Unbiased H_∞ filtering for a class of stochastic systems with time-varying delay

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Abstract: This paper presents the unbiased H_∞ filter design for a class of stochastic systems with time-varying delay. The aim is to design an unbiased filter assuring exponential stability in mean square and a prescribed H_∞ performance level for the filtering error system. Based on the application of the descriptor model transformation and free weighting matrices, delay-dependent sufficient conditions for stochastic systems with time-varying delay are proposed respectively in terms of linear matrix inequalities(LMIs). Numerical examples demonstrate the proposed approaches are effective and are an improvement over existing methods.

Key Words: unbiased H_∞ filtering, stochastic systems, exponential stability in mean square, linear matrix inequalities(LMIs), time-varying delay

1 INTRODUCTION

During the last few decades, estimation of dynamic systems has attracted a lot of attentions and has found many practical applications. It has been recognized that one of the most popular estimation approaches is the H_∞ filtering. One of its main advantages is the fact that it is insensitive to the exact knowledge of the statistics of the noise signals. The estimation procedure ensures that the L_2-induced gain from the noise signals to the estimation error will be less than a prescribed level, where the noise signals are arbitrary energy-bounded signals. See [1] and [2] for a discrete-time investigation; [3] and [4] for linear H_∞ filtering investigation; [5] and [6] for a general nonlinear H_∞ filtering investigation, And optimal filtering approaches for linear or nonlinear systems [7]-[8] are also considered by the researchers. Occasionally, the unbiased condition [9] is taken into account, and under this condition, the average value of the estimation error keeps zero at any time point provided that both the average value of the disturbance and the initial estimation error are equal to zero. By using Riccati equation approaches or linear matrix inequality techniques, the unbiased H_∞ filtering problems are respectively considered for linear systems [10]-[11]. Recently, the H_∞ filtering and control problems of stochastic systems whose models expressed by $It\tilde{o}$-type stochastic differential equations have gained extensive attentions and achieved comprehensive applications in many fields, such as aircraft design, chemical production, distributed network and control systems, and so on. Robust H_∞ estimation problems for continuous- time linear and nonlinear stochastic systems were discussed by [12]-[14], respectively. However, [12] and [13] didn't consider the effect of time delay. And time delay is commonly encountered in control field, it usually cause instability and poor performance of systems. [13]-[14] used the matrix constraint $P \leq \alpha I$ ($\alpha > 0$ is a scalar, P is Lyapunov matrix, I is identity matrix), [16] researched the L_2-L_∞ filtering for stochastic systems with time delay and [16] gave the delay-dependent sufficient conditions, but the stability criteria for the filtering error dynamic system in [16] is asymptotically stable. These results are rather conservative. Moreover, as far as aforementioned filtering methods are concerned, the order of filter error system is the twice as great as the original system, in the process of seeking the filter parameters, which directly result in the complexity and computational burden. These motivate us to investigate the unbiased filter design problem for stochastic systems with time-varying delay.

This paper discusses unbiased H_∞ state estimation for a class of continuous-time linear stochastic systems with time-varying delay. Here, we focus on the design of an unbiased filter such that the estimation error system is stochastically exponentially stable in mean square, whose L_2-induced gain with respect to uncertain disturbance signal is less than a prescribed level γ. First, by constructing an unbiased filter such that the order of filter error dynamic systems is the same as the original system, in contrast with the traditional H_∞ filtering approaches, which result in the order of filter error systems is the twice as great as the original system, to a great extent, the unbiased filter reduce the complexity and computational burden of the real-time filtering process. Second, based on the equivalent descriptor stochastic systems, the descriptor terms $q(t)$ and $g(t)$ as state vectors are introduced to the Lyapunov-Krasovskii

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functional. Third, by using some zero equations, some free weighting matrices are introduced, thus, in the process of design, it is not necessary to do any constraint for Lyapunov matrix, which reduce the conservative of filter design. Furthermore, to guarantee the existence of desired robust H_∞ filters, based on LMI algorithm, delay-dependent sufficient conditions for stochastic systems are proposed, and the minimum γ obtained is less conservative than the existing results. Numerical simulation examples show the methods are effective and are an improvement over existing methods.

For convenience, we adopt the following notations:

$trace(A)(A^T)$ Trace (transpose) of the matrix A.

$A \geq 0 (A > 0)$ Positive semi-definite (positive definite) matrix A.

$L^2_F([0,\infty), R^n)$ space of nonanticipative stochastic processes $\phi(t)$ with respect to filtration F_t satisfying

$$\|\phi(t)\|^2_{L^2_F} = E\int_0^\infty \|\phi(t)\|^2 dt < \infty$$

$L^2_{F_0}([-h,0], R^n)$ the family of R^n −valued stochastic processes $\eta(s)$, $-h \leq s \leq 0$ such that $\eta(s)$ is F_0- measurable for every second and $\int_{-h}^0 E\|\eta(s)\|^2 ds < \infty$

$E\{\cdot\}$ mathematical expectation operator with respect to the given probability measure P.

2 PROBLEM FORMULATION

Consider the following stochastic linear time-delay system

$$\begin{aligned} dx(t) = [A_0x(t) + A_{0d}x(t-h(t)) + B_0v(t)]dt \\ +[C_0x(t) + C_{0d}x(t-h(t))]d\beta(t) \\ x(t) = \varphi(t), t \in [-\tau, 0] \\ y(t) = A_1x(t) + A_{1d}x(t-h(t)) + B_1v(t) \\ z(t) = Lx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^m$ is the measurement output, $\varphi(t)$ is a continuous vector-valued initial function and $\varphi := \{\varphi(s) : -h \leq s \leq 0\} \in L^2_{F_0}([-h,0], R^n)$, $z(t) \in R^r$ is the state combination to be estimated, $v(t) \in L^2_F([0,\infty), R^n)$ stands for the exogenous disturbance signal. A_i, A_{id}, B_i $(i = 0,1)$, C_0, C_{0d} and L are known constant matrices with appropriate dimensions.Where the stochastic variable $\beta(t)$ is zero-mean real scalar Wiener processes defined on a complete probability space (Ω, F, P) with a natural filtration $\{F_t\}_{t\geq 0}$ and satisfies :

$$E\{d\beta(t) = 0,\ E\{d\beta(t)^2\} = dt \tag{2}$$

the time delay $h(t)$ is a time-varying continuous function that satisfies

$$\tau_1 \leq h(t) \leq \tau_2 \tag{3}$$

and

$$\dot{h}(t) \leq \mu \tag{4}$$

Remark 1. Here time delay $h(t)$ varies from τ_1 to τ_2 and the derivative of the time delay is also applicable to cases in which $\mu > 1$, it is different from existing literatures [16] and [17]-[18], which are usually assume $0 \leq h(t) \leq \tau_2$ and $\mu \leq 1$. Therefore, in some extent, the results in this paper are less conservative than existing results.

We set up the following unbiased filtering system

$$\begin{aligned} d\hat{x}(t) = [A_f\hat{x}(t) + (A_{0d} - B_fA_{1d})\hat{x}(t-h(t))]dt \\ +B_fdy(t) + [C_0\hat{x}(t) + C_{0d}\hat{x}(t-h(t))]d\beta(t) \\ \hat{x}(0) = 0 \\ \hat{z}(t) = C_f\hat{x}(t) \end{aligned} \tag{5}$$

where $\hat{x}(t) \in R^n$ is the filter state, $\hat{z}(t) \in R^r$, the constant matrices A_f, B_f, C_f are filter parameters to be designed. Denote $x_e(t) = x(t) - \hat{x}(t)$ and $z_e(t) = z(t) - \hat{z}(t)$, then, we obtain the following filtering error system :

$$\begin{aligned} dx_e(t) = [A_fx_e(t) + (A_0 - A_f - B_fA_1)x(t) + (A_{0d} \\ -B_fA_{1d})x_e(t-h(t) + (B_0 - B_fB_1)v(t)]dt \\ +[C_0x_e(t) + C_{0d}x_e(t-h(t))]d\beta(t) \\ z_e(t) = C_fx_e(t) + (L - C_f)x(t) \end{aligned} \tag{6}$$

Unbiasedness of the filter requires that the estimation error system be independent of the system state x, that is the following conditions ([10] - [11]) are satisfied :

$$A_f = A_0 - B_fA_1,\ C_f = L \tag{7}$$

then we obtain an unbiased filter and the filtering error system can be described as

$$\begin{aligned} dx_e(t) = [A_ex_e(t) + A_{ed}x_e(t-h(t) + B_ev(t)]dt \\ +[C_0x_e(t) + C_{0d}x_e(t-h(t))]d\beta(t) \\ z_e(t) = C_fx_e(t) \end{aligned} \tag{8}$$

where

$$\begin{aligned} A_e = A_0 - B_fA_1, A_{ed} = A_{0d} - B_fA_{1d} \\ B_e = B_0 - B_fB_1 \end{aligned} \tag{9}$$

Remark 2. Eq.(8) implies that the average value of $x_e(t)$ and $z_e(t)$ vanish provided that both the average value of $v(t)$ and the initial condition $x_e(0)$ are zero. Thus, (7) can be called the unbiased condition for the H_∞ filter. Under these conditions, the state variables $x(t)$ of system (1) do not influence Eq.(8). Especially, $A_e = A_f$ shows that the stability of filtering error system is independent of the system (1), therefore, the unbiased filtering technique is also valid for unstable stochastic system.

Remark 3. Here the unbiased filter (5) such that the order of filter error dynamic systems (8) is the same as the original system (1), in contrast with the traditional H_∞ filtering approaches, which result in the order of filter error dynamic system is the twice as great as the original system, to a great extent, the unbiased filter reduce the complexity and computational burden of the real-time filtering process.

Definition 1. Systems (8) with $v(t) \equiv 0$ are said to be exponentially stable in mean square if there exists a positive constant α such that

$$\lim_{t\to\infty} \sup \frac{1}{t} \log E\|x(t)\|^2 \leq -\alpha \tag{10}$$

The objective of this paper is to seek the filter parameters A_f, B_f and C_f such that the augmented system(8) with $v(t) \equiv 0$ is exponentially stable in mean square, More specifically, for a prescribed disturbance attenuation level $\gamma > 0$, such that the performance index

$$\int_0^\infty \|z_e(t)\|^2 dt \le \gamma^2 \int_0^\infty \|v(t)\|^2 dt \qquad (11)$$

is satisfied for zero initial conditions and all nonzero $v(t) \in L_F^2((0,\infty), R^n)$.

2.1 Unbiased H_∞ Filtering for Stochastic System

Before presenting the main results, we first give the following lemmas.

Lemma 1. *Given scalars $\tau_1, \tau_2 > 0$ and μ, the system (8) with $v(t) = 0$ is exponentially stable in mean square, if there exist symmetric positive definite matrices $P > 0, Z > 0, Q_r \ge 0, R > 0, r = 1,2,3, Z > 0, R > 0$, and appropriately dimensioned matrices N_j, T_j and $Y_j, j = 1, \cdots, 6$, such that the following LMI holds.*

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} & \Gamma_{16} & Y_1 \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} & \Gamma_{26} & Y_2 \\ * & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35} & \Gamma_{36} & Y_3 \\ * & * & * & \Gamma_{44} & \Gamma_{45} & \Gamma_{46} & Y_4 \\ * & * & * & * & \Gamma_{55} & \Gamma_{56} & Y_5 \\ * & * & * & * & * & \Gamma_{66} & Y_6 \\ * & * & * & * & * & * & \kappa \end{bmatrix} < 0 \qquad (12)$$

where $\kappa = -(\frac{R}{\tau_2 - \tau_1} + Z)$, an ellipsis $*$ denotes a block induced easily by symmetry and

$$\begin{aligned} \Gamma_{11} &= N_1 A_e + A_e^T N_1^T + T_1 C_0 + C_0^T T_1^T \\ &\quad + Q_1 + Q_2 + Q_3 \\ \Gamma_{12} &= N_1 A_{ed} + A_e^T N_2^T + T_1 C_{0d} + C_0^T T_2^T \\ \Gamma_{13} &= A_e^T N_3^T + C_0^T T_3^T + Y_1 \\ \Gamma_{14} &= A_e^T N_4^T + C_0^T T_4^T - Y_1 \\ \Gamma_{15} &= P - N_1 + A_e^T N_5^T + C_0^T T_5^T \\ \Gamma_{16} &= A_e^T N_6^T - T_1 + C_0^T T_6^T \\ \Gamma_{22} &= N_2 A_{ed} + A_{ed}^T N_2^T + T_2 C_{0d} + C_{0d}^T T_2^T - (1-\mu) Q_3 \\ \Gamma_{23} &= A_{ed}^T N_3^T + C_{0d}^T T_3^T + Y_2 \\ \Gamma_{24} &= A_{ed}^T N_4^T + C_{0d}^T T_4^T - Y_2 \\ \Gamma_{25} &= -N_2 + A_{ed}^T N_5^T + C_{0d}^T T_5^T \\ \Gamma_{26} &= -T_2 + A_{ed}^T N_6^T + C_{0d}^T T_6^T \\ \Gamma_{33} &= -Q_1 + Y_5 + Y_5^T \\ \Gamma_{34} &= -Y_5 \\ \Gamma_{35} &= -N_3 + Y_3^T, \ \Gamma_{36} = -T_3 + Y_4^T \\ \Gamma_{44} &= -Q_2 - Y_6 - Y_6^T \\ \Gamma_{45} &= -N_6 - Y_3^T \\ \Gamma_{46} &= -T_6 - Y_4^T \\ \Gamma_{55} &= -N_5 - N_5^T + (\tau_2 - \tau_1) R \\ \Gamma_{56} &= -N_6^T - T_5 \\ \Gamma_{66} &= P - T_6^T - T_6 + (\tau_2 - \tau_1) Z \end{aligned} \qquad (13)$$

Proof. For convenience, set

$$\begin{aligned} q(t) &= A_e x_e(t) + A_{ed} x_e(t-h(t)) + B_e v(t) && (14) \\ g(t) &= C_0 x_e(t) + C_{0d} x_e(t-h(t)) && (15) \end{aligned}$$

then system (8) becomes the following descriptor stochastic system

$$\begin{aligned} dx_e(t) &= q(t)dt + g(t)d\beta(t) && (16) \\ z_e &= C_f x_e(t) \end{aligned}$$

Choose a Lyapunov-Krasovskii functional for system (16) to be

$$V(t) = \sum_{i=1}^5 V_i(t) \qquad (17)$$

in which

$$V_1(t) = x_e(t)^T P x_e(t)$$

$$V_2(t) = \sum_{i=1}^2 \int_{t-\tau_i}^t x_e(s)^T Q_i x_e(s) ds$$

$$V_3(t) = \int_{t-h(t)}^t x_e(s)^T Q_3 x_e(s) ds$$

$$V_4(t) = \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t q^T(s) R q(s) ds d\theta$$

$$V_5(t) = \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t trace[g^T(s) Z (g(s)] ds d\theta$$

where P, Q_i, Q_3, Z, R are symmetric positive definite matrices with appropriate dimensions. L be the weak infinitesimal operator of (16), the Newton-Leibniz formula provides

$$x_e(t-\tau_1) - x_e(t-\tau_2) = \varkappa + \varsigma \qquad (18)$$

where $\varkappa = \int_{t-\tau_2}^{t-\tau_1} q(s)ds, \varsigma = \int_{t-\tau_2}^{t-\tau_1} g(s)d\beta(s)$.

For appropriately dimensioned matrices N_j, T_j and Y_j ($j = 1,2,3,4,5,6$), equations in (14) - (15) and (18) ensure that

$$\begin{aligned} &2[x_e^T(t)N_1 + x_e^T(t-h(t))N_2 + x_e^T(t-\tau_1)N_3 \\ &+ x_e^T(t-\tau_2)N_4 + q^T(t)N_5 + g^T(t)N_6] * \\ &[A_e x_e(t) + A_{ed} x_e(t-h(t)) + B_e v(t) - q(t)] \equiv 0 \end{aligned} \qquad (19)$$

$$\begin{aligned} &2[x_e^T(t)T_1 + x_e^T(t-h(t))T_2 + x_e^T(t-\tau_1)T_3 \\ &+ x_e^T(t-\tau_2)T_4 + q^T(t)T_5 + g^T(t)T_6] \\ &* [C_0 x_e(t) + C_{0d} x_e(t-h(t)) - g(t)] \equiv 0 \end{aligned} \qquad (20)$$

$$\begin{aligned} &2[x_e^T(t)Y_1 + x_e^T(t-h(t))Y_2 + x_e^T(t-\tau_1)Y_3 \\ &+ x_e^T(t-\tau_2)Y_4 + q^T(t)Y_5 + g^T(t)Y_6] \\ &* [x_e(t-\tau_1) - x_e(t-\tau_2) - \varkappa - \varsigma] \equiv 0 \end{aligned} \qquad (21)$$

$$(\tau_2 - \tau_1)\Lambda - \int_{t-\tau_2}^{t-\tau_1} \Lambda ds \ge 0 \qquad (22)$$

where $\Lambda = \eta^T(t) \tilde{Y} R^{-1} \tilde{Y}^T \eta(t)$.

Adding the terms on the left of (19)-(22) to $L_{v=0}V$, Moreover, by Lemma [19], for any matrix $Z > 0$

$$-2\eta^T(t)\tilde{Y}\varsigma \leq \eta^T(t)\tilde{Y}Z^{-1}\tilde{Y}^T\eta(t) + \varsigma^T Z\varsigma \qquad (23)$$

then $L_{v=0}V$ can be expressed as

$$L_{v=0}V \leq \eta^T(t)\Xi\eta(t) - \int_{t-\tau_2}^{t-\tau_1} \Theta R^{-1}\Theta^T ds$$

$$-\int_{t-\tau_2}^{t-\tau_1} trace\left[g^T(t)Zg(t)\right] ds + \varsigma^T Z\varsigma \qquad (24)$$

where

$$\Xi = \Gamma + \tilde{Y}(\frac{R}{\tau_2 - \tau_1} + Z)^{-1}\tilde{Y}^T \qquad (25)$$

$$\Theta = \eta^T(t)\tilde{Y} + q^T(s)R \qquad (26)$$

$$\eta(t) = \begin{bmatrix} x_e(t) \\ x_e(t-h(t)) \\ x_e(t-\tau_1) \\ x_e(t-\tau_2) \\ q(t) \\ g(t) \end{bmatrix}, \tilde{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \end{bmatrix} \qquad (27)$$

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} & \Gamma_{16} \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} & \Gamma_{26} \\ * & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35} & \Gamma_{36} \\ * & * & * & \Gamma_{44} & \Gamma_{45} & \Gamma_{46} \\ * & * & * & * & \Gamma_{55} & \Gamma_{56} \\ * & * & * & * & * & \Gamma_{66} \end{bmatrix} (28)$$

Since

$$E\left(\varsigma^T Z\varsigma\right) = E\int_{t-h(t)}^{t} trace\left[g^T(t)Zg(t)\right] ds \qquad (29)$$

It follows that

$$EL_{v=0}V(t) \leq E\eta^T(t)\Xi\eta(t) \qquad (30)$$

By Schur's complement, $\Xi < 0$ is equivalent to LMI (12). By [20] a scalar α_0 exist, such that

$$\lim_{t\to\infty} \sup \frac{1}{t} \log E\left\|x_e(t)\right\|^2 \leq -\alpha_0 \qquad (31)$$

which implies system (8) is exponentially stable in mean square. The proof of Lemma 1 is completed.

Theorem 1. *Consider the system (1) with (3) and (4). Given scalars $\tau_1, \tau_2 > 0$ and μ , for a prescribed $\gamma > 0$, the H_∞ performance $\hat{J} < 0$ holds for all nonzero $v(t) \in L_F^2((0,\infty), R^n)$, if there exist symmetric positive definite matrices $P > 0, Q_r \geq 0, r = 1,2,3, Z > 0, R > 0$, scalars $h_i > 0$ and appropriately dimensioned matrices S, X, T_i and Y_i $(i = 1,2,3,4,5,6)$ satisfying the following LMI*

$$\begin{bmatrix} \Gamma & \tilde{Y} & K & F \\ * & -(\frac{R}{\tau_2-\tau_1} + Z) & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \qquad (32)$$

If a solution of the LMI exists then the filter that guarantees the estimation error level of γ is given by (5) with $A_f = A_0 - S^{-1}XA_1, B_f = S^{-1}X$ and $C_f = L$.Where Γ and $\tilde{Y}$ are defined in (28) and

$$K = \begin{bmatrix} L^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, F = \begin{bmatrix} N_1B_e \\ N_2B_e \\ N_3B_e \\ N_4B_e \\ N_5B_e \\ N_6B_e \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \end{bmatrix} (SB_0 - XB_1)$$

$$N_i = h_iS, i = 1,2,3,4,5,6$$

Proof. First by the proof of the Lemma 1, we can deduce that the augmented system (8) with $v(t) = 0$ to be robust exponentially stable in mean square. Second, we prove $\int_0^\infty \left\|z_e(t)\right\|^2 dt \leq \gamma^2 \int_0^\infty \left\|v(t)\right\|^2 dt$ for all nonzero $v(t) \in L_F^2((0,+\infty), R^n)$ with $x_e(t,0) = 0$. Note that for any $T > 0$

$$\hat{J}(T) = E\int_0^T (\left\|z_e(t)\right\|^2 - \gamma^2\left\|v(t)\right\|^2)dt$$

$$\leq E\int_0^T [\left\|z_e(t)\right\|^2 - \gamma^2\left\|v(t)\right\|^2 + L_vV(\xi(t),t)]dt$$

$$= E\int_0^T \begin{bmatrix} \eta(t) \\ v(t) \end{bmatrix}^T \Psi \begin{bmatrix} \eta(t) \\ v(t) \end{bmatrix} dt$$

$$-E\int_0^T \int_{t-h(t)}^t \Theta R\Theta^T dsdt$$

Where

$$\Psi = \begin{bmatrix} \tilde{\Xi} & F \\ * & -\gamma^2 I \end{bmatrix} \qquad (33)$$

$$\tilde{\Xi} = \Gamma + \tilde{Y}^T(\frac{R}{\tau_2 - \tau_1} + Z)^{-1}\tilde{Y}^T + K^TK \qquad (34)$$

Therefore, if $\Psi < 0$, then

$$\hat{J}(T) \leq -\lambda_{\min}(-\Psi)E\int_0^T (\left\|x_e(t)\right\|^2 + \left\|v(t)\right\|^2)dt$$

$$\leq -\lambda_{\min}(-\Psi)E\int_0^T \left\|v(t)\right\|^2)dt < 0 \quad (35)$$

for any nonzero $v(t) \in L_F^2((0,+\infty), R^n)$, which yields $\hat{J}(t) < -\lambda_{\min}(-\Psi)E\int_0^t \left\|v(t)\right\|^2)dt < 0$ and implies $\int_0^\infty \left\|z_e(t)\right\|^2 dt \leq \gamma^2 \int_0^\infty \left\|v(t)\right\|^2 dt$. Substituting (27) and (28) into (33), and letting $SB_f = X$ then $\Psi < 0$ is equivalent to (32). From our assumption, $B_f = S^{-1}X$ and an H_∞ filter is constructed as in the form of (5). The proof of Theorem 1 is completed.

Remark 4. The smallest γ attenuation level such that the conditions of Theorem 1 hold can be readily obtained from the optimal solution of the following LMI optimization problem:

$$\min_{X,\,S,\,T_i\;Y_i, i=1,2,3,4,5,6} \delta \qquad (36)$$

$$s.t.(32) \text{ with } \gamma^2 := \delta$$

the minimum value of γ is given by $\gamma^* = \sqrt{\delta^*}$,where δ^* is the optimal value of δ, and the associated optimal filter is as in (5).

3 NUMERICAL SIMULATION

Consider the stochastic system with time-varying delay and with parameters as follows:

$$\begin{aligned} A_0 &= \begin{bmatrix} -9 & 1 \\ -2 & -10 \end{bmatrix}, B_0 = \begin{bmatrix} -0.3 \\ -0.42 \end{bmatrix} \\ A_1 &= \begin{bmatrix} -8 & 1 \\ 0.5 & -15 \end{bmatrix}, A_{0d} = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & -0.4 \end{bmatrix} \\ A_{1d} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0.4 \\ -0.2 \end{bmatrix} \\ C_0 &= C_{0d} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, L = \begin{bmatrix} 1 & -1 \end{bmatrix} \end{aligned}$$

Our purpose is to design a filter of the form (5) such that the resulting filtering error system (8) is exponentially stable in mean square with a prescribed H_∞ performance level γ. For the simple choice of $h_1 = 1, h_2 = 0.3, h_3 = 0.03, h_4 = 0.3, h5 = 0.012, h6 = 0.0513$, the minimum achievable noise attenuation level obtained in [16] is $\gamma = 7.6 \times 10^{-4}$, however, applying the delay-dependent criterion of Theorem 1, we obtain $\gamma_{\min} = 1 \times 10^{-7}$ (for $t_1 = 0, t_2 = 0.1, \mu = 0.1$), which is far smaller than the result obtained in [16]. When we take $\gamma = 0.5$, the corresponding filter matrices obtained by Theorem 1 and [16] respectively are

$$\begin{aligned} A_f &= \begin{bmatrix} -10.9327 & 2.5498 \\ -1.1575 & -16.8525 \end{bmatrix} \\ B_f &= \begin{bmatrix} -0.2361 & 0.0876 \\ 0.0771 & -0.4517 \end{bmatrix} \\ C_f &= \begin{bmatrix} 1 & -1 \end{bmatrix} \end{aligned} \tag{37}$$

$$\begin{aligned} A_f &= \begin{bmatrix} -6.5950 & 23.5159 \\ -11.4734 & -1.7319 \end{bmatrix} \\ B_f &= \begin{bmatrix} -1.0249 & -1.8207 \\ 1.3648 & -0.7162 \end{bmatrix} \\ C_f &= \begin{bmatrix} -0.4167 & 0.7050 \end{bmatrix} \end{aligned} \tag{38}$$

Set the initial conditions as $x(0) = [0.1, 0]^T$ and $\hat{x}(0) = [0.04, 0]$, respectively. The exogenous disturbance input $v(t)$ is set as random signal and $v(0) = 0.1$. When $v(t) = 0$, the time responses of real state x_1, x_2 and filter state $\hat{x}_1$, $\hat{x}_2$ are displayed in Figure 1. It shows the resulting filtering error system (8) is exponentially stable in mean square with $v(t) = 0$.
Figure 2 gives the responses of the function $w(t) = \int_0^\infty \|\tilde{z}(t)\|^2 dt \Big/ \int_0^\infty \|v(t)\|^2 dt$. It is seen that the value of this function is less than 0.0012, which reveals that the maximum value of H_∞ performance level $\sqrt{0.0012} = 0.0346$ is much less than the prescribed level $\gamma = 0.5$. Therefore, we can see that the designed H_∞ filter meets the specified requirements. The trajectories of real state

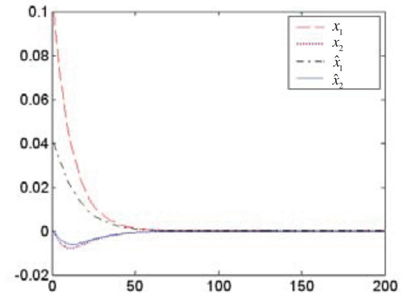

Figure 1: The time responses of real state x and filter state $\hat{x}$

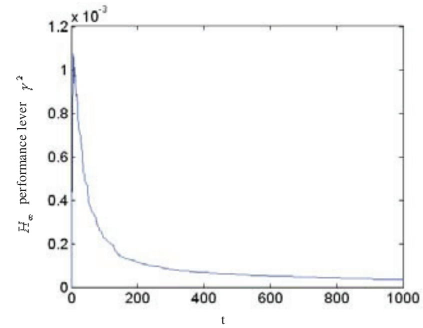

Figure 2: The time responses of H_∞ performance function

x_1, x_2 and filter state $\hat{x}_1$, $\hat{x}_2$ for the filter (37) and (38) are displayed in Figure 3. The simulation results imply that the desired goal is well achieved. Moreover, the effect of the filtering by our method is better than the method in [16].

4 CONCLUSION

The problem of unbiased H_∞ filtering for a class of linear stochastic systems with time-varying delay has been addressed in this paper. LMI-based technique, an exponentially stable in mean square unbiased filter is designed which guarantee L_2 gain (from the external disturbance to the estimation error) to be less than a prescribed level $\gamma > 0$. Delay-dependent sufficient conditions for stochastic systems with time-varying delay are presented. By constructing unbiased filter, the computational complexity is reduced greatly in the process of seeking the filter parameters. And by applying descriptor model transformation of the system and introducing some free weighting matrices, the results obtained are less conservative than the existing results. Numerical examples have clearly demonstrated that our design than the existing with less conservatism. Our approaches to the problem can be applied to various problems with time-varying state delay, including control and estimation of stochastic systems with polytopic uncertainties and general H_∞ output-feedback control.

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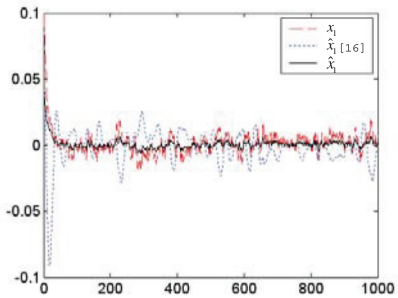
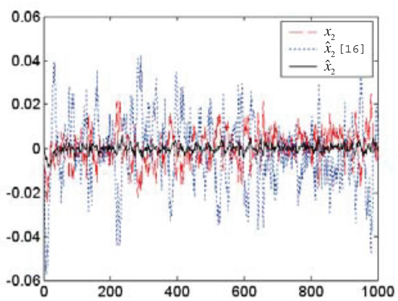


(a)



(b)

Figure 3: The trajectories of real state x and filter state $\hat{x}$ for the filter (37) and the filter (38)

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