3} \tag{10}$$

with $\gamma > 0$. (10) can be rewritten as:

$$s = \dot{x}_{1,3} - \dot{x}_{r1,r3} \tag{11}$$

where:

$$\dot{x}_{r1,r3} = \dot{x}_{d1,d3} - \gamma\tilde{x}_{1,3} \tag{12}$$

Note 1. It has been shown that the filtered error described by (10) has the following properties: (i) the equation $s(t) = 0$ defines the time-varying hyperplane in R^n, on which the tracking error vector $\tilde{x}_{1,3}$ decays exponentially to zero.(ii) if $\tilde{x}_{1,3}(0) = 0$ and $|s(t)| \leq \varepsilon$ with constant ε, then $\tilde{x}_{1,3}(t) \in \Omega_\varepsilon = \{\frac{\tilde{x}_{1,3}(t)}{\tilde{x}_{1,3}} \leq 2^{i-1}\gamma^{i-2}\varepsilon, i = 1,2\}$ for $\forall t \geq 0$ and (iii) if $\tilde{x}_{1,3}(0) \neq 0$ and $|s(t)| \leq \varepsilon$ then $\tilde{x}_{1,3}(t)$ will converge to Ω_ε within a time constant of $\frac{(n-1)}{\gamma}$ [18].
Taking the time derivative of (11), we obtain:

$$\dot{s} = \ddot{x}_{1,3} - \ddot{x}_{r1,r3} \tag{13}$$

Despite the robustness of the SMC, a possible chattering may deteriorate the control performance and may even drive the system to be unstable. Therefore, a modified filtered error [19] is introduced that can be expressed as:

$$s_\varepsilon = s - \varepsilon tanh(\frac{s}{\varepsilon}) \tag{14}$$

Let's define $k(t) = (k_1(t), ..., k_n(t))^T$ and $d(t) = (d_1(t), ..., d_n(t))^T$. All joints stiffness and damping are assumed to be bounded. That is:

$$|k_i(t)| \leq k_{ui} \tag{15}$$

and

$$|d_i(t)| \leq d_{ui} \tag{16}$$

Suppose that $\bar{KG}$ and $\bar{DG}$ are the upper bounds of KG and DG respectively. We will design the control strategy relying on the modified filtered error (14). However, before we proceed in explaining the suggested control strategy, below assumptions are needed to be satisfied:
A1. The signals $x_{1,3}$, $\dot{x}_{1,3}$, and $\ddot{x}_{1,3}$ are available for measurement.
A2. The signals $x_{d1,d3}$, $\dot{x}_{d1,d3}$, and $\ddot{x}_{d1,d3}$ are bounded and piecewise continuous.
A3. All joints stiffness and damping parameters are bounded.

4 Robust Direct Adaptive Fuzzy Control (RDAFC) Design

Since the dynamics of the robot is assumed to be unknown then $F(X)$, G, $K(t)$, and $D(t)$ would be unknown. We use the fuzzy logic in approximating $F(X)$ and we will denote such an approximation as $\hat{F}(X|\theta)$. Suppose that the approximation error is w, that is:

$$w = \hat{F}(X|\theta) - F(X) \tag{17}$$

The minimum approximation error w^* is defined to be:

$$w^* = \hat{F}(X|\theta^*) - F(X) \tag{18}$$

Where θ^* is the optimal parameter vector of θ that is defined as:

$$\theta^* = \text{arg min}_{|\theta| \in M_\theta}[\sup_{X \in M_X} \hat{F}(X|\theta) - F(X)] \tag{19}$$

and

$$\hat{F}(X|\theta) = \theta^T h(q,\dot{q}) \tag{20}$$

Let's introduce the following control action:

$$\begin{aligned}\tau_m = \hat{G}^{-1}[&-(\hat{w} + \hat{\bar{KG}}|x_1 - x_3| + \hat{\bar{DG}}|\dot{x}_1 \\ &-\dot{x}_3|)tanh(\frac{s_\varepsilon}{\varepsilon}) - K_d s - F(\hat{X}|\theta) + \dot{X}_d]\end{aligned} \tag{21}$$

with $K_d = \text{diag}(k_{d1}, ..., k_{d2n})$; $k_{d1}, ..., k_{d2n}$ are positive constants. The control action (21), with parameters update laws described by (22)-(26) below, can be shown to provide globally stable performance for the FJR described by (5). The parameters update laws are:

$$\dot{\hat{\theta}} = \eta_1 s_\varepsilon h(q,\dot{q}) \tag{22}$$

$$\dot{\hat{G}} = \eta_2 s_\varepsilon \tau_m \tag{23}$$

$$\dot{\hat{w}} = \begin{cases} \eta_3|s_\varepsilon| & \text{if}(|\hat{w}| < M_w) \text{ or} \\ & (|\hat{w}| = M_w \text{ and} \\ & \eta_3|s_\varepsilon| \leq 0) \\ P(\eta_3|s_\varepsilon|) & \text{if}(|\hat{w}| = M_w \text{ and} \\ & \eta_3|s_\varepsilon| > 0) \end{cases} \tag{24}$$

$$\dot{\hat{\bar{KG}}} = \begin{cases} \eta_4|s_\varepsilon| & \text{if}(|\hat{\bar{KG}}| < M_{KG}) \text{ or} \\ & (|\hat{\bar{KG}}| = M_{KG} \text{ and} \\ & \eta_4|s_\varepsilon| \leq 0) \\ P(\eta_4|s_\varepsilon|) & \text{if}(|\hat{\bar{KG}}| = M_{KG} \text{ and} \\ & \eta_4|s_\varepsilon| > 0) \end{cases} \tag{25}$$

$$\dot{\hat{\tilde{D}G}} = \begin{cases} \eta_5|s_\varepsilon| & \text{if}(|\hat{\tilde{D}G}| < M_{KG}) \text{ or} \\ & (|\hat{\tilde{D}G}| = M_{DG} \text{ and} \\ & \eta_5|s_\varepsilon| \leq 0) \\ P(\eta_5|s_\varepsilon|) & \text{if}(|\hat{\tilde{D}G}| = M_{KG} \text{ and} \\ & \eta_5|s_\varepsilon| > 0) \end{cases} \quad (26)$$

with M_w, M_{KG}, and M_{DG} are design parameters that specify the bounds of $\hat{w}$, $\hat{\tilde{K}G}$, and $\hat{\tilde{D}G}$ respectively. The stable performance of the strategy above can be shown through considering the Lyapunov candidate $V = \frac{1}{2}(s_\varepsilon^T s_\varepsilon + \frac{1}{2\eta_1}\tilde{\theta}^T\tilde{\theta} + \frac{1}{2\eta_2}\tilde{G}^T\tilde{G} + \frac{1}{2\eta_3}\tilde{w}^T\tilde{w} + \frac{1}{2\eta_4}\tilde{\tilde{K}G}^T\tilde{\tilde{K}G} + \frac{1}{2\eta_5}\tilde{\tilde{D}G}^T\tilde{\tilde{D}G})$. Then it can be proved that the time derivative of this Lyapunov candidate is decreasing along the RDAFC control strategy above, i.e. $\dot{V} \leq -s_\varepsilon^T K_d s_\varepsilon$. Hence all closed loop signals are bounded with the modified filter to be zero within a small time constant as detailed in Note 1. Nextly, we will modify the RDAFC and make it suitable for one of the industrial robots which is the KUKA LWR.

5 RDAFC Strategy for The LWR

The dynamics of the KUKA LWR can be described by [21]:

$$f(q,\dot{q},\ddot{q}) = \tau + K(t)(q_{FRI} - q) + D(t)(\dot{q}_{FRI} - \dot{q}) \quad (27)$$

$f(q,\dot{q},\ddot{q})$ is the unknown dynamics function and q_{FRI} is the desired position values. For the case of time-varying joints stiffness and damping, one can write (27) as:

$$f(q,\dot{q},\ddot{q}) = \tau + K(t)(q_d - q) + D(t)(\dot{q}_d - \dot{q}) \quad (28)$$

In order to modify the RDAFC strategy to be applicable to the KUKA LWR, we introduce the error signal to be $\tilde{q} = q - q_d$ and the filtered error to be $s = \dot{\tilde{q}} + \gamma\tilde{q}$. Likewise to the RDAFC strategy derived in section 4, we can say that $s = \dot{q} - \dot{q}_r$ with $\dot{q}_r = \dot{q}_d - \gamma\tilde{q}$. Furthermore, we introduce the modified filtered error to be $s_\varepsilon = s - \varepsilon tanh(\frac{s}{\varepsilon})$. Now, let's suppose that the control strategy that stabilizes (28) is described by τ^*. We propose a fuzzy controller τ_f that approximates such a stabilizing controller with w to be the error between τ_f and τ^*. That is:

$$w = \tau_f(q,\dot{q}|\theta) - \tau_m^* \quad (29)$$

The minimum approximation error w^* is defined to be:

$$w^* = \tau_f(q,\dot{q}|\theta^*) - \tau_m^* \quad (30)$$

Where θ^* is the optimal parameter vector of θ that is defined as:

$$\theta^* = \arg\min_{|\theta|\in M_\theta}[\sup_{q\in M_q,\dot{q}\in M_{\dot{q}}}\tau_f(q,\dot{q}|\theta^*) - \tau_m^*] \quad (31)$$

and

$$\tau_f(q,\dot{q}|\theta) = \theta^T h(q,\dot{q}) \quad (32)$$

with M_q and $M_{\dot{q}}$ are the allowable sets of q and $\dot{q}$ respectively. Let's introduce $\hat{k}_u$ and $\hat{d}_u$ to be parameter vectors compensating for k_u and d_u respectively. We will assume that $|\hat{w}| \leq M_w$, $|\hat{k}_u| \leq M_k$, and $|\hat{d}_u| \leq M_d$, i.e. the parameter vectors $\hat{w}$, $\hat{k}_u$ and $\hat{d}_u$ are required to remain within prescribed sets.
Let's consider the control action to be composed of two terms; a fuzzy control action τ_f and a bounding term τ_b, that is:

$$\tau = \tau_f + \tau_b \quad (33)$$

Where:

$$\tau_b = -K_d s(t) - \Gamma(\hat{k}_u + \hat{d}_u + \hat{w}) \quad (34)$$

$K_d = \text{diag}(k_{d1}, k_{d2}, ..., k_{dn})$ with k_{d1}, k_{d2}, ...,k_{dn} are positive constants, and $\Gamma = \text{diag}(tanh(\frac{s_1}{\varepsilon_1}), tanh(\frac{s_2}{\varepsilon_2}), ..., tanh(\frac{s_n}{\varepsilon_n}))$. Therefore, the need for knowing the robot dynamics is relaxed through the use of the control action (33). In order to guarantee the stable performance for the suggested RDAFC strategy, the parameters vectors $\hat{k}_u$, $\hat{d}_u$, θ, and $\hat{w}$ are updated according to the following laws:

$$\dot{\hat{k}}_u = \begin{cases} \eta_1|Zs_\varepsilon| & \text{if}(|\hat{k}_u| < M_k) \text{ or} \\ & (|\hat{k}_u| = M_k \text{ and} \\ & \eta_1|Zs_\varepsilon| \leq 0) \\ P(\eta_1|Zs_\varepsilon|) & \text{if}(|\hat{k}_u| = M_k \text{ and} \\ & \eta_1|Zs_\varepsilon| > 0) \end{cases} \quad (35)$$

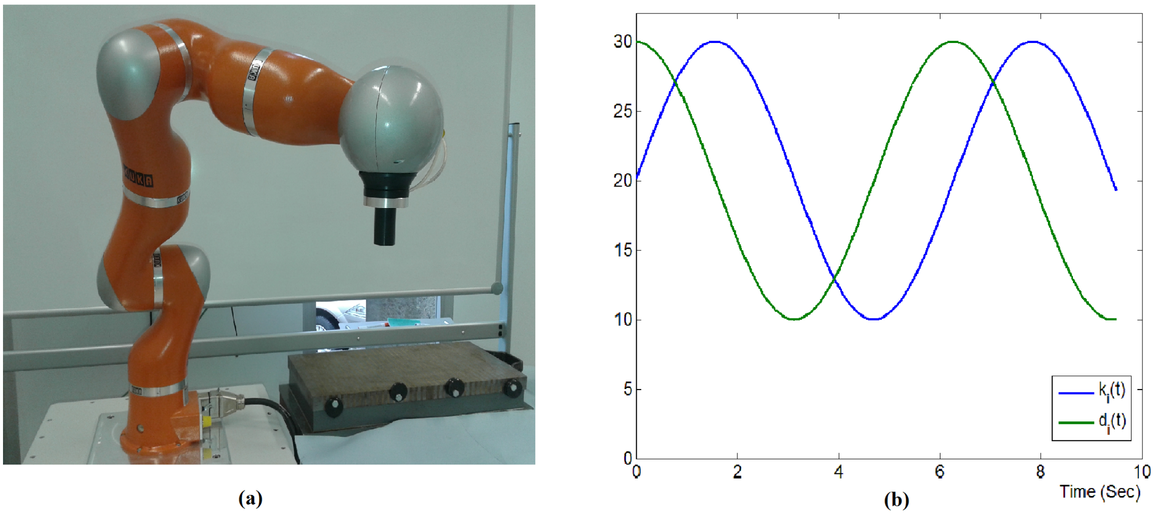
$$\dot{\hat{d}}_u = \begin{cases} \eta_2|\dot{Z}s_\varepsilon| & \text{if}(|\hat{d}_u| < M_d) \text{ or} \\ & (|\hat{d}_u| = M_d \text{ and} \\ & \eta_2|\dot{Z}s_\varepsilon| \leq 0) \\ P(\eta_2|\dot{Z}s_\varepsilon|) & \text{if}(\hat{d}_u = M_d \text{ and} \\ & \eta_2|\dot{Z}s_\varepsilon| > 0) \end{cases} \quad (36)$$

$$\dot{\theta} = -\eta_3 s_\varepsilon^T h(q,\dot{q}) \quad (37)$$

$$\dot{\hat{w}} = \begin{cases} \eta_4|s_\varepsilon| & \text{if } (|\hat{w}| < M_w) \text{ or } (|\hat{w}| = M_w \text{ and} \\ & \eta_4|s_\varepsilon| \leq 0) \\ P(\eta_4|s_\varepsilon|) & \text{if } (|\hat{w}| = M_w \text{ and } \eta_3|s_\varepsilon| > 0) \end{cases} \quad (38)$$

where $\eta_1, \eta_2, \eta_3, \eta_4 > 0$, $Z \in R^{n\times n}, Z = \text{diag}(q_{d1} - q_1, ..., q_{dn} - q_n)$ and $P(.)$ is the projection function, that is:

$$P(\eta_1|Zs_\varepsilon|) = \eta_1|Zs_\varepsilon| - \eta_1|Zs_\varepsilon|(\frac{\hat{k}_u^T\hat{k}_u}{|\hat{k}_u|^2})$$



(a) (b)

Figure 1: (a) KUKA Lightweight Robot (LWR); (b)The waveforms of the KUKA LWR i^{th} joint stiffness (blue) and damping (green).

$$P(\eta_2|\dot{Z}s_\varepsilon|) = \eta_2|\dot{Z}s_\varepsilon| - \eta_2|\dot{Z}s_\varepsilon|(\frac{\hat{d}_u^T\hat{d}_u}{|\hat{d}_u|^2})$$

$$P(\eta_4|s_\varepsilon|) = \eta_4|s_\varepsilon| - \eta_4|s_\varepsilon|(\frac{\hat{w}^T\hat{w}}{|\hat{w}|^2})$$

The stability of the RDAFC strategy can be ascertained through considering the Lyapunov candidate $V = \frac{1}{2}s_\varepsilon^T M(q)s_\varepsilon + \frac{1}{2\eta_1}\tilde{k}_u^T\tilde{k}_u + \frac{1}{2\eta_2}\tilde{d}_u^T\tilde{d}_u + \frac{1}{2\eta_3}\tilde{\theta}^T\tilde{\theta} + \frac{1}{2\eta_4}\tilde{w}^T\tilde{w}$, with $\tilde{k}_u = \hat{k}_u - k_u$, $\tilde{d}_u = \hat{d}_u - d_u$, $\tilde{\theta} = \theta - \theta^*$ and $\tilde{w} = \hat{w} - w^*$ and it can be shown that the time derivative of V is decreasing along the strategy above. Therefore, $s_\varepsilon \to 0$ as $t \to \infty$ and from (37) we can say that $\dot{\theta} \to 0$ as $t \to \infty$. Hence, θ will be always bounded and there is no need to use the projection function used for the update laws of the other parameters. For (35), (36), and (38) the right hand side is greater than or equal to zero that may cause the proliferation of their parameters with time. Therefore, the use of the projection function was inevitable to force those parameters for remaining within a certain bound.

6 Experimental Results

In order to see the performance of the suggested RDAFC strategy, we used it in controlling a KUKA Lightweight Robot (LWR) with time-varying joints stiffness and damping parameters. The KUKA LWR is a 7-DOF industrial robot and the key features of the KUKA LWR is detailed in [22]. For research purposes, a Fast Research Interface (FRI) is available in the robot hardware that makes its joints control strategy customizable by the user through a C++ platform hence allowing researchers to apply their own control schemes in controlling the robot joints [20]. Through its platform, we programmed the joints stiffness and damping to vary in sine and cosine waveforms respectively, that is for the i^{th} joint $k_i(t) = 20 + 10sin(t)$ and $d_i(t) = 20 + 10cos(t)$. Figure 1.a shows the KUKA LWR that we used in our experiments and Figure 1.b shows the waveforms of the variations of the joints stiffness and damping parameters.
Figure 2.a-f show the desired pose signals of the KUKA LWR end effector and the corresponding joint position signals are shown in Figure 3.a-g.
The control actions when using the RDAFC strategy in commanding the joints of the given robot are shown in Figure 3.h-n. The RDAFC strategy was used with the following details:
$K_d = \text{diag}(15, 12, 12, 15, 10, 5, 4)$
$\varepsilon^T = [0.005, 0.07, 0.06, 0.05, 0.04, 0.09, 0.04]^T$
$\gamma^T = [80, 80, 80, 50, 12, 12, 5]^T$
$M_k^T = [2.3, 3.3, 2.0, 3.4, 2.4, 2.2, 2.2]^T$
$M_d^T = [2.1, 3.0, 3.0, 2.0, 2.0, 2.0, 1.7]^T$
$M_w^T = [0.002, 0.017, 0.007, 0.0005, 0.0007, 0.00012$
$, 0.0007]^T$
$\eta_1 = 0.001$, $\eta_2 = 0.01$, $\eta_3 = 0.0001$, and $\eta_4 = 0.0001$.
Gauss membership functions of the form:

$$A_{ij}(u_j) = exp(-\frac{(u_j - c)^2}{2\sigma^2}) \quad (39)$$

are used in the premise of the i^{th} If-Then rule of the RDAFC. c and σ are the center and width of the gauss membership function. The joints position and velocity, say q and $\dot{q}$ respectively, are considered the input variables for the fuzzy logic controller, and each one of those state variables is assigned with two membership functions for the premise part of the if-then rules. For simplicity, we will describe each Gauss membership function described by (39), with an ordered pair (c, σ). Below are the parameters of the fuzzy sets of the variables considered in the RDAFC, say q and $\dot{q}$:
$q_1 : (-0.7, 0.0849)$ and $(-0.9, 0.0849)$
$q_2 : (-0.4, 0.0849)$ and $(-1, 0.0849)$
$q_3 : (0.3, 0.0849)$ and $(0.2, 0.0849)$
$q_4 : (1.6, 0.0849)$ and $(1.2, 0.0849)$

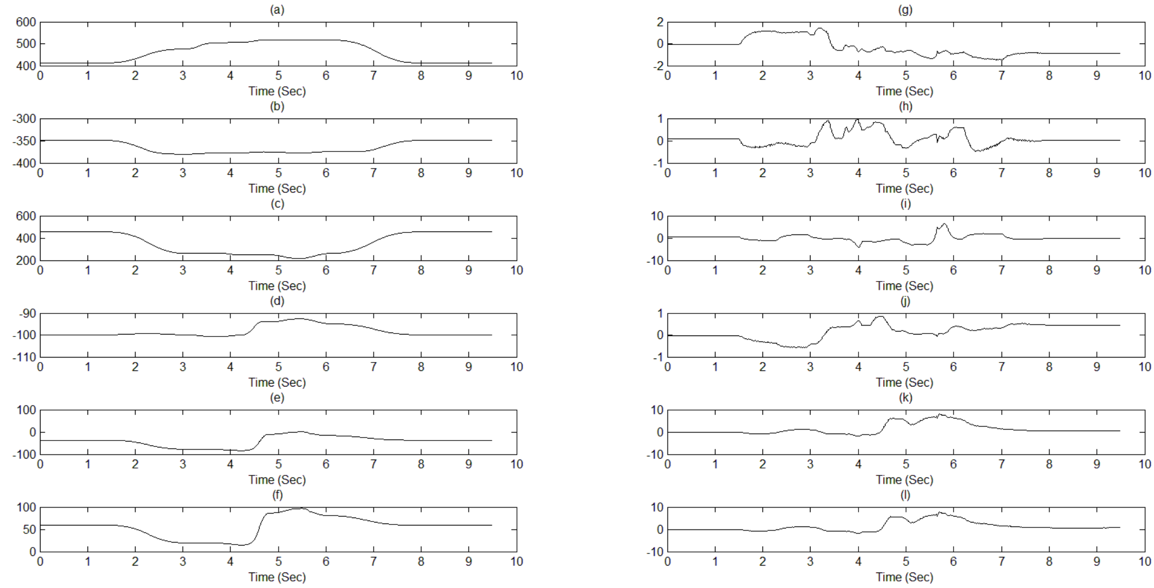


Figure 2: The manipulated object signals: (a) x (in mm); (b) y (in mm); (c) z (in mm); (d) Θ (in degree); (e) Ψ (in degree); (f) Φ (in degree); (g) e_x (in mm); (h) e_y (in mm); (i) e_z (in mm); (j) e_Θ (in degree); (k) e_Ψ (in degree); (l) e_Φ (in degree).

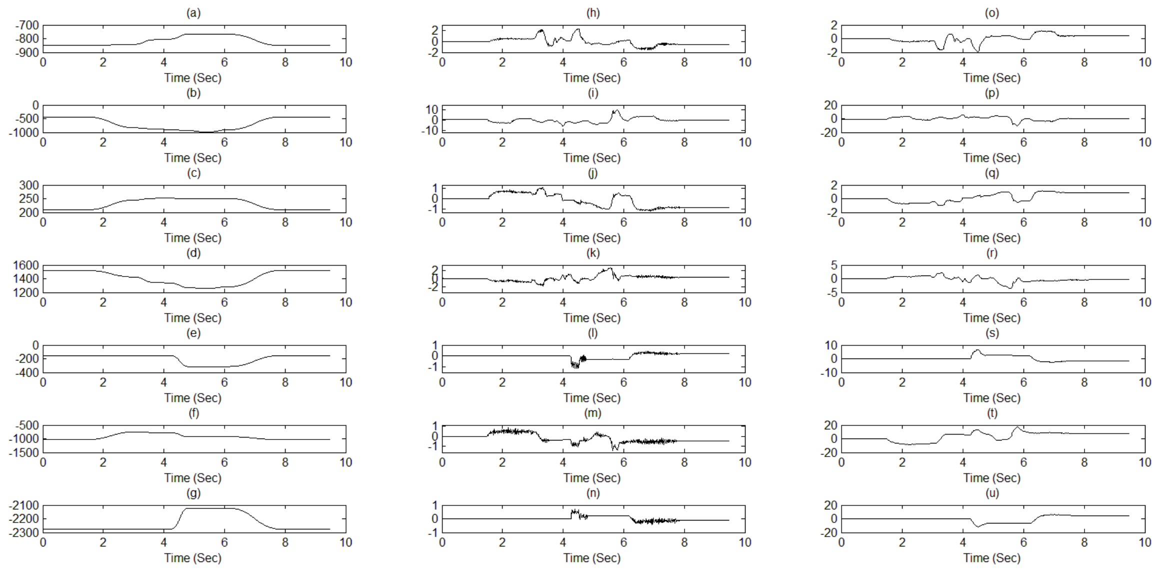


Figure 3: Joints position and velocity signals: (a) q_{d1} (in mrad); (b) q_{d2} (in mrad);(c) q_{d3} (in mrad); (d) q_{d4} (in mrad); (e) q_{d5} (in mrad); (f) q_{d6} (in mrad); (g) q_{d7} (in mrad); (h) τ_1 (in N.m); (i) τ_2 (in N.m); (j) τ_3 (in N.m); (k) τ_4 (in N.m); (l) τ_5 (in N.m); (m) τ_6 (in N.m); (n) τ_7 (in N.m); (o) $\dot{\tilde{q}}_1$ (in mrad); (p) $\dot{\tilde{q}}_2$ (in mrad); (q) $\dot{\tilde{q}}_3$ (in mrad); (r) $\dot{\tilde{q}}_4$ (in mrad); (s) $\dot{\tilde{q}}_5$ (in mrad); (t) $\dot{\tilde{q}}_6$ (in mrad); (u) $\dot{\tilde{q}}_7$ (in mrad).

$q_5 : (-0.1, 0.0849)$ and $(-0.4, 0.0849)$
$q_6 : (-0.7, 0.0849)$ and $(-1.1, 0.08494)$
$q_7 : (-2, 0.0849)$ and $(-2.3, 0.0849)$
$\dot{q}_1 : (0.14, 0.0849)$ and $(-0.1, 0.0849)$
$\dot{q}_2 : (0.6, 0.0849)$ and $(-0.5, 0.0849)$
$\dot{q}_3 : (0.05, 0.0849)$ and $(-0.06, 0.0849)$
$\dot{q}_4 : (0.3, 0.0849)$ and $(-0.3, 0.0849)$
$\dot{q}_5 : (0.3, 0.0849)$ and $(-0.6, 0.0849)$
$\dot{q}_6 : (0.4, 0.0849)$ and $(-0.6, 0.0849)$
$\dot{q}_7 : (0.6, 0.0849)$ and $(-0.2, 0.0849)$

As per using the suggested RDAFC control strategy in section 5, the joints control actions were graphed in Figure 3.h-n that resulted in the joints position error signals shown in Figure 3.o-u. We can see that the RDAFC strategy is of excellent joint space performance. Figure 2.g-l show the pose error signals and we can notice that the excellent joint space tracking performance had a direct reflection on that of the task space. Furthermore, we graphed the filtered error and the modified filtered error and as shown in Figure 4. Figure 5 shows the estimate of the i^{th} joint approximation error $\hat{w}_i$, stiffness bound k_{ui}, and damping bound d_{ui}. We can see that all signals involved in the design are bounded and excellent tracking performance is achieved despite the time variance of the

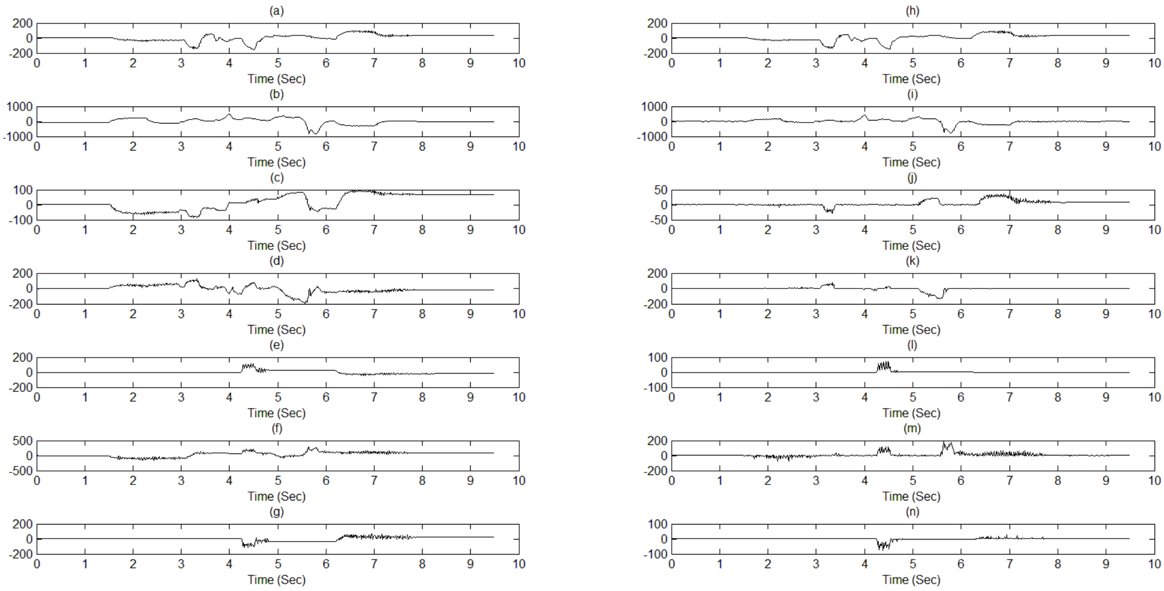


Figure 4: Joints filtered error and modified filtered error: (a)s_1; (b) s_2; (c) s_3; (d) s_4; (e) s_5; (f) s_6; (g) s_7; (h) $s_{\varepsilon 1}$; (i) $s_{\varepsilon 2}$; (j) $s_{\varepsilon 3}$; (k) $s_{\varepsilon 4}$; (l) $s_{\varepsilon 5}$; (m) $s_{\varepsilon 6}$; (n) $s_{\varepsilon 7}$;

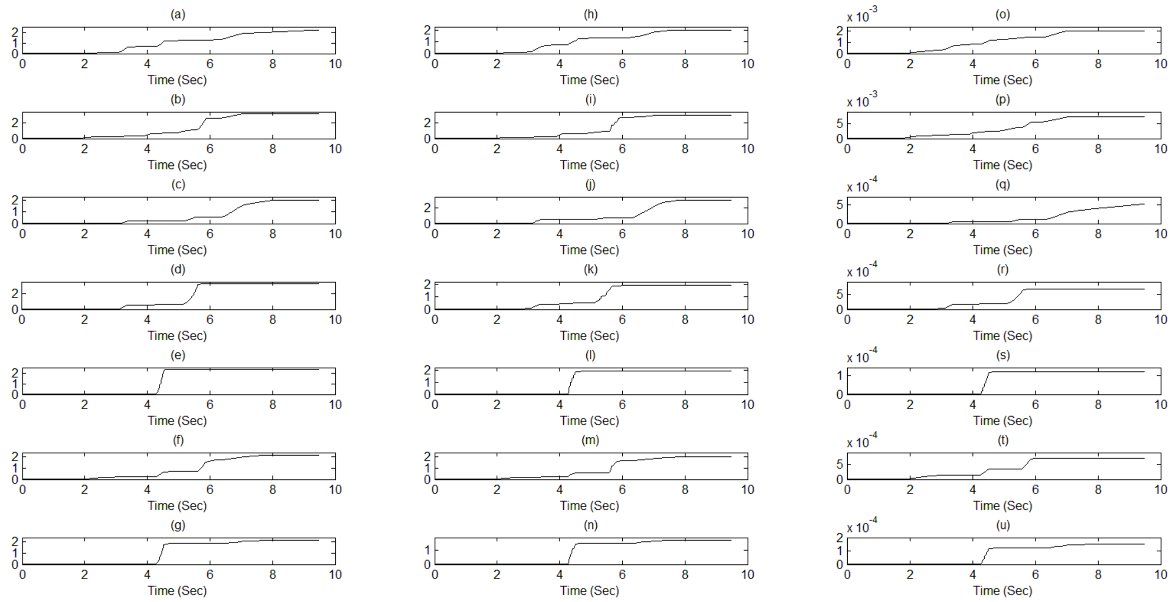


Figure 5: (a)$\hat{w}_1$; (b) $\hat{w}_2$; (c) $\hat{w}_3$; (d) $\hat{w}_4$ (e) $\hat{w}_5$; (f) $\hat{w}_6$; (g) $\hat{w}_7$; (h) $\hat{k}_{u1}$; (i) $\hat{k}_{u2}$; (j) $\hat{k}_{u3}$; (k) $\hat{k}_{u4}$; (l) $\hat{k}_{u5}$; (m) $\hat{k}_{u6}$; (n) $\hat{k}_{u7}$; (o) $\hat{d}_{u1}$; (p) $\hat{d}_{u2}$; (q) $\hat{d}_{u3}$; (r) $\hat{d}_{u4}$; (s) $\hat{d}_{u5}$; (t) $\hat{d}_{u6}$; (u) $\hat{d}_{u7}$.

joints stiffness and damping parameters.

7 Conclusion

The control problem of the Flexible Joint Robots (FJR), with unknown dynamics and unknown time-varying joints stiffness and damping, was addressed and a Robust Direct Adaptive Fuzzy Control (RDAFC) strategy is proposed for such robot systems. RDAFC strategy is a synergy of the concepts of fuzzy logic approximation and the Sliding Mode Control (SMC). The fuzzy approximator relaxes the need for knowing the robot dynamics and the SMC accommodates the variations of the joints elasticity parameters. Then we modify the RDAFC strategy to be suited for the KUKA Lightweight Robot (LWR) with the joints stiffness and damping parameters to be time-variant. The stable performance of the suggested RDAFC strategy is shown and experiment is performed on the KUKA LWR with varying the joints stiffness and damping in sine and cosine waveforms and the efficiency of the suggested approach is shown.

8 Acknowledgement

This work is supported by the Fonds Nationale de Recherche (FNR) in Luxembourg under grant no. AFR

2955286.

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