Faculty of Sciences, Technology and Communication

UNIVERSITÉ DU LUXEMBOURG
Research Unit in Engineering Sciences (RUES)

Reducing the Mesh-burden and Computational Expense in Multi-scale Free Boundary Engineering Problems

Stéphane p. a. ßordas (Spaß), Pierre Kerfriden + team
University of Luxembourg and Cardiff University

AICES Graduate School
RWTH-Aachen University
I3MS Seminar Series
20140512

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UNIVERSITÉ DU LUXEMBOURG

279578 - REALTCUT - Towards real time multiscale simulation of cutting in non-linear materials with applications to surgical simulation and computer guided surgery

UNIVERSITÉ DU LUXEMBOURG
Research Unit in Engineering Sciences (RUES)

CARDIFF UNIVERSITY
PRIFYSGOL CAERDYDD

- A small, young, dynamic university
- 3 languages (English, German, French); bilingual and trilingual degrees
- Strong mathematics and Comp. Sc.
- RUES: 3 professors in computational mechanics, 30 collaborators
- Computational sciences priority 1
- Strong local industry
- Strong and supportive national funding
- 7 EU projects in engineering, of which RealTcut: ERC Starting Grant (Bordas)

- A large, established university (1883)
- 95% 3 or 4* at RAE2008 in Civil
- Over 100 EU projects awarded of which ITN: INSIST
- Mechanics Research: 40 researchers, 14 faculty members
- Advanced manufacturing and characterisation

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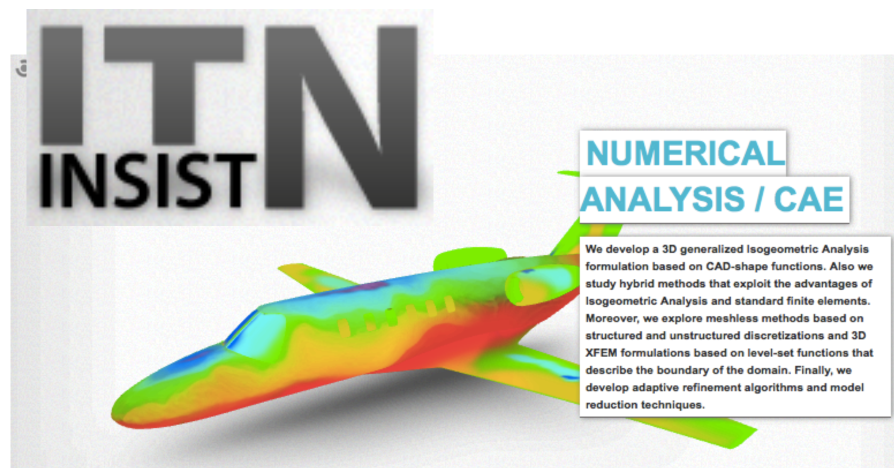


Institute of mechanics and advanced materials

Prof. Stéphane Bordas, Director.
Extended FEM/ Meshless

Prof. Feodor Borodich
Theoretical/Nano mechanics, contact, adhesion

Prof. Pwt Evans
Contact mechanics, tribology

Dr. Paul Howson
Transcendental eigenvalue problems

Prof. Bhushan Karihaloo
Advanced materials, theoretical mechanics

Prof. David Kennedy
Eigenvalue problems, advanced numerical methods

Dr. Pierre Kerfriden
Multiscale, model order reduction, fracture

Dr. Siva Kulasegaram
Meshless methods

Prof. Ray Snidle
Contact mechanics, tribology

Dr. Lars Beex
Multiscale methods

Dr. Hanxing Zhu
Theoretical mechanics, cellular materials

The institute

- 6 professors, 6 lecturers/senior lecturers
- 10 post-doc fellows
- 17 PhD students
- ~ £1.0M funding annually

IMAM
Institute of Mechanics & Advanced Materials

Theoretical & Computational

Tribology & Contact Mechanics

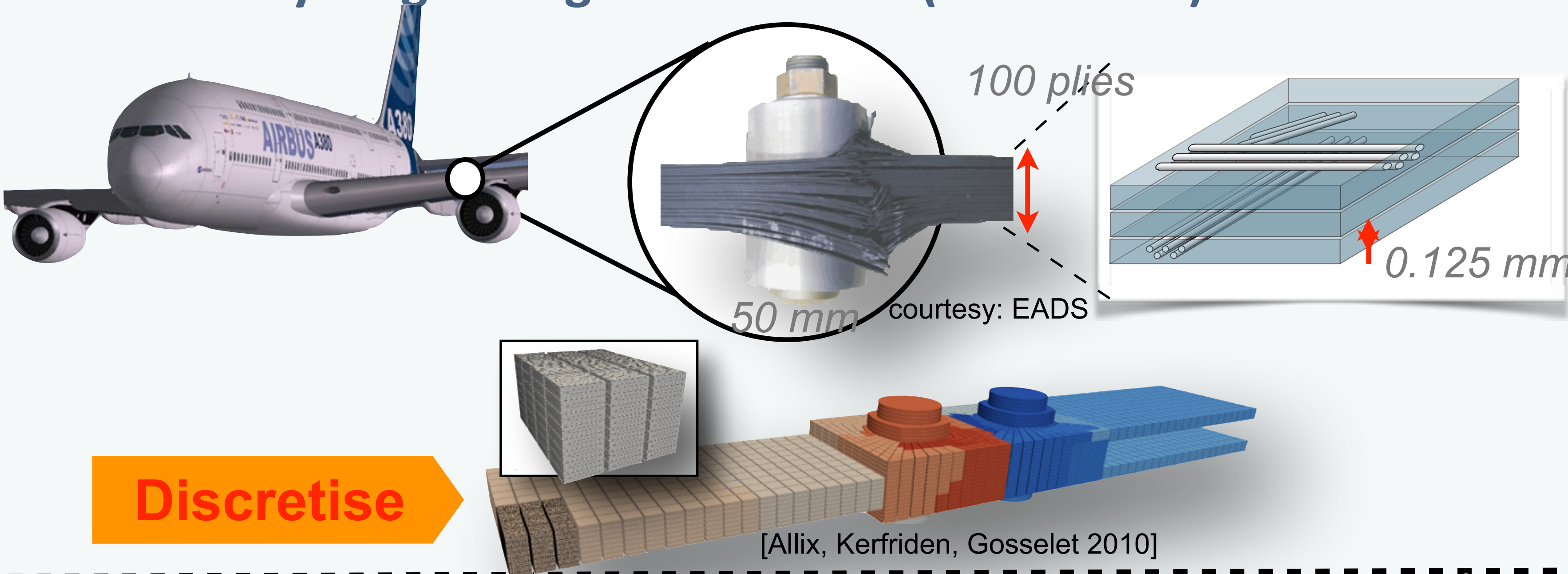
Experiments

Theory

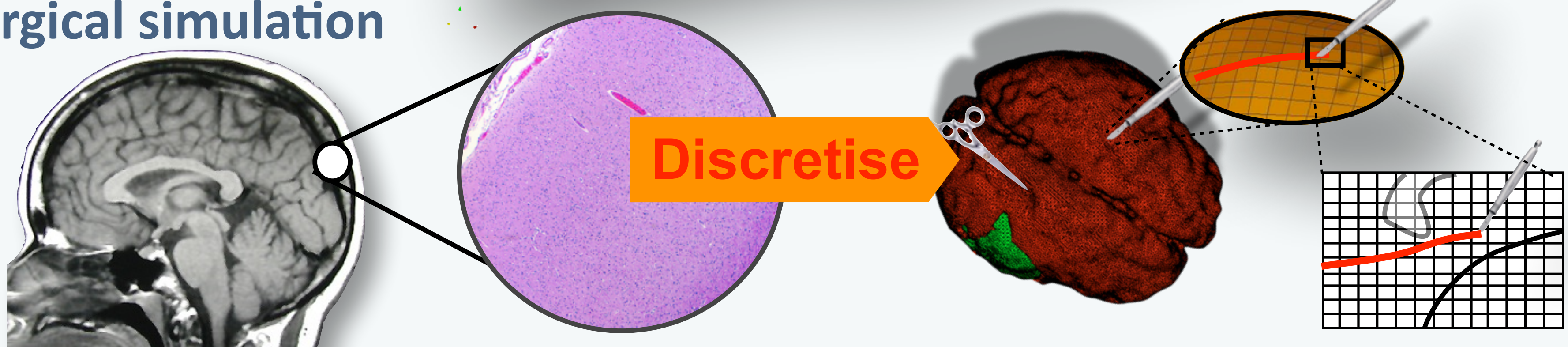
Computational Mechanics

Motivation: multiscale fracture/cutting

Practical early-stage design simulations (interactive)



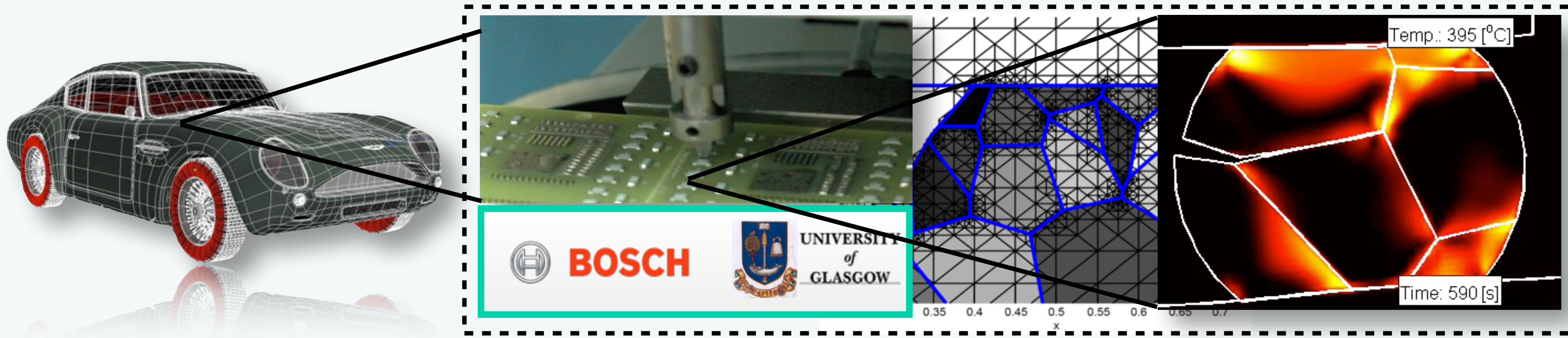
Surgical simulation

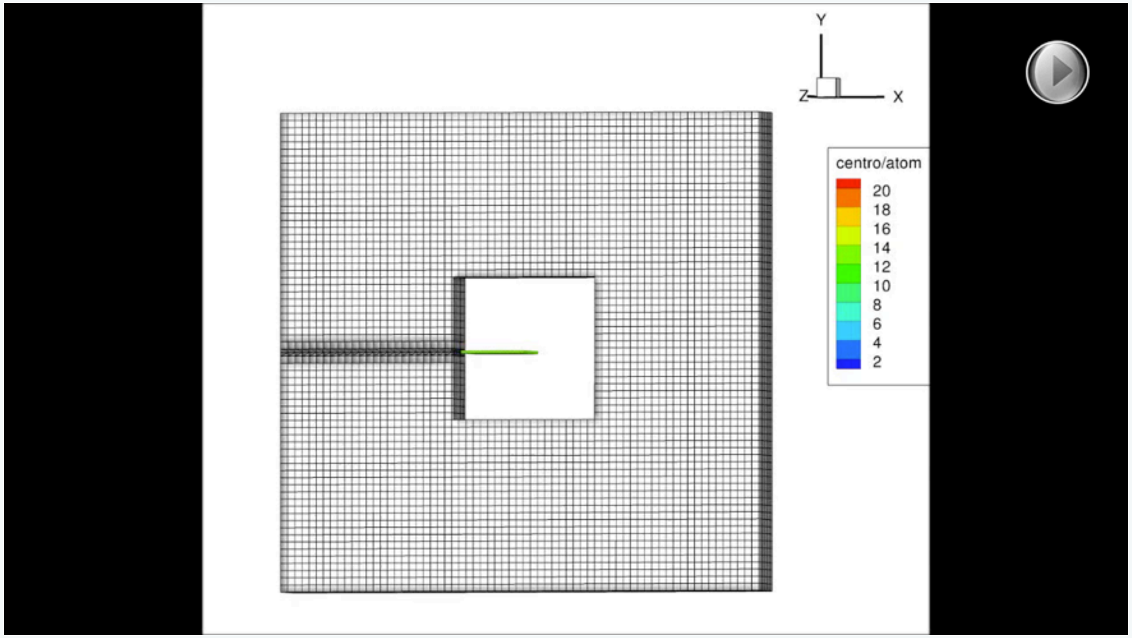


- Reduce the problem size while controlling the error (in QoI) when solving very large (multiscale) mechanics problems

Motivation: multiscale fracture - Example

Solder joint durability (microelectronics), Bosch GmbH



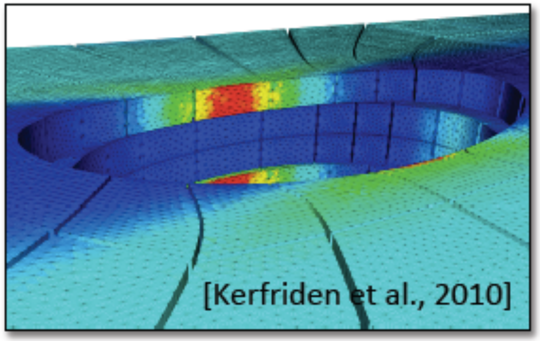


- Efficient numerical prediction of material and structural failure

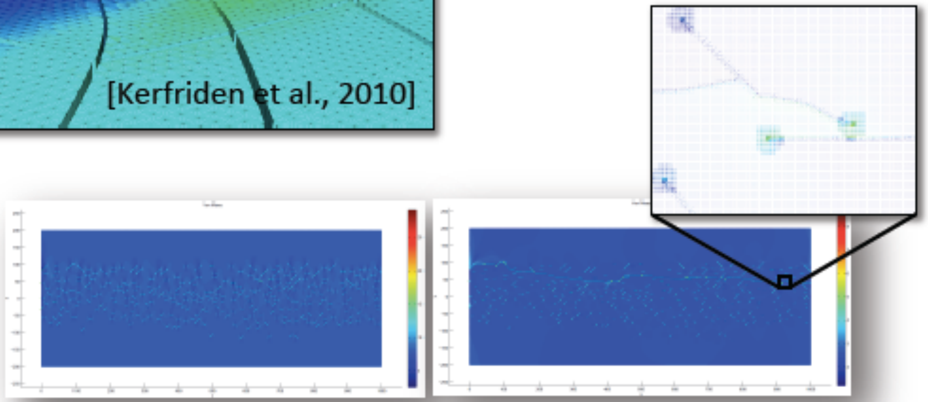
L. Beex

S.P.-A. Bordas

P. Kerfriden



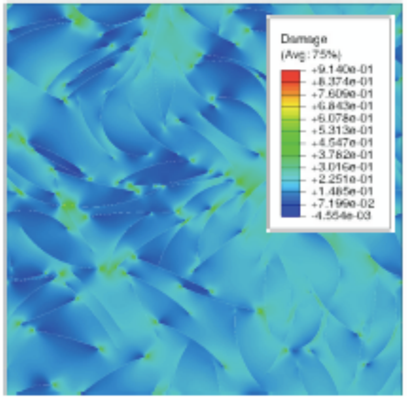
[Kerfriden et al., 2010]



Initial crack distribution

Final fracture

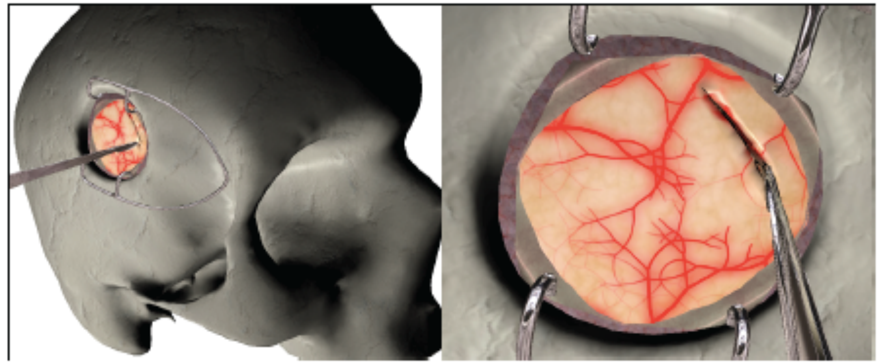
[Sutula et al., 2013]



[Silani et al., 2013]

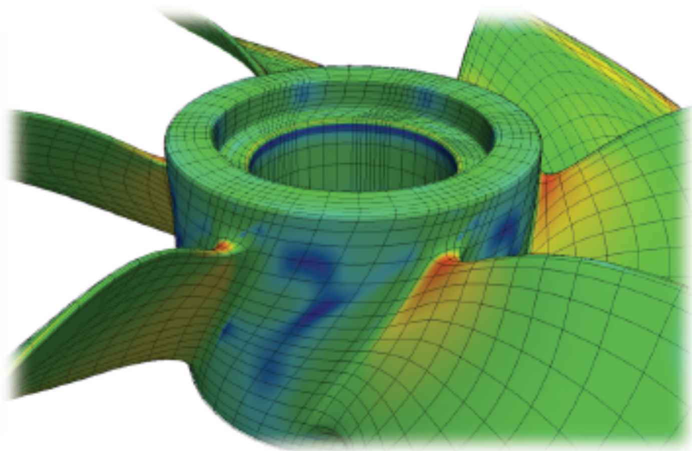
- Characterisation and optimisation of composites

- Interactive simulations of biological structures

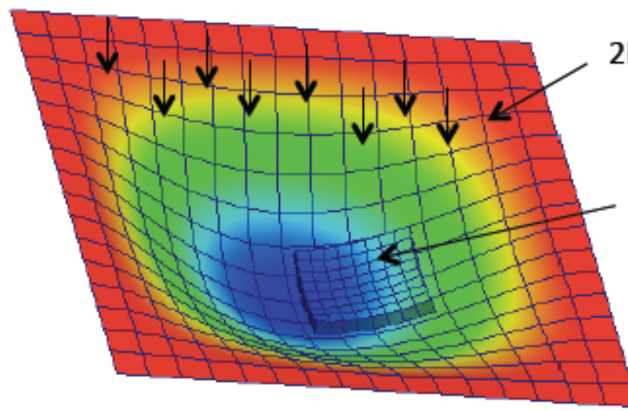


[Courtecuisse et al., 2013]

- Simplified Link between CAD/CT scans and analysis



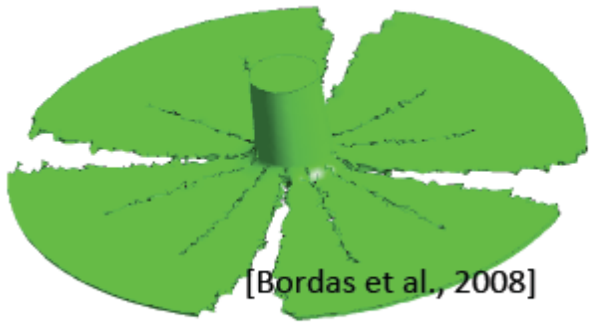
[Scott et al., 2013]



[Nguyen et al., 2013]

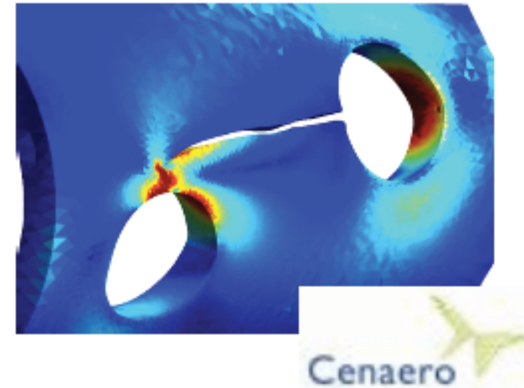
- Advanced discretization techniques for complex PDEs

  - XFEM/meshfree



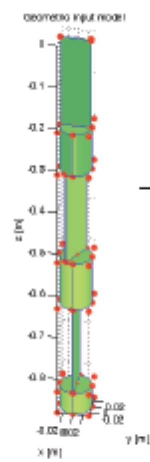
[Bordas et al., 2008]

Taylor bar problem
(dynamic fragmentation)

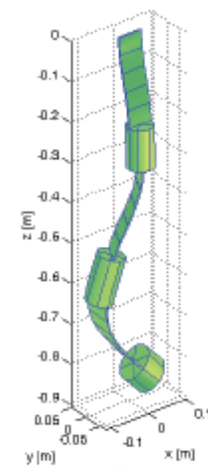


  - Isogemetric analysis

Model simplification (CAD)



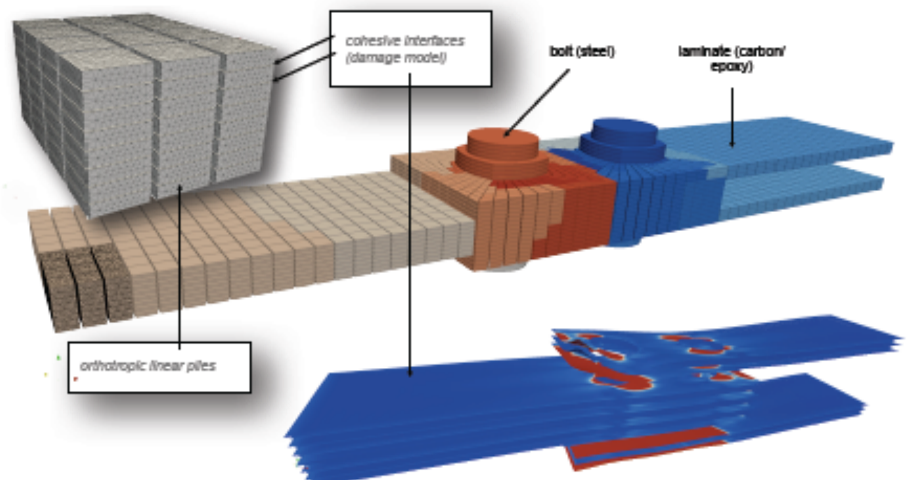
IGA



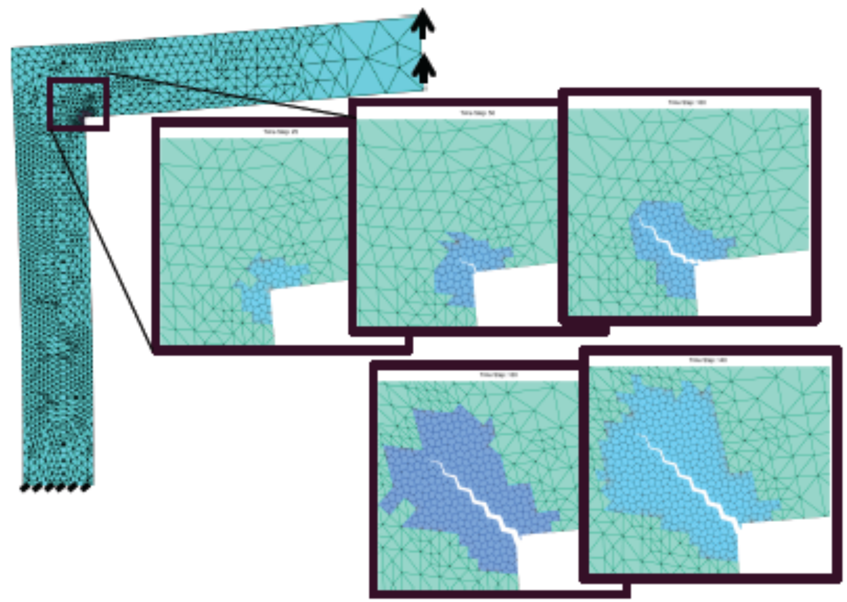
[Tornincasa et al., 2013]

- Multilevel methods to reduce CPU time by orders of magnitude and devise robust, efficient code/model coupling

  - HPC Adaptive multiscale models/solvers with controlled accuracy



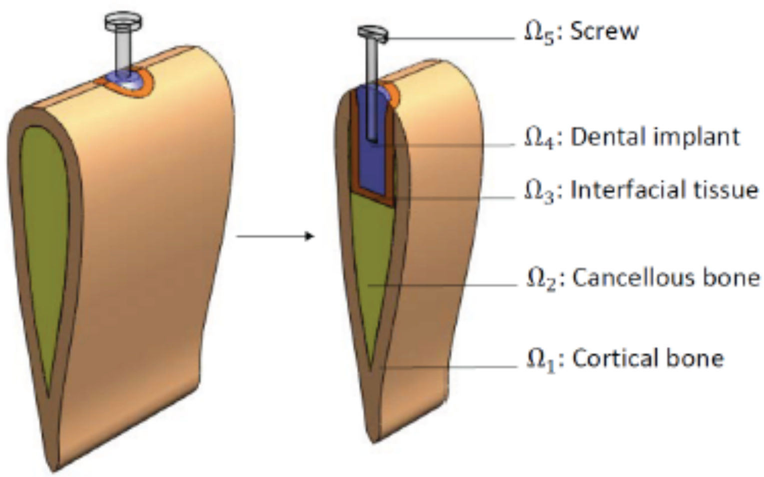
[Kerfriden et al., 2010]



[Akbari et al., 2013]

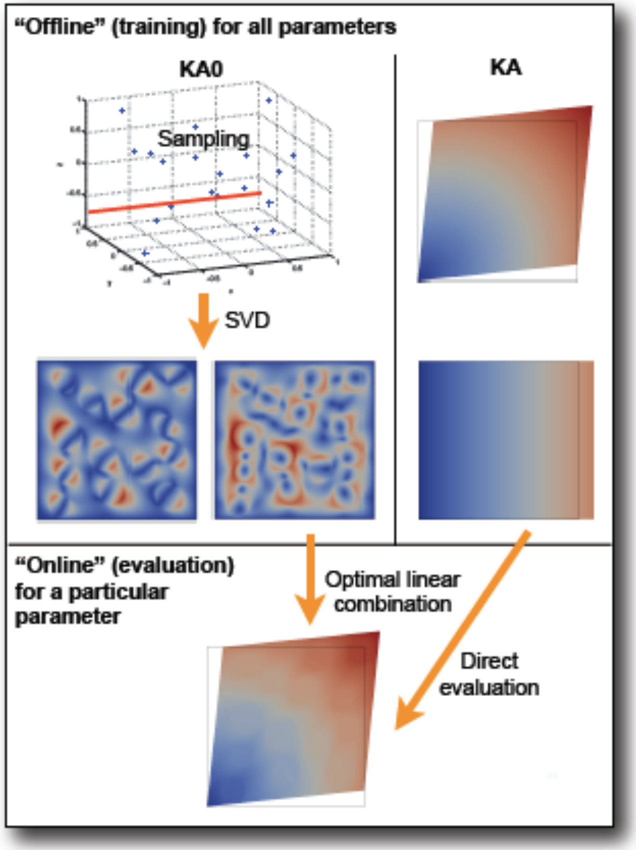
- Multilevel methods to reduce CPU time by orders of magnitude and devise robust, efficient code/model coupling

  - Virtual chart with controlled accuracy via ROM for multiscale modelling and real-time optimisation



[Hoang et al., 2013]

"offline" / "online" strategy



[Kerfriden et al., 2013]

Model reduction

✓ multi-scale & homogenisation
✓ algebraic model reduction (using POD
✓ Newton-Krylov, “local/global”, domai
decomposition

Discretization

➡ partition of unity enrichment
✓ (enriched) meshless methods
✓ level sets

➡ isogeometric analysis
➡ implicit boundaries

Error control

✓ XFEM: goal-oriented error estimates
- used by CENAERO (Morfeo XFEM)

✓ meshless methods for fracture
✓ error estimation for reduced models

Part 0. An adaptive method for fracture - application to polycrystalline failure

Ahmad Akbari, Pierre Kerfriden, Spaß

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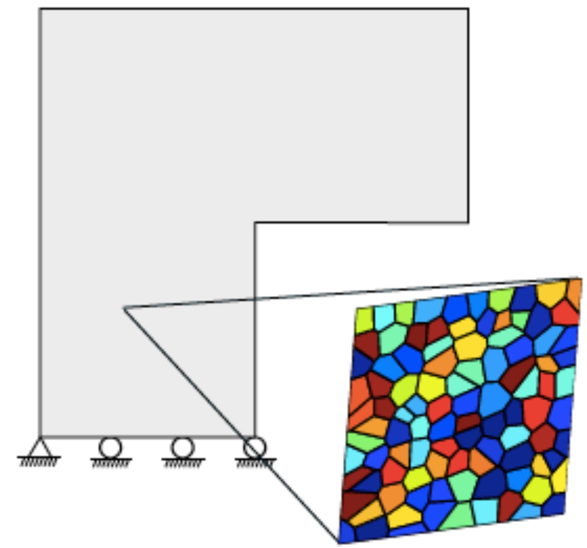
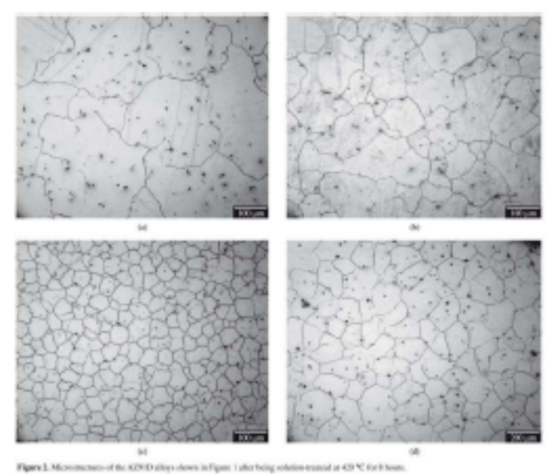
Multiscale modelling

- Bottom-up view: **replace heterogeneous subscale model by an equivalent**, smoother, **model** at the scale where predictions are required (i.e. macroscopic scale)

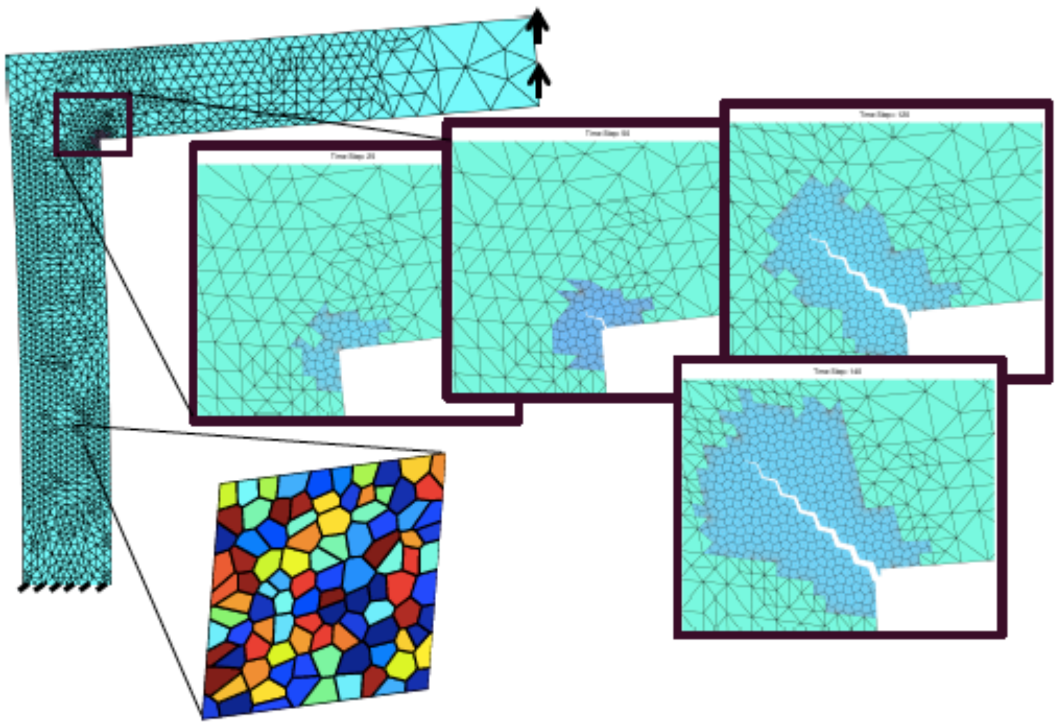
- When is scale-bridging necessary?
  - Derive predictive macroscopic models that are difficult to obtain using phenomenological approaches
  - Optimise subscale properties to obtain better overall characteristics
  - Observations at microscale but approximations required away from region of interest to remain tractable



[Chen et al. 2011]

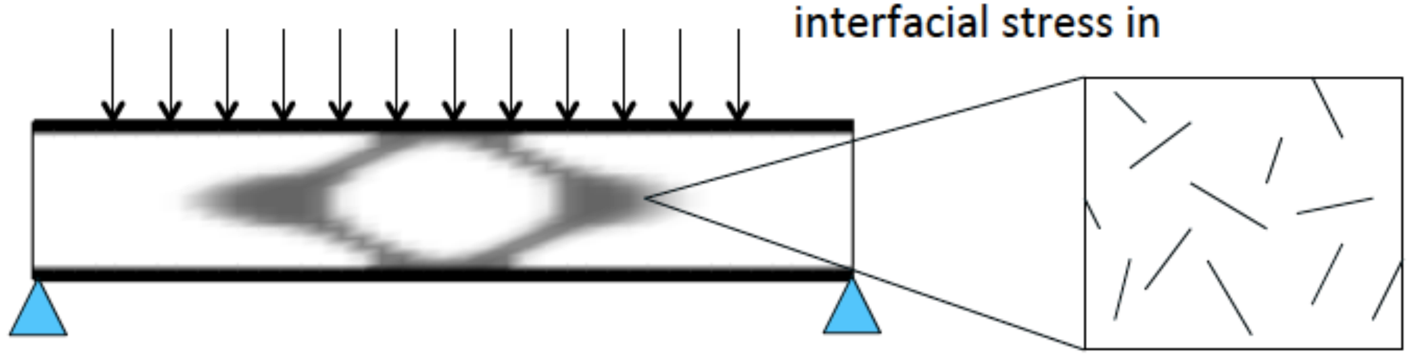


Examples



Approximation of the behaviour of polycrystalline materials away from macroscopic cracks

Optimisation of fiber content in sandwich beams to minimise interfacial stress in

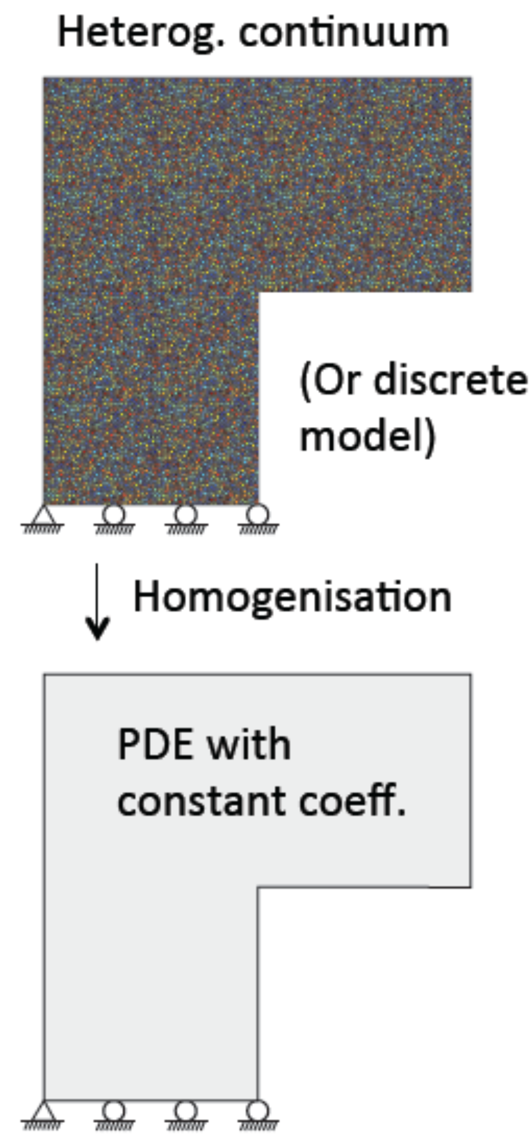


Mathematical formulation

- Knowing the governing equations at the microscale, can we find homogeneous governing equations at the macroscale s. t.:
  - The solution of the macroscale problem converges to the solution of the microscale problem when the scale ratio tends to zero

- Hopefully: the solution of the macroscale problem is a good approximation of the solution of the microscale problem (in some sense) even **when the scale ratio is not very small.**

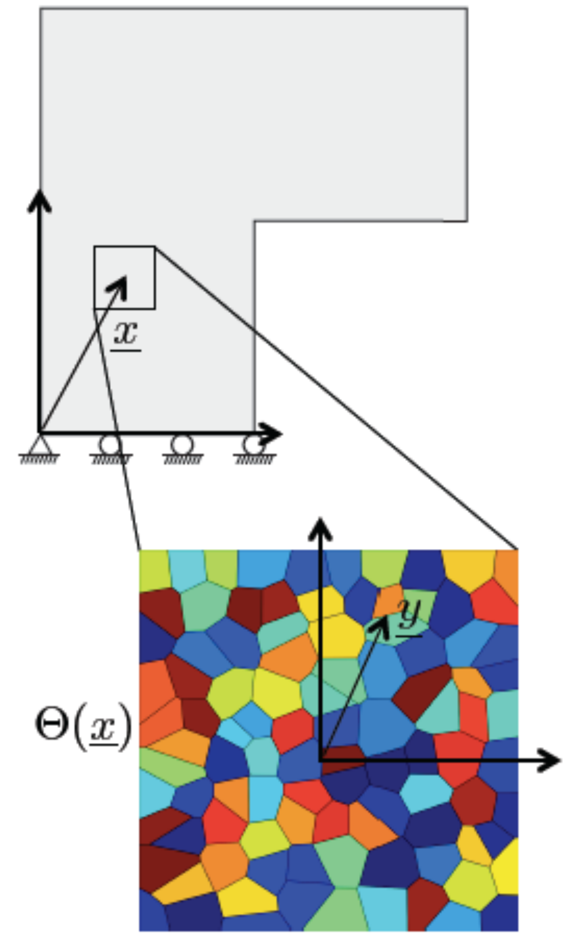
Error in QoI macroscopic < Tolerance
Cost of solving macromodel << subscale model



Classical homogenisation in linearised elasticity

- Heterogeneous microstructure undergoing moderate deformations, observations at macroscale, slow loading, scale separability

- Macroscale candidate model: lin. elasticity
  - Equilibrium
    $$\operatorname{div} \underline{\underline{\sigma}}^{\mathrm{M}} + \underline{f} = \underline{0} \qquad \text{in } \Omega$$
    $$\underline{\underline{\sigma}}^{\mathrm{M}} \cdot \underline{n} = \underline{0} \qquad \text{in } \partial\Omega_{\mathrm{f}}$$
  - Kinematic equations
    $$\underline{u}^{\mathrm{M}} = \underline{U}_{\mathrm{d}} \qquad \text{in } \partial\Omega_{\mathrm{u}}$$
    $$\underline{\underline{\epsilon}}^{\mathrm{M}} = \frac{1}{2}\left(\underline{\underline{\operatorname{grad}}}\,\underline{u}^{\mathrm{M}} + \underline{\underline{\operatorname{grad}}}\,\underline{u}^{\mathrm{M}^T}\right) \qquad \text{in } \Omega$$
  - Constitutive relation by classical micromechanics
    $$\underline{\underline{\sigma}}^{\mathrm{M}} = \mathcal{S}^{\mathrm{M}}\left(\underline{\underline{\epsilon}}(\underline{u}^{\mathrm{M}})\right) \qquad \text{in } \Omega$$



Classical homogenisation in linearised elasticity

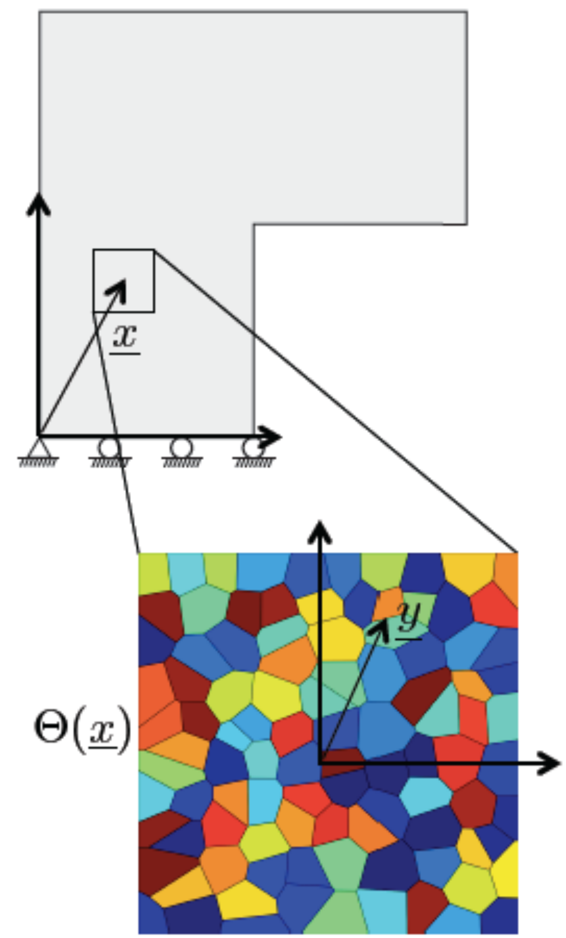
- Attach a representative volume element to the material point: volume of material large enough to represent the statistics of the distribution of material properties (unit cell in periodic case)

  $\underline{\underline{\sigma}}^{\mathrm{m}} = \underset{\sim}{D}(\underline{y}) : \underline{\underline{\epsilon}}^{\mathrm{m}}$

- Suppose that the RVE is mechanically equilibrated: $\underline{\mathrm{div}}\,\underline{\underline{\sigma}}^{\mathrm{m}} = \underline{0}$

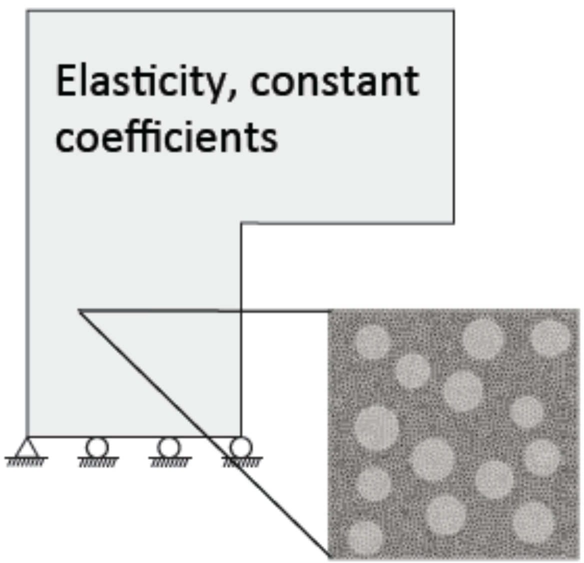
- The effective constitutive law the/a relationship between average stress and average strain

  $< \underline{\underline{\sigma}}^{\mathrm{m}} >= \mathcal{S}^{\mathrm{M}}\left(< \underline{\underline{\epsilon}}^{\mathrm{m}} >\right)$



Computational homogenisation

- Obtain $<\underline{\underline{\sigma}}^{\mathrm{m}}>=\mathcal{S}^{\mathrm{M}}\left(<\underline{\underline{\epsilon}}^{\mathrm{m}}>\right)$ by solving RVE problem numerically



Homogeneous strain (macroscopic part)

"micro" fluctuation

$$\bar{\underline{u}}(\underline{y}) = \underline{\underline{\epsilon}}^{\mathrm{M}} \cdot \underline{y} + \tilde{\underline{u}}(\underline{y})$$

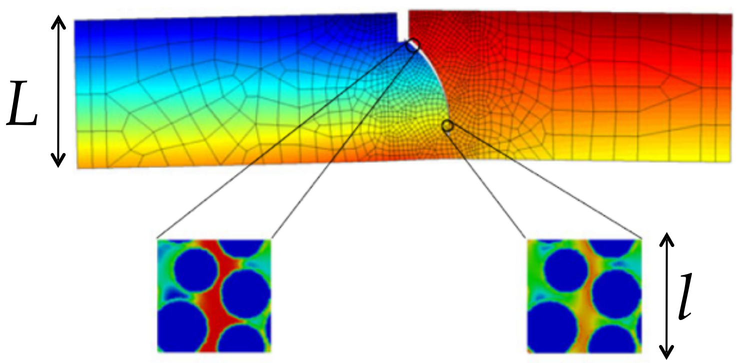
- Ill-posed, requires BC for fluctuation compatible with $<\underline{\underline{\epsilon}}(\widetilde{\underline{u}}(\underline{y}))>$
- One possibility: Dirichlet problem, fluctuation vanishes on boundary

$\rightarrow$ Very expensive too solve

Multiscale methods for Fracture

- Non-concurrent

Damage zone is modelled by a macroscopic cohesive crack that homogenises the failure zone.

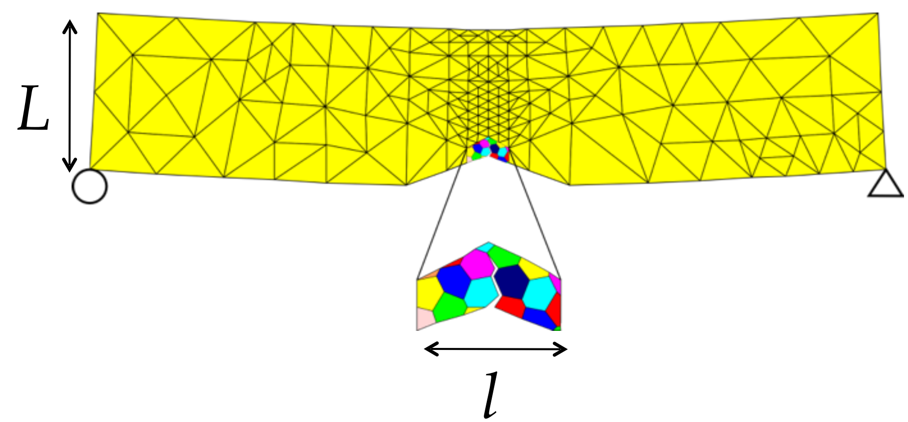


V.P. Nguyen 2012

$L/l >> 1$

- Concurrent

Damage zone is modelled directly at the microscale and coupled to the coarse scale.



$L/l > 1$

Ways to reduce the fracture models

- Homogenisation (FE^2, etc.) - Hierarchical
- Concurrent (bridging domain, ARLEQUIN, etc.)
- Enrichment (PUFEM, XFEM, GFEM)
- Model reduction

Fine Scale: micro-structure

➢Microscale problem:

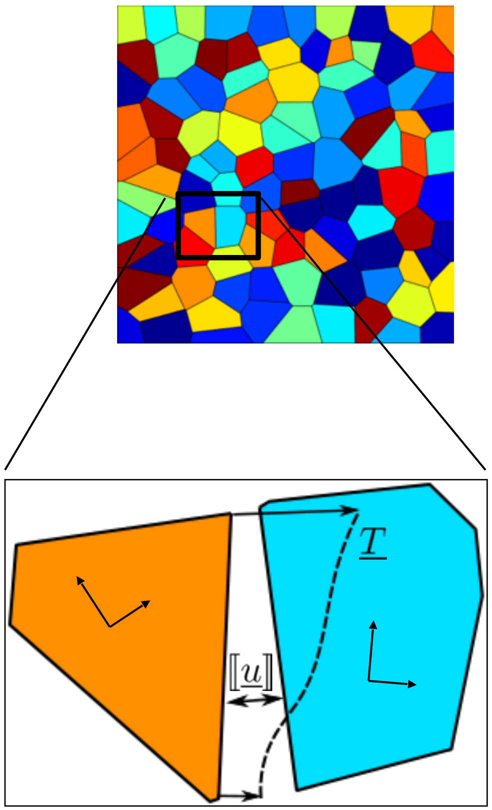
$$\int_{\Omega/\Gamma_c} \boldsymbol{\sigma}(\mathbf{u}) : \delta\boldsymbol{\varepsilon} \, \mathrm{d}\Omega + \int_{\Gamma_c} \mathbf{T} \cdot [\![\delta \mathbf{u}]\!] \mathrm{d}\Omega = \int_{\partial\Omega} \mathbf{f} \cdot \delta \mathbf{u} \mathrm{d}\Gamma$$

- Orthotropic grains

$$\forall \mathbf{x} \in \Omega/\Gamma_c, \quad \boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon}$$

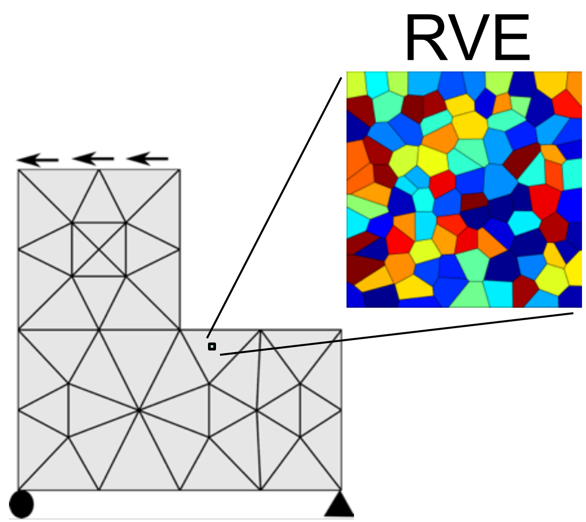
- Cohesive interface

$$\forall \mathbf{x} \in \Gamma_c, \quad \mathbf{T}_{|t} = T\left(\left([\![\mathbf{u}]\!]_{|\tau} \right)_{\tau \leq t} \right)$$



Coarse Scale

➢Macroscale problem:

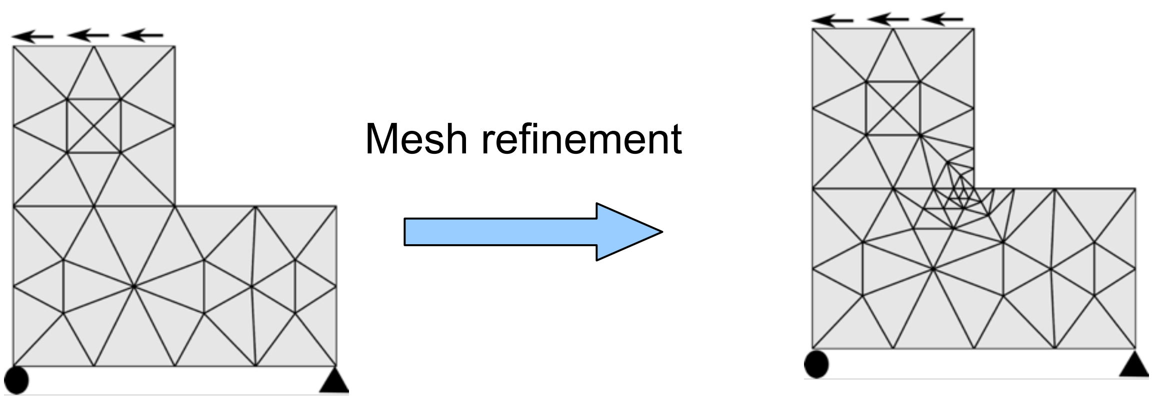


- FE^2 Method

Based on averaging theorem (computational homogenisation)

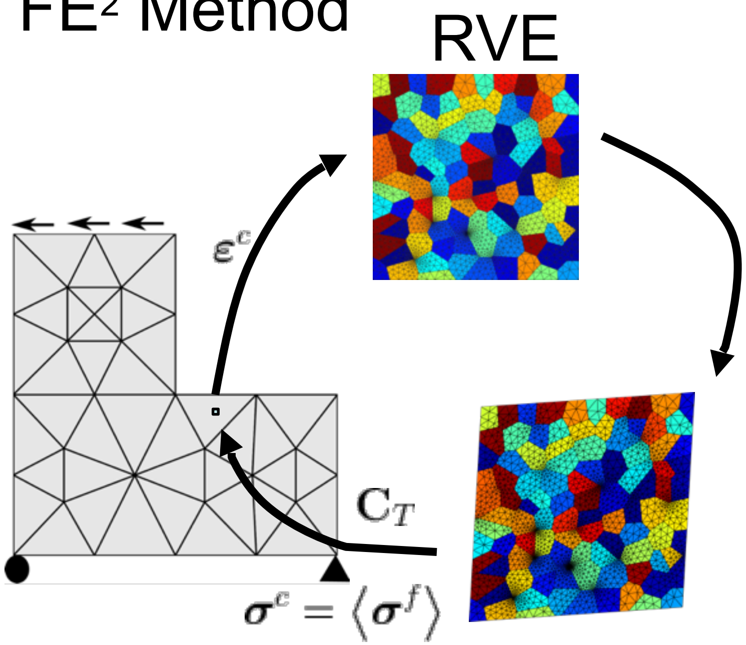
- Adaptive mesh refinement

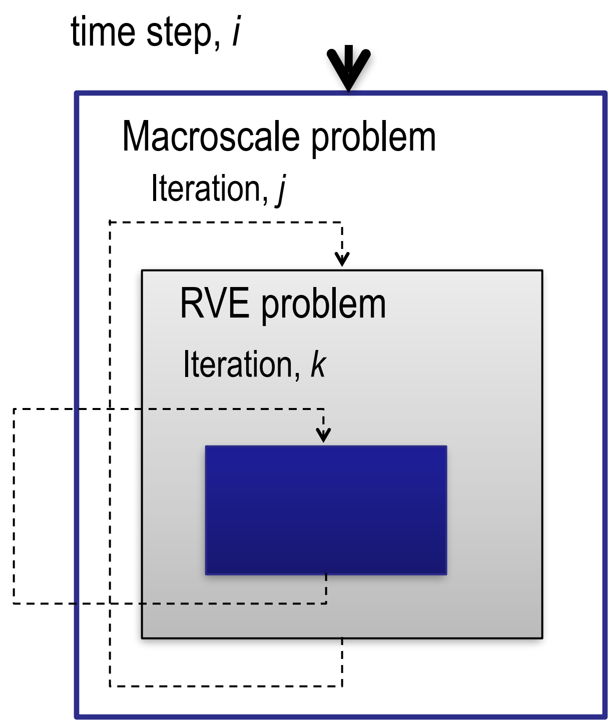
Error estimation by Zienkiewicz-Zhu-type recovery technique



Coarse Scale: FE2

- The FE² Method

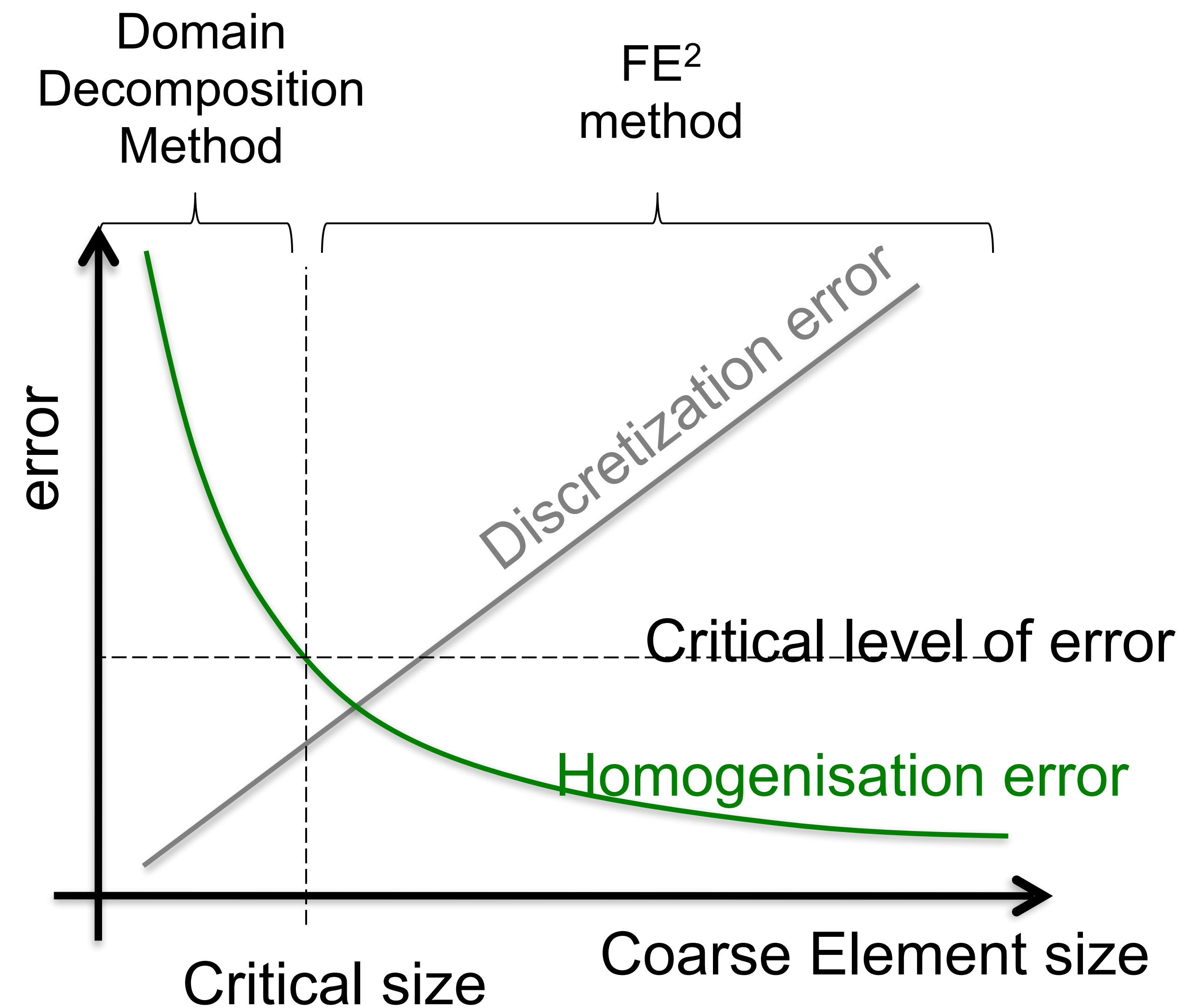




❖ Shortcoming of the FE² Method :

<u>Lack of scale separation</u>
RVE cannot be found in the **softening regime**

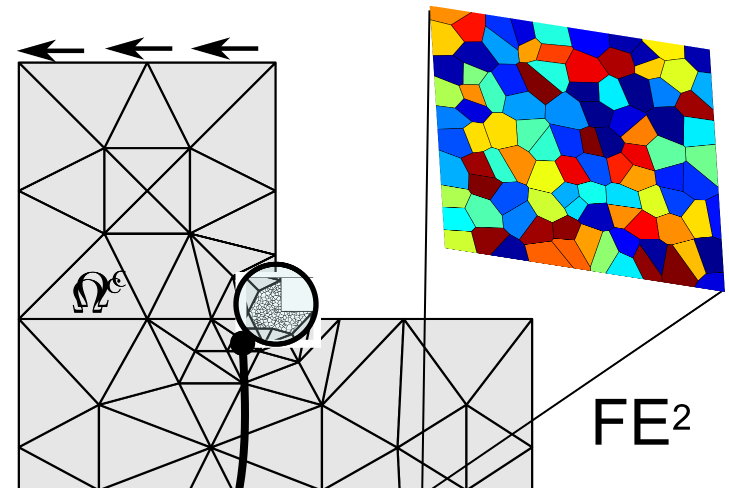
Error control in multiscale modelling

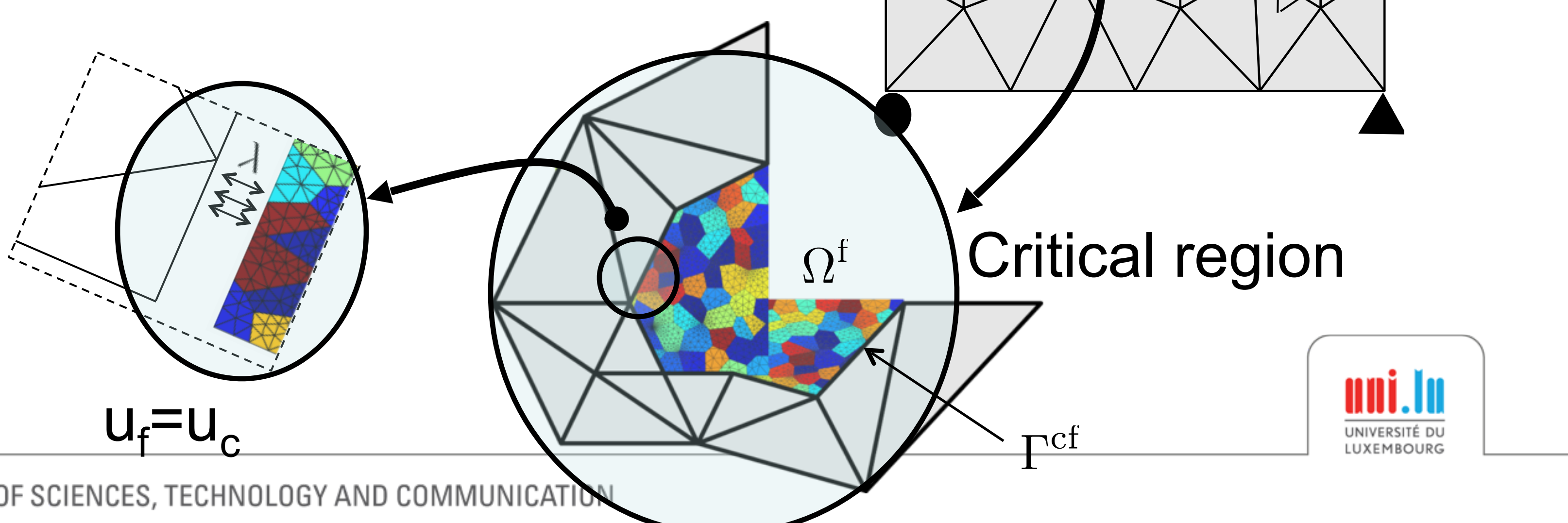


Fine-Coarse scales Coupling

Solution beyond FE^2 :
"Hybrid Multiscale Method"

- FE^2 for non-critical region (hierarchical multiscale)
- Domain decomposition for critical region (concurrent multiscale)

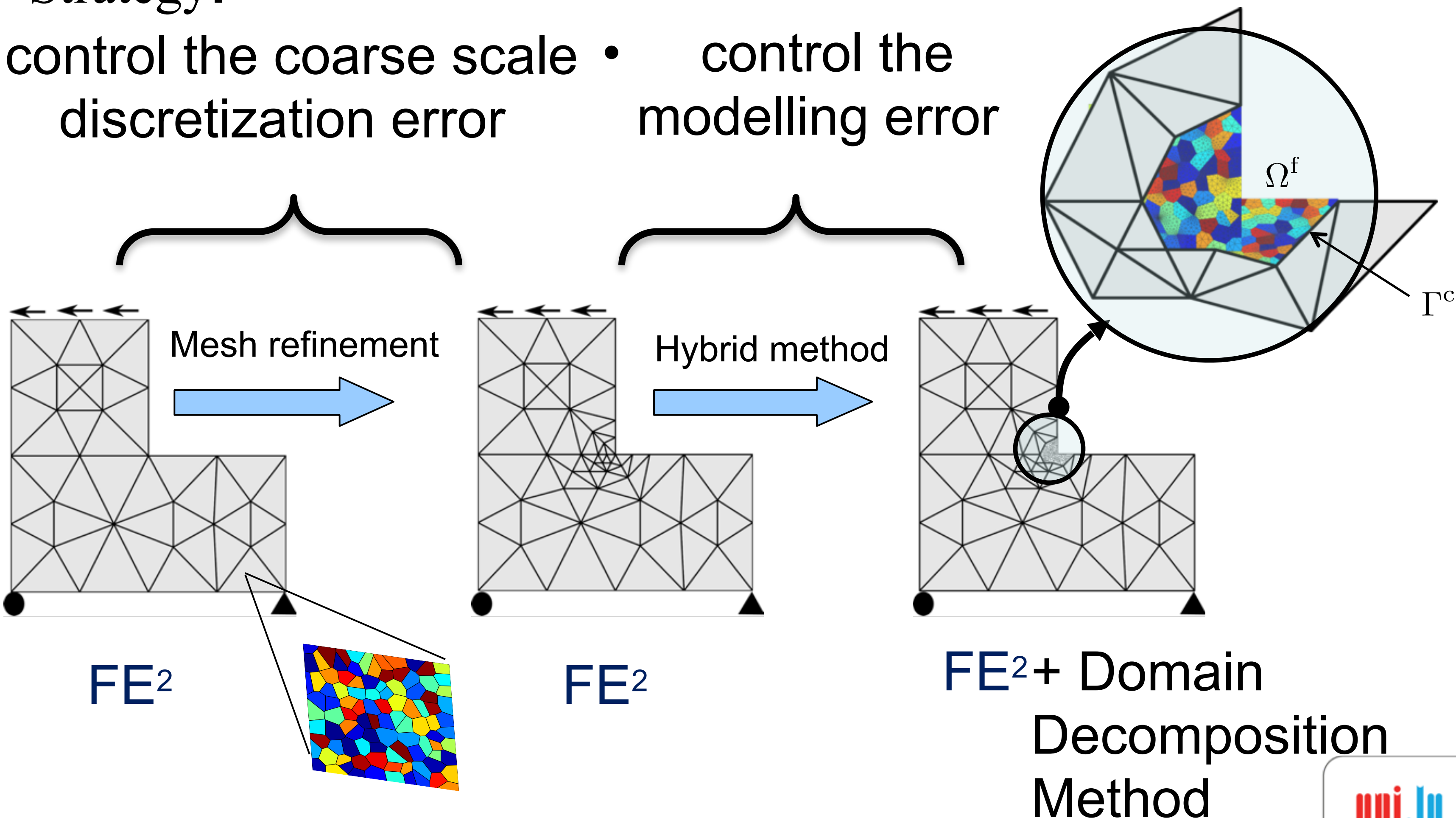




Adaptive multiscale method: A Concurrent approach

➢Strategy:

- control the coarse scale discretization error
- control the modelling error



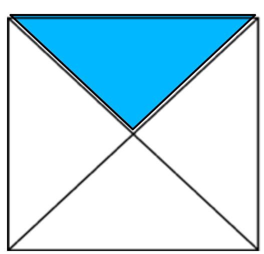
Coarse Scale: Adaptive mesh refinement

➢Coarse scale Adaptive mesh refinement

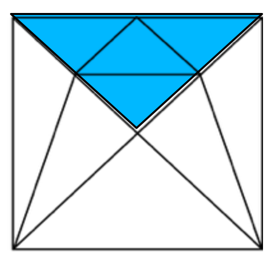
- Error estimation by Zienkiewicz-Zhu-type recovery technique

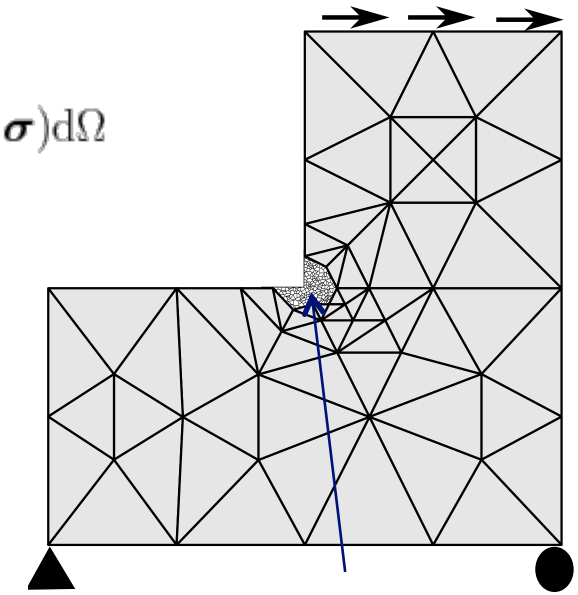
$$\|e\| = \int_{\Omega_c} (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}) : \left(\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}}\bigg|_{\mathbf{u}^c} \right)^{-1} : (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}) \mathrm{d}\Omega$$

Element to refine



Refined mesh

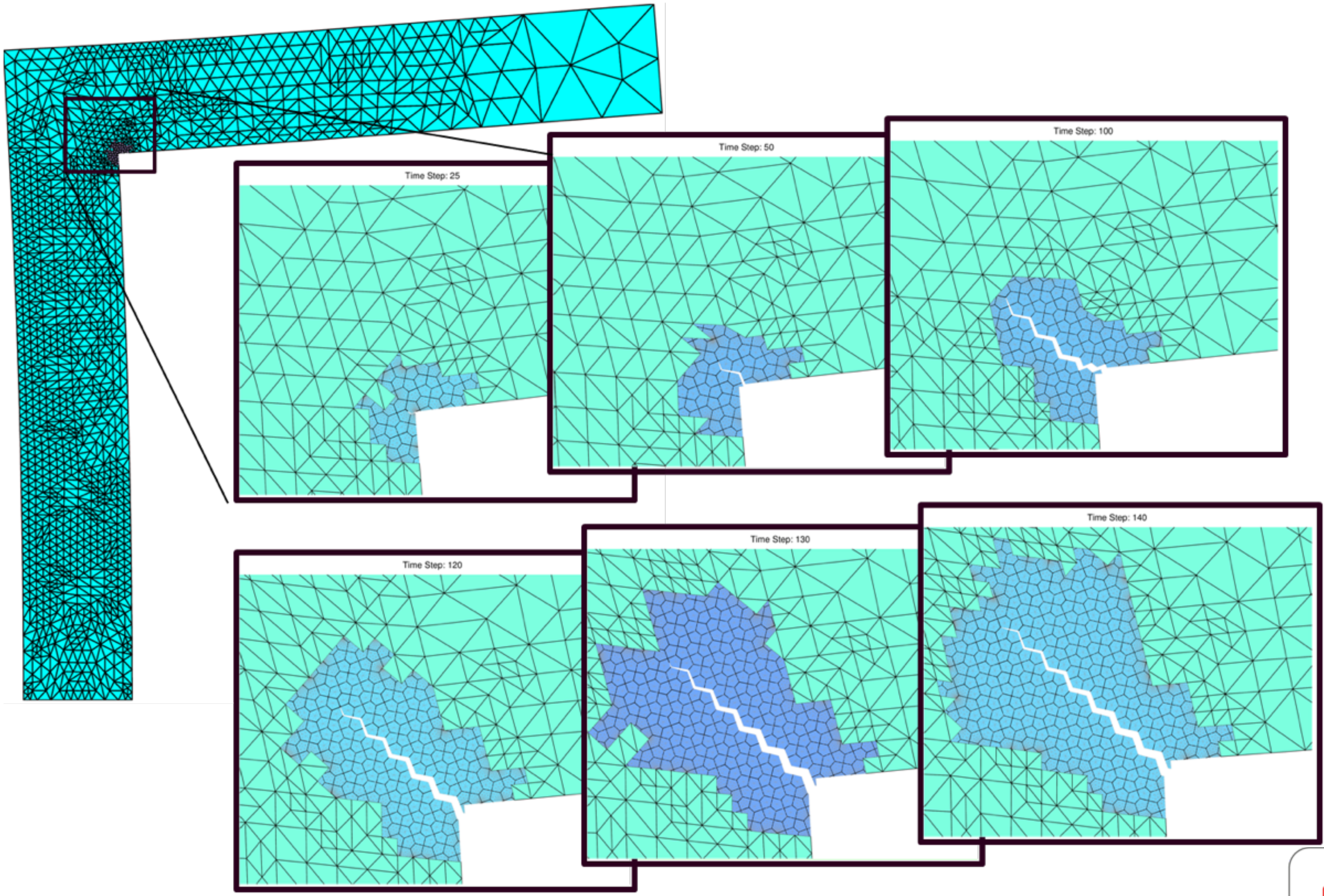




Error due to the discretisation of Ω^{f} neglected

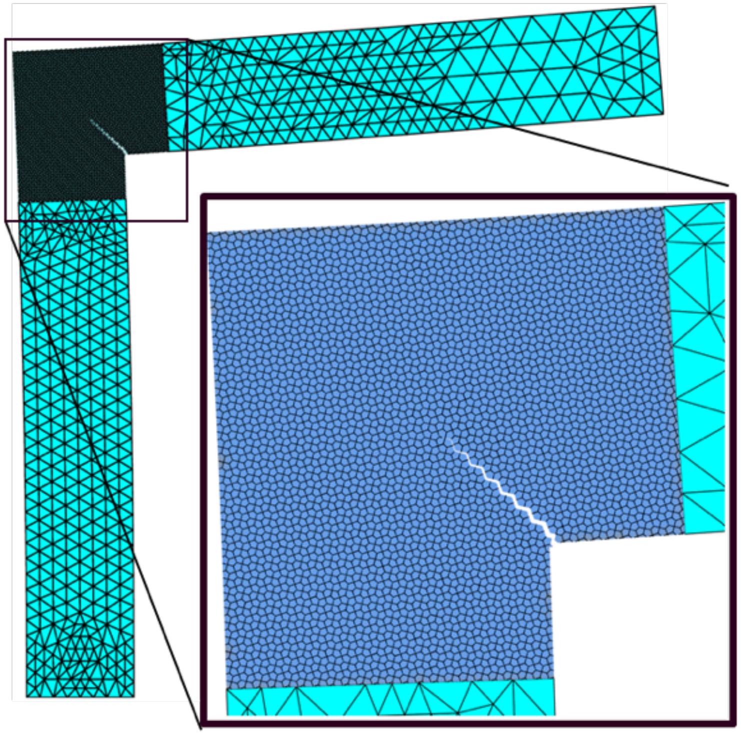
•Convergence criterion: $\dfrac{\|e\|}{\|\sigma\|} < Tol$

Results: L-shape

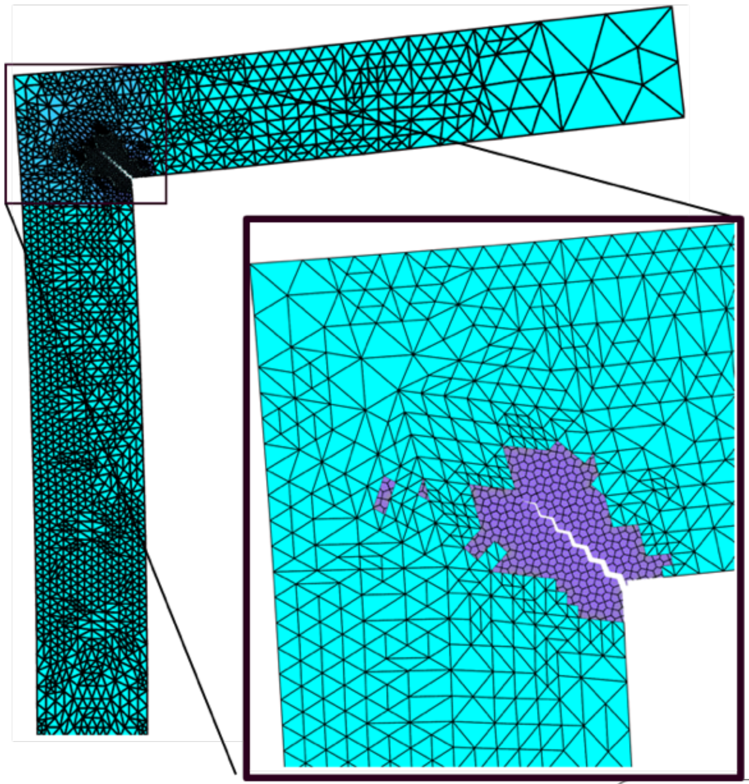


Results: L-shape

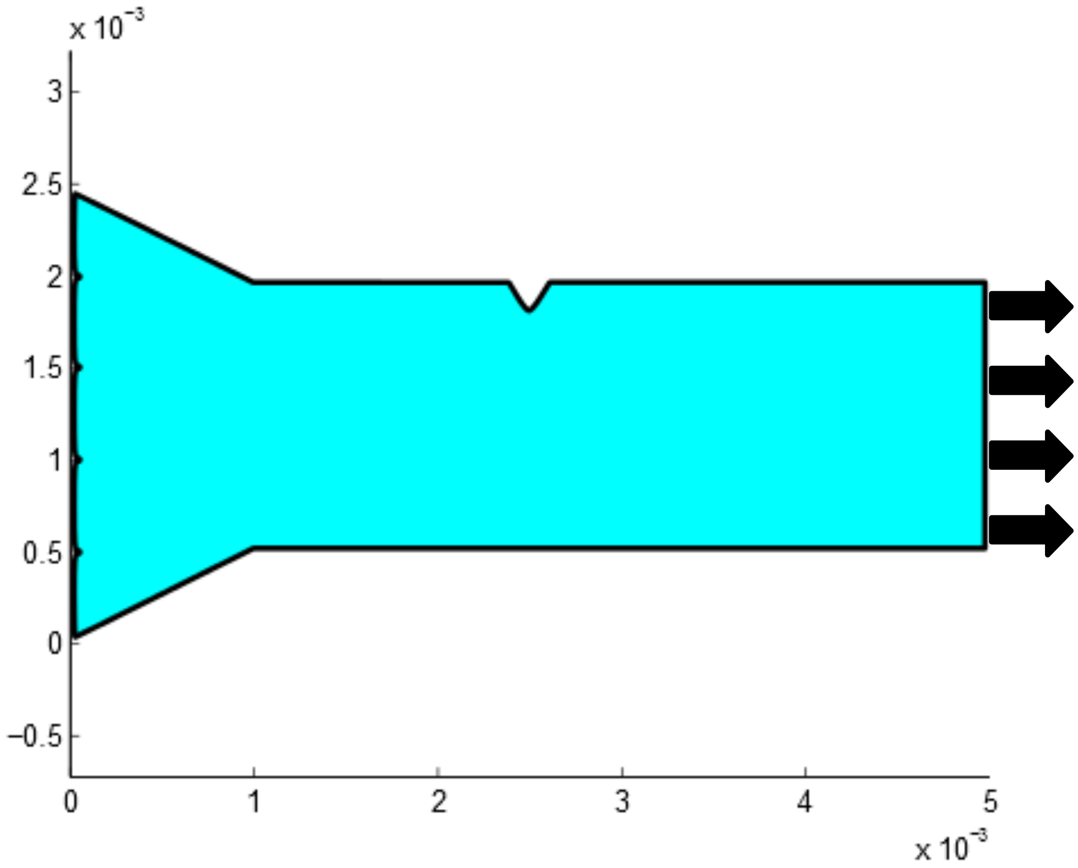
Direct Numerical Solution



Adaptive Multiscale method



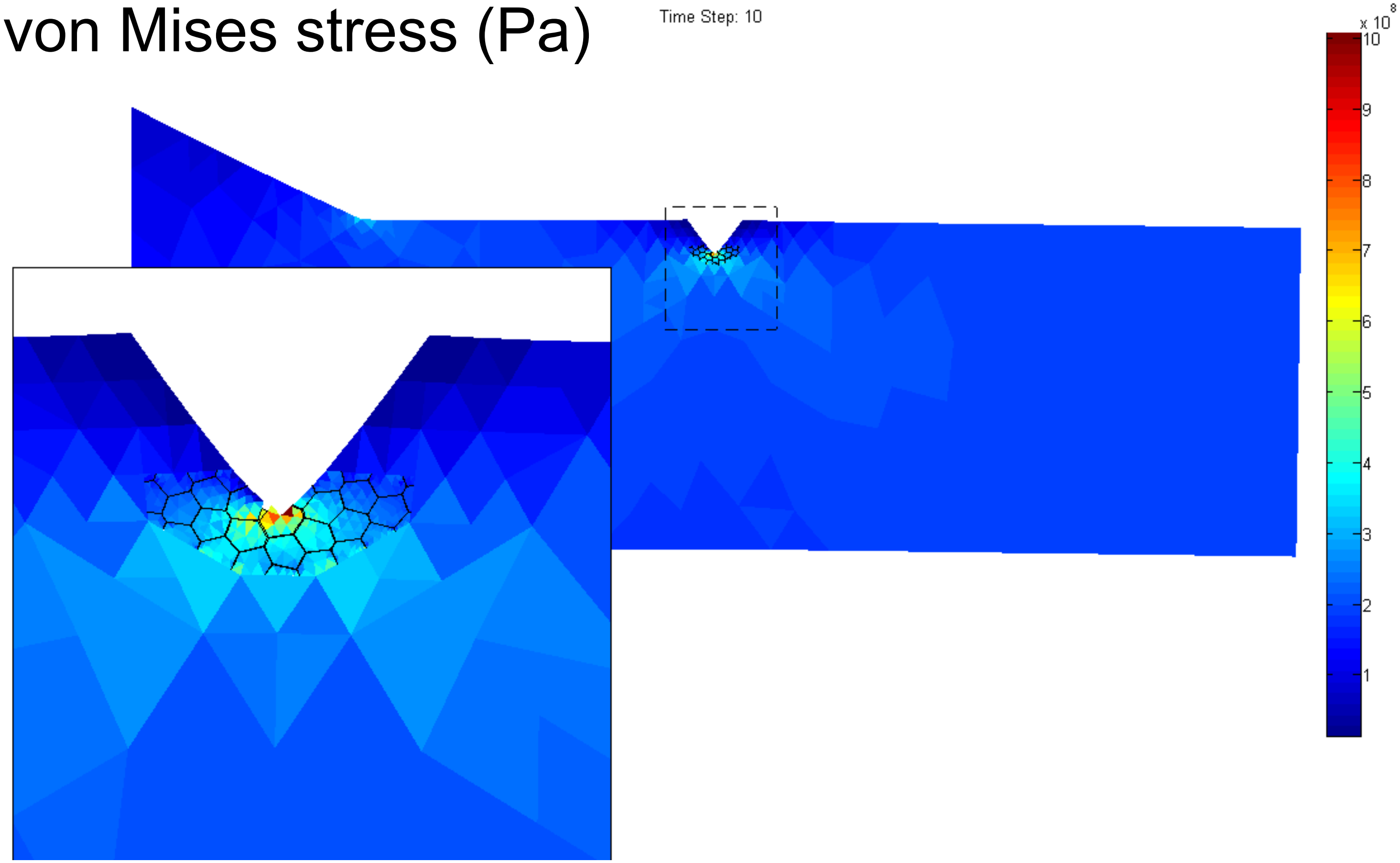
Results: uni-axial tension



❖ Sizes are in mm

Results: uni-axial tension

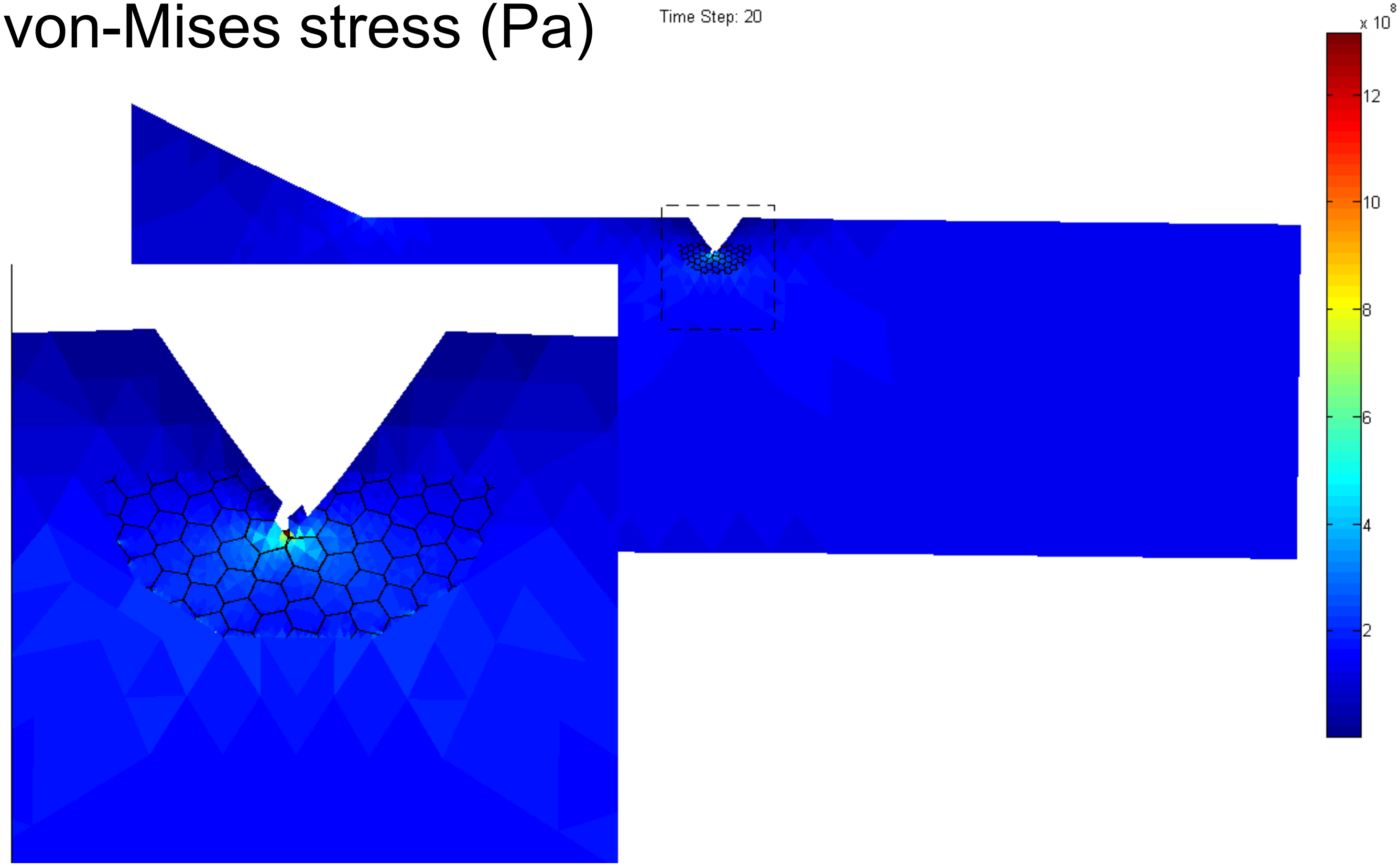
von Mises stress (Pa)



❖ 100X (magnification of displacement)

Results: uni-axial tension

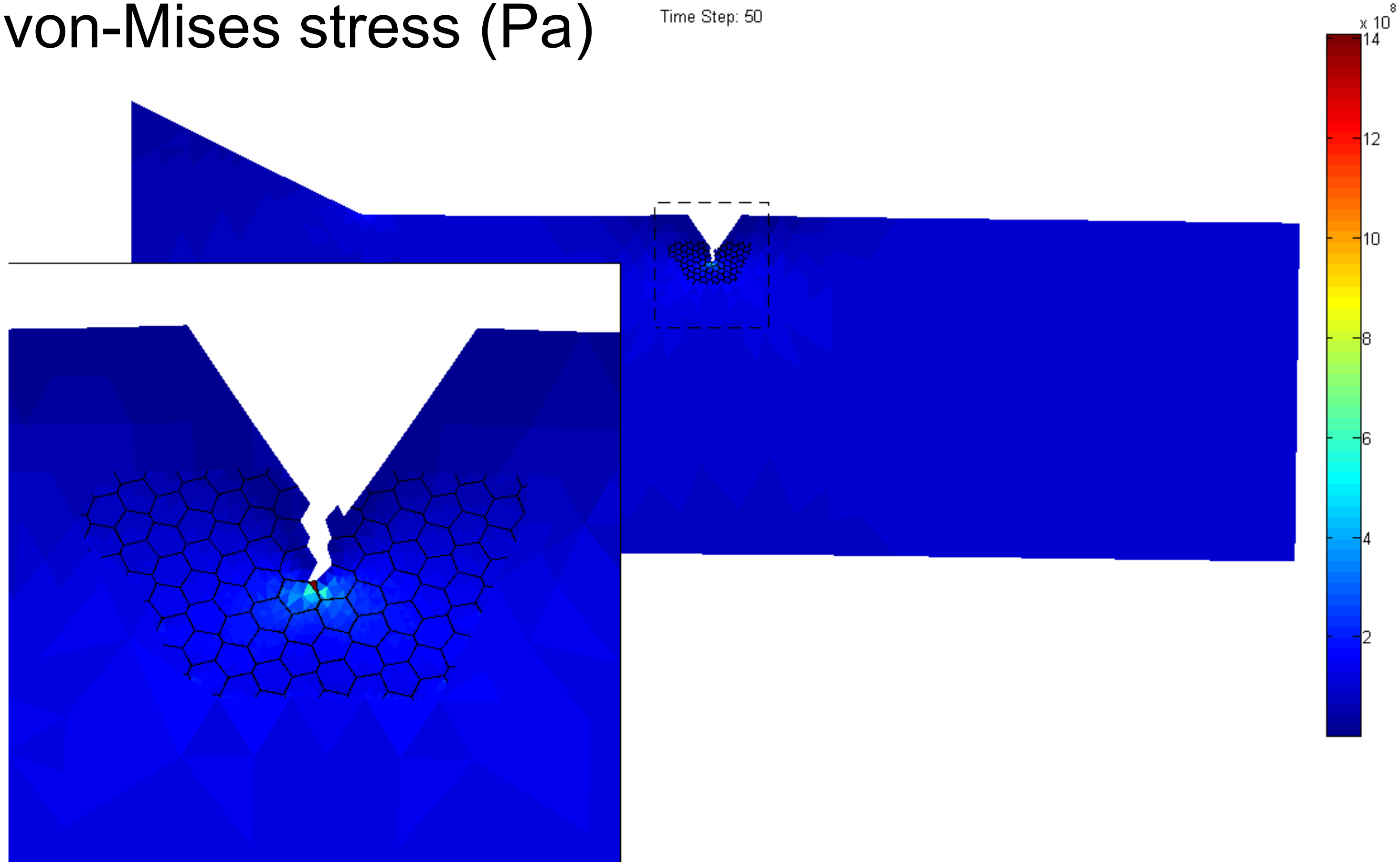
von-Mises stress (Pa)



❖ 100X (magnification of displacement)

Results: uni-axial tension

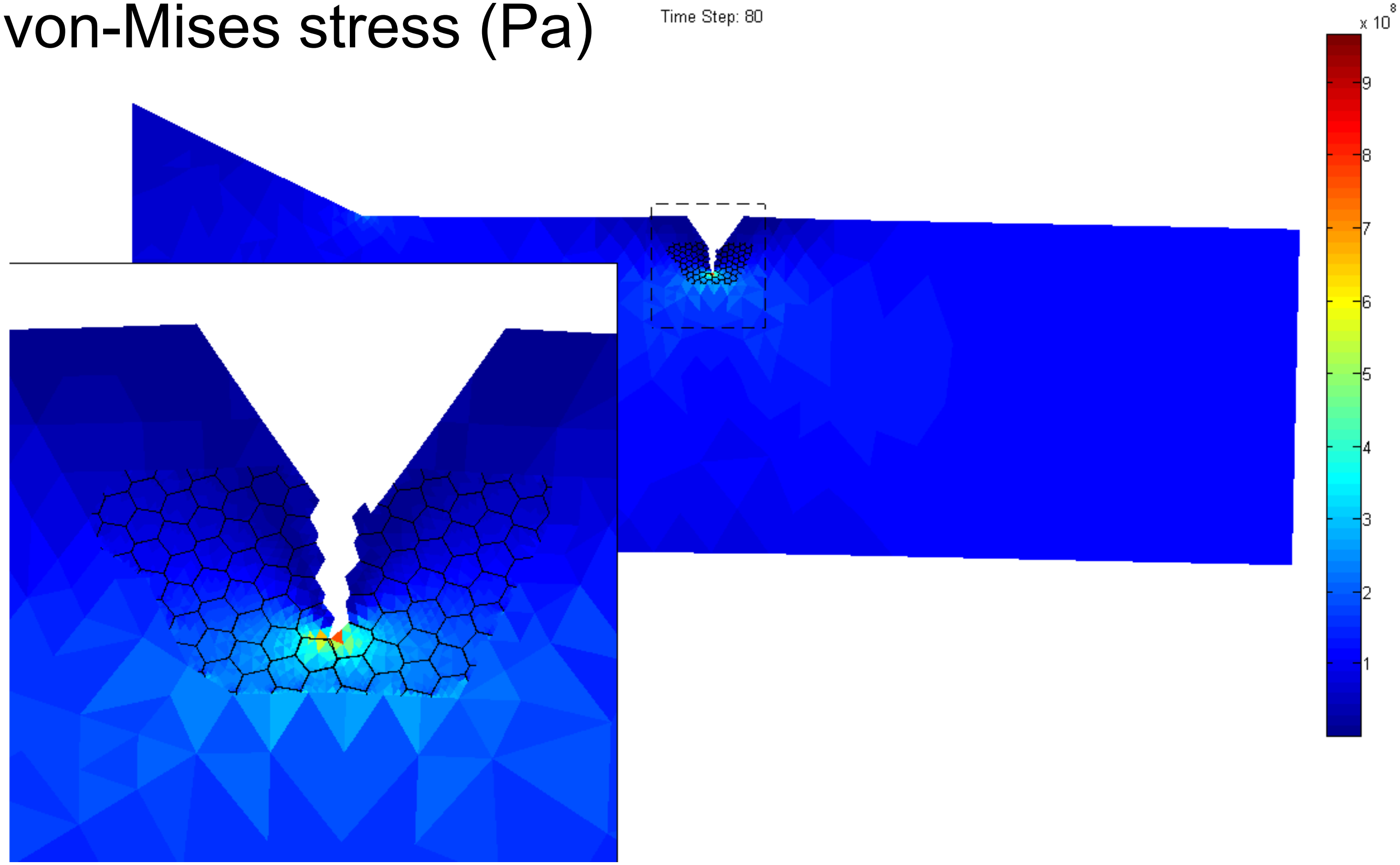
von-Mises stress (Pa)



❖ 100X (magnification of displacement)

Results: uni-axial tension

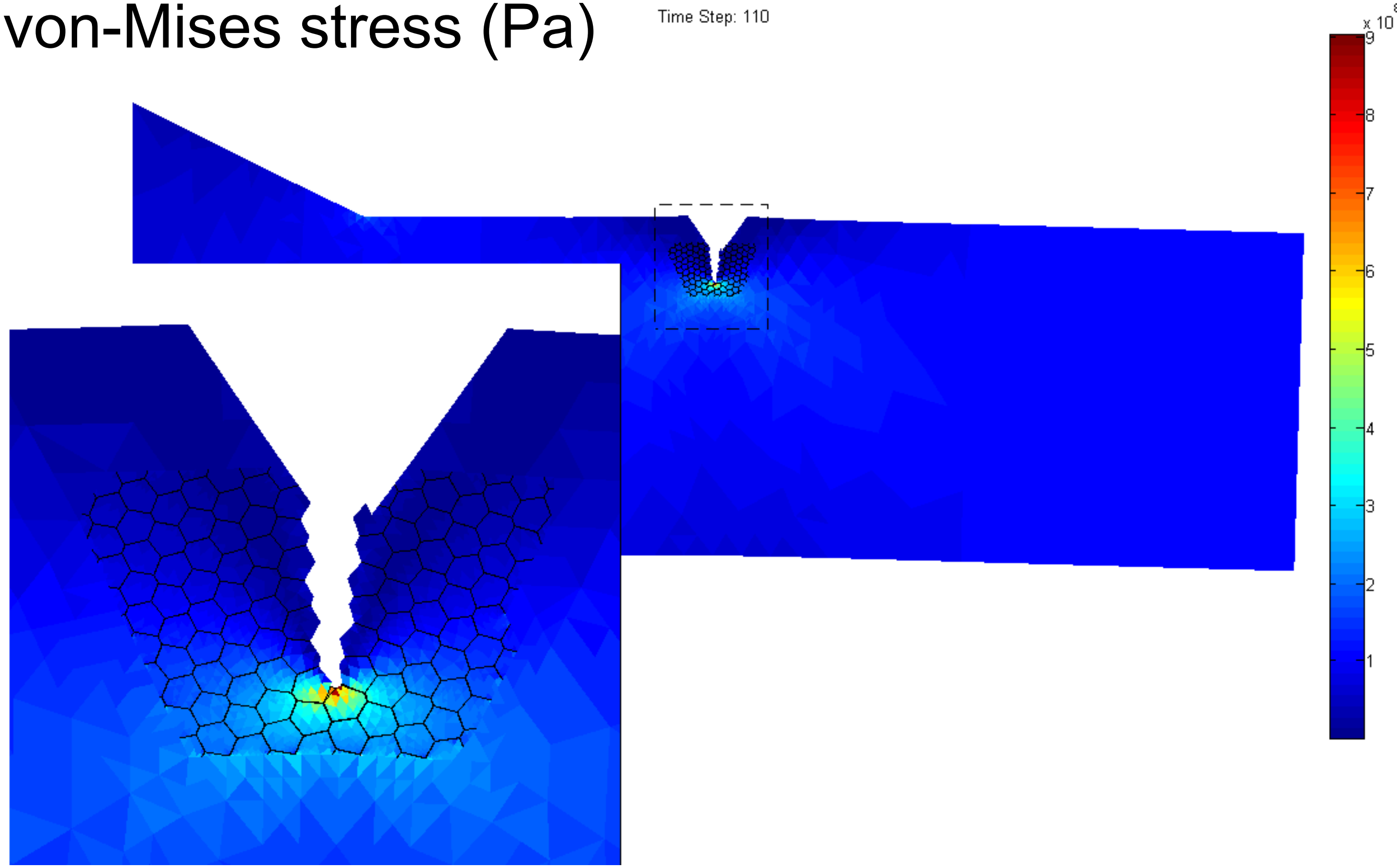
von-Mises stress (Pa)



❖ 100X (magnification of displacement)

Results: uni-axial tension

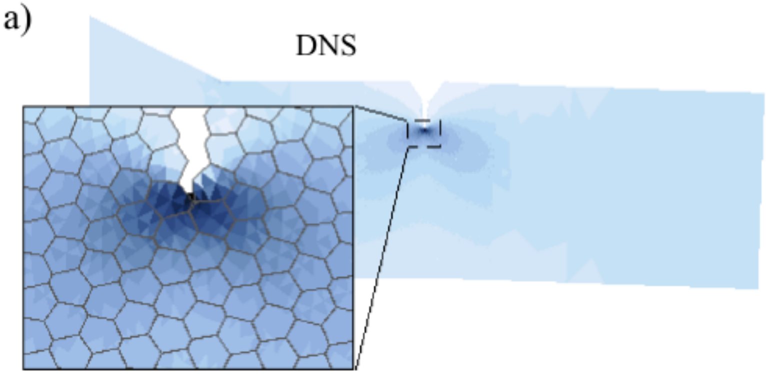
von-Mises stress (Pa)



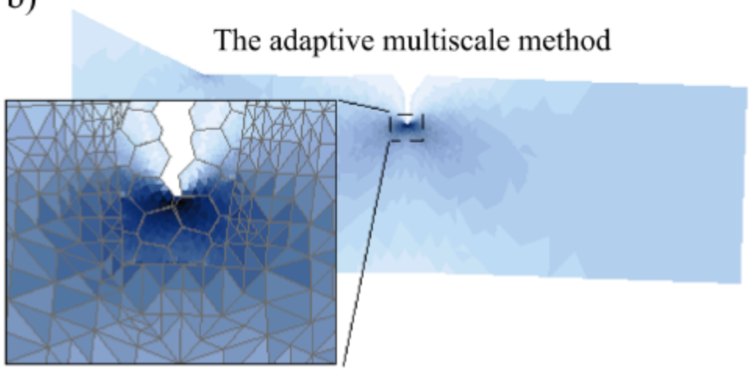
❖ 100X (magnification of displacement)

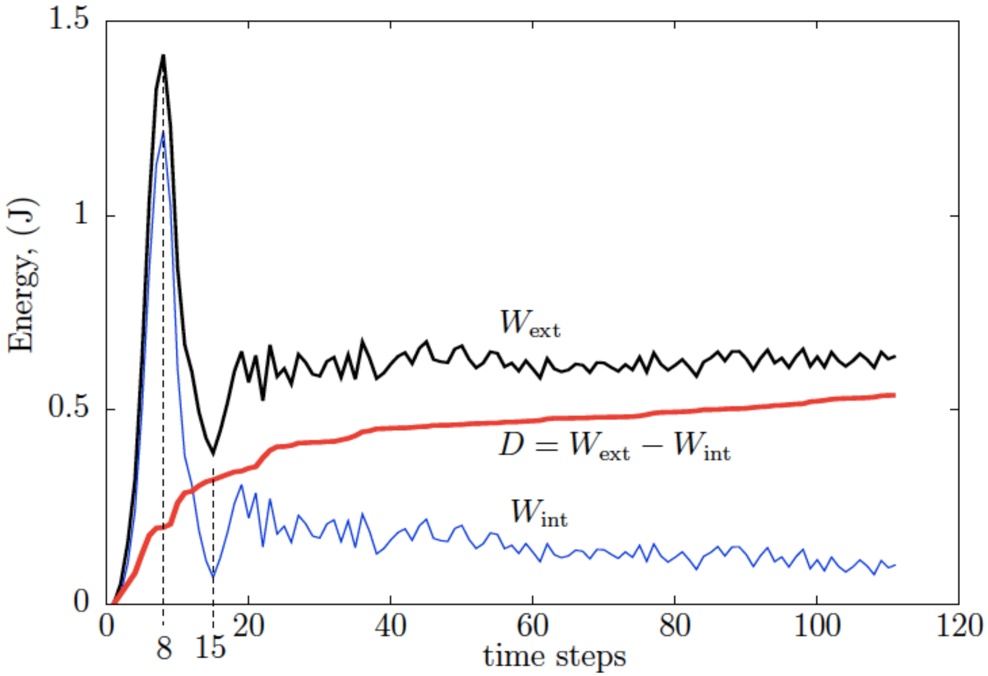
Verification

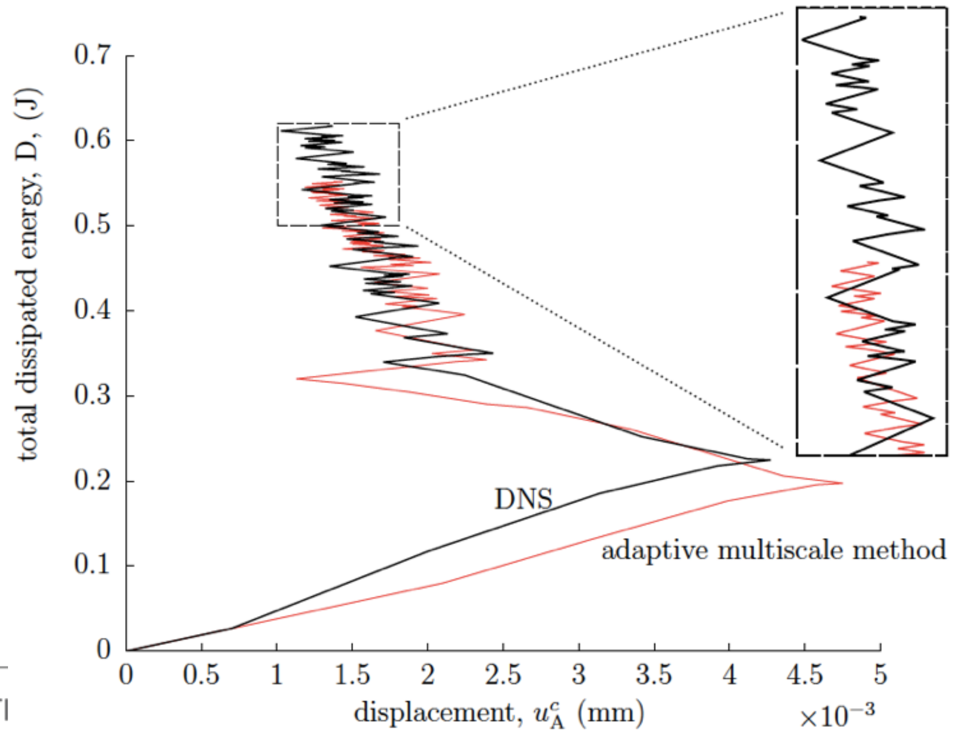
a)



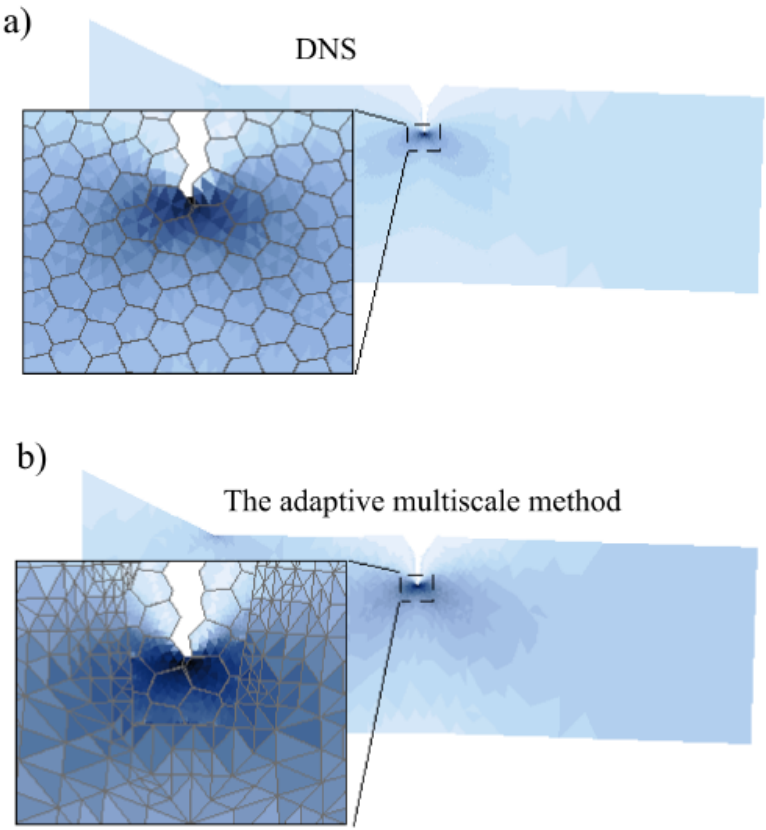
b)

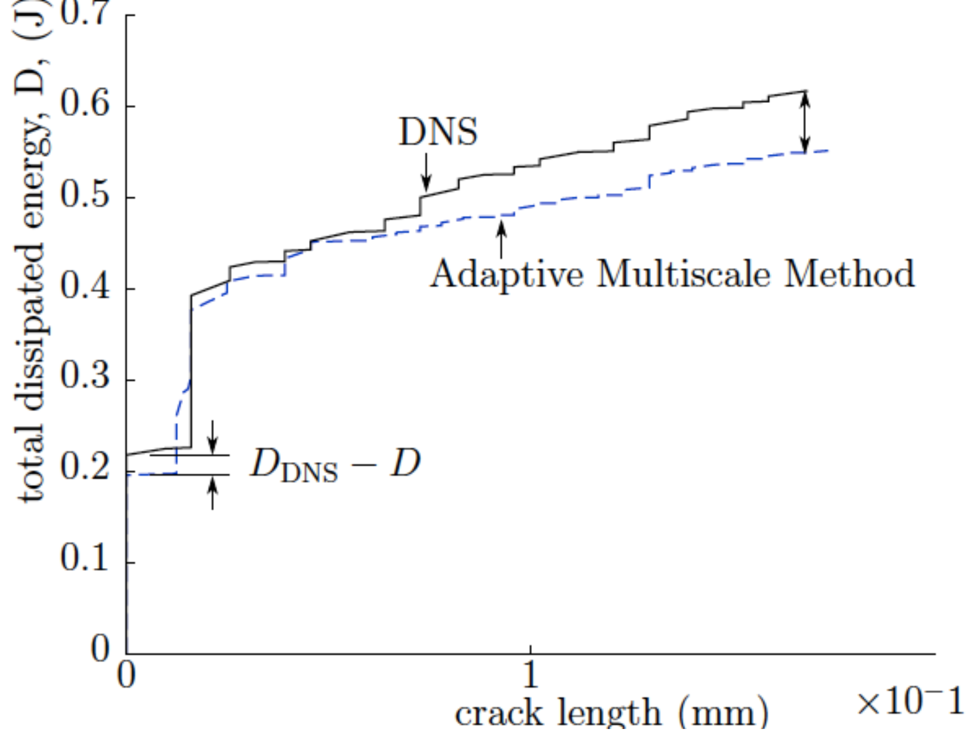


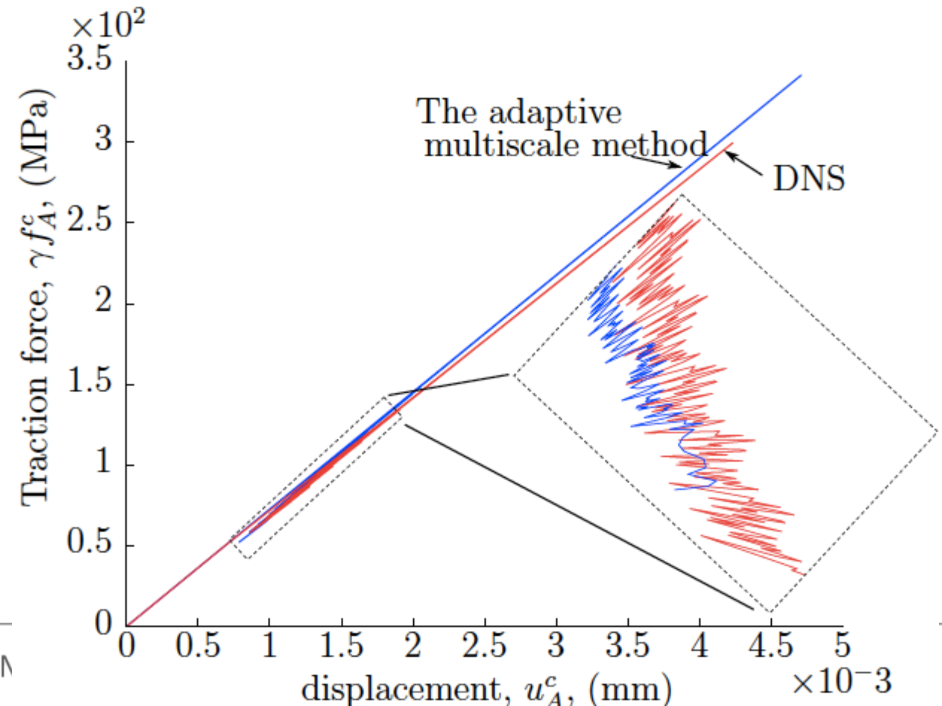




Verification

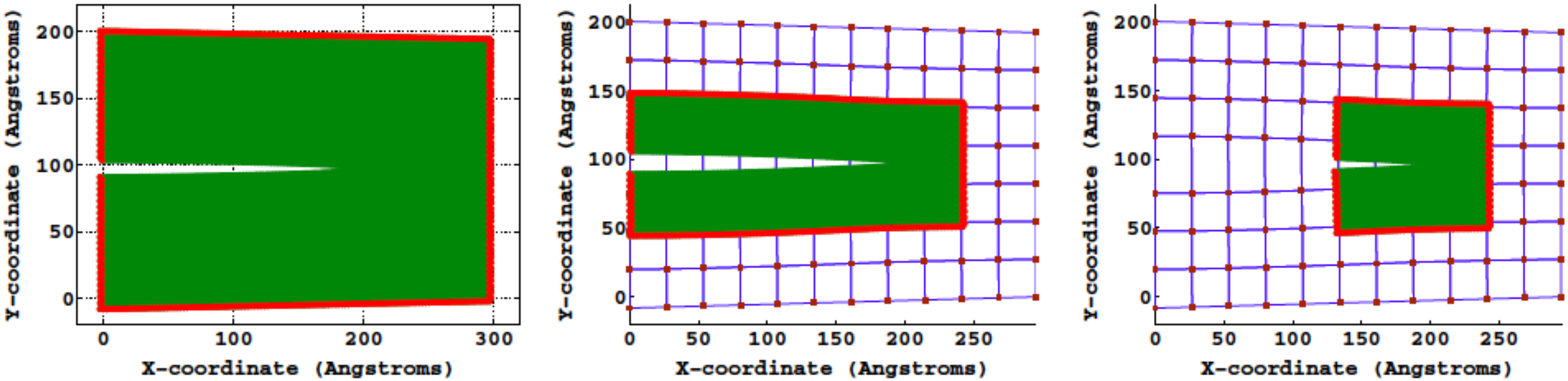






Perspectives

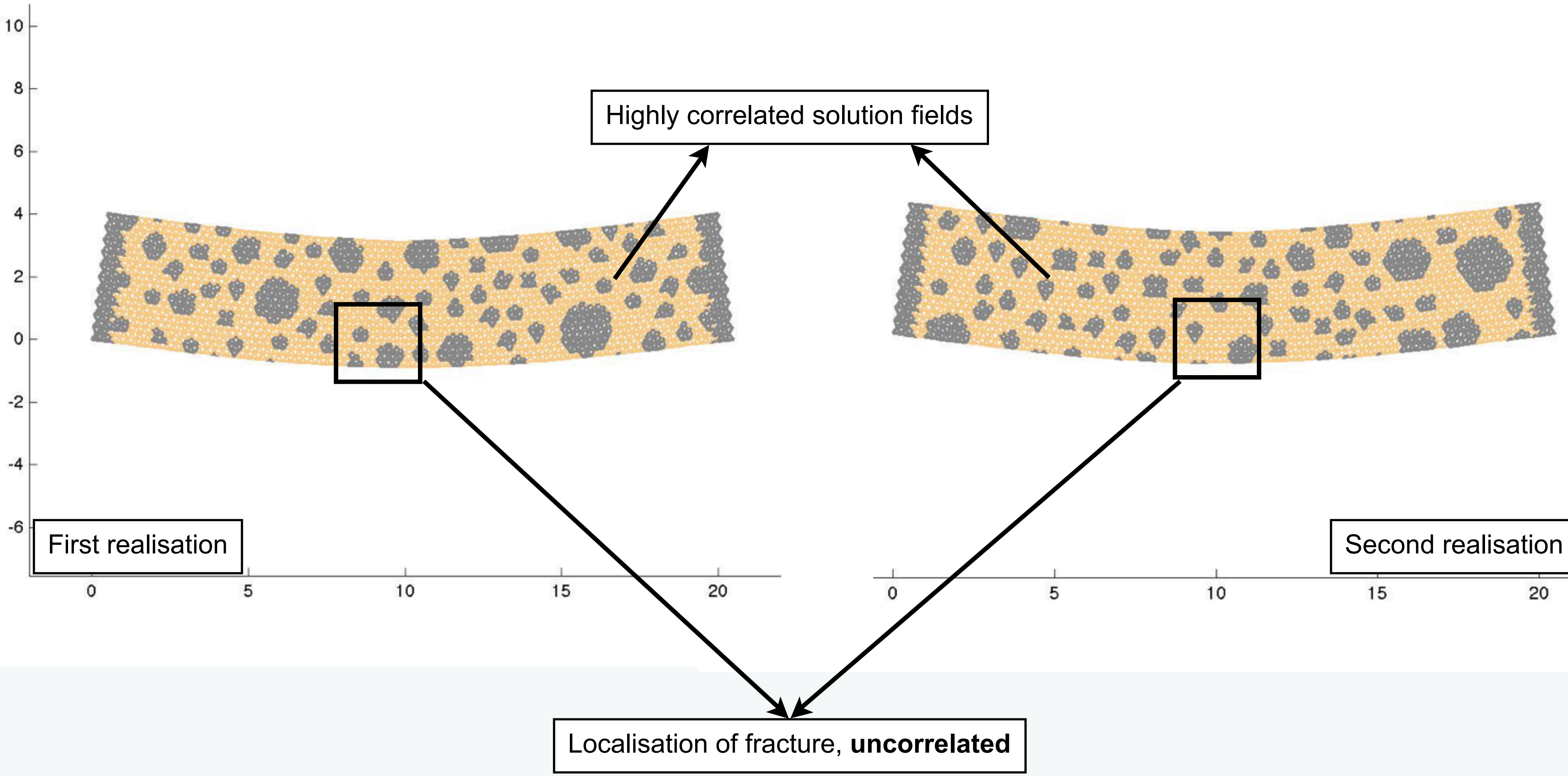
- coarsening once the crack is open
- molecular dynamics at the fine scale



- real-life problems! :)
- coupling with algebraic model reduction (POD)

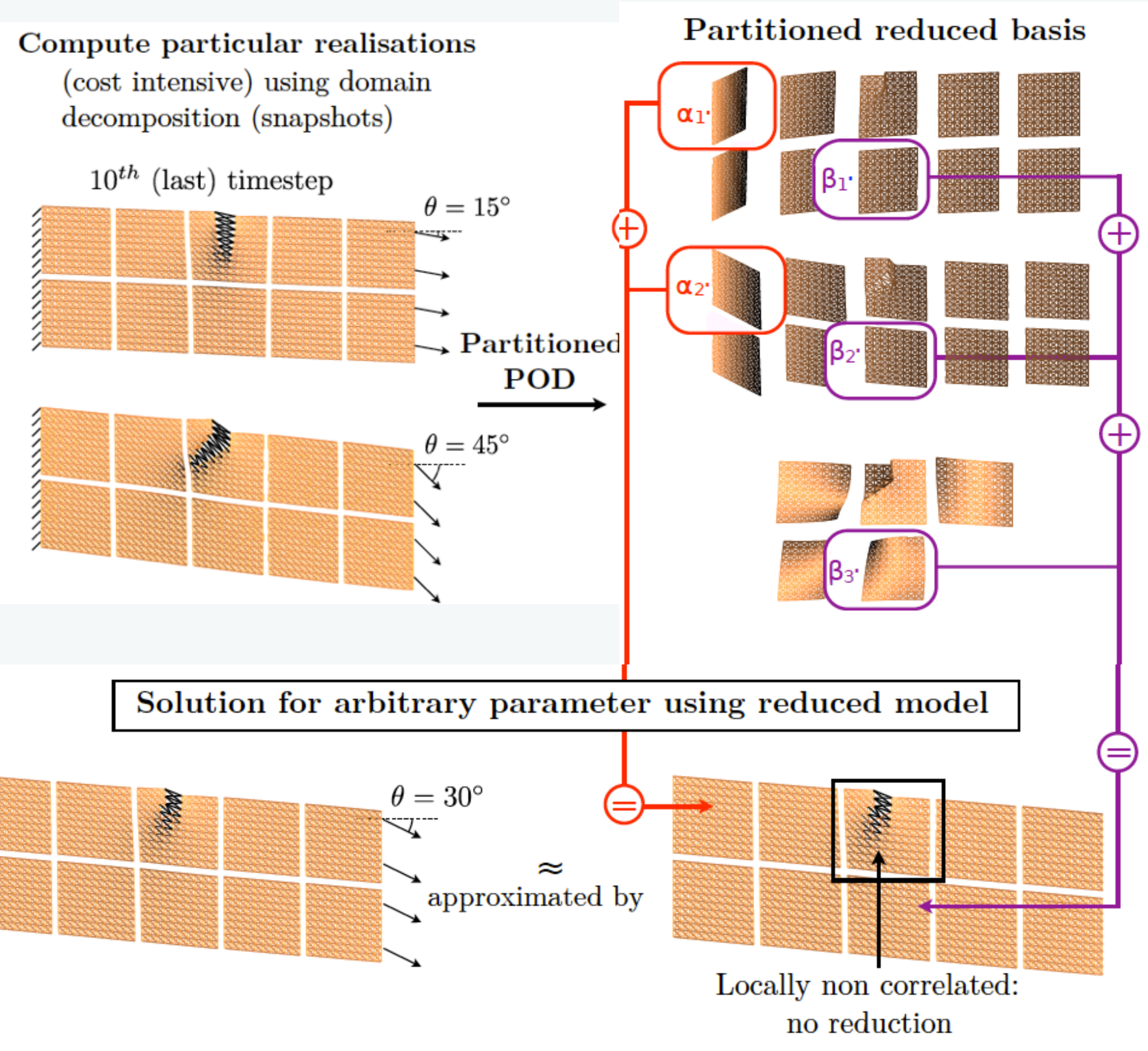
Link with algebraic model reduction (Proper Orthogonal Decomposition)

Parametric / stochastic multiscale fracture mechanics



- Direct numerical simulation: efficient preconditioner?
- Reduced order modelling?
- Adaptive coupling?

Reduced DDM-POD



- Decompose the structure into subdomains
- Perform a reduction in the highly correlated region
- Couple the reduced to the non-reduced region by a primal Schur complement

Publications

http://hdl.handle.net/10993/16347

http://orbilu.uni.lu/handle/10993/14475

http://orbilu.uni.lu/handle/10993/10207

http://orbilu.uni.lu/handle/10993/10066

http://orbilu.uni.lu/handle/10993/12454

http://orbilu.uni.lu/handle/10993/16323

http://orbilu.uni.lu/handle/10993/12012

http://orbilu.uni.lu/handle/10993/12014

Part I. Streamlining the CAD-analysis transition

Part II. Some advances in enriched FEM

Part III. Application to H cutting of Si wafers

Part IV. Interactive cutting sim.

Faculty of Sciences,
Technology
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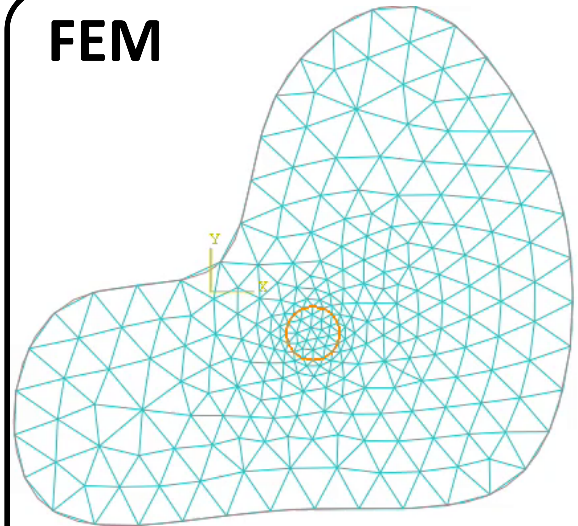
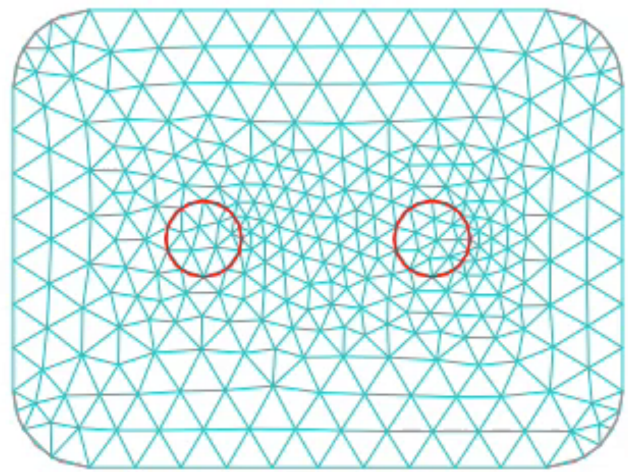
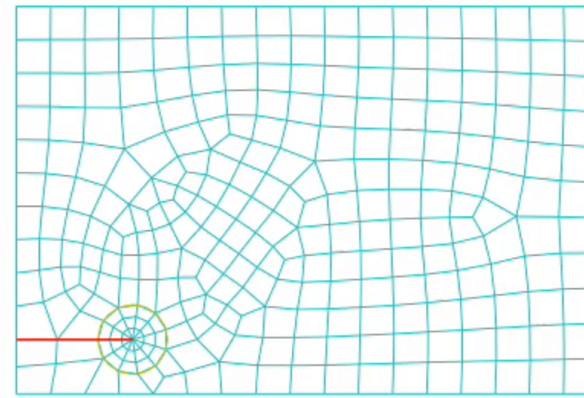
Part I. Streamlining the CAD-analysis transition

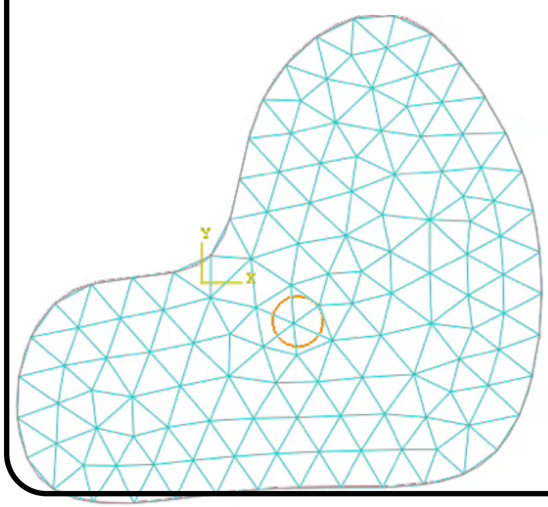
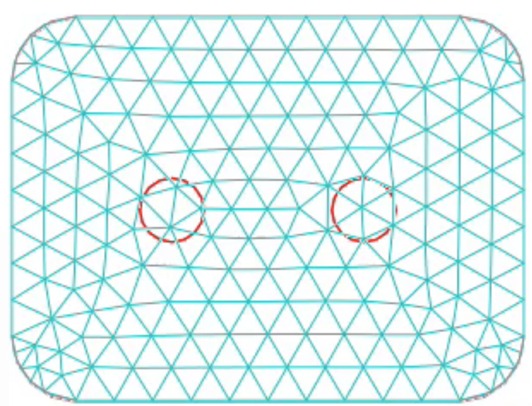
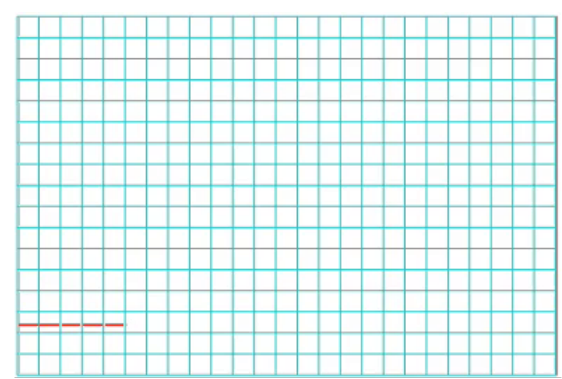
Coupling, or decoupling?

Motivation: free boundary problems - mesh burden

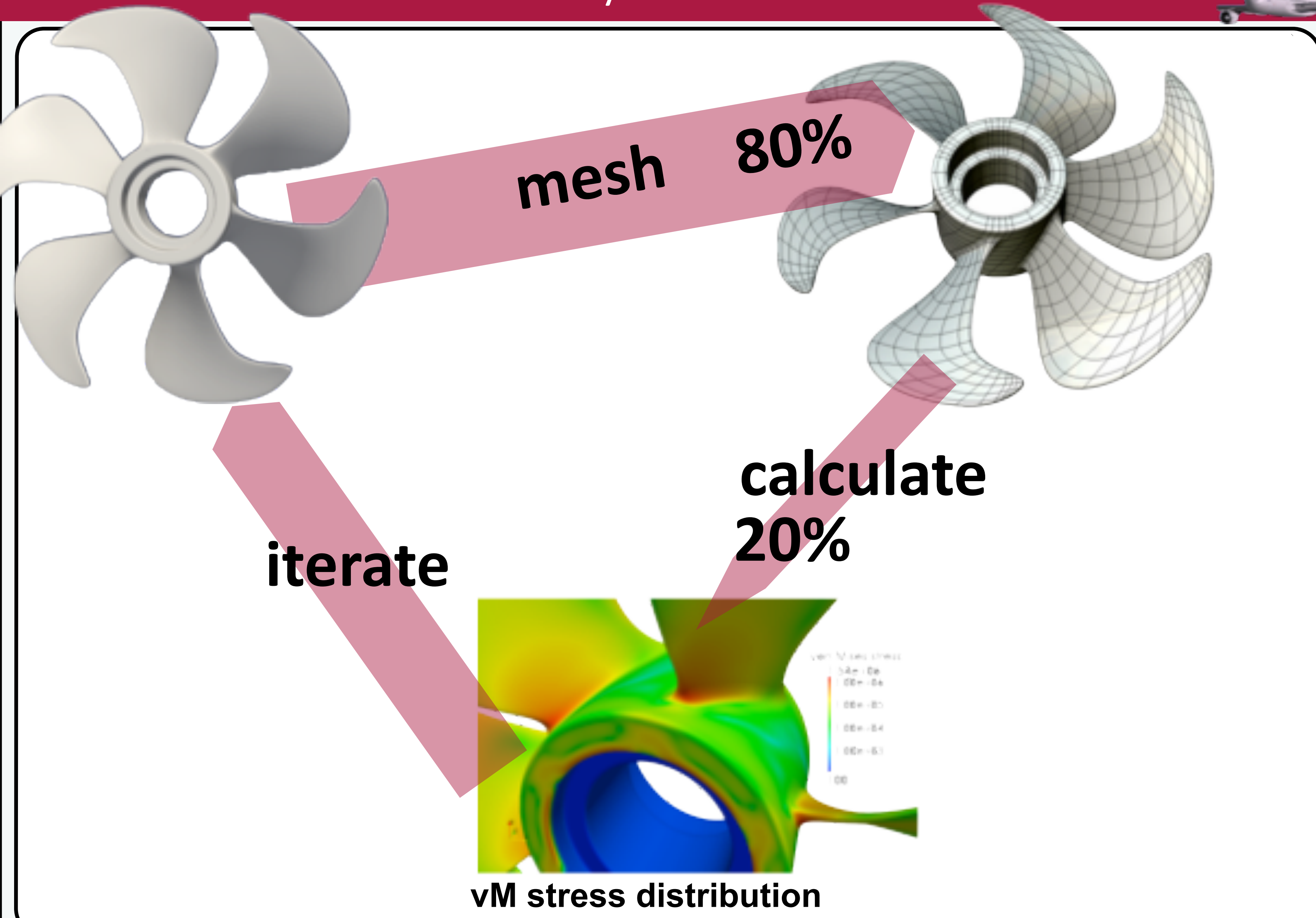
FEM

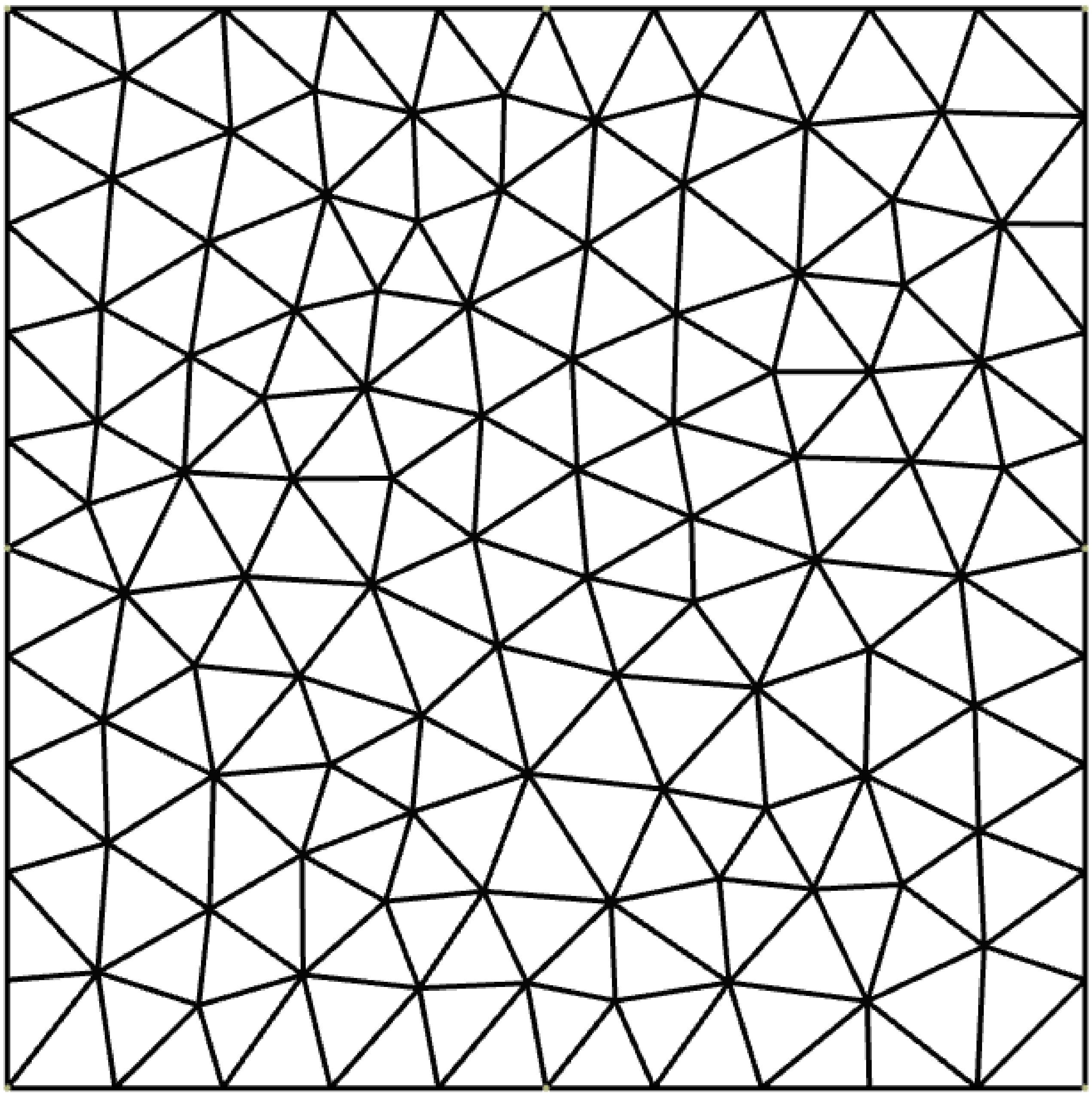
XFEM

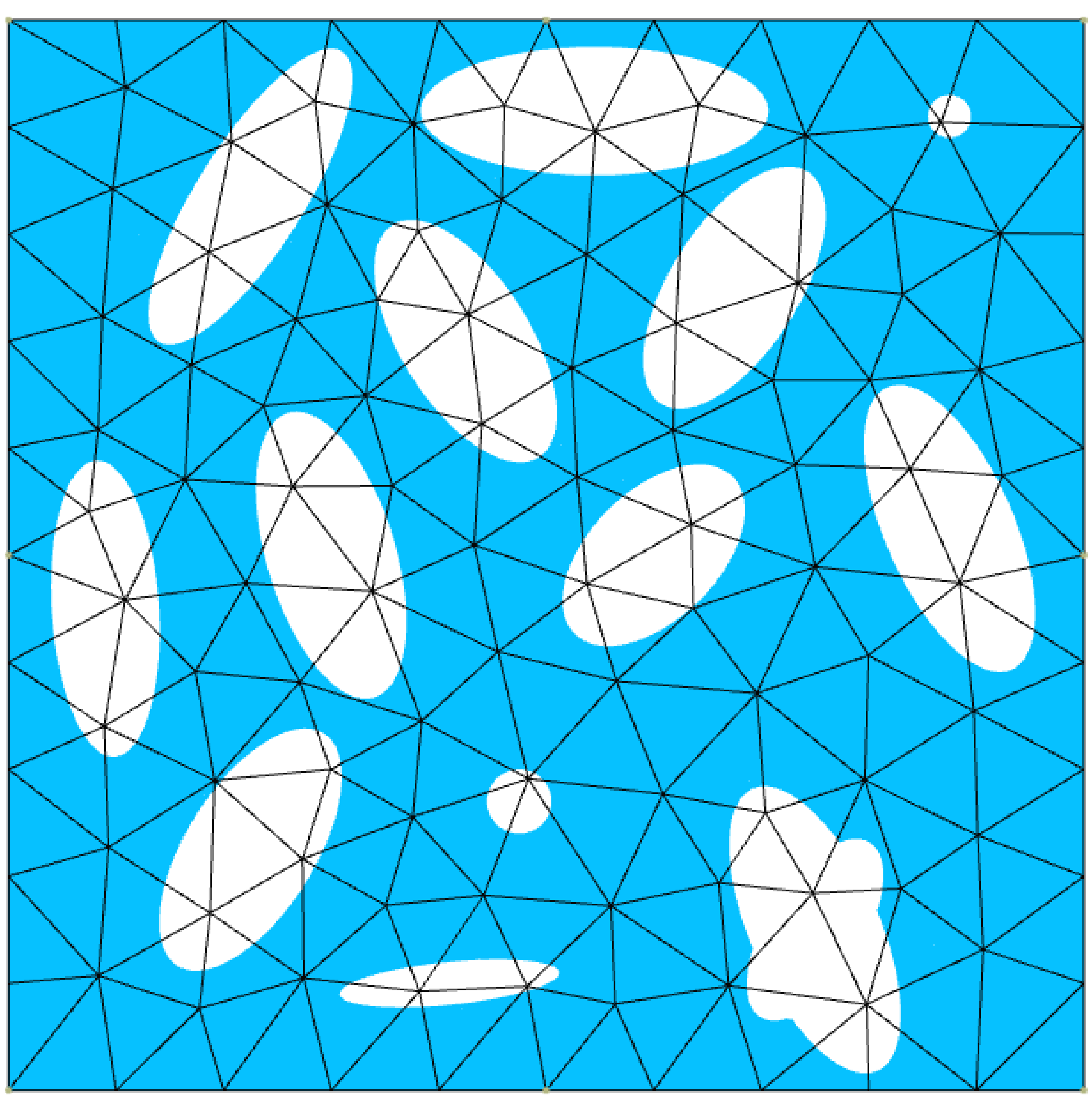
CAD to Analysis



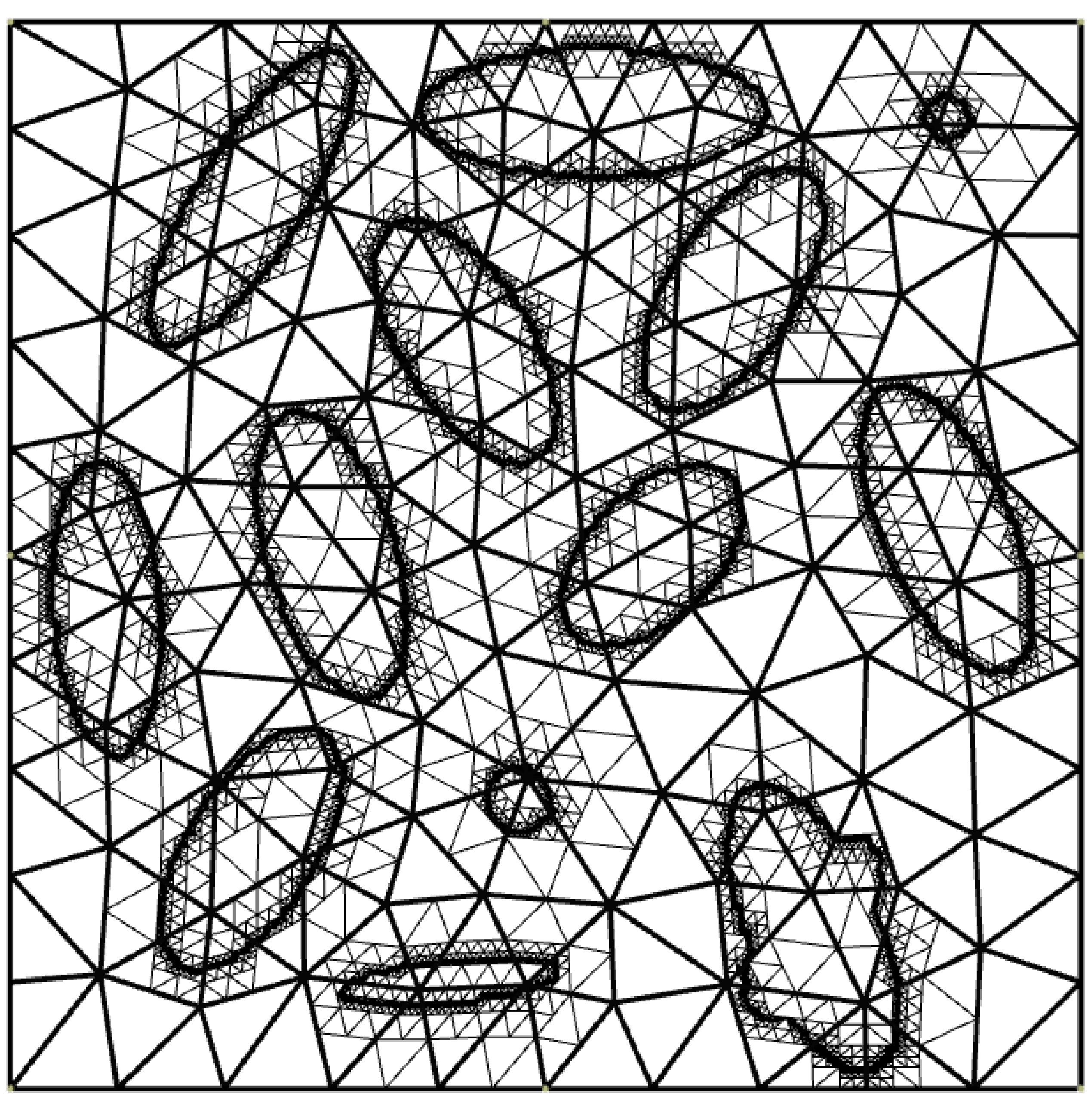
One would like to be able to use such a mesh



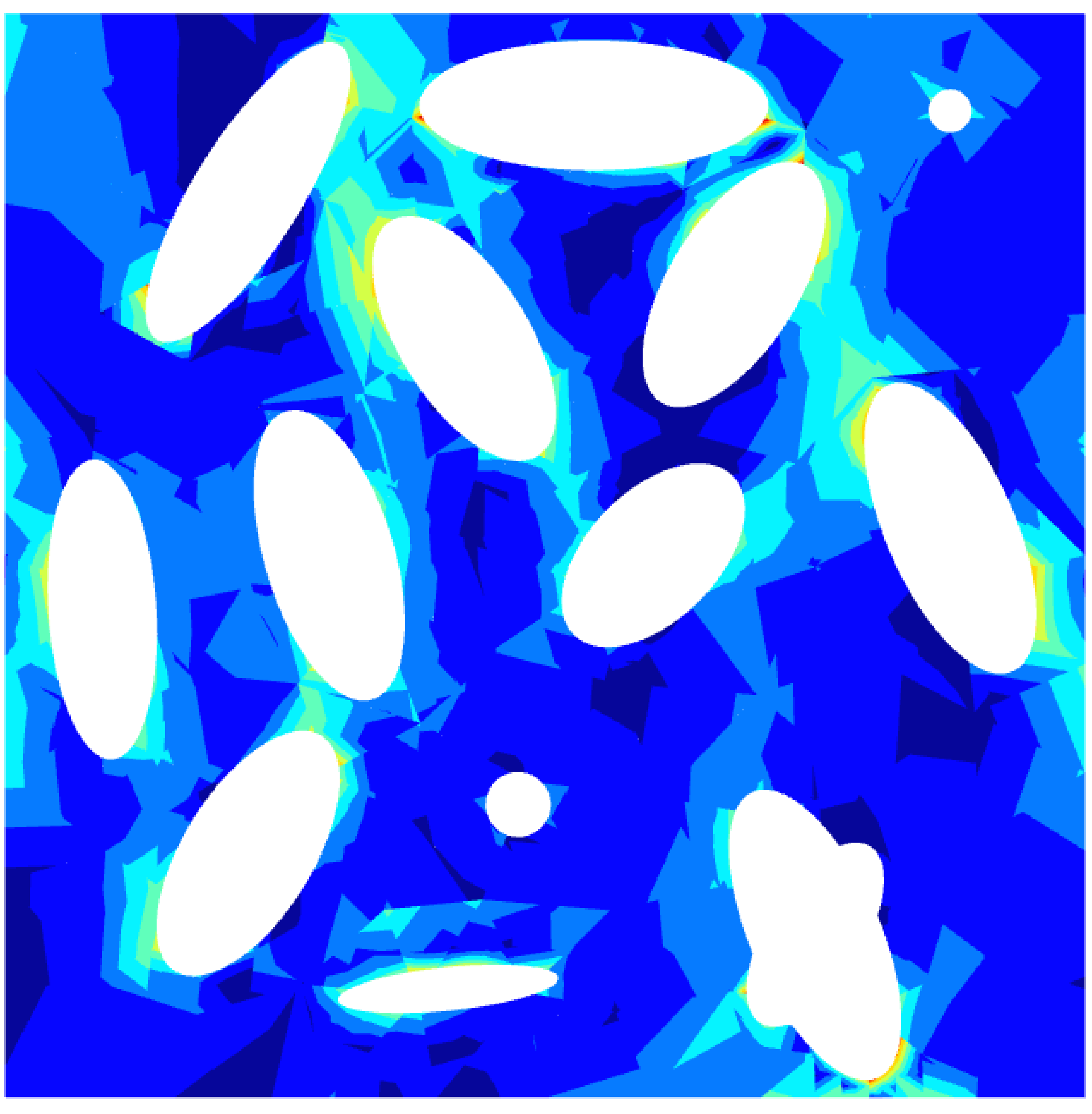
Superimpose the geometry onto an arbitrary background mesh



Compute interactions between the geometry and the mesh

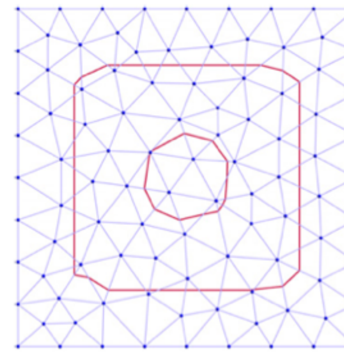


Perform the analysis



Paradigm 1: Separate field and boundary discretisation

- Immersed boundary method (Mittal, *et al.* 2005)
- Fictitious domain (Glowinski, *et al.* 1994)
- Embedded boundary method (Johansen, *et al.* 1998)
- Virtual boundary method (Saiki, *et al.* 1996)
- Cartesian grid method (Ye, *et al.* 1999, Nadal, 2013)

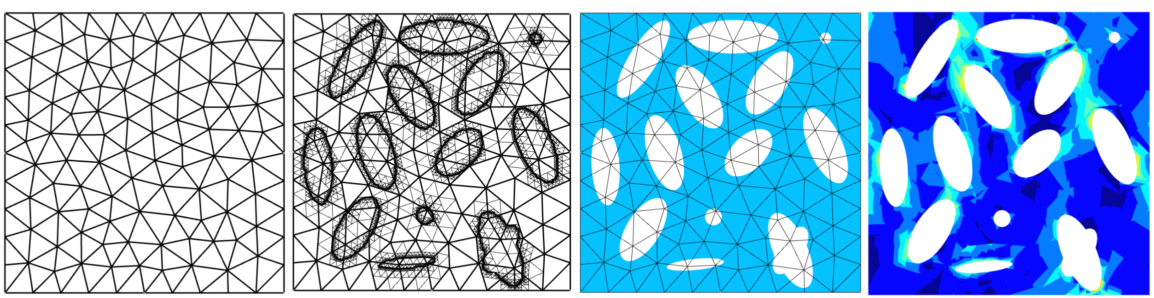


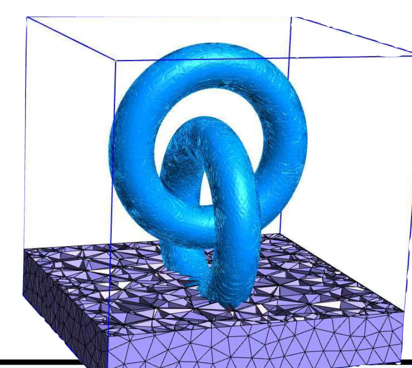
✓ Easy adaptive refinement + error estimation (Nadal, 2013)

✓ Flexibility of choosing basis functions

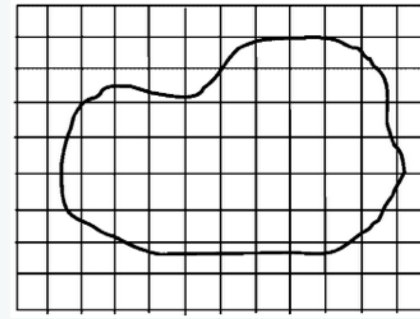
- Accuracy for complicated geometries? BCs on implicit surfaces?

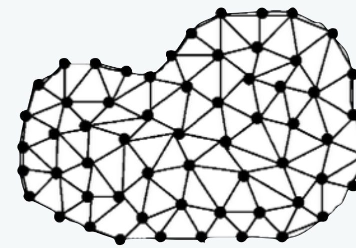
➡ An accurate and implicitly-defined geometry from arbitrary parametric surfaces including corners and sharp edges (Moumnassi, *et al.* 2011)





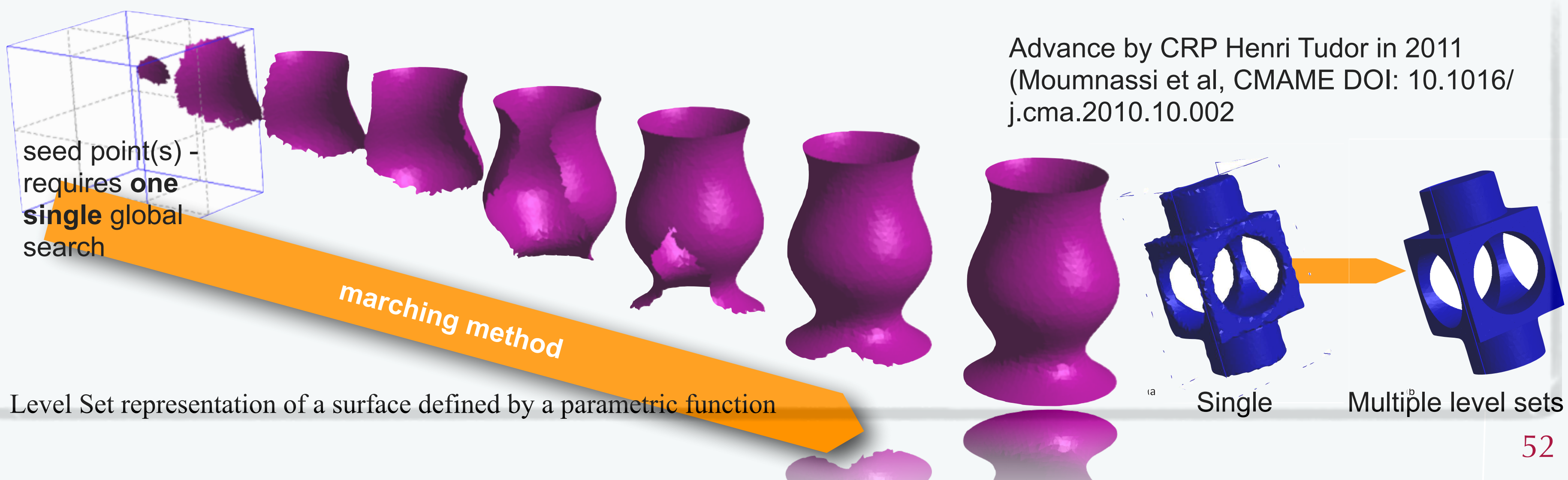
Ex: Moumnassi et al, CMAME DOI:10.1016/j.cma.2010.10.002





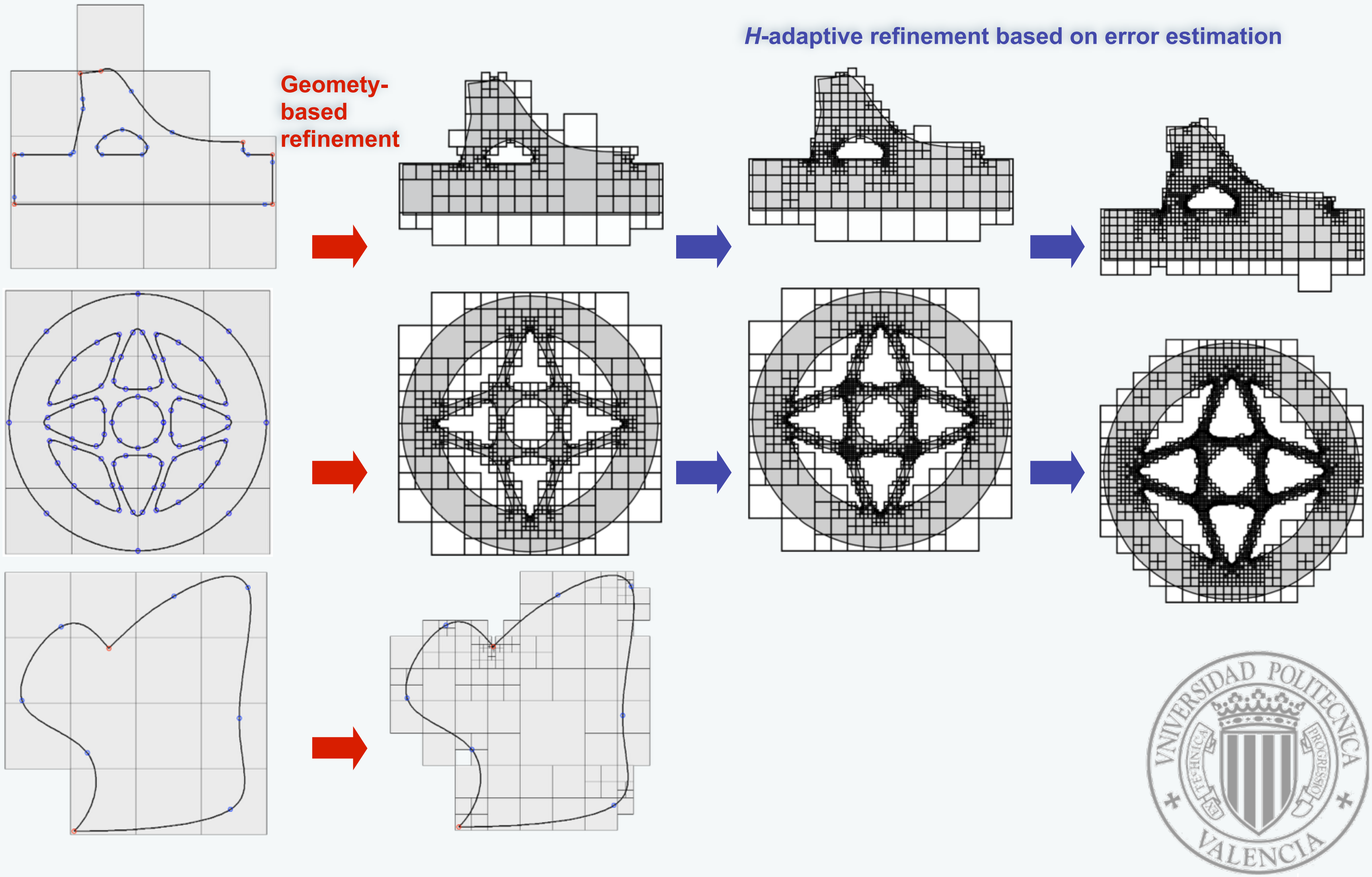
- Objectives
  - insert surfaces in a structured mesh
    - without meshing the surfaces (boundary, cracks, holes, inclusions, etc.)
    - directly from the underlying CAD model
    - model arbitrary solids, including sharp edges and vertices
  - keep as much as possible of the mesh as the CAD model evolves, i.e. reduce mesh dependence of the implicit boundary representation
  - maintain the convergence rates and implementation simplicity of the FEM

Advance by CRP Henri Tudor in 2011 (Moumnassi et al, CMAME DOI: 10.1016/ j.cma.2010.10.002

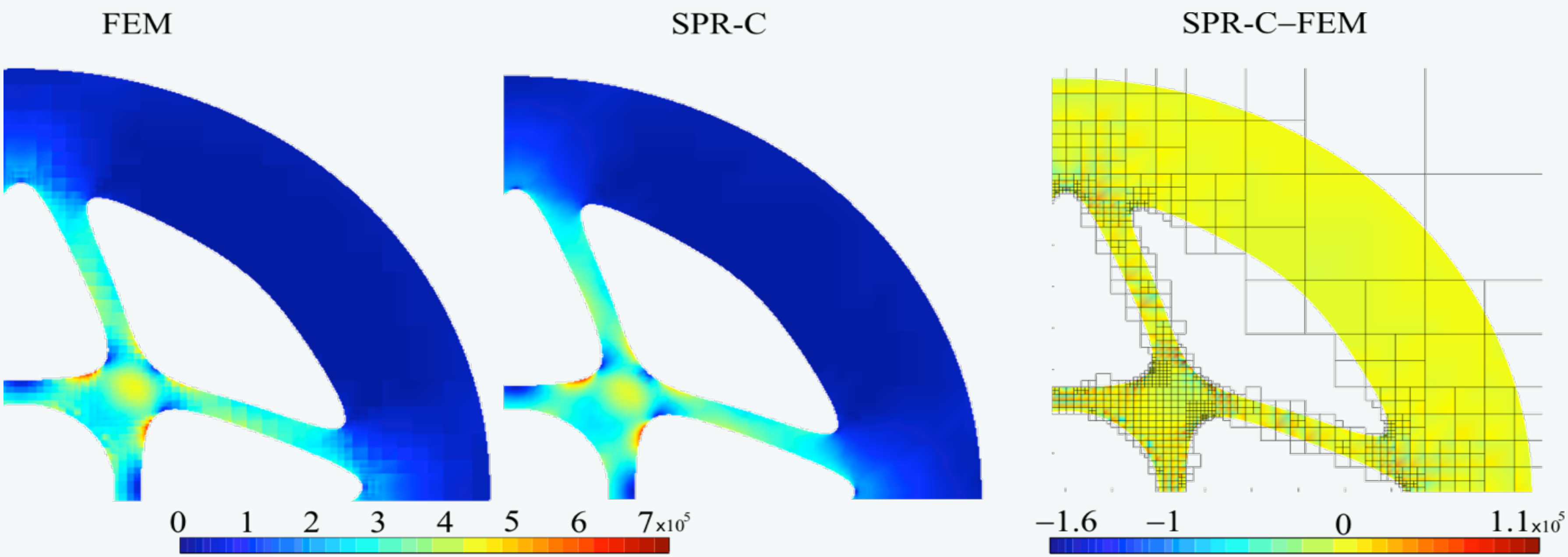


Level Set representation of a surface defined by a parametric function

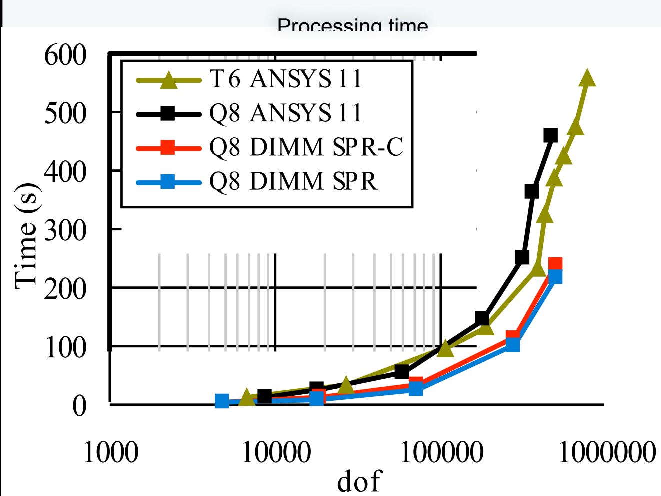
Pixel/Voxel-based FEA on Cartesian grids (Valencia)

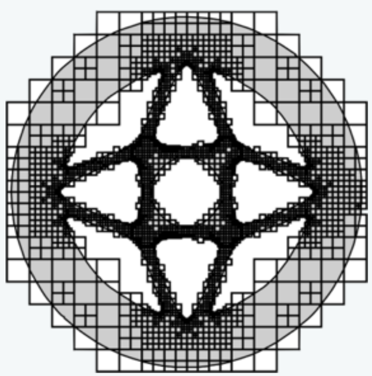


Pixel/Voxel-based FEA on Cartesian grids (Valencia)



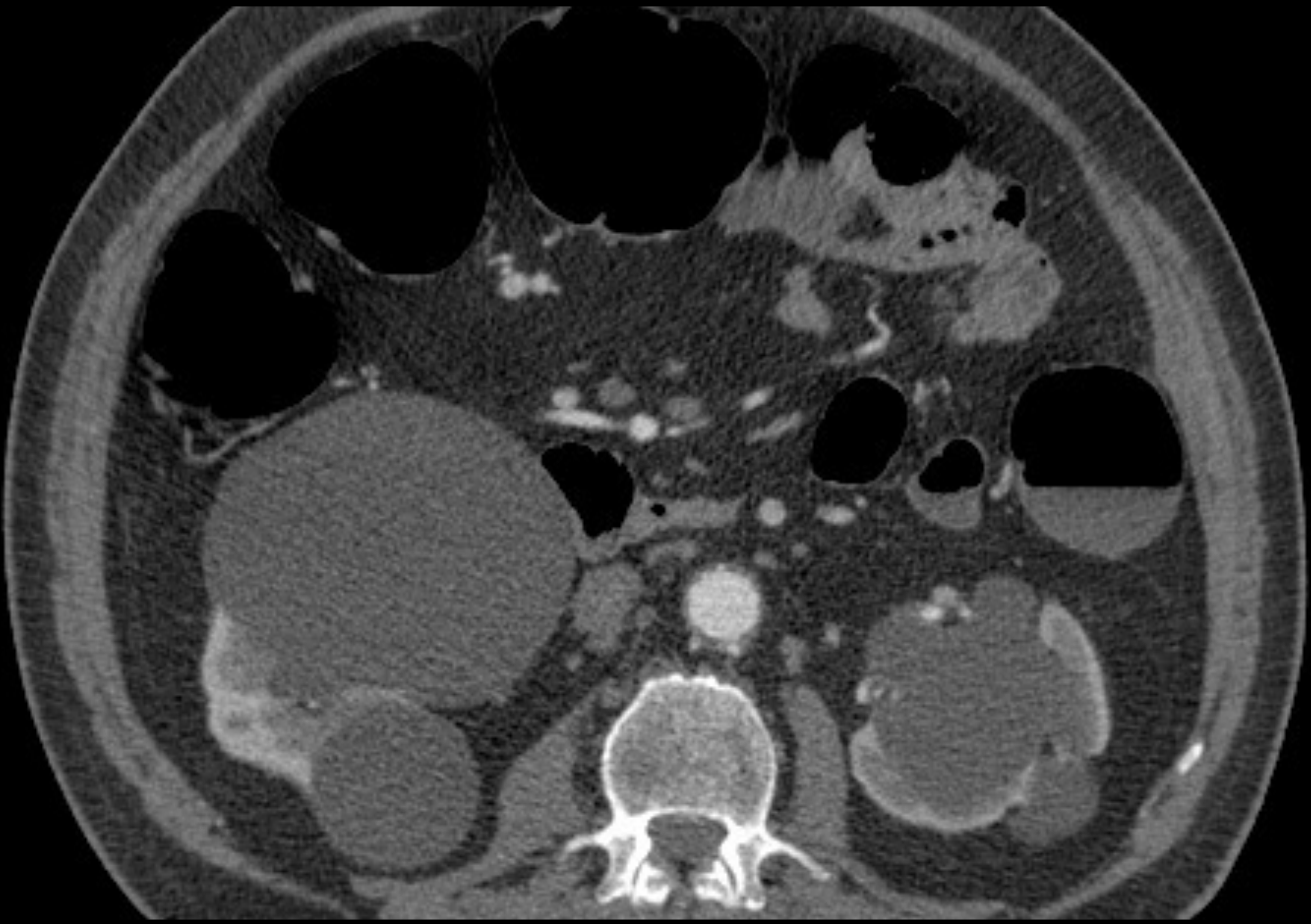
Quad8 uniform refinement





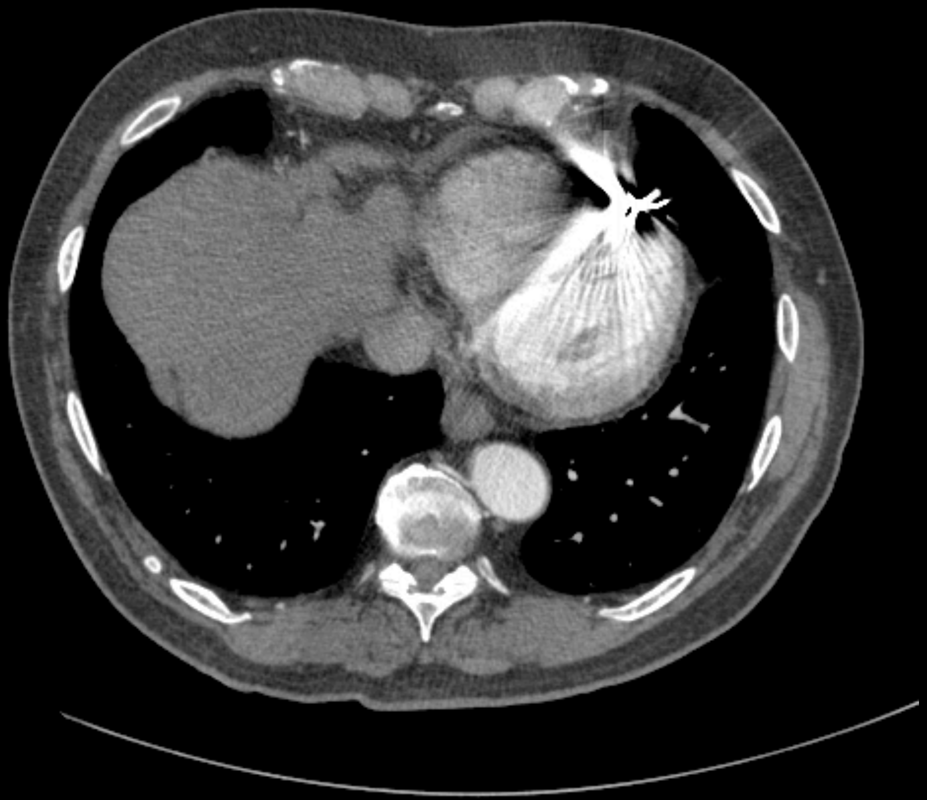
EXTENSION TO IMAGE TO MESH TRANSITION UNDER WAY

How can we move from an image…



COLONIX, OSIRIX

…or perhaps a series of images…

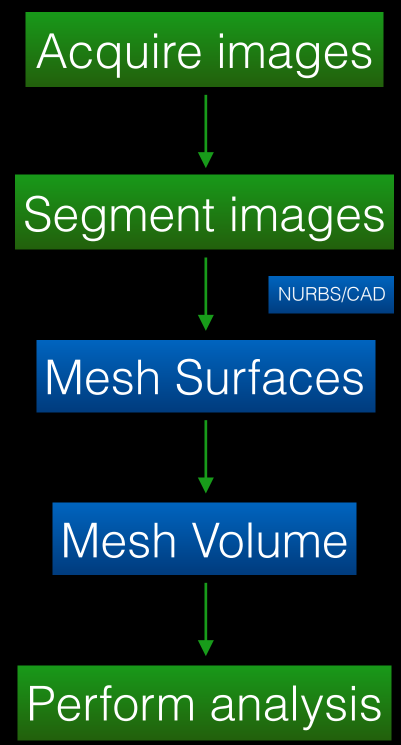


Source: COLONIX, OSIRIX

to a full mechanical analysis?

Pipeline to analysis

Traditional



Each voxel j is a 32-bit floating point measurement

z

Planes (64)

x

Rows (64)

y

Cols (64)

Image: CGAL Project

Soft segmentation

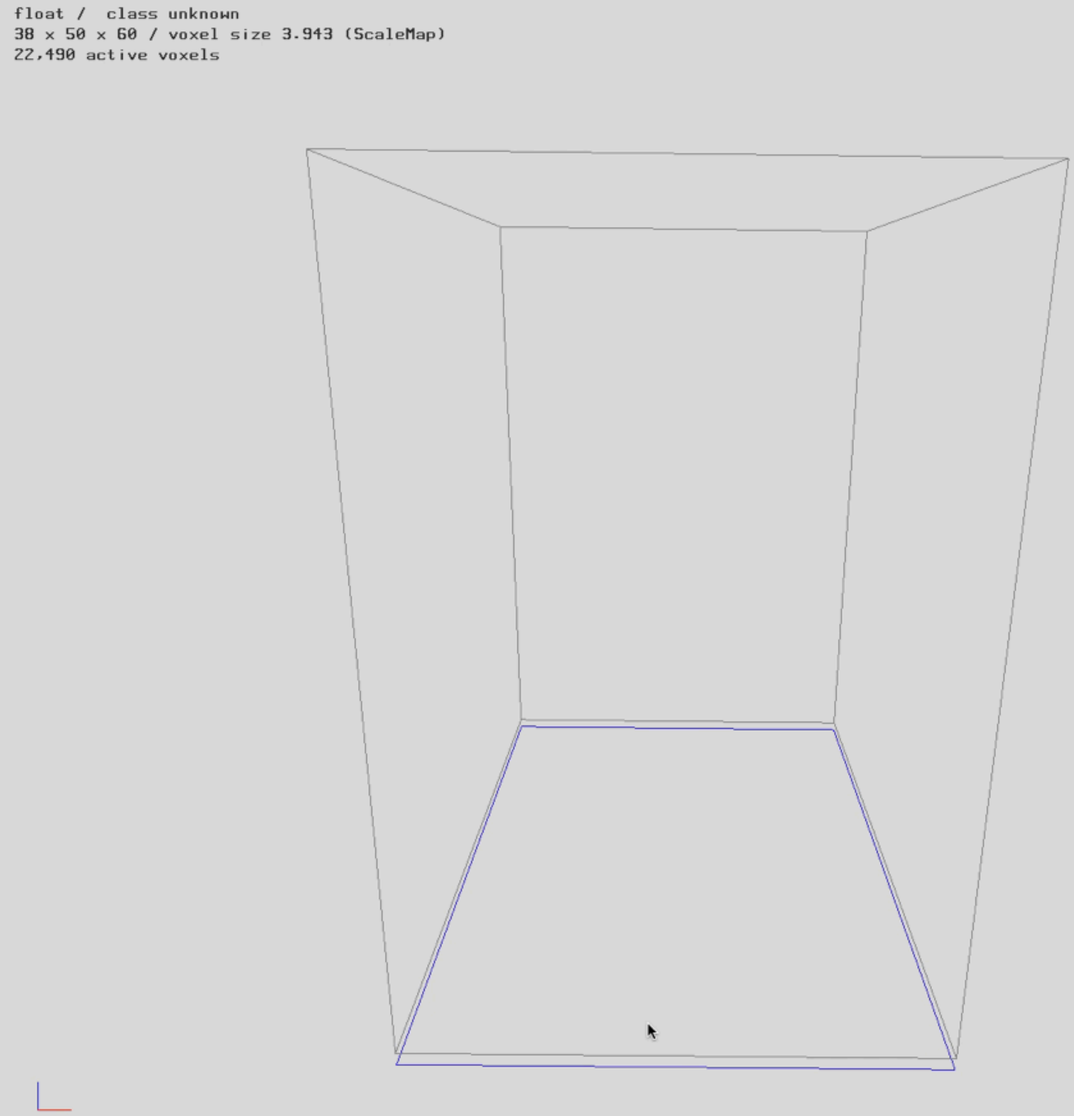
$$0 < m_k(j) < 1 \quad \forall j,\, k$$

$$\sum_{k=1}^{K} m_k(j) = 1 \quad \forall j$$

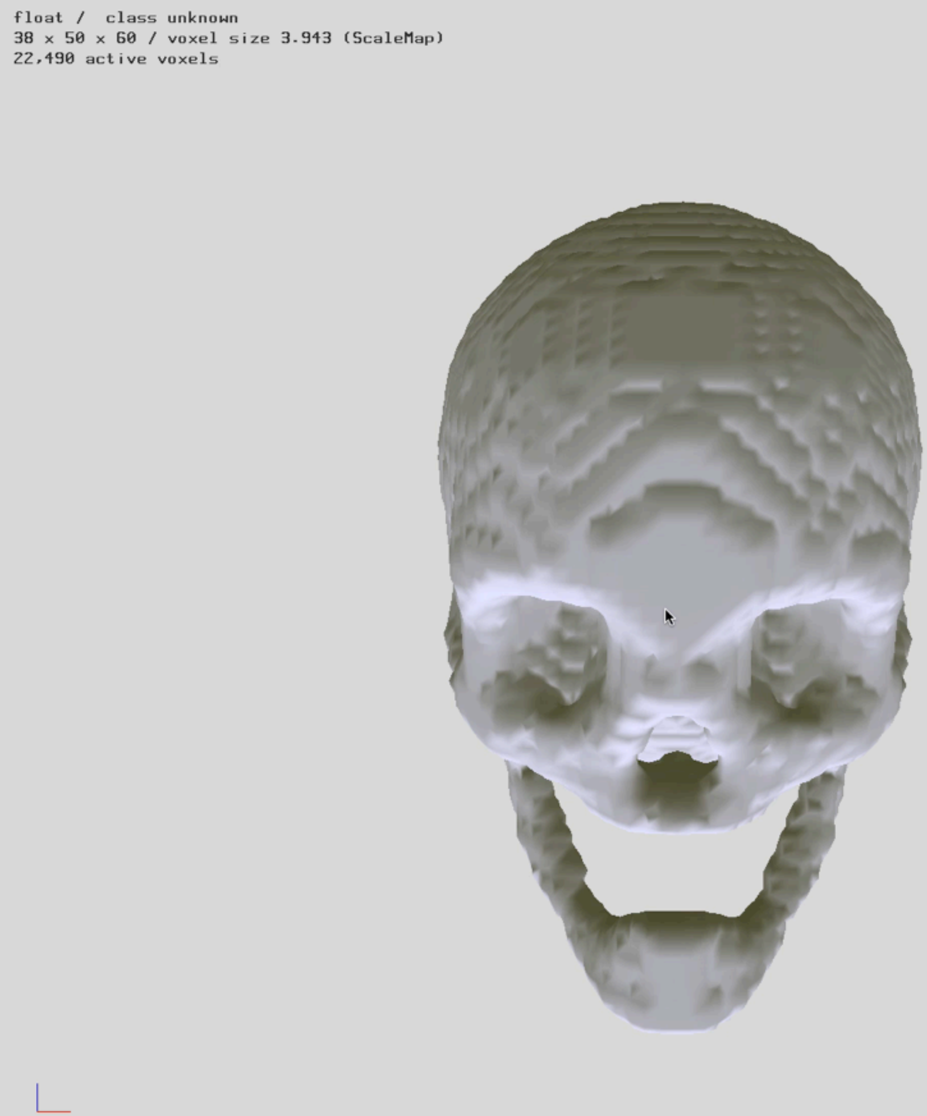
Hard segmentation

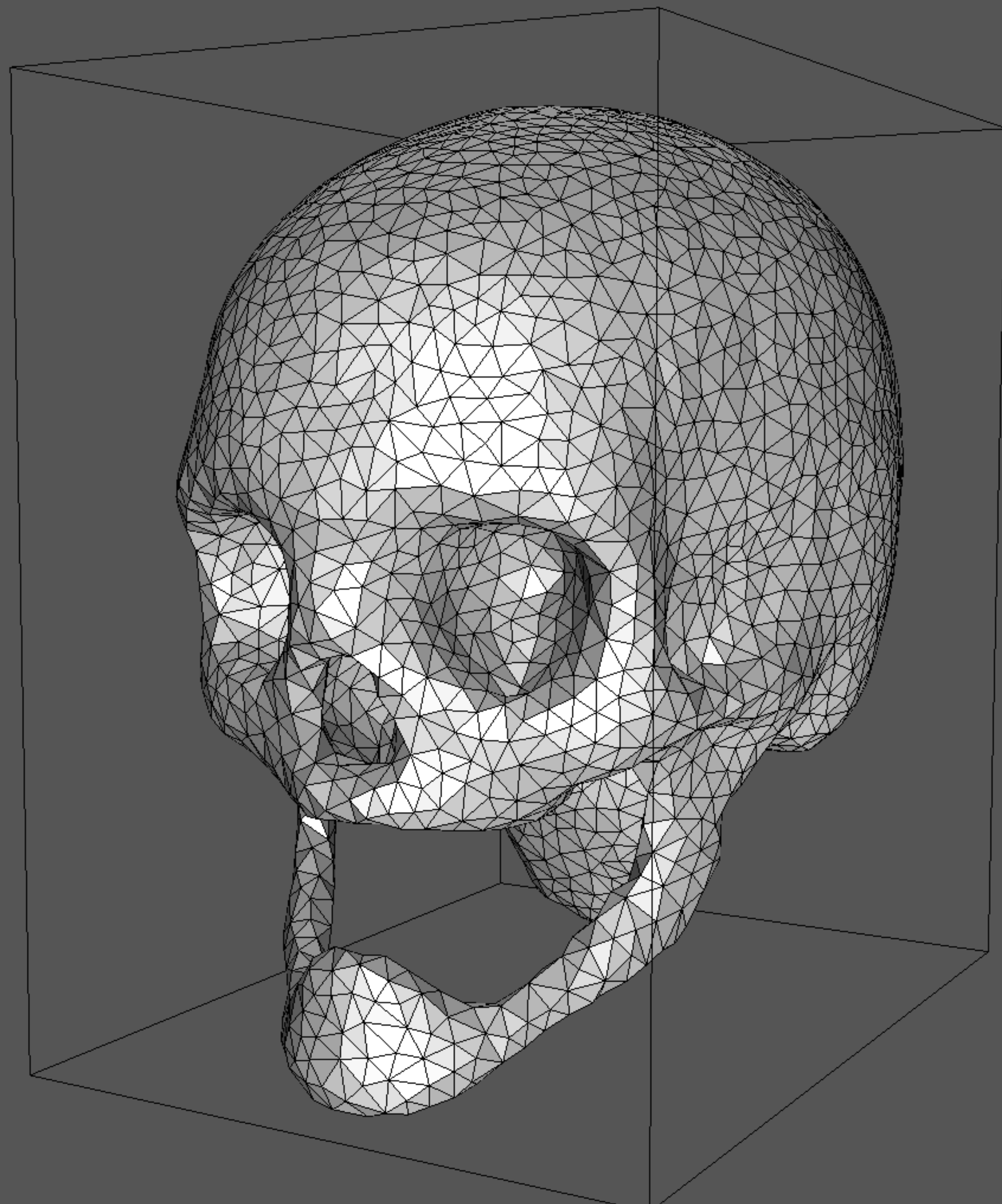
$$\Omega = \bigcup_{k=1}^{K} S_k \qquad S_k \cap S_j = \varnothing \quad \forall k \neq j$$

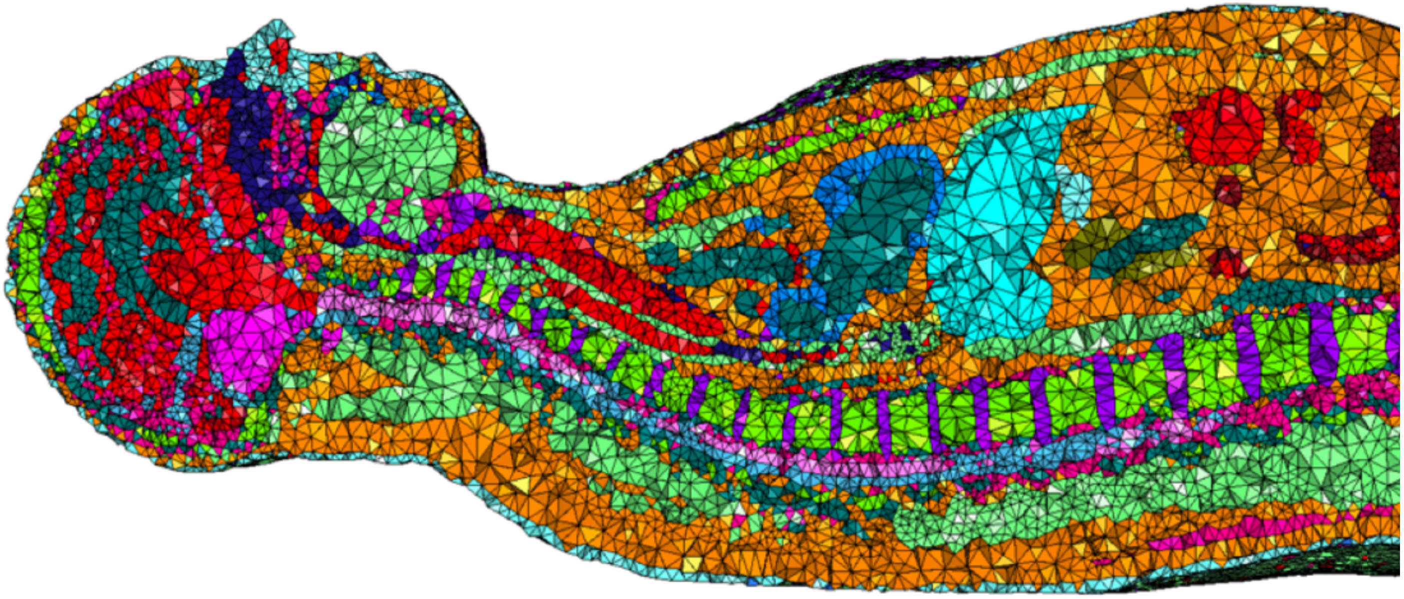
Hard Segmentation at 0.2f



Hard Segmentation at 0.2f with CGAL and OpenVDB







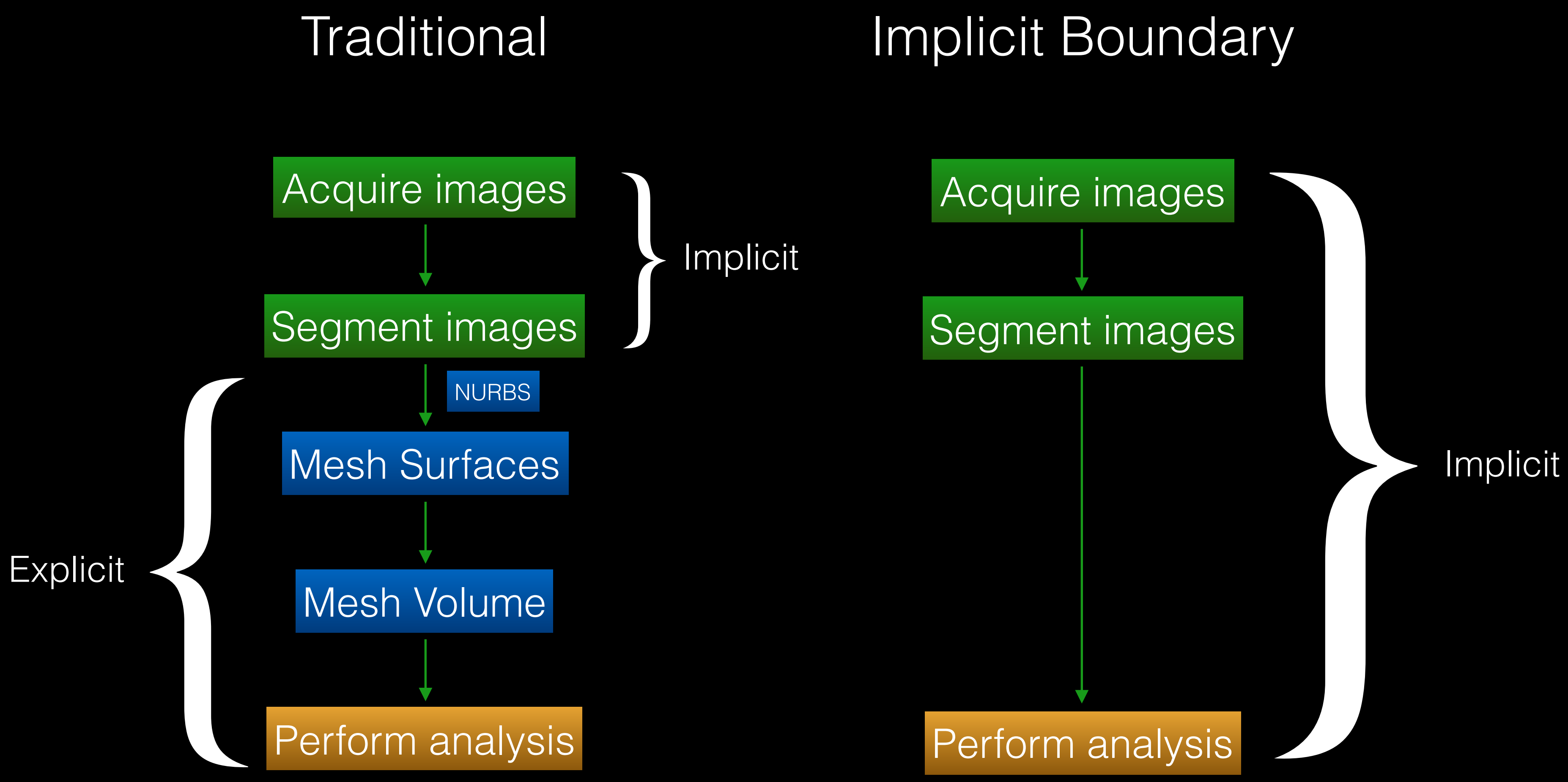
Visible Human

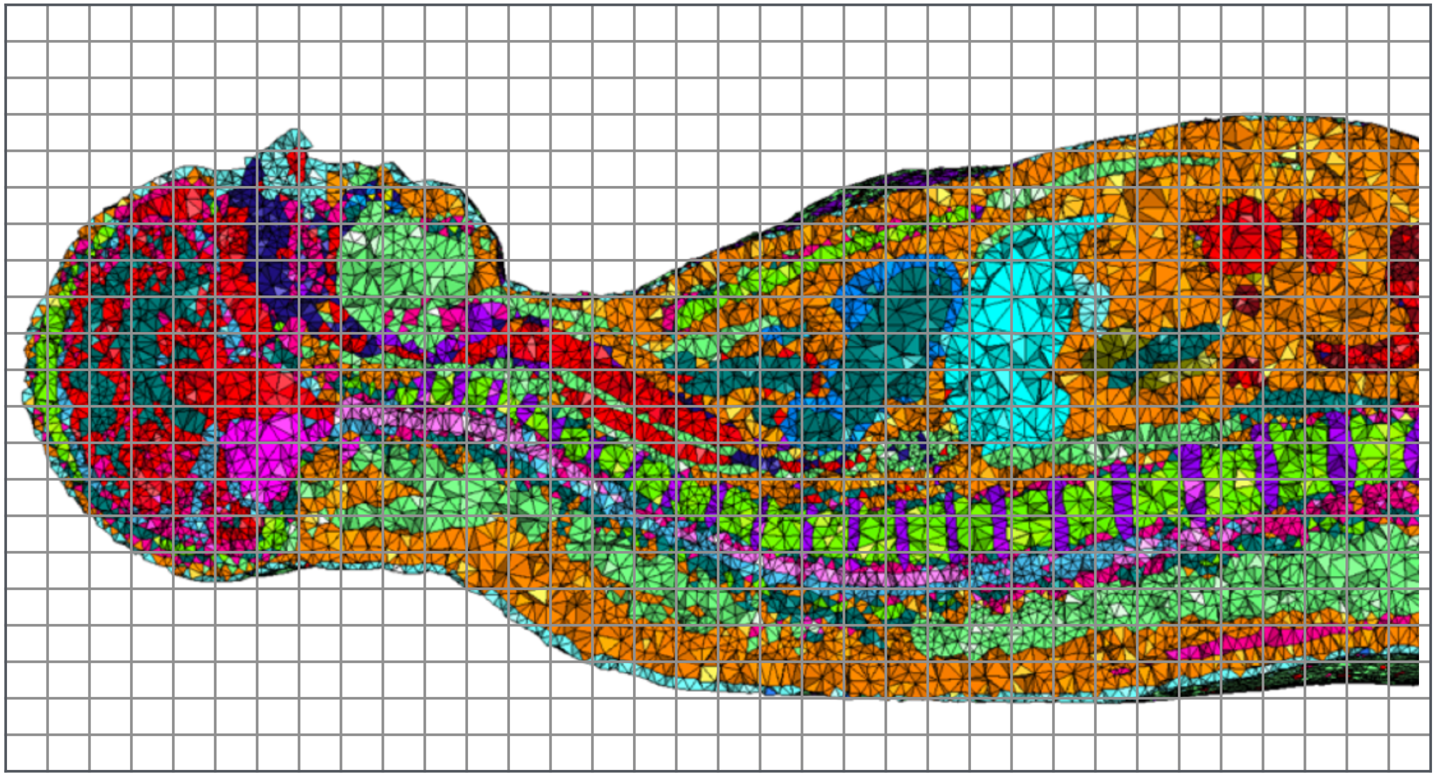
Stephane Lanteri (INRIA) and France Telecom

Problems

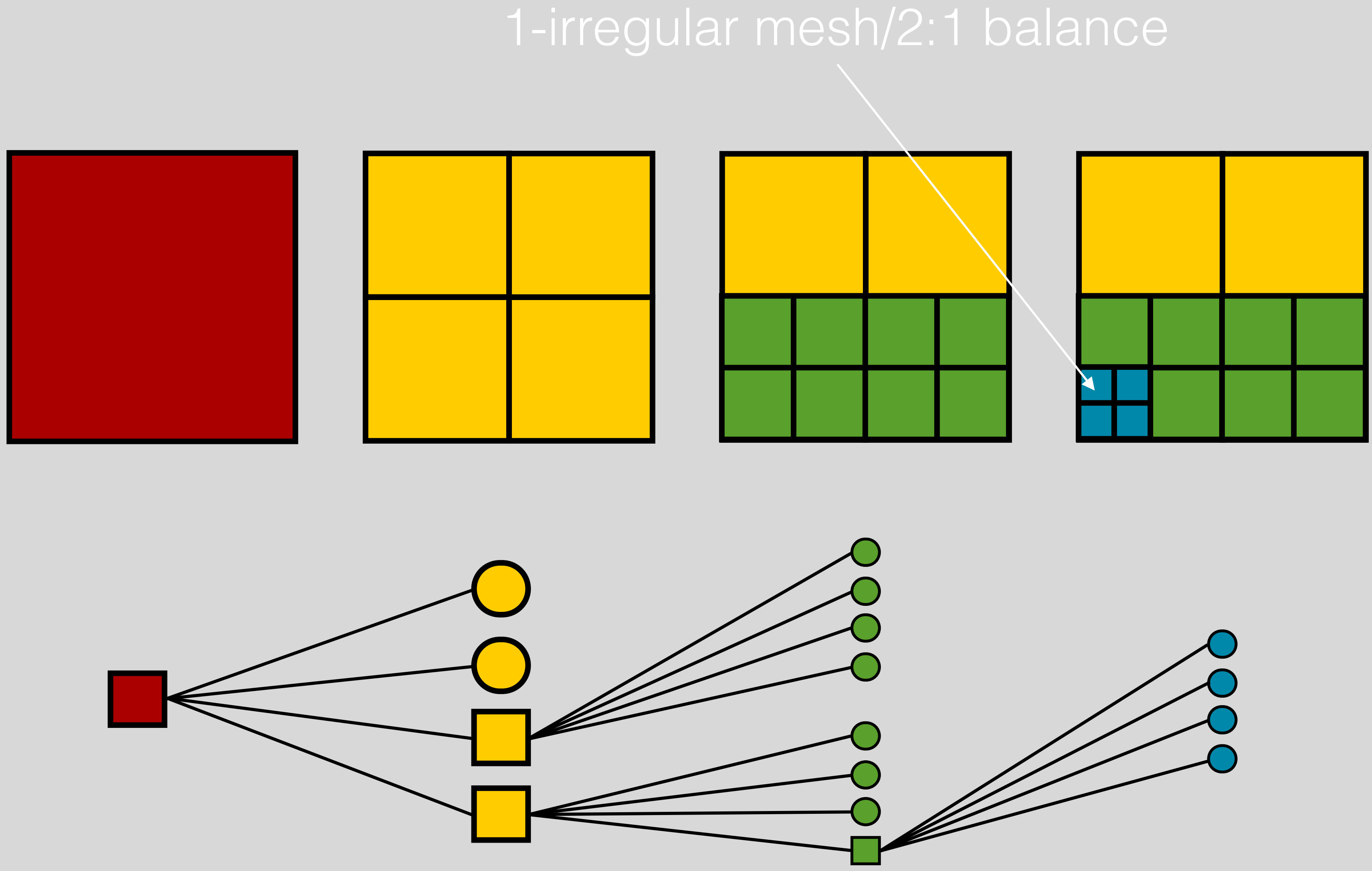
- **Core problem:** Geometry is tightly coupled with discretisation.
- How will we deal with:
  - Dynamic topology eg. cutting.
  - Clinical environments.
  - Refinement.
  - Complex microstructures.

Pipelines to analysis





The method

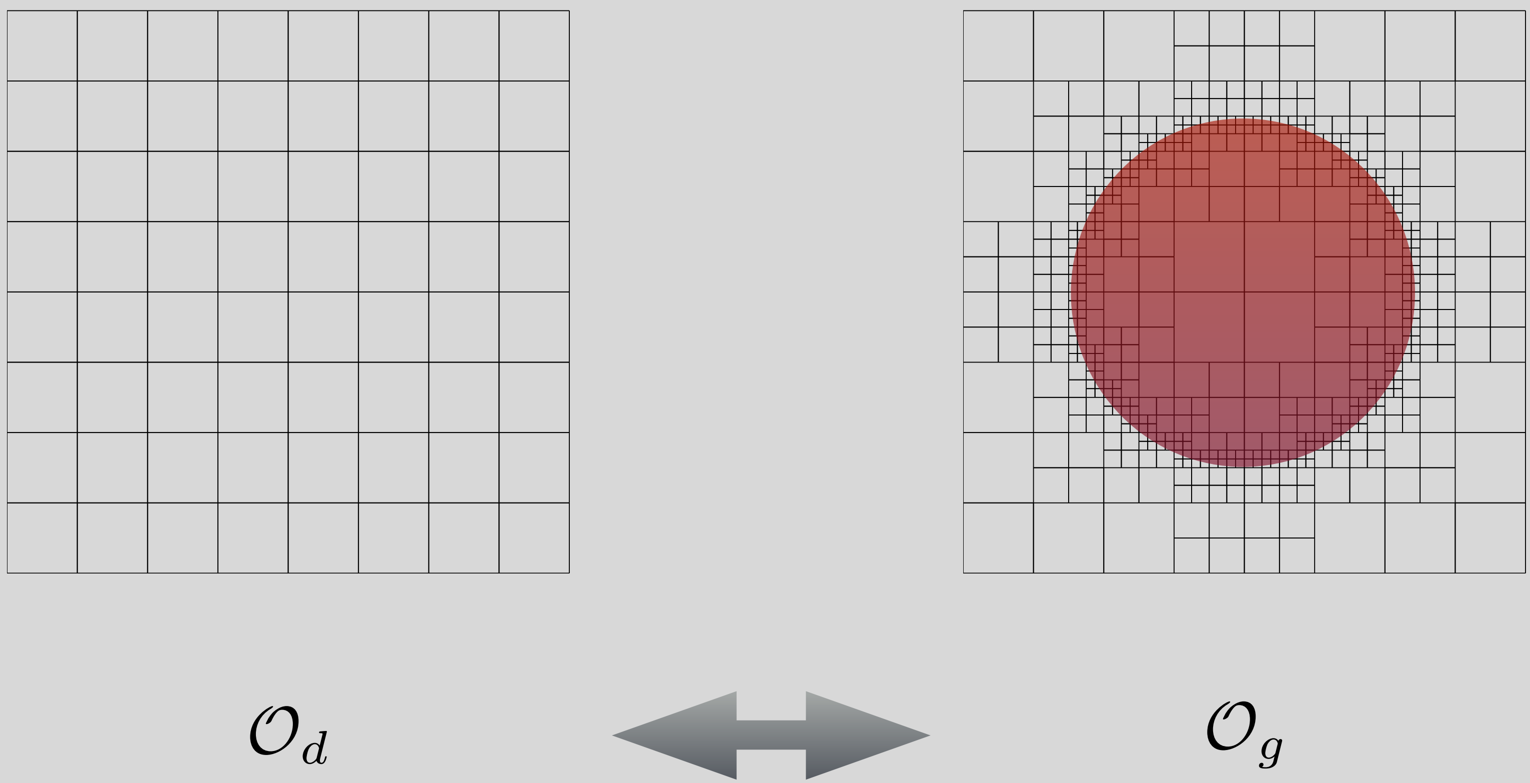


Octree data structure

Nested Octree

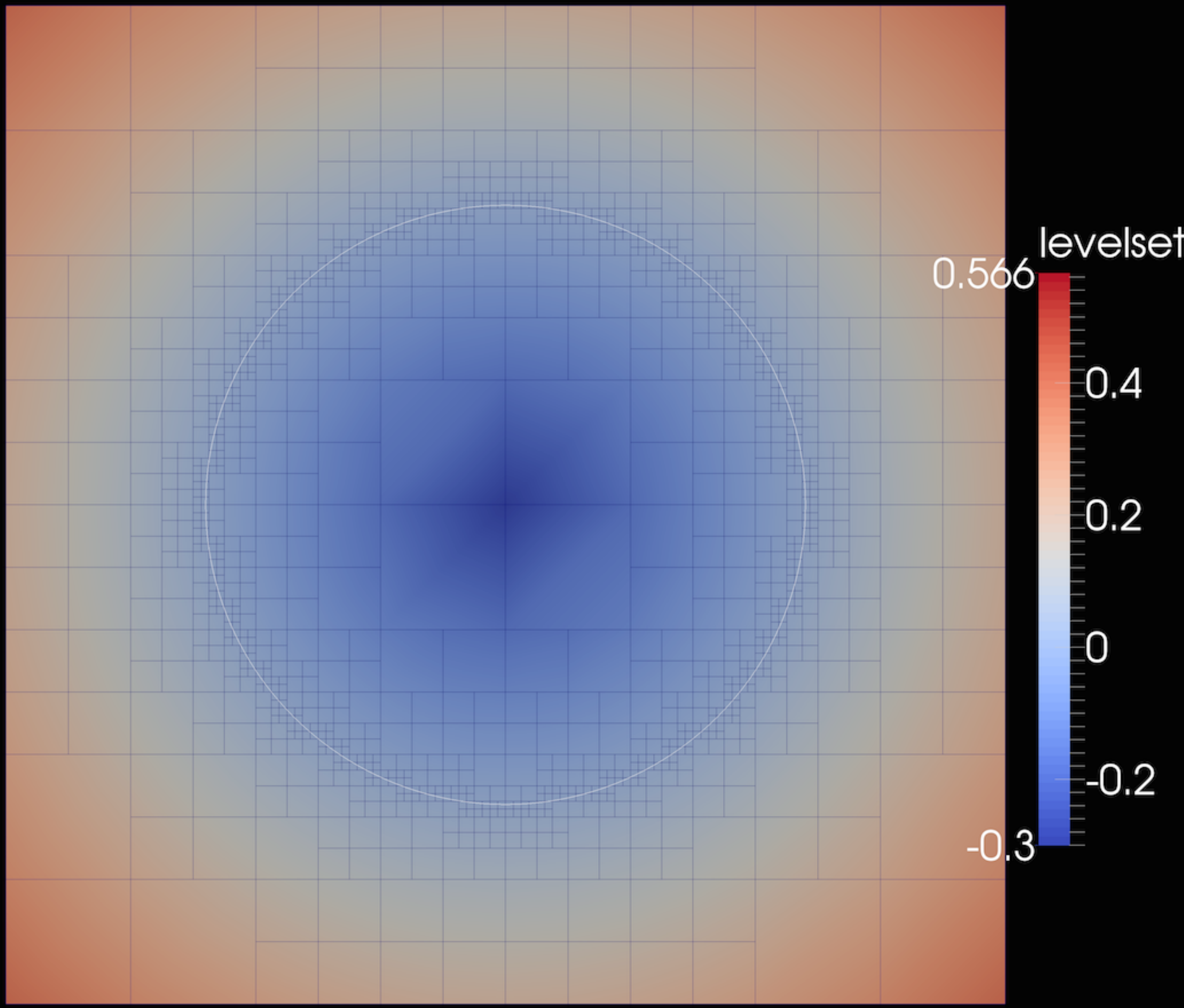
Discretisation

Geometry

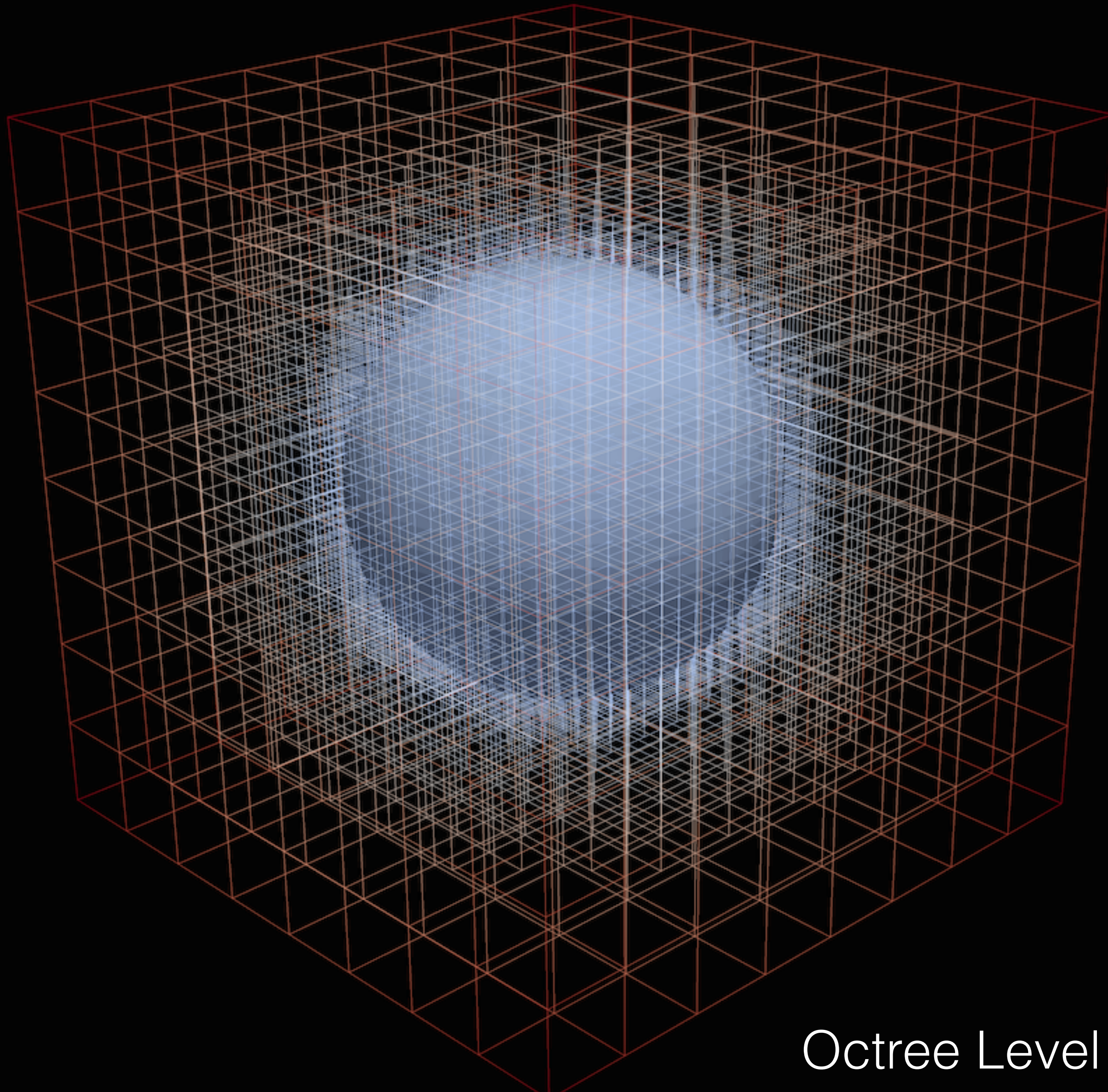


$\mathcal{O}_d$ $\longleftrightarrow$ $\mathcal{O}_g$

$\mathcal{M}$



Quadtree Level 7/Level 4

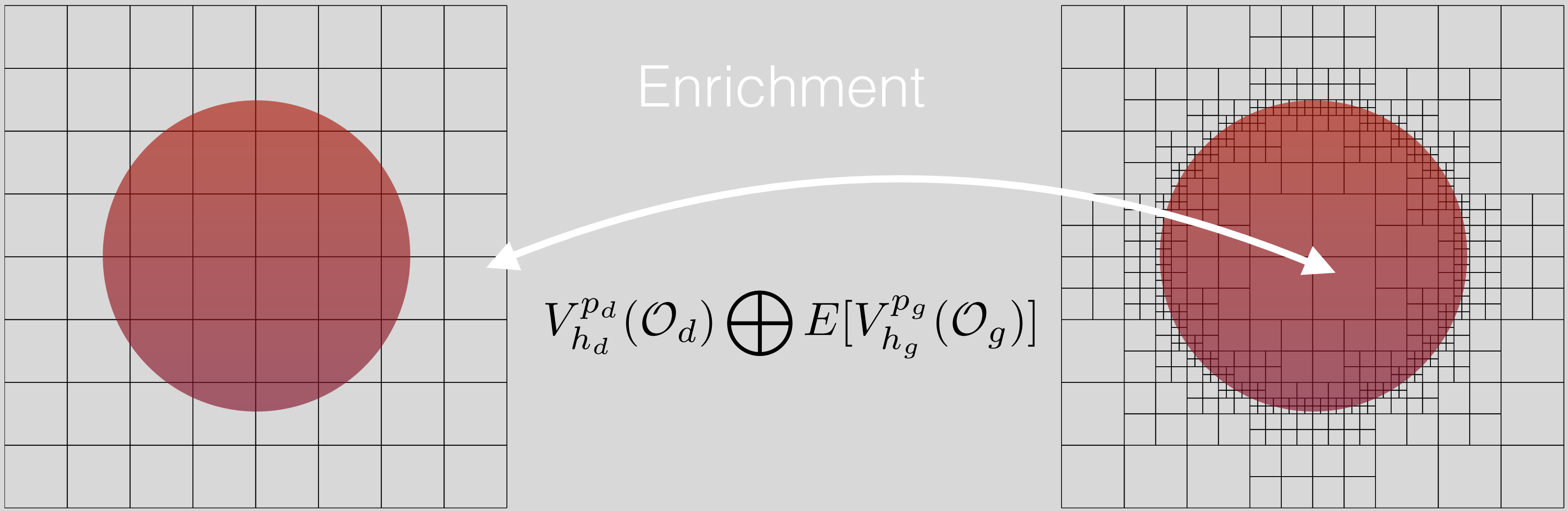


Octree Level 5/Level 3

6a2e86c

Surface

How to transfer geometric information back to the discretisation?

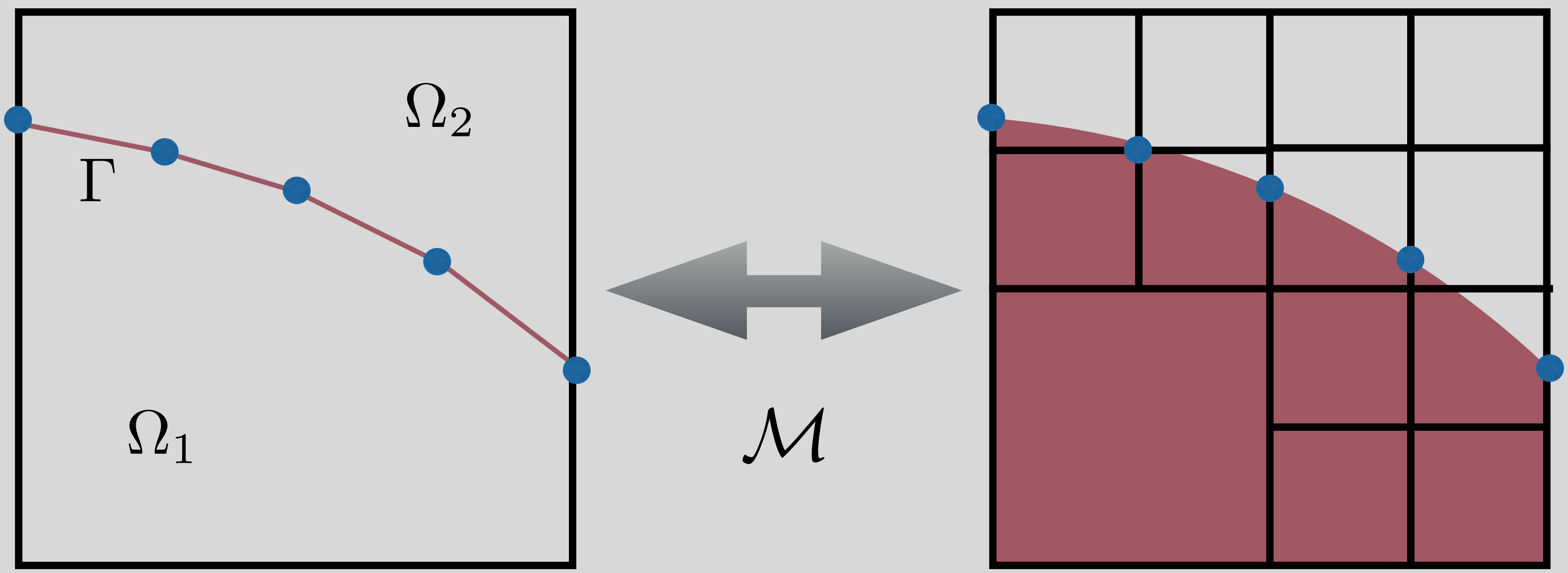


$$V_{h_d}^{p_d}(\mathcal{O}_d) \bigoplus E[V_{h_g}^{p_g}(\mathcal{O}_g)]$$

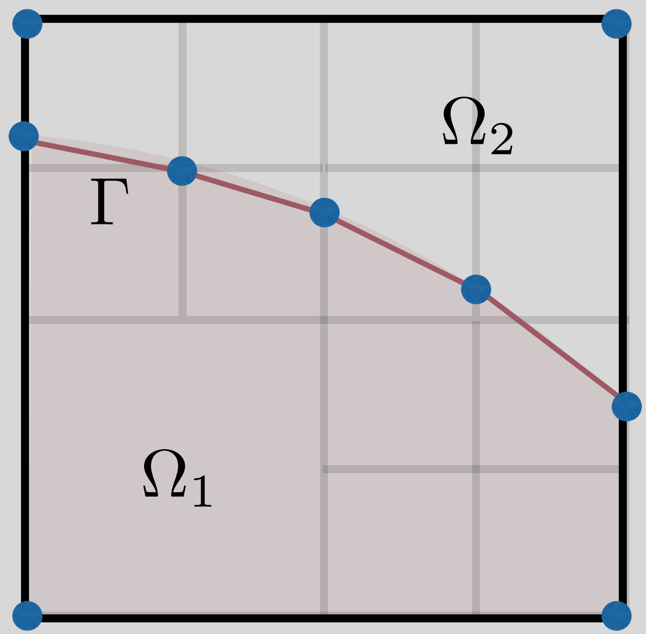
$$V_{h_d}^{p_d}(\mathcal{O}_d) \qquad p_d > p_g \qquad V_{h_g}^{p_g}(\mathcal{O}_g)$$

$$h_d > h_g \qquad p_g = 1$$

For each enriched cell in the discretisation…

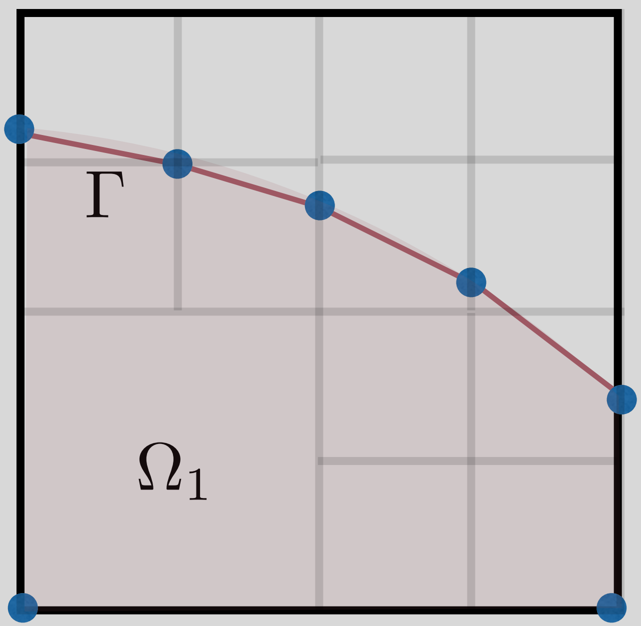


generate local Delaunay triangulation…



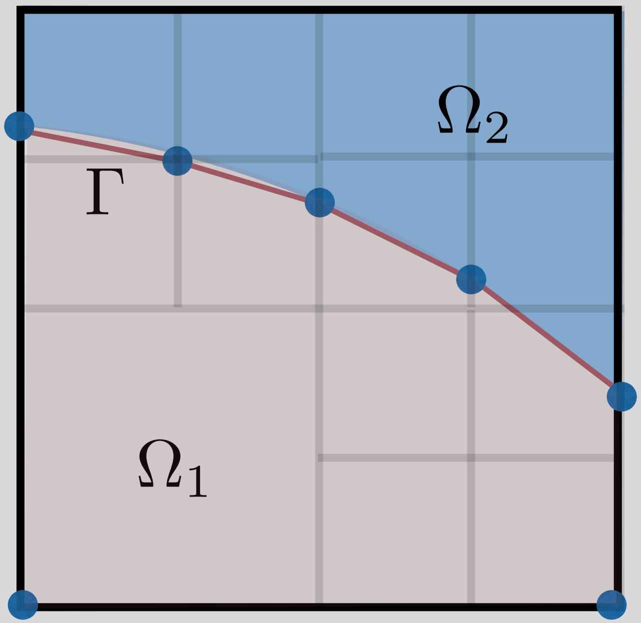
Case 1: boundary

finite cell method, implicit boundary method…



Case 2: inclusion

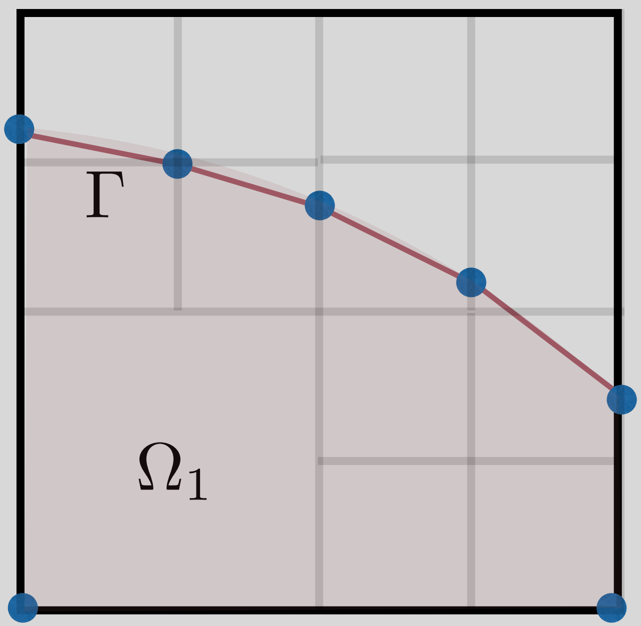
XFEM, PUM…



$$\mathbf{u}_h(\mathbf{x}) = \sum_{i=1}^{N} \mathbf{N}_i u_i + \sum_{i=1}^{N} \mathbf{N}_i \sum_{j=1}^{M} \psi_j(\mathbf{x}) a_i^j$$

Case 3: Dirichlet Boundary

Nitsche's method, Lagrange multipliers…



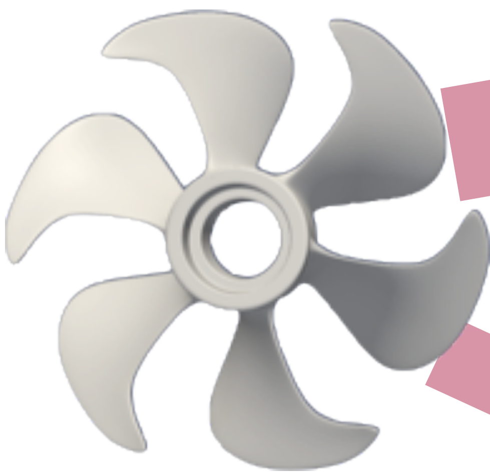
Faculty of Sciences, Technology and Communication

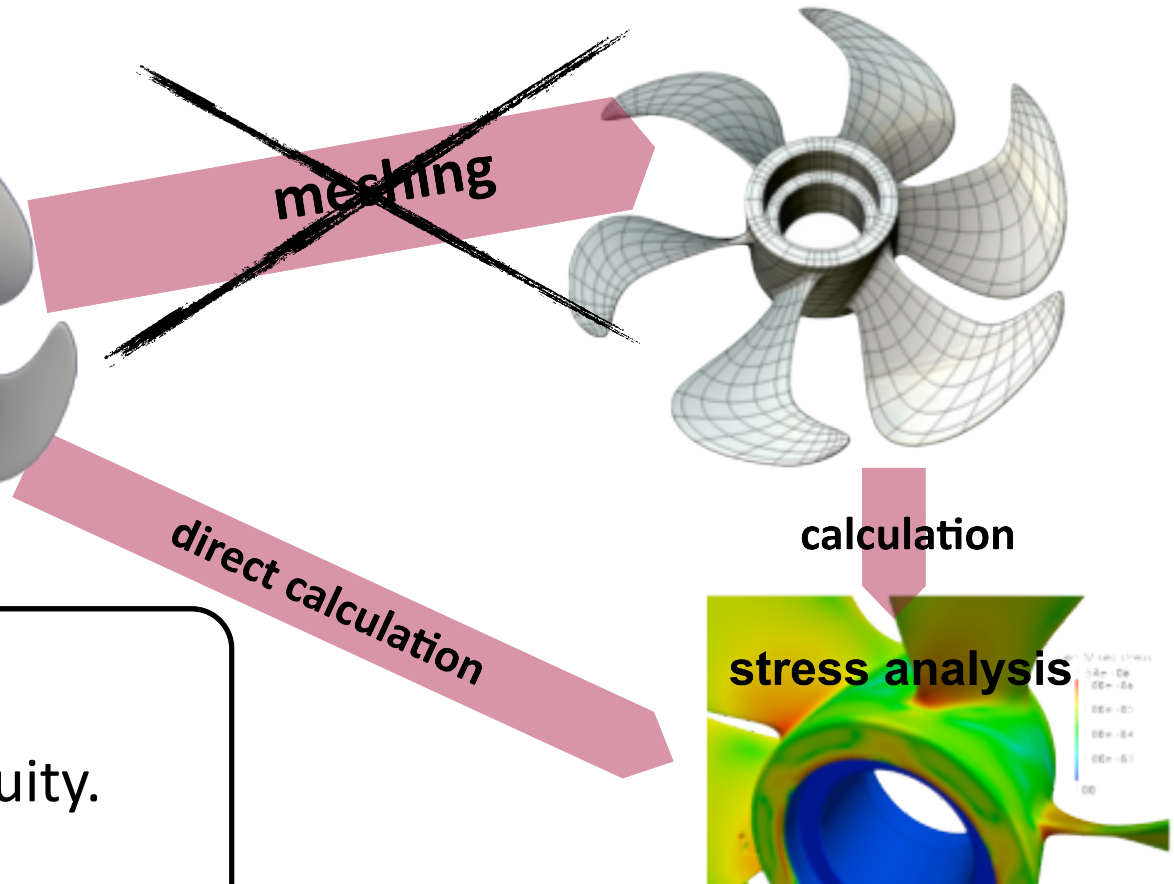
Paradigm 2 : IGA

Couple Geometry and Approximation

UNIVERSITÉ DU LUXEMBOURG

Approximate the unknown fields with the same basis functions (NURBS, T-splines …) as that used to generate the CAD model





- Exact geometry.
- High order continuity.
- *hpk*-refinement

1. Generate a **volume** discretization using the **surface** geometry only?

2. Realistic solids can in general not be represented by only one volume (patch) and multiple **patch**es must be **glue**d together to avoid "leaks" (Nitsche, T-splines, PHT-splines, RL/LR-splines)

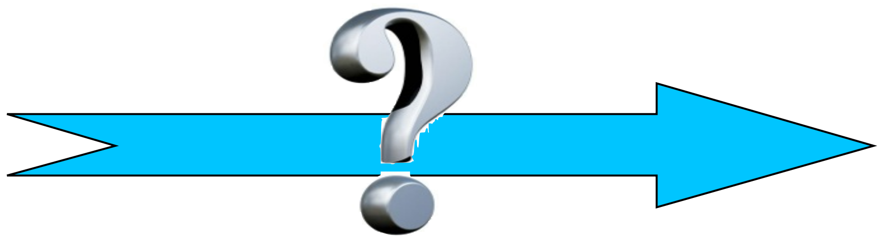
3. Refinement must be done everywhere in the domain (T, PHT… splines)

3 KEY QUESTIONS FOR IGA

With Gang Xu: Generalized IGA - Field-independent geometry approximation

Isogeometric Analysis with BEM

Boundary representation



Domain representation

1. IGABEM with NURBS for 2D elastic problems (Simpson, *et al*. CMAME, 2011).

2. IGABEM with T-splines for 3D elastic problems (Scott, *et al.* CMAME, 2012).

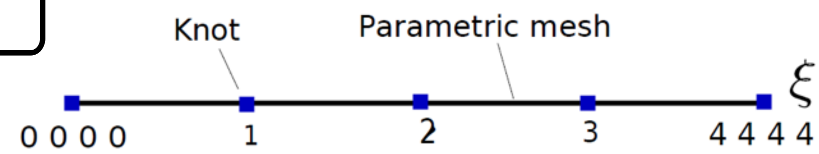
3. IGABEM with T-splines for 3D acoustic problems (Simpson, *et al*. 2013 - MAFELAP2013 TH1515).

Difficulties in dealing with nonlinear problems and non-homogeneous materials.

Knot vector

a non-decreasing set of coordinates in the parametric space.

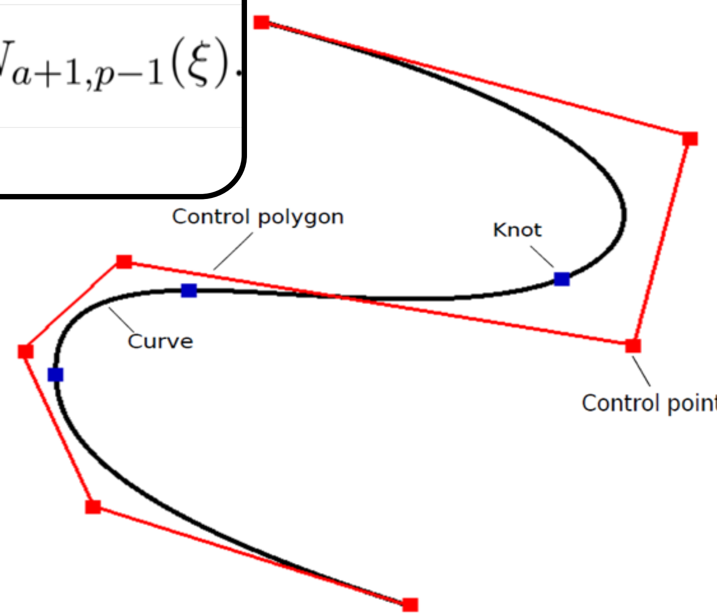
$$\Xi = \{\xi_1,\ \xi_2,\ \cdots,\ \xi_{n+p+1}\}$$



B-spline basis function

$$N_{a,0}(\xi) = \begin{cases} 1, & \text{if } \xi_a \le \xi < \xi_{a+1} \\ 0, & \text{otherwise.} \end{cases}$$

$$N_{a,p}(\xi) = \frac{\xi - \xi_a}{\xi_{a+p} - \xi_a} N_{a,p-1}(\xi) + \frac{\xi_{a+p+1} - \xi}{\xi_{a+p+1} - \xi_{a+1}} N_{a+1,p-1}(\xi).$$



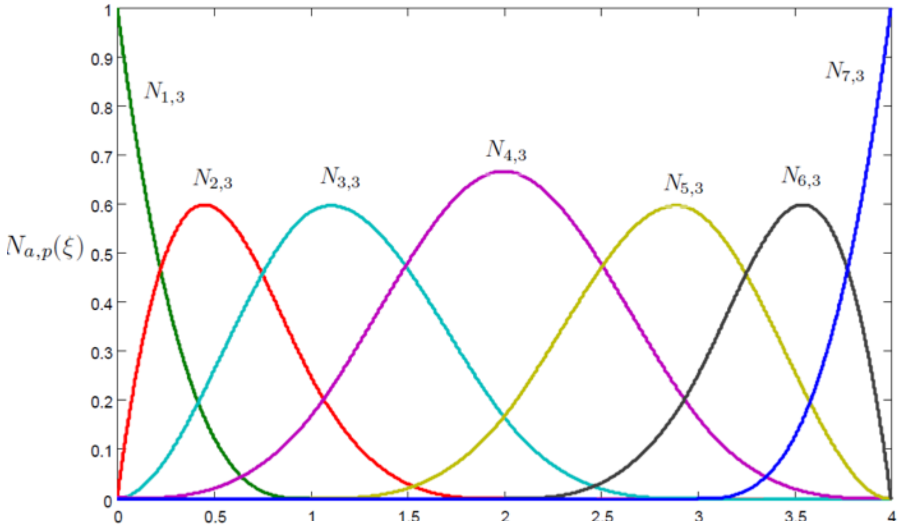
NURBS basis function

$$R_{a,p}(\xi) = \frac{N_{a,p}(\xi) w_a}{W(\xi)} = \frac{N_{a,p}(\xi) w_a}{\sum_{\hat{a}=1}^{n} N_{\hat{a},p} w_{\hat{a}}},$$

Properties of NURBS

- Partition of Unity

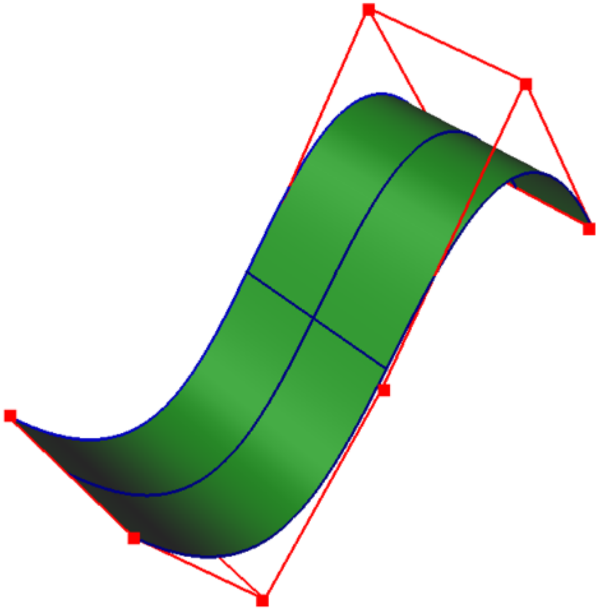
$$\sum_{i=1}^{n} R_{i,p}(\xi) = 1$$



- Non-negative

- *p-1* continuous derivatives

- Tensor product property



$$\mathbf{S}(\xi,\eta) = \sum_{i=1}^{n}\sum_{j=1}^{m} R^{1}{}_{i,p}(\xi) R^{2}{}_{j,q}(\eta) \mathbf{B}_{i,j}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m} R^{1}{}_{i,p}(\xi) R^{2}{}_{j,q}(\eta) = \left(\sum_{i=1}^{n} R^{1}{}_{i,p}(\xi)\right)\left(\sum_{j=1}^{m} R^{2}{}_{j,q}(\eta)\right)$$

NURBS to T-splines

www.tsplines.com

(NURBS geometry)

www.tsplines.com

(T-splines geometry)

NURBS

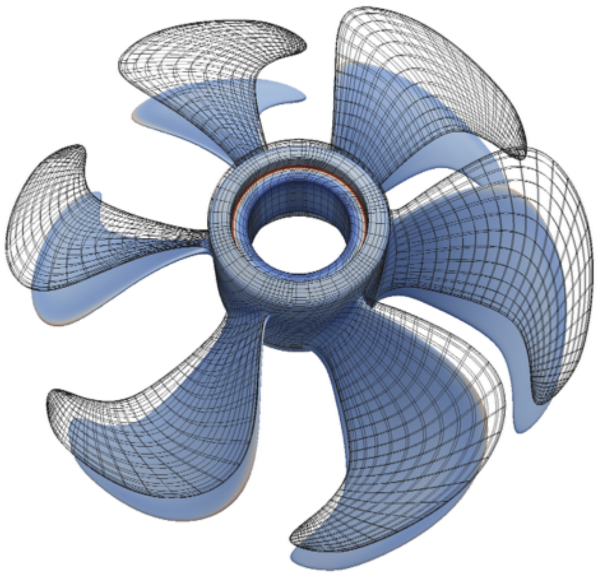
- No watertight geometry
- No local refinement scheme

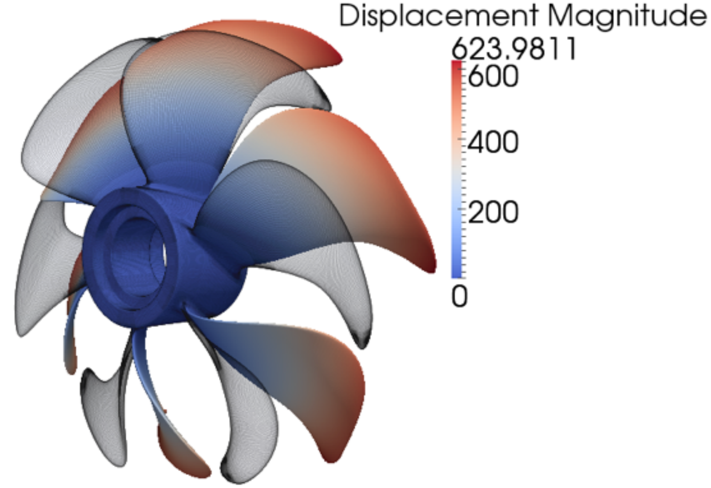
T-splines

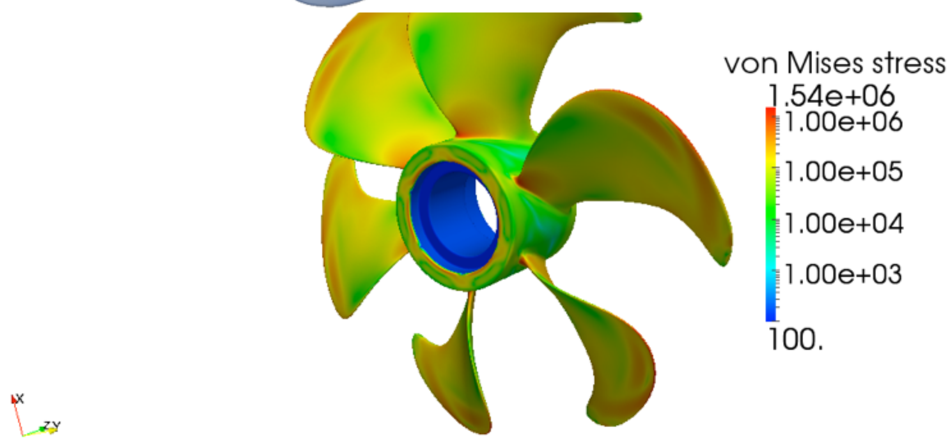
- Local knot vector (as Point-based splines)
- Global topology

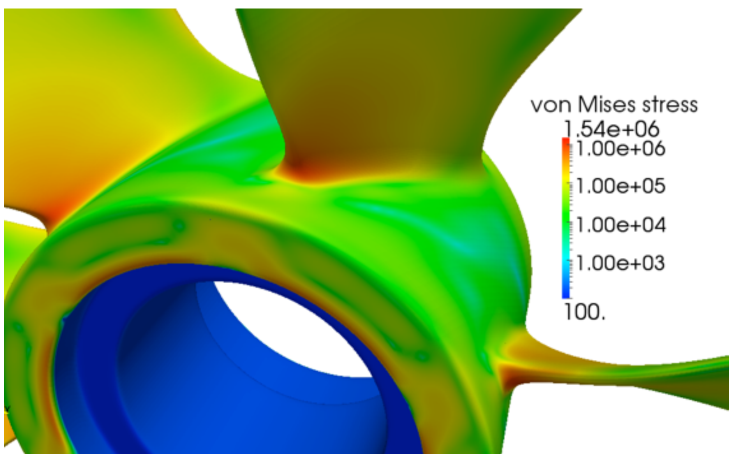
Y. Bazilevs, V.M. Calo, J.A. Cottrell, J.A. Evans, T.J.R. Hughes, S. Lipton, M.A. Scott, and T.W. Sederberg. Isogeometric analysis using T-splines. CMAME, 199(5-8):229–263, 2010.

Propeller: NURBS would require several patches - single patch T-splines









Isogeometric boundary element analysis using unstructured T-splines
MA Scott, RN Simpson, JA Evans, S Lipton, SPA Bordas, TJR Hughes, TW Sederberg
CMAME, 2013. http://orbilu.uni.lu/handle/10993/11850

Part II. Some recent advances in enriched FEM

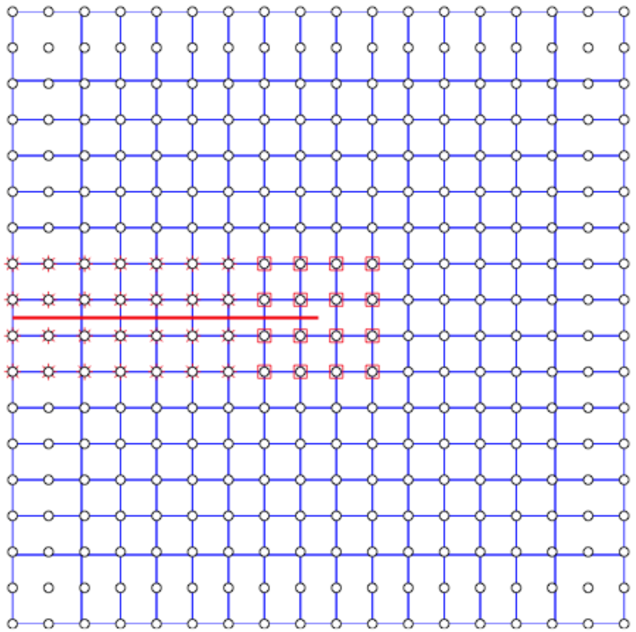
Faculty of Sciences,
Technology
and Communication

Handling discontinuities in isogeometric formulations

with Nguyen Vinh Phu, Marie Curie Fellow

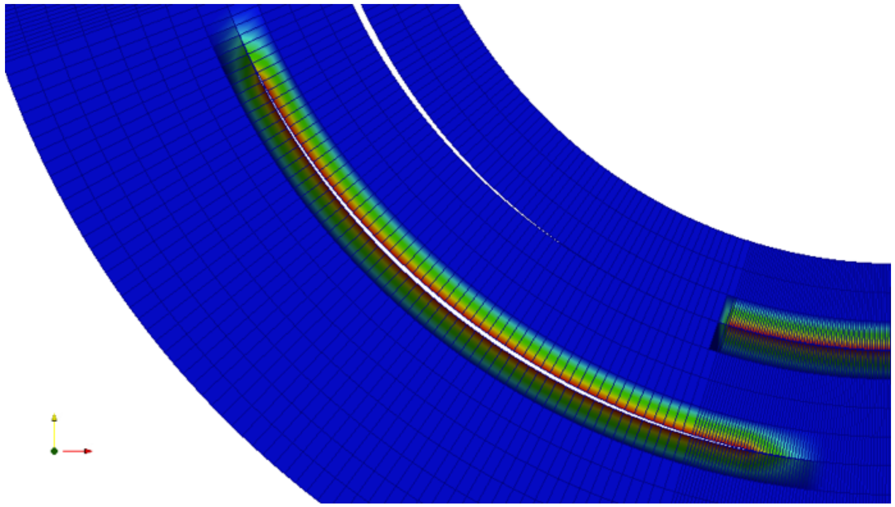
Discontinuities modeling

PUM enriched methods



- IGA: link to CAD and accurate stress fields
- XFEM: no remeshing

Mesh conforming methods



- IGA: link to CAD and accurate stress fields
- Apps: delamination

PUM enriched methods (XIGA)

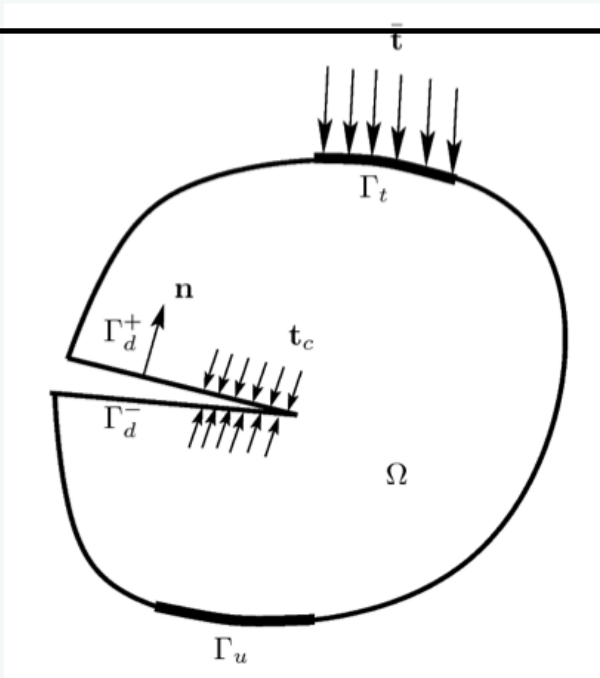
$$\mathbf{u}^h(\mathbf{x}) = \sum_{I \in \mathcal{S}} R_I(\mathbf{x}) \mathbf{u}_I + \sum_{J \in \mathcal{S}^c} R_J(\mathbf{x}) \Phi(\mathbf{x}) \mathbf{a}_J$$

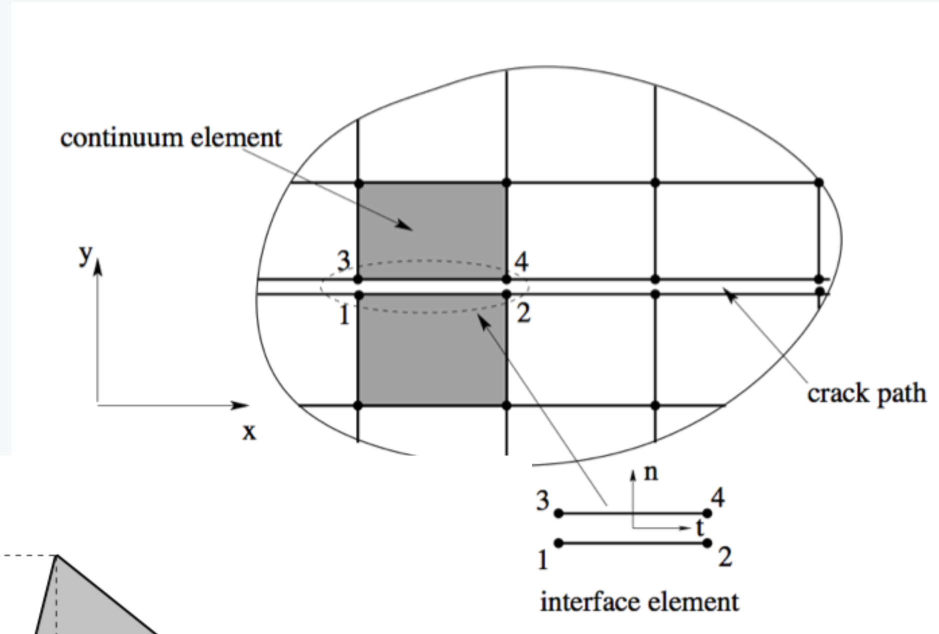
NURBS basis functions

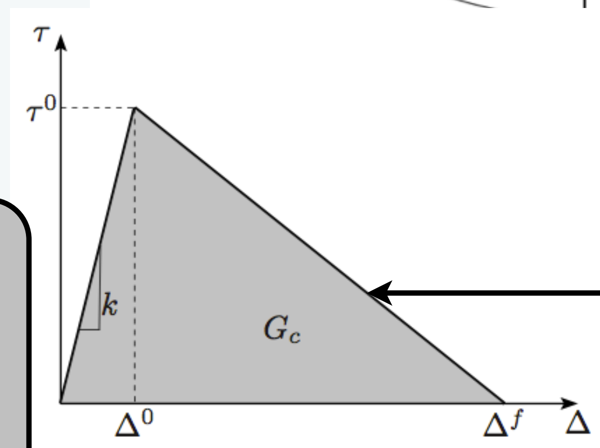
enrichment functions

1. E. De Luycker, D. J. Benson, T. Belytschko, Y. Bazilevs, and M. C. Hsu. X-FEM in isogeometric analysis for linear fracture mechanics. IJNME, 87(6):541–565, 2011.
2. S. S. Ghorashi, N. Valizadeh, and S. Mohammadi. Extended isogeometric analysis for simulation of stationary and propagating cracks. IJNME, 89(9): 1069–1101, 2012.
3. D. J. Benson, Y. Bazilevs, E. De Luycker, M.-C. Hsu, M. Scott, T. J. R. Hughes, and T. Belytschko. A generalized finite element formulation for arbitrary basis functions: From isogeometric analysis to XFEM. IJNME, 83(6):765–785, 2010.
4. A. Tambat and G. Subbarayan. Isogeometric enriched field approximations. CMAME, 245–246:1 – 21, 2012.

Delamination analysis with cohesive elements (standard approach)



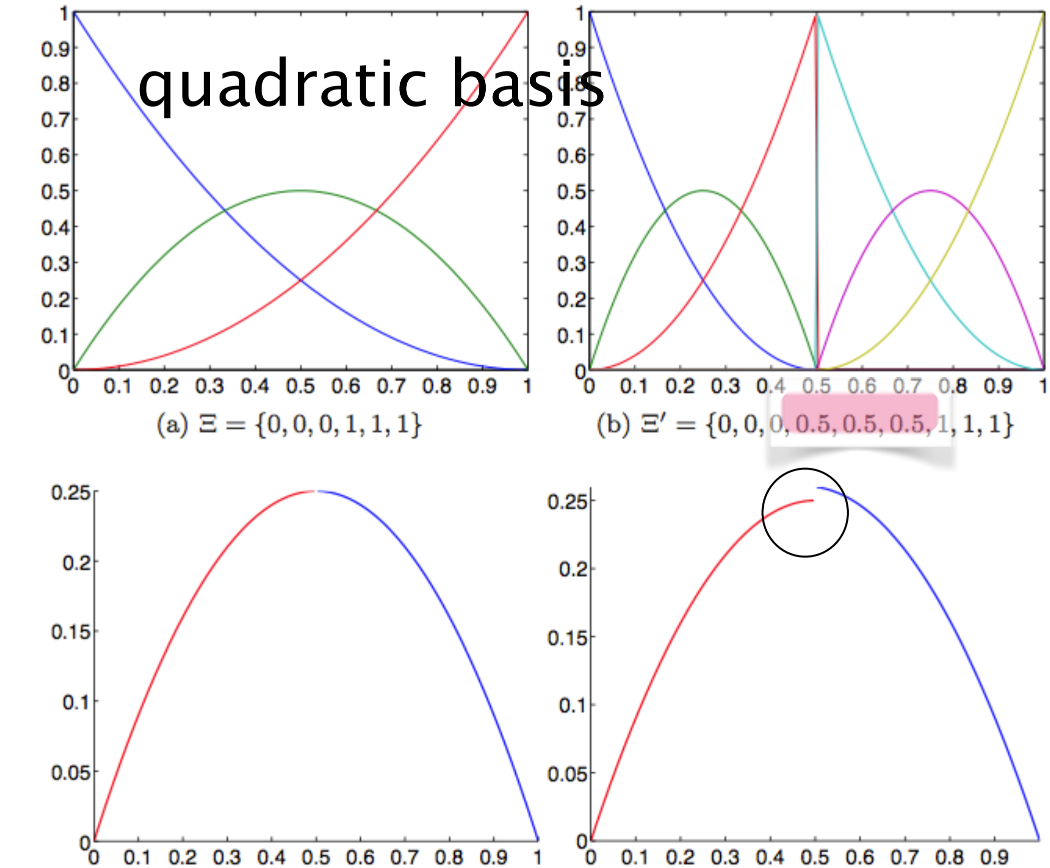




- No link to CAD
- Long preprocessing
- Refined meshes

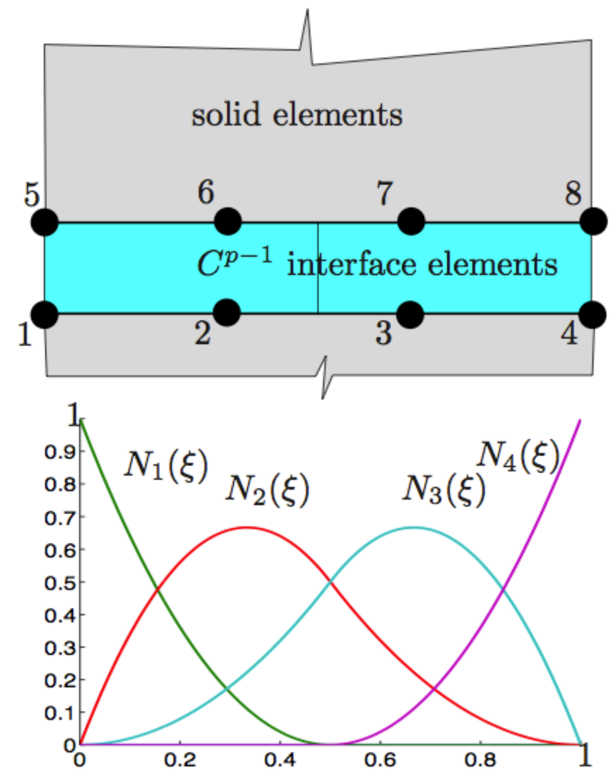
$$\int_\Omega \delta \mathbf{u} \cdot \mathbf{b} \mathrm{d}\Omega + \int_{\Gamma_t} \delta \mathbf{u} \cdot \bar{\mathbf{t}} \mathrm{d}\Gamma_t = \int_\Omega \delta \boldsymbol{\epsilon} : \boldsymbol{\sigma}(\mathbf{u}) \mathrm{d}\Omega + \int_{\Gamma_d} \delta [\![\mathbf{u}]\!] \cdot \mathbf{t}^c([\![\mathbf{u}]\!]) \mathrm{d}\Gamma_d$$

Isogeometric cohesive elements



(a) $\Xi = \{0, 0, 0, 1, 1, 1\}$

(b) $\Xi' = \{0, 0, 0, 0.5, 0.5, 0.5, 1, 1, 1\}$

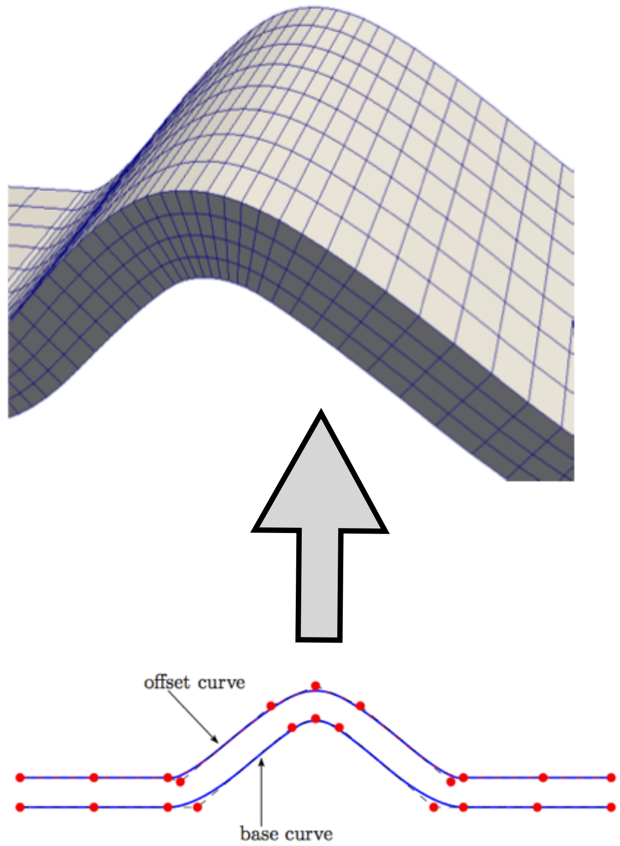


Knot insertion

1. C. V. Verhoosel, M. A. Scott, R. de Borst, and T. J. R. Hughes. An isogeometric approach to cohesive zone modeling. IJNME, 87(15):336–360, 2011.
2. V.P. Nguyen, P. Kerfriden, S. Bordas. Isogeometric cohesive elements for two and three dimensional composite delamination analysis, 2013, Arxiv.

Isogeometric cohesive elements: advantages

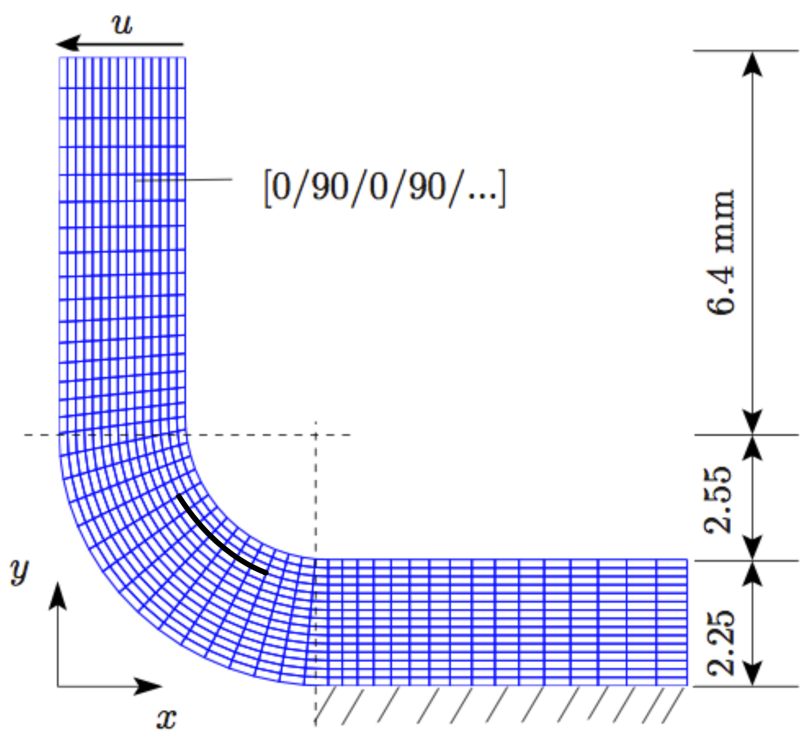
- Direct link to CAD
- Exact geometry
- Fast/straightforward generation of interface elements
- Accurate stress field
- Computationally cheaper

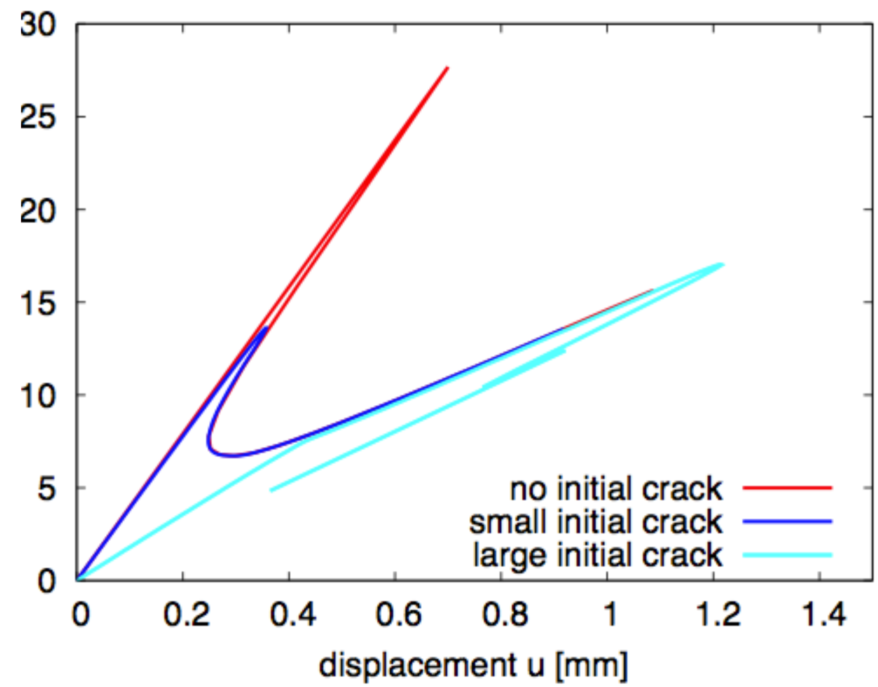


- 2D Mixed mode bending test (MMB)
- 2 x 70 quartic-linear B-spline elements
- Run time on a laptop 4GBi7: 6 s
- Energy arc-length control

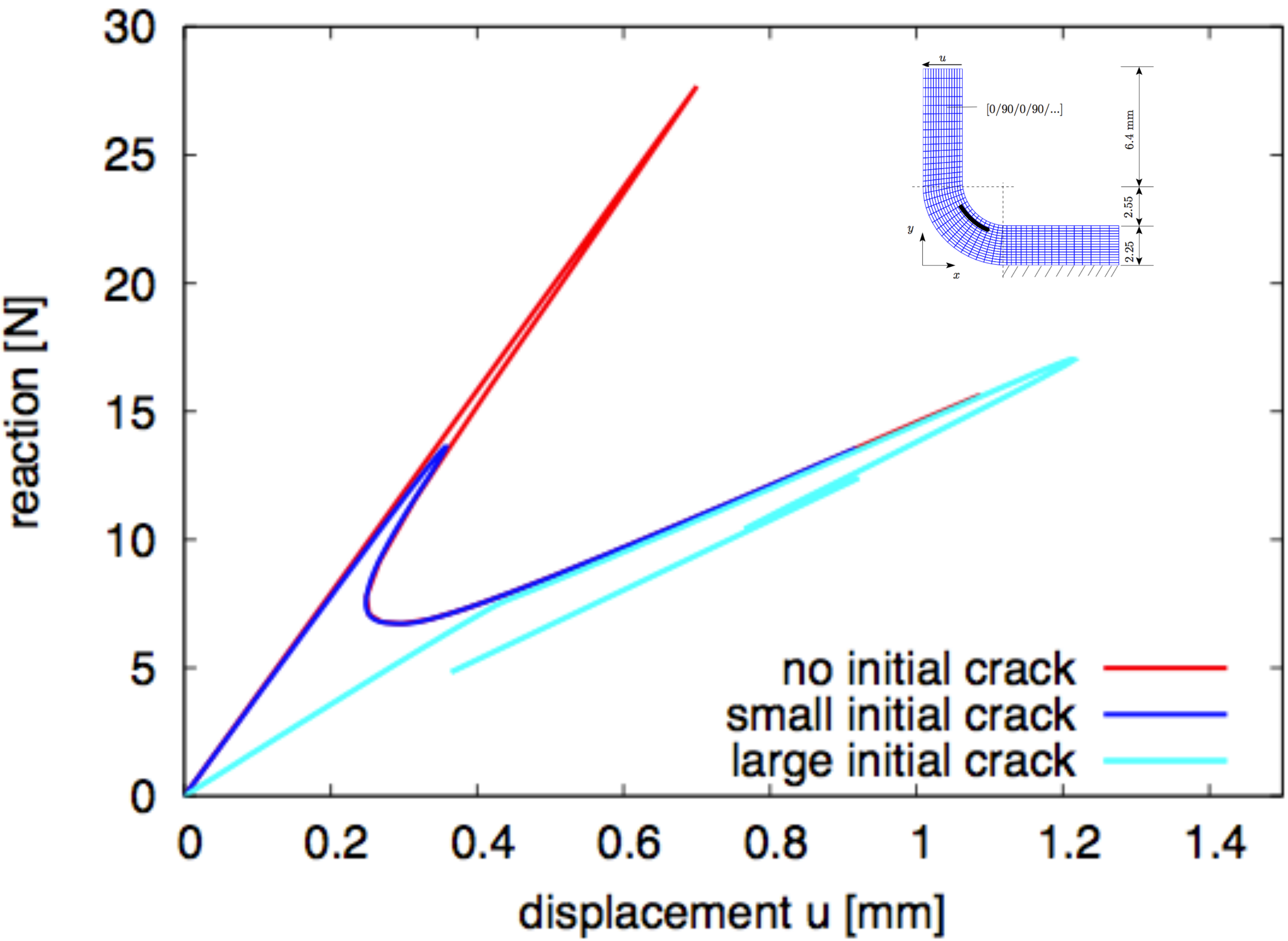
V. P. Nguyen and H. Nguyen-Xuan. High-order B-splines based finite elements for delamination analysis of laminated composites. Composite Structures, 102:261–275, 2013.

Isogeometric cohesive elements: 2D example



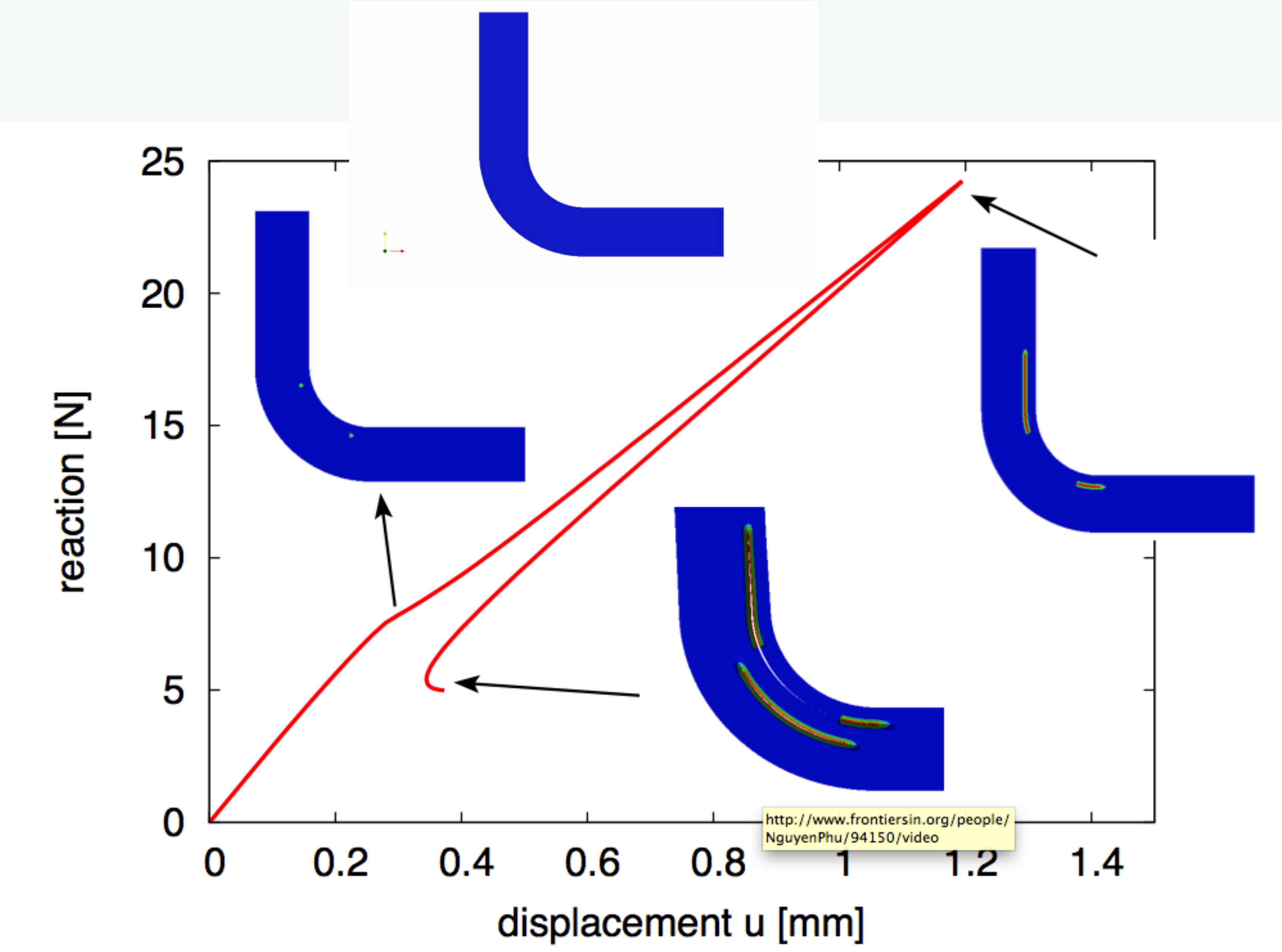


- Exact geometry by NURBS + direct link to CAD
- It is straightforward to vary
  (1) the number of plies and
  (2) # of interface elements:
- Suitable for parameter studies/design
- Solver: energy-based arc-length method (Gutierrez, 2007)

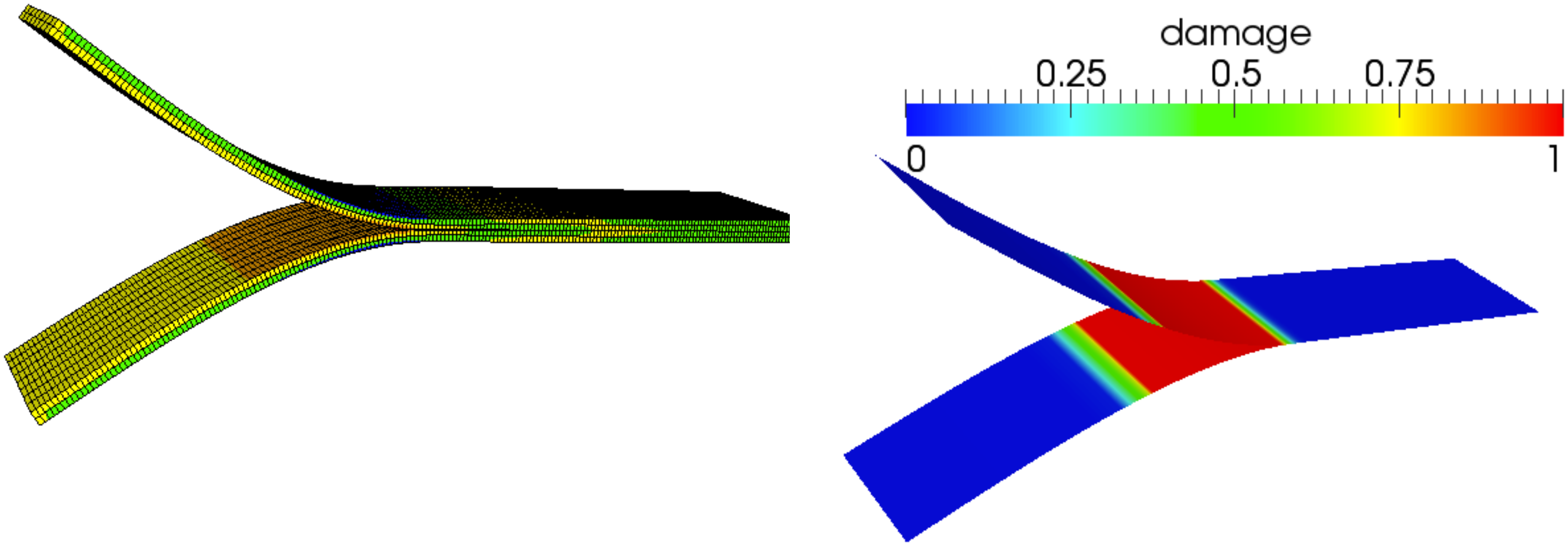


30
25
20
15
10
5
0
reaction [N]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
displacement u [mm]
no initial crack
small initial crack
large initial crack
u
[0/90/0/90/...]
6.4 mm
2.55
2.25
y
x

Isogeometric cohesive elements: 2D example

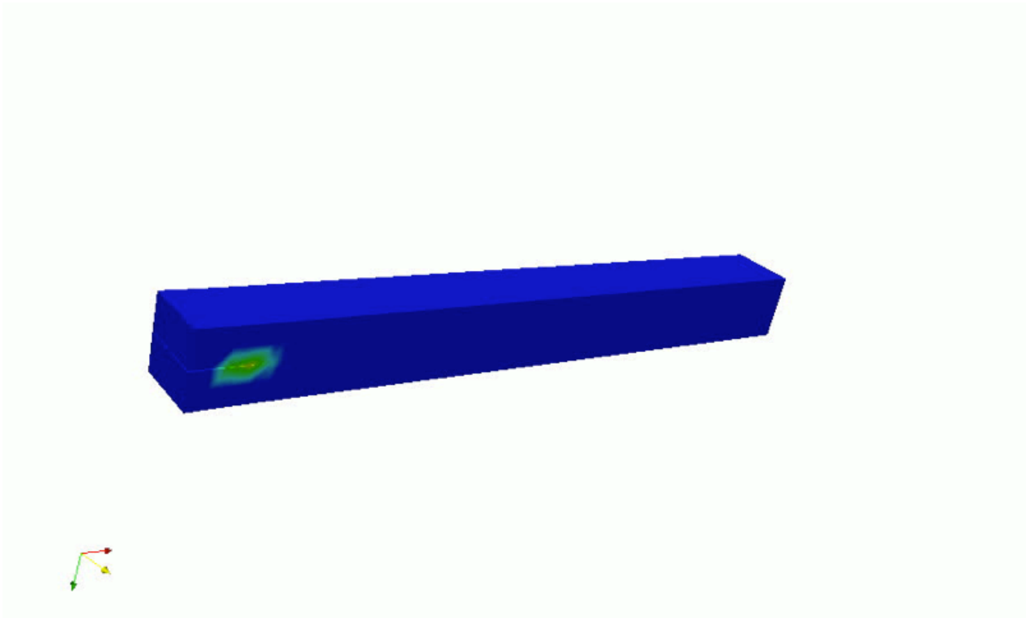


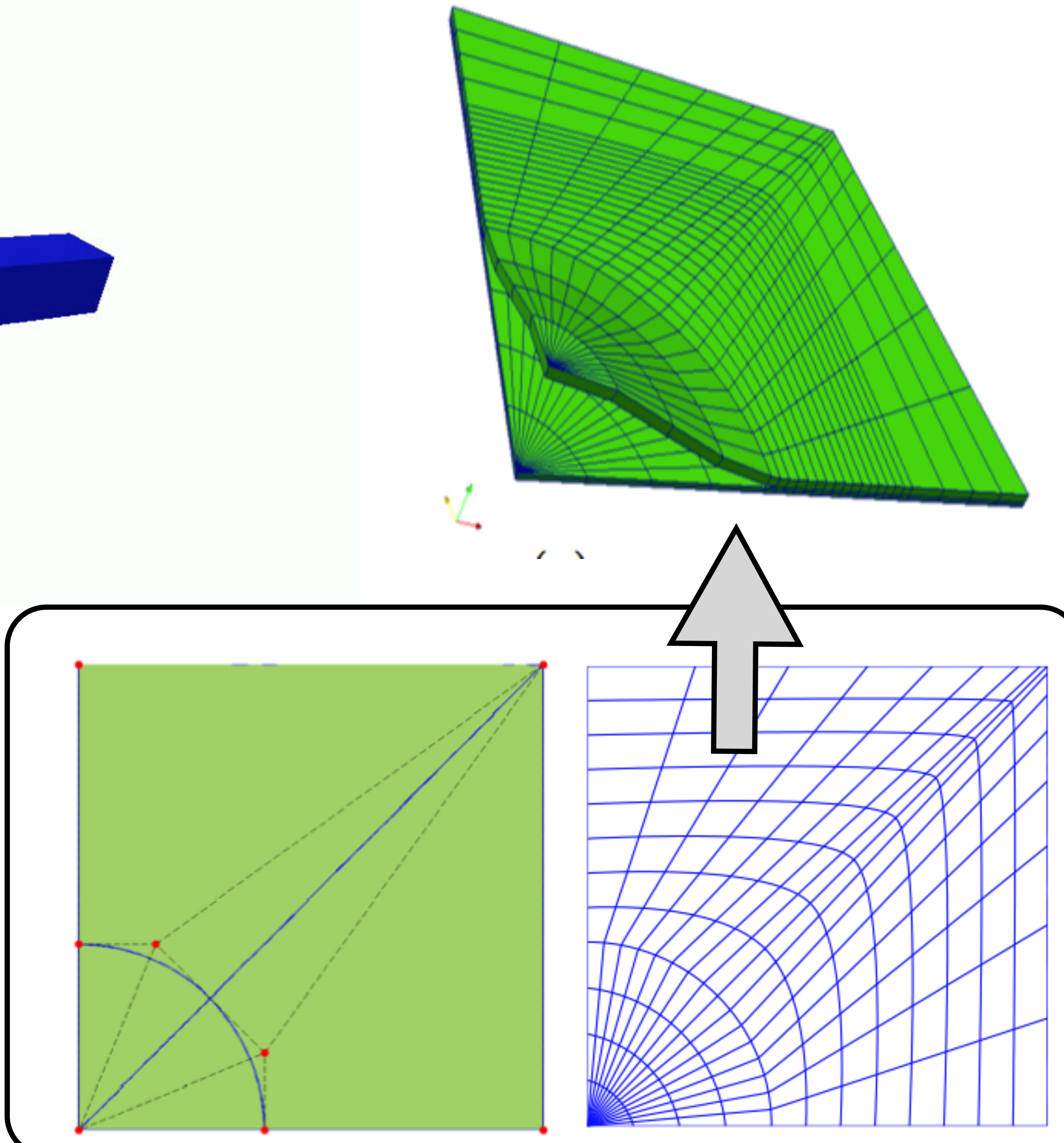
Isogeometric cohesive elements: 3D example with shells



- Rotation free B-splines shell elements (Kiendl et al. CMAME)
- Two shells, one for each lamina
- Bivariate B-splines cohesive interface elements in between

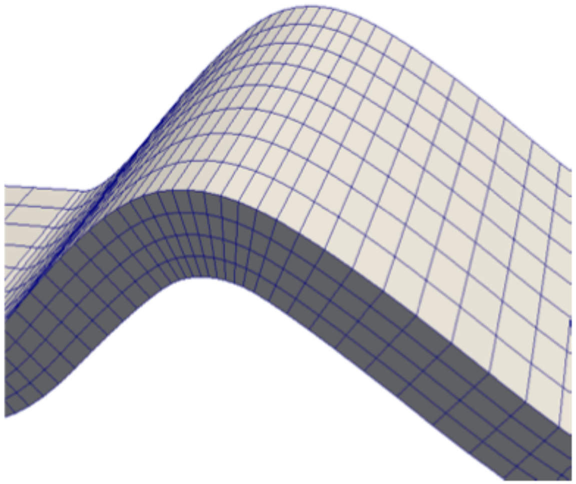
Isogeometric cohesive elements: 3D examples



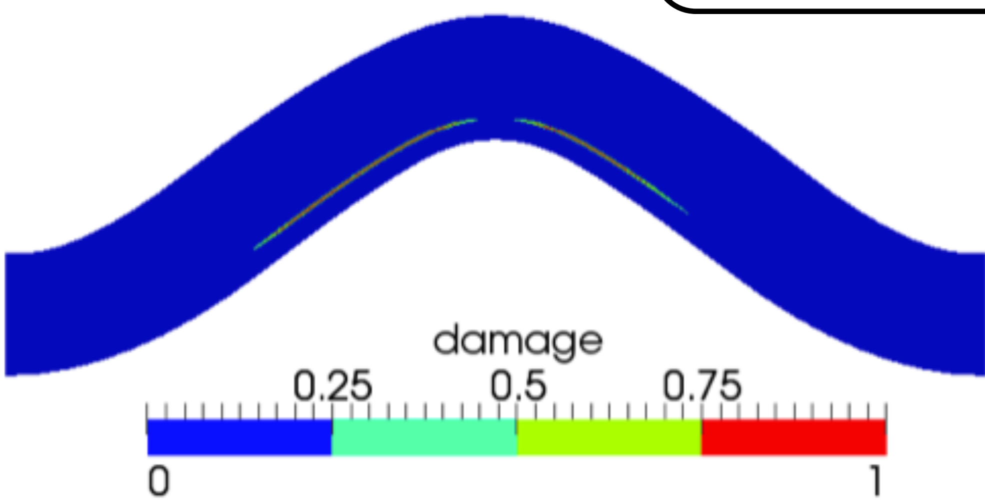


- cohesive elements for 3D meshes the same as 2D
- large deformations

Isogeometric cohesive elements



- singly curved thick-wall laminates
- geometry/displacements: NURBS
- trivariate NURBS from NURBS surface(*)
- cohesive surface interface elements



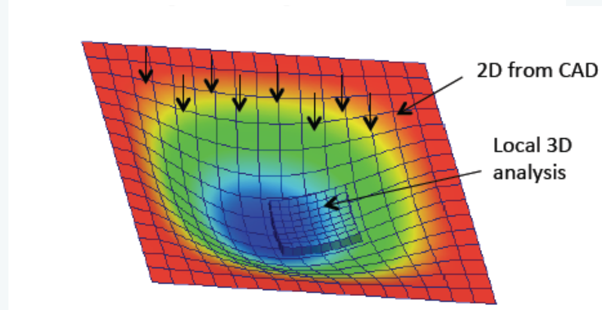
(*)V. P. Nguyen, P. Kerfriden, S.P.A. Bordas, and T. Rabczuk. An integrated design-analysis framework for three dimensional composite panels. Computer Aided Design, 2013. submitted.

Future work: model selection (continuum, plate, beam, shell?)

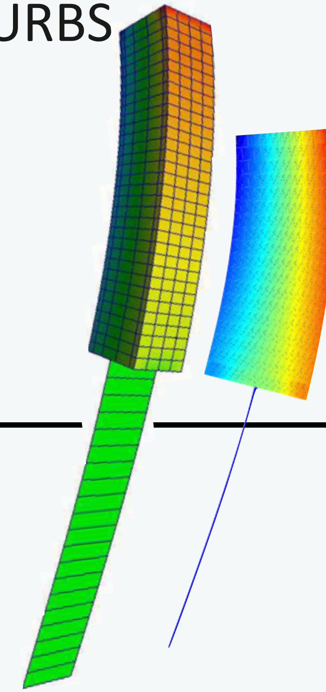
Model selection

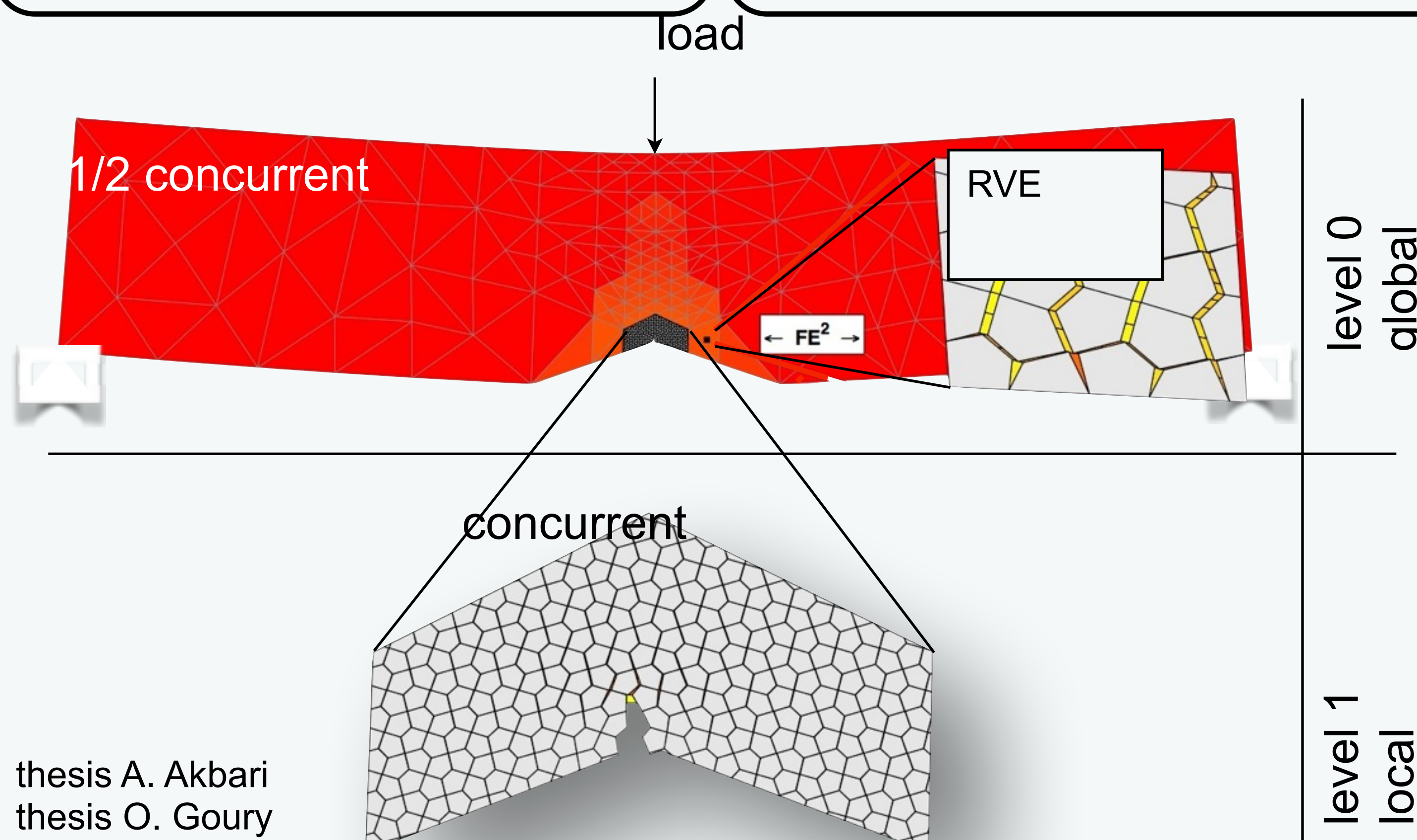
- Model with shells
- Identify “hot spots” - dual
- Couple with continuum
- Coarse-grain

- Nitsche coupling - NURBS-NURBS



[Nguyen et al., 2013]





thesis A. Akbari
thesis O. Goury

Part III. Application to multi-crack propagation

with Danas Sutula, President Scholar

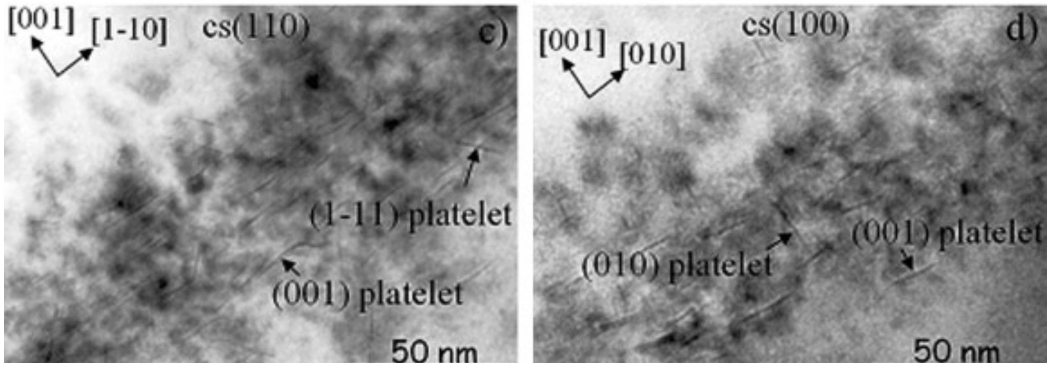
Faculty of Sciences,
Technology
and Communication

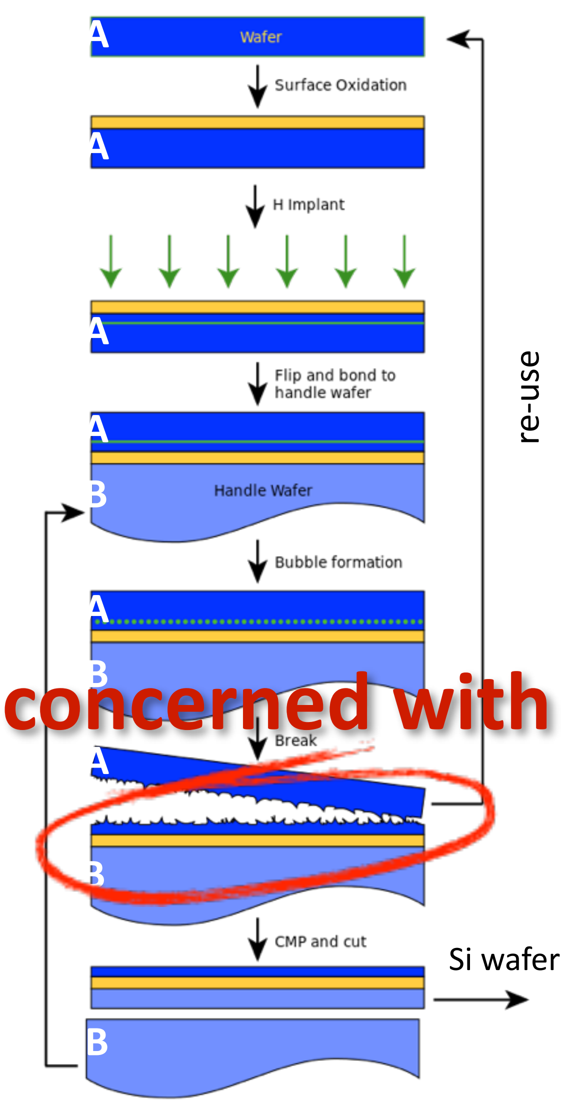
Numerical Modeling of SOI Wafer Splitting

Physical process

Manufacturing process: *SmartCut™*

- H^+ ionization of a thin surface of Si
- Bonding to a handle-wafer (stiffener)
- High temperature thermal annealing
- Nucleation and growth of cavities filled with H_2
- Pressure driven micro crack growth
- Coalescence and post-split fracture roughness





Objectives

Determine:

- micro crack nucleation points and direction
- multiple crack paths until coalescence
- time to complete fracture
- final surface roughness

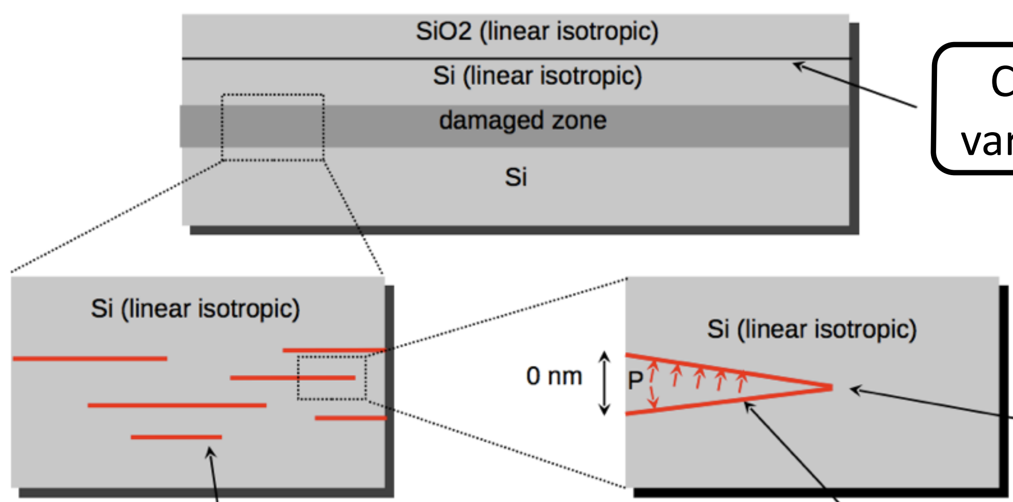
Model

Modeling cavities by zero thickness surfaces

- discontinuities in the displacement field

Linear elastic fracture mechanics (LEFM)

- infinite stress at crack tip, i.e. *singularity*



Cohesive interface with variation in surface energy

fracture criterion at the discontinuity tip

statistically distributed discontinuities

discontinuity subjected to H_2 pressure

Discretization: XFEM

Extended Finite Element Method (XFEM)

- Introduced by Ted Belytschko (1999) for elastic problems

Fracture of "XFEM" using XFEM

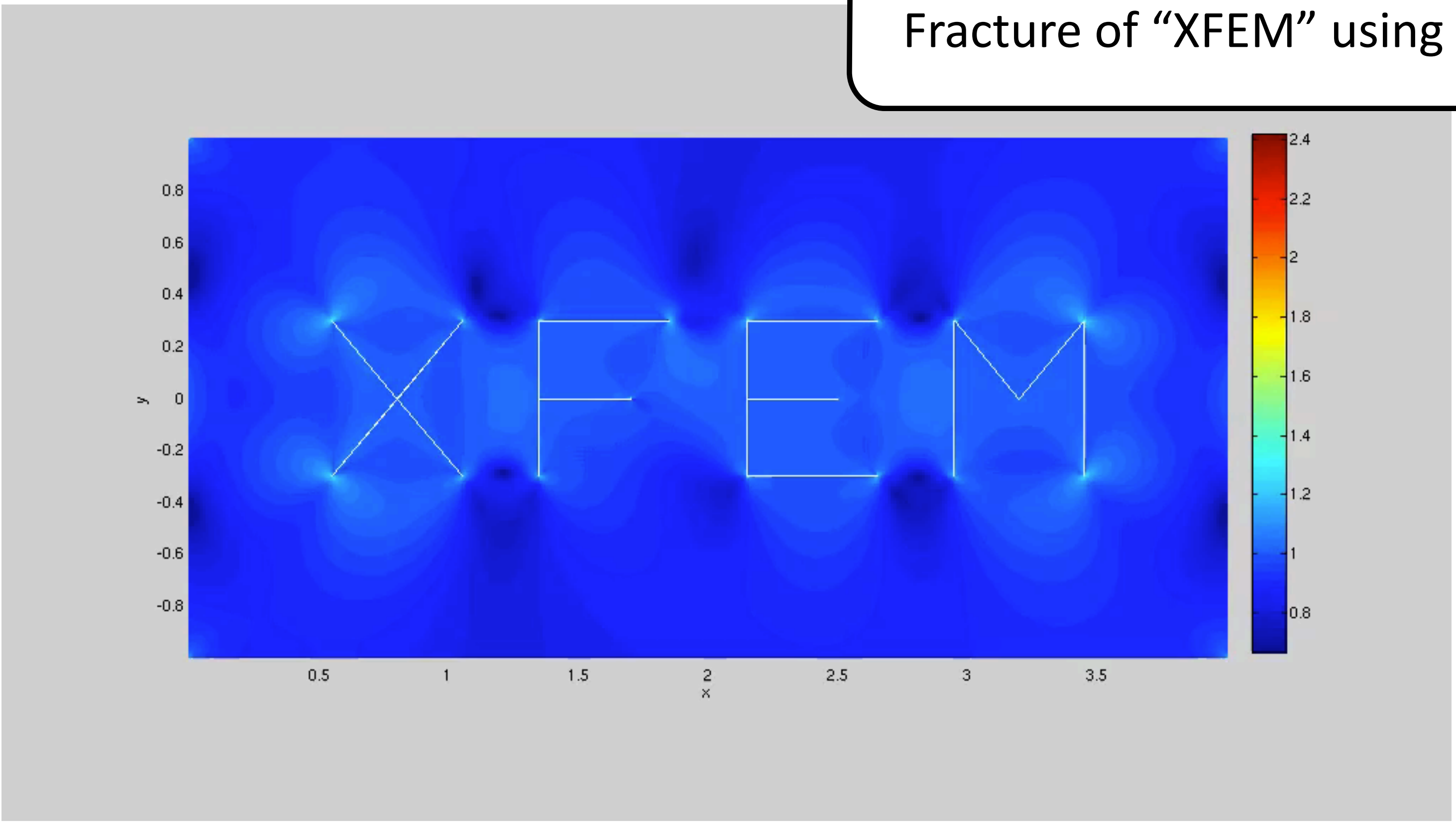
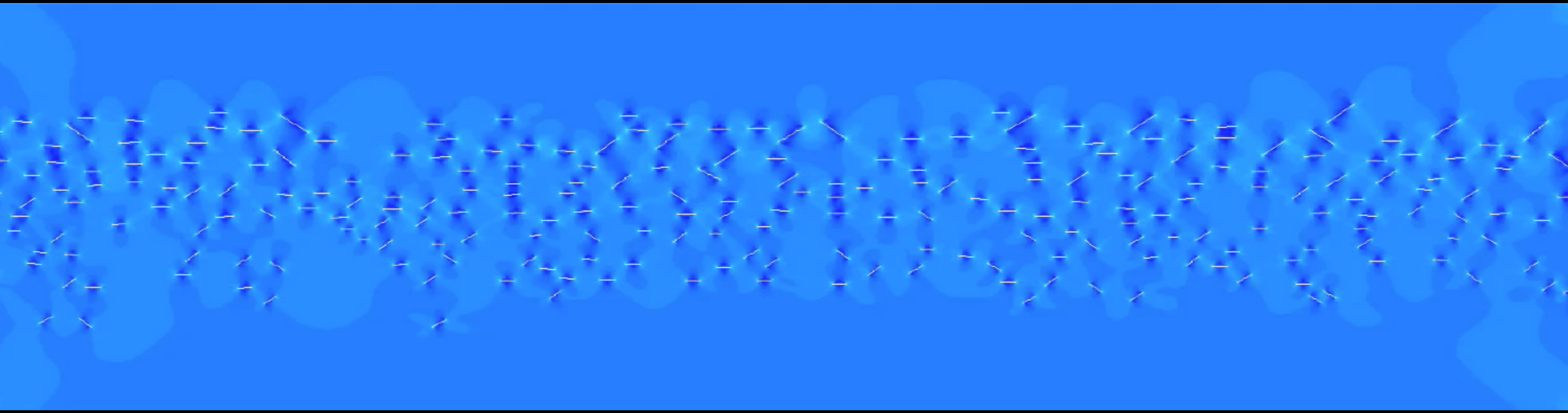


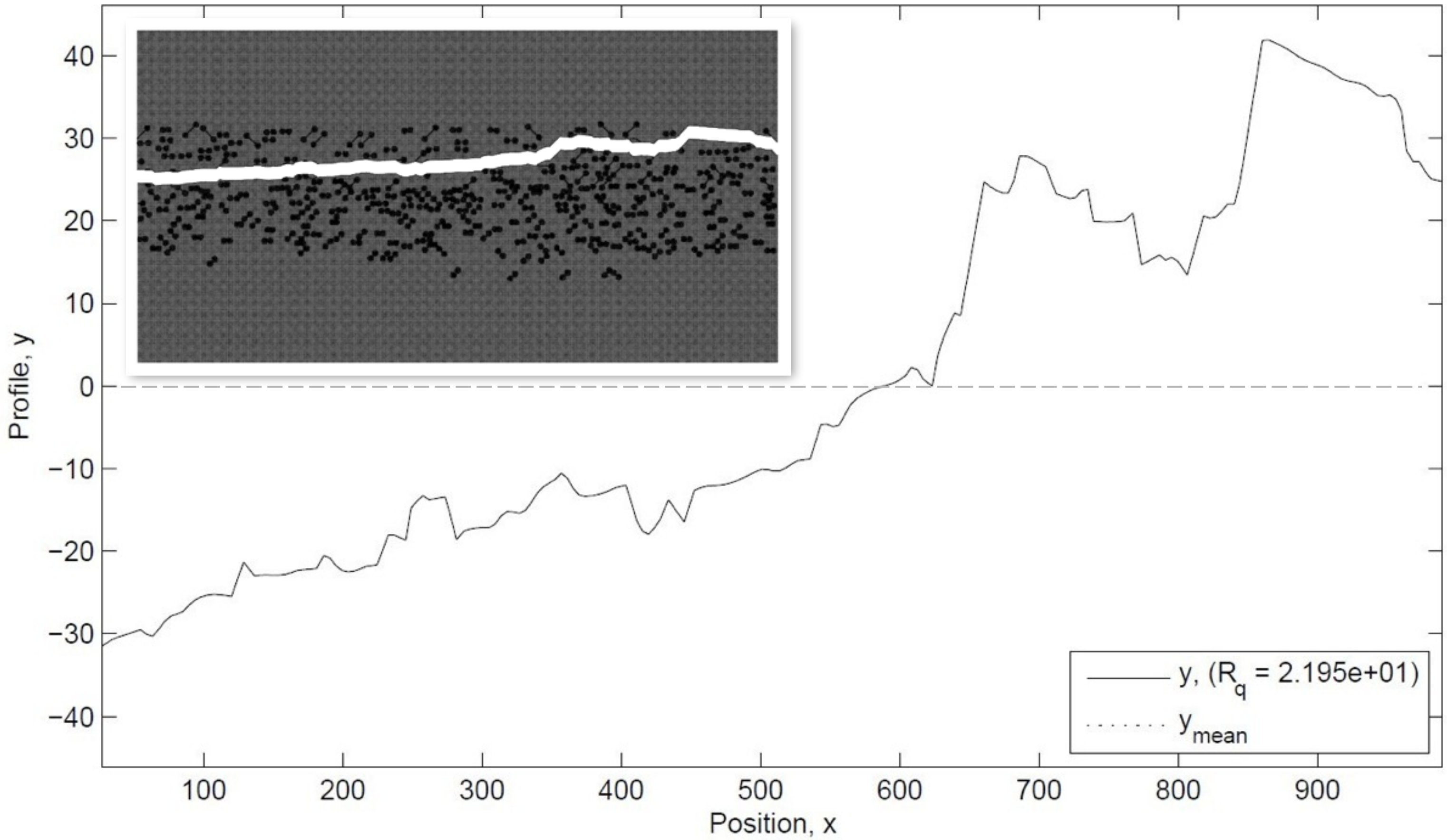
Plate with 300 cracks - vertical extension BCs



Example #1

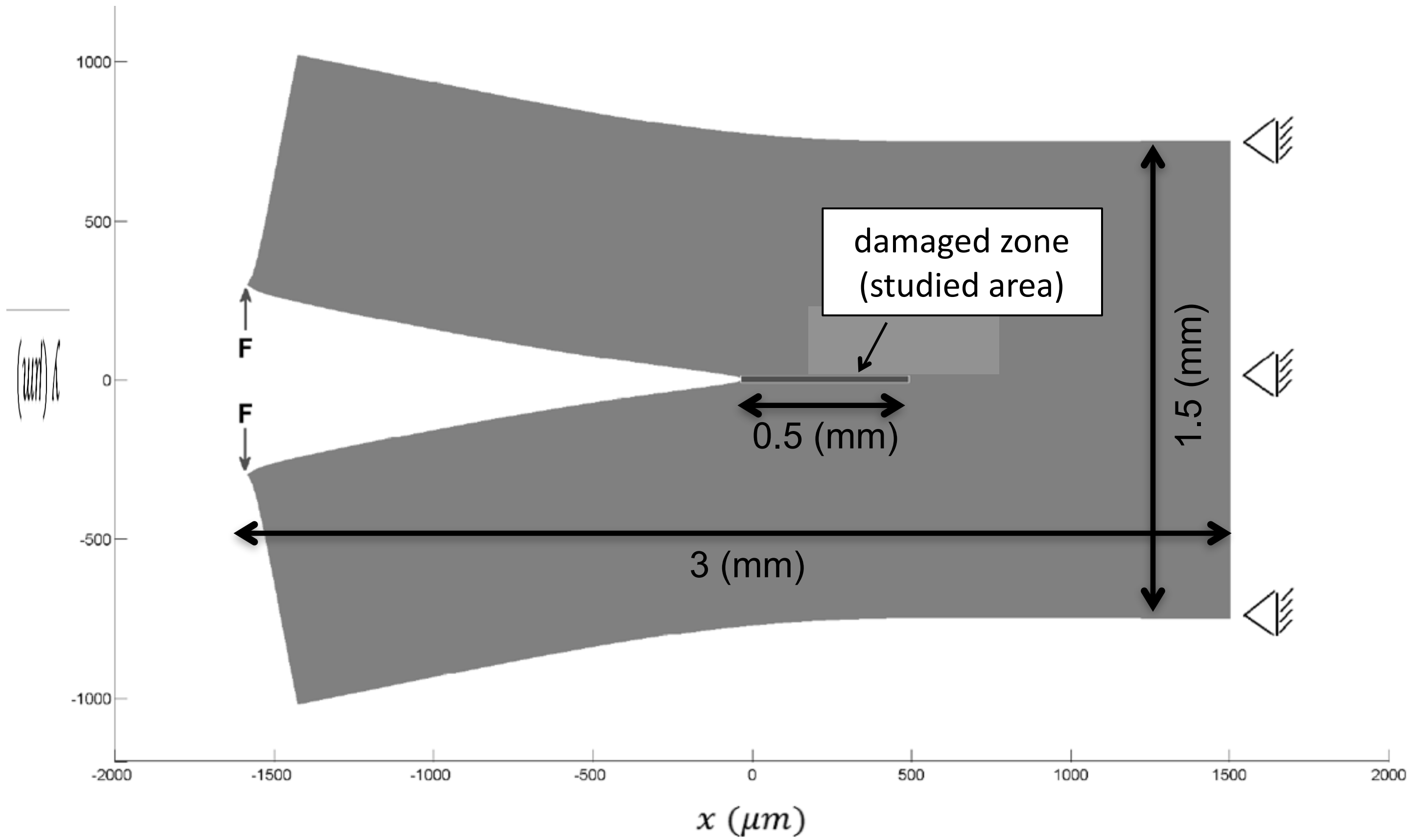
Vertical extension of a plate with 300 cracks

Post-split roughness



Mechanical splitting of a wafer sample

- Post-split roughness as a function of micro crack distribution

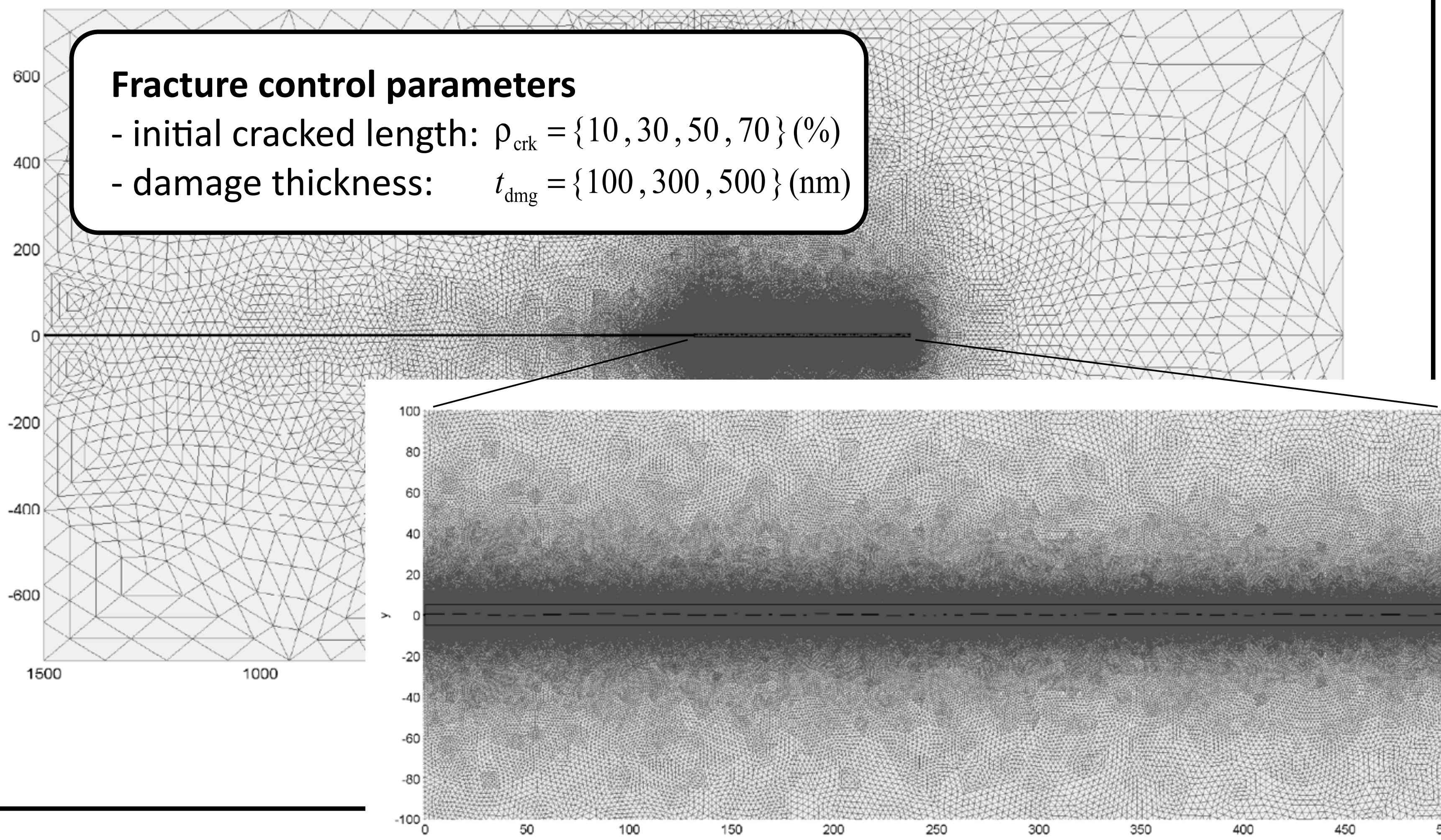


Mechanical splitting of a wafer sample

- Discretisation (≈1mln. DOF, h_e = 150 nm)

Fracture control parameters

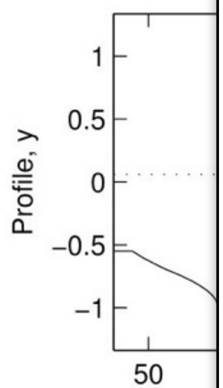
- initial cracked length: $\rho_{crk} = \{10, 30, 50, 70\}\ (\%)$
- damage thickness: $t_{dmg} = \{100, 300, 500\}\ (\text{nm})$



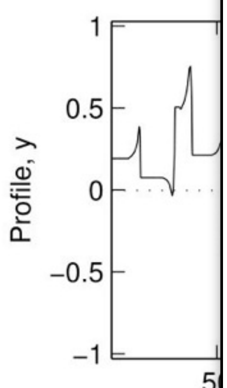
Example #2

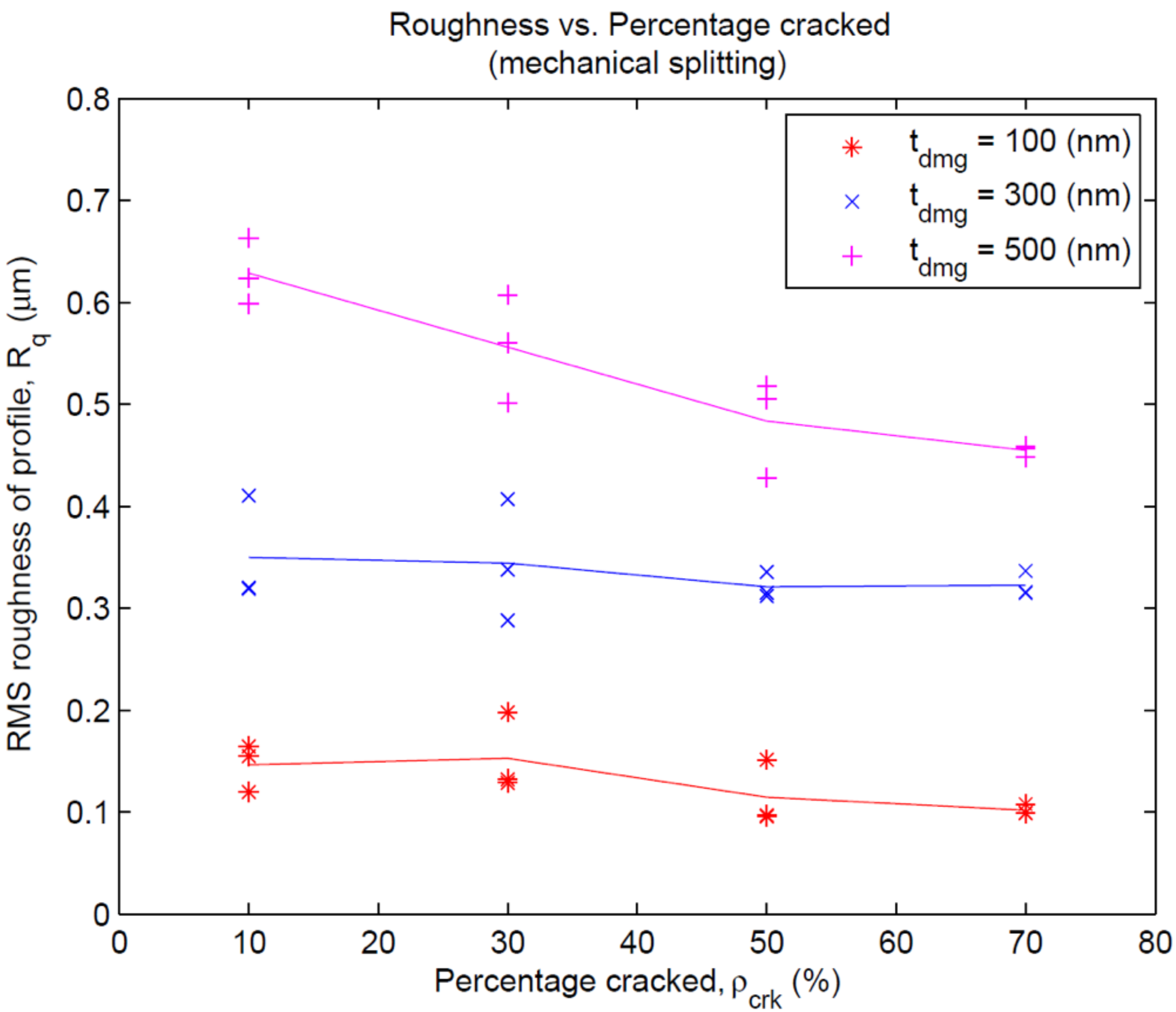
Fracture rou

- Case exar



- Case exar



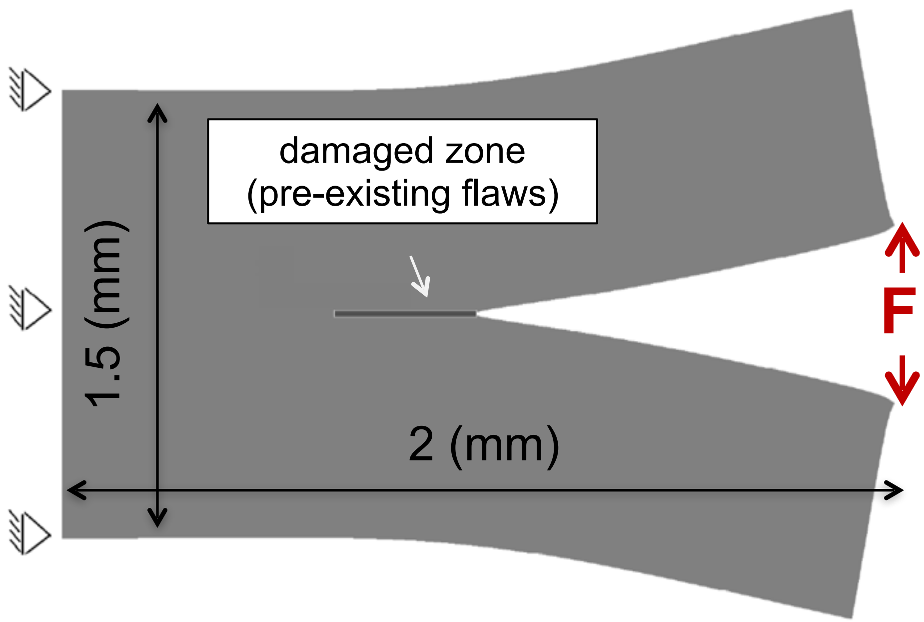


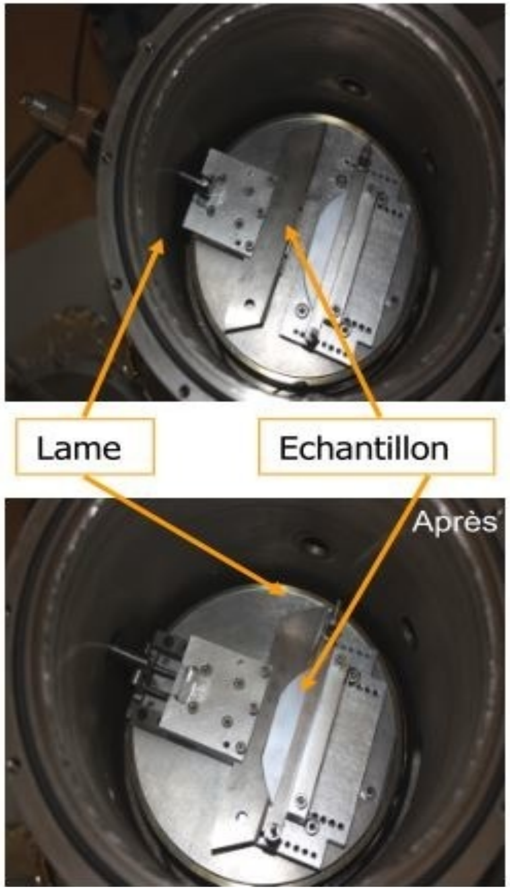
Application to Si-wafer splitting

Soitec

Mechanical splitting of a wafer

- Post-split roughness as a function of micro crack distribution
- Consider a representative material sample
- BC: blade loading = fixed displacements (RHS)
- 20 initial micro cracks within the damage zone





Physical experiment

Application to Si-wafer splitting

Mechanical splitting of a wafer

- Fracture path comparison: *max-hoop crit.* VS. *energy min*.
- NOTE: non-uniform scaling of axis, y / x = 400

Si-wafer splitting using a wedge blade
(comparison of two growth criteria)

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Part IV. Application to surgical simulation

with Institue of Advanced Studies (iCube, University of Strasbourg, France: Hadrien Courtecuisse), INRIA, SHACRA Team (Stéphane Cotin, Christian Duriez); Karol Miller, UWA.

Faculty of Sciences, Technology and Communication

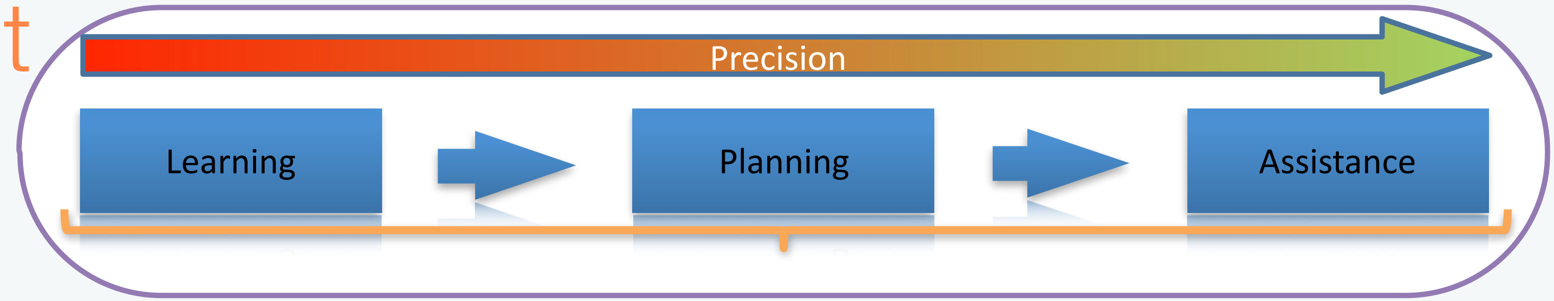
erc

RealTcut

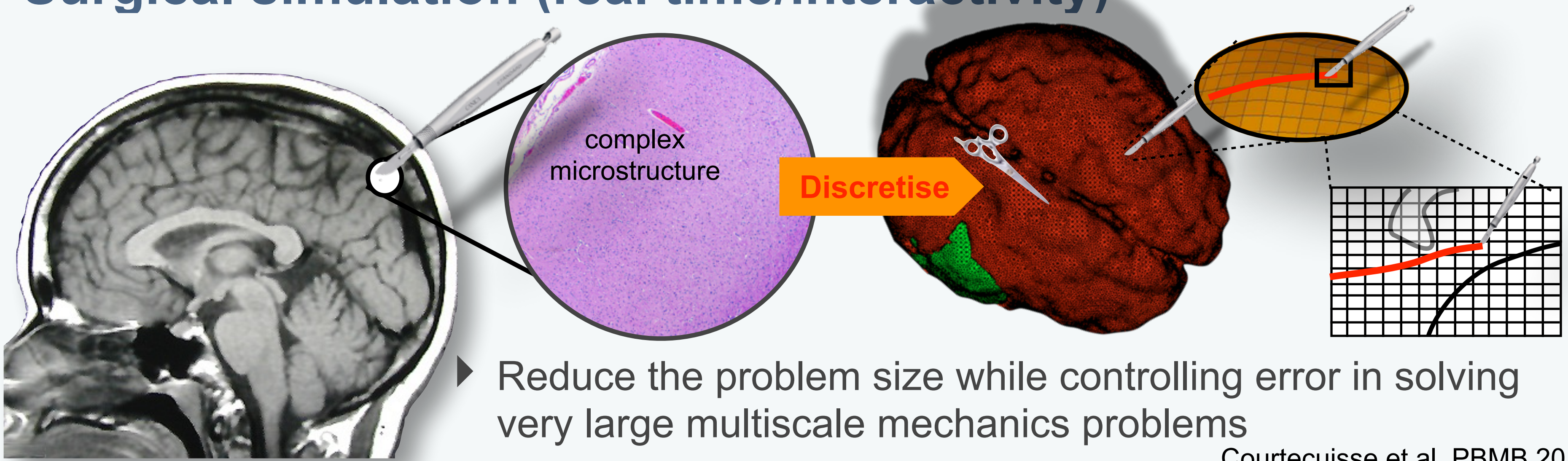
Interactive multiscale cutting simulations

UNIVERSITÉ DU LUXEMBOURG

RealTcut

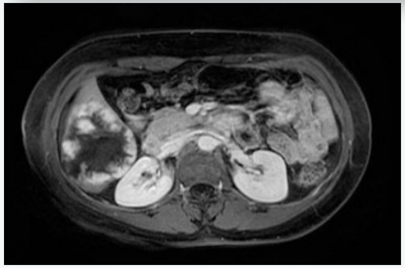


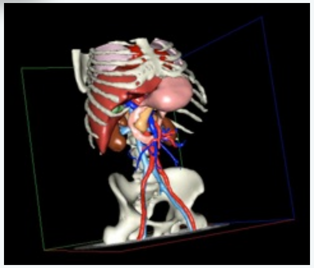
Surgical simulation (real time/interactivity)

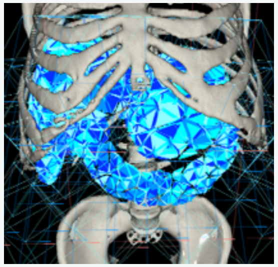


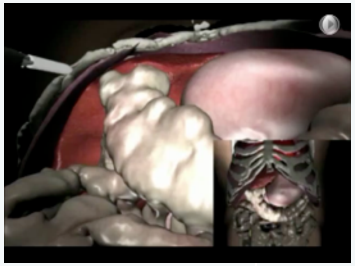
- Reduce the problem size while controlling error in solving very large multiscale mechanics problems

Courtecuisse et al. PBMB 2011







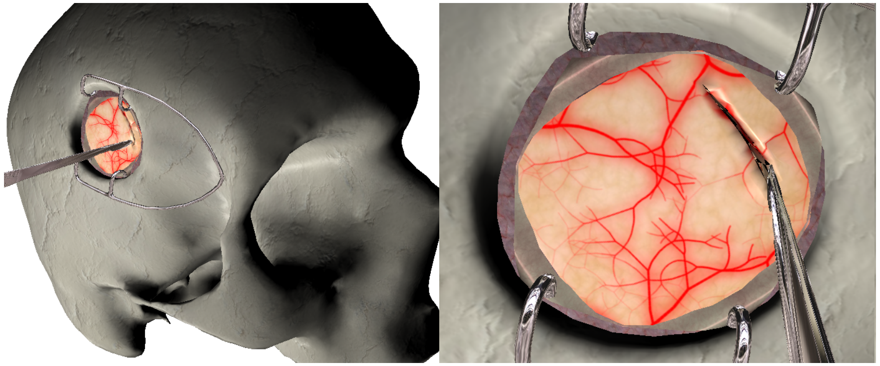


Concrete objective: compute the response of organs during surgical procedures (including cuts) in real time (50-500 solutions per second)

Two schools of thought

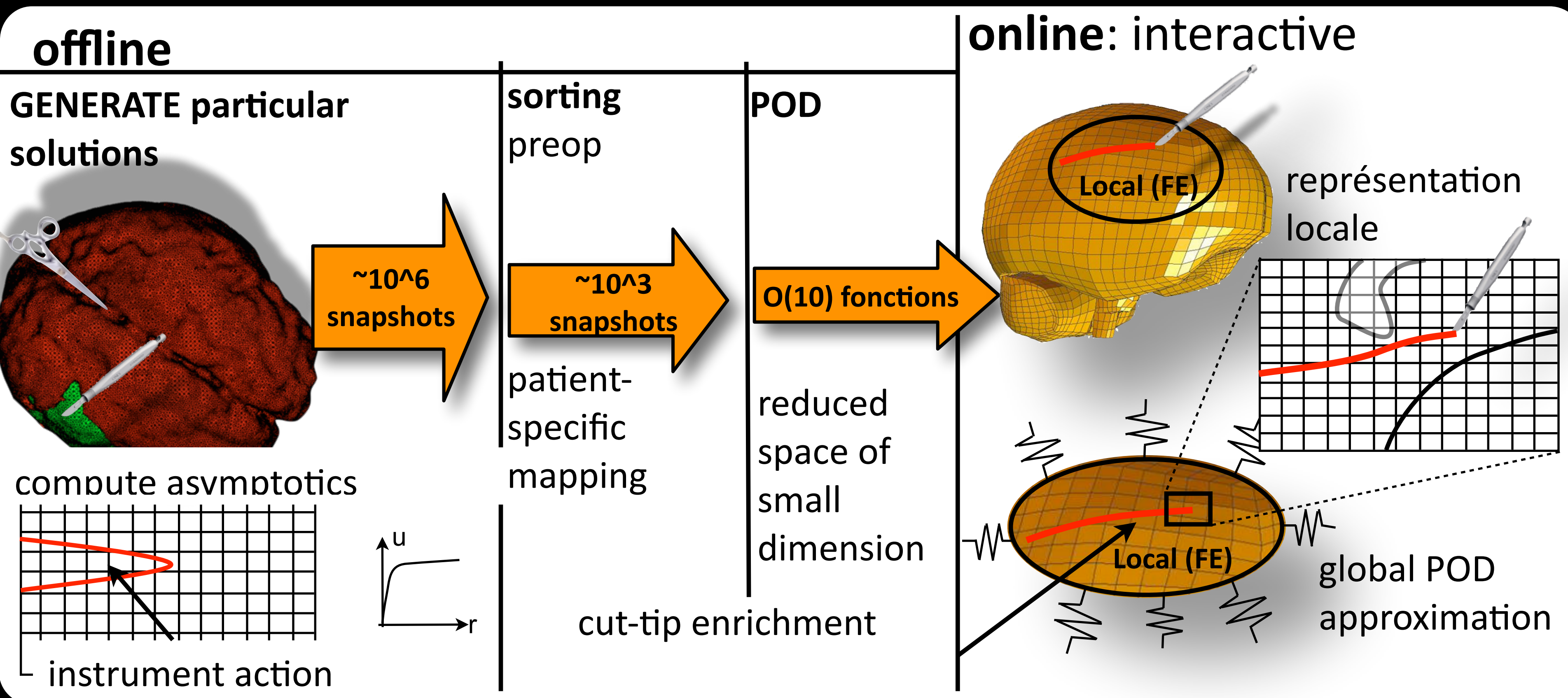
- constant time
  - ➡ accuracy often controlled visually only
- model reduction or "learning"
  - ➡ scarce development for biomedical problems
  - ➡ no results available for cutting

First implicit, interactive method for cutting with contact



[Courtecuisse et al., MICCAI, 2013]
Collaboration INRIA

Proposed approach: maximize accuracy for given computational time. Error control


offline
GENERATE particular solutions
~10^6 snapshots
compute asymptotics
u
r
instrument action
sorting
preop
~10^3 snapshots
patient-specific mapping
POD
O(10) fonctions
reduced space of small dimension
cut-tip enrichment
online: interactive
Local (FE)
représentation locale
Local (FE)
global POD approximation

A semi-implicit method for real-time deformation, topological changes, and contact of soft tissues

Paper ID : 269

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Acknowledgements

EPSRC
Pioneering research and skills

The Royal Academy of Engineering

THE ROYAL SOCIETY

Inria
INVENTEURS DU MONDE NUMÉRIQUE

hpc CYMRU WALES

SEVENTH FRAMEWORK PROGRAMME

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PERMIX: Multiscale, XFEM, large deformation, coupled 2 LAMMPS, ABAQUS, OpenMP - Fortran 2003, C++

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