Seasonal asymmetric persistence in volatility: an extension of GARCH models

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Abstract

In this paper, we study non-linear dynamics in the CAC 40 stock index. Our empirical results, suggest combining seasonality, persistence and asymmetric effects to model the conditional volatility. We observe that seasonality can have an asymmetric impact on the volatility. In particular, we show that negative shocks observed on Mondays have a greater impact on the volatility than the other days. Then we construct a seasonal asymmetric GARCH model. It consists to add seasonal terms in the variance equation of a GJR-GARCH (1,1) model.
Keywords: non linearity, conditional volatility, asymmetry, seasonal processes, GJR-GARCH model.

1 Introduction

Mandelbrot [19] and Fama [10] both reported evidence that large (small) changes in the price are often followed by other large (small) changes. This autocorrelation of the volatility of returns was modeled by Engle [9] within the framework of ARCH (Autoregressive conditional Heteroskedasticity) processes extended to GARCH models (Generalized Autoregressive Conditionally Heteroskedasticitic) by Bollerslev [4]. Different studies have revealed that the ARCH and GARCH processes are unsuitable to take into account effects of asymmetry often noticed on the conditional volatility of stock returns. It seems that the conditional volatility reacts more at the announcements of bad news. In particular, Black [3] observes the existence of a negative correlation between the current return and the future volatility. Volatility asymmetry may be captured using a GJR–GARCH (1,1) model introduced by Glosten, Jagannathan, Runkel, [14]. In this model the conditional volatility depend on the sign and on the amplitude of the past estimation errors.

In a general case, ARCH models explain a part of the leptokurtic effect noticed in financial series, but not at all.

During the past decade, some studies have shown that big fluctuations could be inherent to the market structure. Numerous researches concerning the microstructure of the markets have been developed like weekend effects and other anomalies. In particular, the day of the week effect has been studied in a number of papers: French [13] Hamon and Jacquillat [15]. In these papers, Monday returns are found to be negative while the returns on Friday tended to be higher than the other days. Not only do the average returns on Monday tend to differ, Bessembinder and Hertzel [2] show that returns on Mondays are positively correlated with Fridays' returns while returns on Tuesdays are negatively correlated with Mondays' returns. Then, these authors propose a periodic autoregressive model (PAR) in their empirical studies.

Additionally, there is evidence that the volatility vary with the day of the week, see Foster and Viswanathan [11]. To take into account these latter empirical observations, Bollerslev and Ghysels [5] use a periodic GARCH model (PGARCH). Franses and Paap [12] observe positive autocorrelation on Monday and day of the week variation in the persistence of volatility. Then, they combine the PAR model for the returns with the PGARCH model for the volatility.

In this paper, we observe that seasonality can have an asymmetric impact in the conditional variance equation. In our empirical study, we show that negative shocks observed on Mondays have a greater impact on the volatility than the other days. Then we propose an asymmetric seasonal GARCH process to model asymmetric and seasonal effects jointly. We study the seasonal effect both in the returns and the volatility in the case of the CAC 40 stock index series from 1987 to 2002. The paper is organized as follows. First, we give some statistics for the returns of the CAC 40. Preliminary results are mentioned. Then, we present methodology and empirical results. The paper finalizes with some conclusions.

2 Data and Statistical analysis

The data used are the daily index series (CAC 40) of the French Stock Exchange during the period 09/14/1987- 10/01/2002 (3920 observations). The Phillips Perron (PP) [21] unit root test shows that one unit root exists in the CAC 40 series (the PP value is 0.6496, which is greater than the critical value at 5%). We take the log difference of the value of the index so as to convert the data into continuously compounded returns. The PP value for this series is now -60.97, which is less than the critical value at 5%. Some summary statistics on the returns are presented in table 1.

Table 1: summary statistics of CAC40 returns

Average	Standard Errors	skewness	kurtosis	Jarque bera	LB(30)*	LB^2(30)*
0.00016	0.0135	-0.3758 (-9.635)**	7.5961 (58.93)**	3543.4 (5.99)**	53.5	1607.2

** the critical values are compared with 1.96 ; *The Ljung Box test is compared with $\chi^2(29) = 42.56$

As the table 1 shows, the index has a small positive average return. The daily variance is 0.00018. The skewness coefficient indicates that the returns distribution is substantially negatively skewed. Furthermore, the excess of kurtosis gives evidence of a strong probability of negative extreme returns for the index CAC 40. The conclusion is that the assumption of normality for the returns index is rejected.

Autocorrelation is revealed applying the statistics of Ljung Box [18] calculated with 30 lags LB (30) to the return and the squared of returns. This test is a first indication on the presence of a strong heteroscedasticity and on a linear or non-linear structure in the series of index returns. To comfort this result, non-linearity tests are applied using the routine proposed by Ashley and Patterson [1]. After prewhitening the data, we routinely bootstrap the significance levels, as well as computing them based on asymptotic theory. We draw 1000 T samples at random from the empirical distribution of the observed T- sample of data. The Brock, Dechert, Scheinkman (BDS,[6]), McLeod-Li [20], Engle [9] and Tsay [24] tests are implemented in Toolkit, a Windows-based computer program presented in Ashley and Patterson [1]. The hypothesis of non-linearity is accepted if the thresholds of probability are lower than 0.05. Results of the tests are presented in table 2.

Table 2: Non linearity tests on the returns

Tests	McLeod-Li (L=24)	Engle (P=5)	Tsay (K=5)	BDS (M=2,3,4) (ε/σ=0.5,1,2)
Bootstrap	0.000	0.000	0.00	0.000
Asymptotic	0.000	0.000	0.00	0.000

All of the tests appear to have high power to detect non-linearity in the data. We conclude in favour of non-linear structures but we cannot specify what kind of non–linear process can be used to model returns series.

Tests of Time Reversibility (TR) can complement the existing tests. In particular, the TR test of Chen Chou and Kuan [7] (the CCK test) is powerful against asymmetry in volatility while the BDS test is not. In effect, time series that exhibit asymmetric behaviours are typically time irreversible. When ε_t is

time reversible, it can be shown that for each k = 1,2 , the distribution of ε_t - ε_{t-k} is symmetric (about the origin). If this symmetric condition fails, there is some asymmetric dependence between ε_t and $\varepsilon_{t-k.}$ In view of this property, non-linear time series are time irreversible in general.
In table 3 we report the statistics of the CCK test. We consider β = 0.5 and 1 and we take k = 1,2,3,4,5 as the empirical applications of the authors.

Table 3: CCK test of the daily returns

TR test ($C_{exp,k}$)	k	β = 0.5	β = 1
	1	-1.96*	-1.98*
	2	-3.89*	-4.65*
	3	-3.53*	-3.79*
	4	-2.35*	-3.83*
	5	-1.36	-3.81*

significance at 5% level

The CCK tests are significant in all cases except for k =5 and beta = 0.5.
The results indicate that the data are time irreversible and take a first indication on the potential asymmetry in the returns series.

The application of these different tests has permitted to show the presence of non-linearity in the series. However it can be possible that other effects explain the structure of the returns like deterministic events. Some authors have shown their existence in the mean and volatility characteristics, and have studied the effects of seasonality observed in the returns. To test if a weekend effect exists in the average returns of the CAC 40 during our period of observations, we use the regression between the index returns and the days of the week. Table 4 confirms the existence of a Monday effect for the CAC 40 returns, and a seasonal effect on Tuesday.

Table 4: seasonalities in CAC 40 returns

	Monday	Tuesday	Wednesday	Thursday	Friday	R^2
Returns (t - statistic)	-0.0009* (-1.96)	0.0010* (2.12)	-2.97E-05 (-0.06)	0.0005 (1.14)	0.0004 (0.93)	0.0025

*Significance at 5 % level

Then, to characterize the mean equation, we construct an autoregressive seasonal model. This model consists to add the seasonal dummies in an autoregressive process. We suggest an AR(3) and an MA(1) processes to take into account the autocorrelation in the index returns. We obtain the following equation:

$$r_t = \phi_3 r_{t-3} + \phi_1 \varepsilon_{t-1} + \delta_1 D_{1,t} + \delta_2 D_{2,t} + \varepsilon_t \,, \quad \varepsilon_t \sim \text{iid normal } (0,\sigma_\varepsilon) \qquad (1)$$

with $D_{1,t}$ and $D_{2,t}$ being dummies for Monday and Tuesday.

Table 5 gives the results of the estimation.

Table 5: Seasonal autoregressive model

	Return	t - statistic	LB (30)*
δ_1	-0.000951	-1.977610	
δ_2	0.001070	2.225086	38.337
ϕ_3	-0.048251	-3.013129	
θ_1	0.028204	1.764096	

*The Ljung Box test is compared with the value equal 38.88

Even if the effects of seasonality are not very important, (see the R^2 statistic in table 4), this model can be accepted since the hypothesis of autocorrelation is rejected by the Ljung Box test applied on the residuals.

3 Methodology and empirical results

Little Work has ever been devoted to linking the weekend effect with heteroscedasticity and /or to a seasonal behaviour of market volatility. Most studies that consider weekend effect for the returns assume that the volatility does not vary with the day of the week. As Franses and Paap [12] have suggested, it seems important to take account of both features jointly. This weekend effect on volatility can be explained by the fact that there is a concentration to publish all kinds of bad news on the weekends. The consequence on the market will be a lower return and higher volatility on Monday. This phenomenon sometimes ascribed to a leverage effect is completely ignored in the GARCH processes, the sign of returns playing any role on the volatility. We verify that the returns on the index are not symmetric as indicate the negative values of the cross correlogram between the squared residuals and the residuals of the model. An additional stage is to test that according to the days of the week, the potential asymmetric responses of volatility can be different. For that, we use regressions defined by :

$$\varepsilon_t^2 = c + w_s S_{t-1}^- \varepsilon_{t-1} D_{s,t} + e_t \quad , \quad e_t \sim \text{ iid normal } (0,\sigma_e) \tag{2}$$

where $S_{t-1}^- = 1$ when $\varepsilon_{t-1} < 0$ and $S_{t-1}^- = 0$ otherwise and $D_{s,t}$ represent the days of the week. s = 1,2,....,5.

Applying eqn.(2), we observe that the Monday effect has an asymmetric impact on the volatility since w is negative, even if this asymmetry feature is only significant at 10%:

$$\varepsilon_t^2 = \underset{(23.62)}{0.000178} - \underset{(-1.90)}{0.003577}\, S_{t-1}^- \varepsilon_{t-1} D_{1,t} \tag{3}$$

To model both the seasonality and leverage effect on the volatility, we propose an asymmetric seasonal GARCH (1,1) model. The conditional volatility of the index CAC 40 is set as :

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \delta_2 D_{2,t} + w_1 S_{t-1}^- \varepsilon_{t-1}^2 D_{1,t} + \beta \sigma_{t-1}^2 \tag{4}$$

In comparison with the GJR GARCH (1,1) model, we add seasonal terms in the variance equation. The potential seasonality on Tuesday is represented by the coefficients δ_2 while w_1 estimate the asymmetric seasonal impact on the conditional variance. The effect of a positive shock is represented by the coefficient α_1 and of a negative shock by (α_1 +w_1). So, in this model the impact of shocks depends on the Monday effect. In table 6 we report estimates of the model.

Table 6: Estimates for the seasonal asymmetric GARCH model

	coefficients	t statistic*
δ_1	-0.000992	-2.373342
δ_2	0.000690	1.933654
ϕ_3	-0.032654	-2.085364
θ_1	0.044400	2.570453
α_0	1.19E-05	9.747532
α_1	0.096825	12.18165
β	0.858666	75.54010
w_1	0.074333	3.003194
δ_2	-2.97E-05	-4.963929

*significance at 5%

Looking at the table 6, we observe that the coefficients in the mean equation are widely significant (at 10% for δ_2). In the variance equation, the seasonal heteroscedasticity is significant on Monday and Tuesday. The results indicate that the sign of the innovation has an influence on the volatility of returns. A positive shock at 1% increases the volatility at 0.09% while a negative shock at 1% increase the volatility at 0.17%. Then the degree of asymmetry is equal of 1.76. The study of the standardized residuals sample statistics of the seasonal asymmetric GARCH model, show significant decrease of kurtosis from 7.5961 to 5.1107, the skewness from -0.3758 to -0.3416 and Jarque Bera [16] from 3543.447 to 803.0970. The Ljung Box [18] test with standardized residuals and squared standardized residuals are employed to verify that there is no autocorrelation and no ARCH effects. As the table 7 shows, our model has taken care of the non-linear dependence and there is no significant autocorrelation.

Table 7: Tests on the standardized residuals

Average	Standard errors	skewness	kurtosis	Jarque bera	LB(30)*	$LB^2(30)$*
0.0001	0.0135	-0.3416	5.1107	803.0970	30.561	21.528

*are compared with $\chi^2(21)$ = 32.67

We can confirm these results by table 8, applying on the standardized residuals, non-linear tests suggested by Ashley and Patterson [1].

Table 8: Non-linearity tests on standardized residuals

Tests	McLeod-Li (L=24)	Engle (P=5)	Tsay (K=5)	BDS (m=2, ε/σ = 1)
Bootstrap	0.254	0.169	0.850	0.134
Asymptotic	0.239	0.165	0.848	0.142

* Significance at 5% level

To evaluate the TR property of model-standardized residuals, the CCK test is not directly applicable. So we use a modified version of the CCK test proposed by Chen [8]. Nevertheless, in table 9, we show that the modified CCK test still detects some non-linear dependence not captured by the BDS test.

Table 9: The Modified CCK test on the standardized residuals

TR test ($C_{exp,k}$)	k	β = 0.5	β = 1
	1	-1.45	-1.43
	2	-2.61*	-3.85*
	3	-2.28*	-3.32*
	4	-1.43	-3.32*
	5	-0.39	-3.32*

* significance at 5% level

For some k, the modified CCK test rejects the model. However, there is a difference between table 3 and table 9. The statistics $C_{exp,k}$ derived of the modified CCK test are all smaller than those for the returns. So, the model has captured some (but not at all) time irreversibility in the return series.

4 Conclusion

The goal of this paper has been to characterize a volatility model by its ability to capture the seasonality in both the conditional mean and the conditional variance equation. We have shown that the Monday effect and seasonality on Tuesday appear in these two equations. Nevertheless, while the seasonalities are introduced in an additive manner in the conditional mean equation, the Monday effect has an asymmetric impact in the conditional volatility. To take into account these features, we propose a seasonal asymmetric GARCH model. This model appears to capture a large part of non- linearities present in the variance, even if it seems to neglect other asymmetries sources. For further research, it would be interesting to test the prediction of the model for forecasting the volatility out of sample. Furthermore, similar applications to larger markets such as those in Europe will be another extension.

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