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H_∞ filtering for stochastic time- delay systems

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***Abstract*— This paper presents the robust H_∞ filter design for stochastic systems with time-varying delay. The aim is to design a stable linear filter assuring exponential stability in mean-square and a prescribed H_∞ performance level for the filtering error system. Based on the application of the descriptor model transformation and free weighting matrices, delay-dependent sufficient conditions are proposed in terms of linear matrix inequalities(LMIs). Numerical example demonstrates the proposed approaches are effective and are an improvement over existing ones.**

***Index Terms*—exponential stability in mean-square, robust H_∞ filter, time-varying delay, linear stochastic system, linear matrix inequalities(LMIs).**

I. Introduction

In recent years, stochastic H_∞ filtering and control problems with system models expressed by $It\tilde{o}$-type stochastic differential equations have become a interesting research topic and has gained extensive attention; (see [1]-[2]), there have been a lot of studies on H_∞ control or state estimation in deterministic systems, (see [3]-[4]). Robust H_∞ estimation problems for linear and nonlinear stochastic systems were discussed by [1] and [5], respectively.

This paper discusses robust H_∞ filtering for stochastic systems with time-varying delay. Here, we focus on the design of a linear state estimator such that the dynamics of the estimation error is stochastically exponentially stable in mean square, whose L_2 -induced gain with respect to uncertain disturbance signal is less than a prescribed level γ. First, by extending the descriptor system approach introduced in [6] for deterministic systems with delay to the stochastic systems case. We represent the stochastic systems in the equivalent descriptor stochastic systems. Based on the equivalent descriptor form representation, we introduced a new type of Lyapunov-Krasovskii functional. Second, we extended the idea introduced in [7] to stochastic systems case. By using some zero equations, we introduce some free weighting matrices. To guarantee the existence of desired robust H_∞ filters, Delay-dependent sufficient conditions are proposed based on LMI algorithm and the minimum γ obtained are less conservative than corresponding results in the literature. Numerical simulation example shows the results are effective and are an improvement over existing methods.

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For convenience, we adopt the following notations:

$Tr(A)(A^T)$ Trace (transpose) of the matrix A.

$A \geq 0 (A > 0)$ Positive semi-definite (positive definite) matrix A.

$L^2_{\mathcal{F}}([0,\infty); R^n)$ space of nonanticipative stochastic processes $\phi(t)$ with respect to filtration $\mathcal{F}_t$ satisfying

$$\|\phi(t)\|^2_{L^2_{\mathcal{F}}} = E\int_0^\infty \|\phi(t)\|^2 dt < \infty$$

$L^2_{\mathcal{F}_0}([-\tau, 0]; R^n)$ the family of R^n −valued stochastic processes $\eta(s)$, $-\tau \leq s \leq 0$ such that $\eta(s)$ is $\mathcal{F}_0-$ measurable for every second and $\int_{-\tau}^0 E\|\eta(s)\|^2 ds < \infty$

$E\{\cdot\}$ mathematical expectation operator with respect to the given probability measure P.

II. Main Results

Consider the following stochastic linear time-delay systems

$$\begin{aligned} dx(t) &= [A_0x(t) + A_{0d}x(t-h(t) + B_0v(t)]dt \quad (1)\\ &\quad +[C_0x(t) + C_{0d}x(t-h(t))]d\beta(t) \\ x(t) &= \varphi(t), t \in [-\tau, 0] \\ dy(t) &= [A_1x(t) + A_{1d}x(t-h(t) + B_1v(t)]dt \quad (2)\\ &\quad +[C_1x(t) + C_{1d}x(t-h(t))]d\beta(t) \\ z(t) &= Lx(t) \quad (3) \end{aligned}$$

where $x(t) \in R^n$ is the state vector, the time delay $h(t)$ is a time-varying continuous function that satisfies

$$0 \le h(t) \le \tau \tag{4}$$

and

$$\dot{h(t)} \le \mu \le 1 \tag{5}$$

$y(t) \in R^m$ is the measurement output.$\varphi(t)$is a continuous vector-valued initial function and $\varphi := \{\varphi(s) : -\tau \le s \le 0\} \in L^2_{F_0}([-\tau, 0], R^n)$.$z(t) \in R^r$ is the state combination to be estimated, $v(t) \in L^2_F([0,\infty); R^n)$ stands for the exogenous disturbance signal. A_i, A_{id}, B_i, C_i, C_{id}, $i = 1, 2$,and L are known constant matrices with appropriate dimensions. Where the variables $\beta(t)$ is 1-D Brownian motion satisfying $E\{d\beta(t)\} = 0, E\{d\beta(t)^2\} = dt$.

We take the following linear filter for the estimation of $z(t)$

$$\begin{aligned} dx_f(t) &= A_f x_f(t)dt + B_f dy(t) \\ x_f(0) &= 0 \\ z_f(t) &= C_f x_f(t) \end{aligned} \tag{6}$$

where $x_f(t) \in R^n, \hat{z}(t) \in R^r$,the constant matrices A_f, B_f, C_f are filter parameters to be designed. denoting $\xi^T(t) = [x^T(t), x_f{}^T(t)]$ and $\tilde{z}(t) = z(t) - z_f(t)$, then, we obtain the following augmented systems:

$$\begin{aligned} d\xi(t) &= \tilde{A}\xi(t) + \tilde{A}_d\xi(t-h(t)) + \tilde{B}v(t)]dt \\ &\quad +[\tilde{C}\xi(t) + \tilde{C}_d\xi(t-h(t))]d\beta(t) \\ \tilde{z} &= \tilde{L}\xi(t) \end{aligned} \tag{7}$$

where

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} A_0 & 0 \\ B_f A_1 & A_f \end{bmatrix}, \tilde{A}_d = \begin{bmatrix} A_{0d} & 0 \\ B_f A_{1d} & 0 \end{bmatrix} \\ \tilde{B} &= \begin{bmatrix} B_0 \\ B_f B_1 \end{bmatrix}, \tilde{L} = \begin{bmatrix} L & -C_f \end{bmatrix} \\ \tilde{C} &= \begin{bmatrix} C_0 & 0 \\ B_f C_1 & 0 \end{bmatrix}, \tilde{C}_d = \begin{bmatrix} C_{0d} & 0 \\ B_f C_{1d} & 0 \end{bmatrix} \end{aligned} \tag{8}$$

Definition 1: System (7) with $v(t) \equiv 0$ is said to be exponentially stable in mean square if there exists a positive constant α such that

$$\lim_{t\to\infty} \sup \frac{1}{t} \log E \|x(t)\|^2 \le -\alpha$$

The objective of this paper is to seek the filter parameters A_f, B_f and C_f such that the augmented system(7) with $v(t) \equiv 0$ is exponentially stable in mean square, More specifically, for a prescribed disturbance attenuation level $\gamma > 0$, such that the performance index

$$\hat{J} = \int_0^\infty (\tilde{z}^T\tilde{z} - \gamma^2 v^T v)dt \tag{9}$$

is negative $\forall 0 \ne v(t) \in L^2_F((0,\infty), R^n)$.

A. Delay-dependent H_∞ filtering

Before presenting the main results, we first give the following lemmas.

Lemma 1: Given scalars $\tau > 0$ and $\mu < 1$, the system (7) with $v(t) = 0$ is exponentially stable in mean square, if there exist symmetric positive definite matrix $P > 0, Q \ge 0, Z > 0, R > 0$, and appropriately dimensioned matrices N_i, T_j and $Y_i, (i = 1,2,3,4; j = 1,2,3,4,5)$ such that the following LMI holds.

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \tau Y_1 & Y_1 \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \tau Y_2 & Y_2 \\ * & * & \Gamma_{33} & \Gamma_{34} & \tau Y_3 & Y_3 \\ * & * & * & \Gamma_{44} & \tau Y_4 & Y_4 \\ * & * & * & * & -\tau R & 0 \\ * & * & * & * & * & -Z \end{bmatrix} < 0 \tag{10}$$

where an ellipsis $*$ denotes a block induced easily by symmetry, and

$$\begin{aligned} \Gamma_{11} &= N_1\tilde{A} + \tilde{A}^T N_1^T + T_1\tilde{C} + \tilde{C}^T T_1^T + Q + Y_1 + Y_1^T \\ \Gamma_{12} &= N_1\tilde{A}_d + \tilde{A}^T N_2^T + T_1\tilde{C}_d + \tilde{C}^T T_2^T - Y_1 + Y_2^T \\ \Gamma_{13} &= P - N_1 + \tilde{A}^T N_3^T + \tilde{C}^T T_3^T + Y_3^T \\ \Gamma_{14} &= \tilde{A}^T N_4^T - T_1 + \tilde{C}^T T_4^T + Y_4^T \\ \Gamma_{22} &= N_2\tilde{A}_d + \tilde{A}_d^T N_2^T + T_2\tilde{C}_d + \tilde{C}_d^T T_2^T - (1-\mu)Q \\ &\quad -Y_2 - Y_2^T \\ \Gamma_{23} &= -N_2 + \tilde{A}_d^T N_3^T + \tilde{C}_d^T T_3^T - Y_3^T \\ \Gamma_{24} &= \tilde{A}_d^T N_4^T - T_2 + \tilde{C}_d^T T_4^T - Y_4^T \\ \Gamma_{33} &= -N_3 - N_3^T + \tau R, \quad \Gamma_{34} = -N_4^T - T_3 \\ \Gamma_{44} &= P - T_4^T - T_4 + \tau Z \end{aligned}$$

Proof: For convenience, set

$$\begin{aligned} q(t) &= \tilde{A}\xi(t) + \tilde{A}_d\xi(t-h(t)) + \tilde{B}v(t) && (11) \\ g(t) &= \tilde{C}\xi(t) + \tilde{C}_d\xi(t-h(t)) && (12) \end{aligned}$$

then system (7) becomes the following descriptor stochastic systems

$$\begin{aligned} d\xi(t) &= q(t)dt + g(t)d\beta(t) \\ \tilde{z} &= \tilde{L}\xi(t) \end{aligned} \tag{13}$$

Choose a Lyapunov-Krasovskii functional for system (13) to be

$$V(t) = \sum_{i=1}^{4} V_i(t)$$

in which

$$V_1(t) = \xi(t)^T P\xi(t)$$

$$V_2(t) = \int_{t-h(t)}^{t} \xi^T(s)Q\xi(s)ds$$

$$V_3(t) = \int_{-\tau}^{0}\int_{t+\theta}^{t} q^T(s)Rq(s)dsd\theta$$

$$V_4(t) = \int_{-\tau}^{0}\int_{t+\theta}^{t} trace[g^T(s)Z(g(s)]dsd\theta$$

where P, Q, Z, R are symmetric positive definite matrices with appropriate dimensions. L be the weak infinitesimal operator of (13), the Newton-Leibniz formula provides

$$\xi(t) - \xi(t-h(t)) = \int_{t-h(t)}^{t} q(s)ds + \int_{t-h(t)}^{t} g(s)d\beta(s) \quad (14)$$

For appropriately dimensioned matrices N_i, Y_i and T_i $(i = 1, 2, 3, 4)$, equations in (11) - (12) ensure that

$$2\left[\xi^T(t)N_1 + \xi^T(t-h(t))N_2 + q^T(t)N_3 + g^T(t)N_4\right] * \left[\tilde{A}\xi(t) + \tilde{A}_d\xi(t-h(t)) + \tilde{B}v(t) - q(t)\right] \equiv 0 \quad (15)$$

$$2\left[\xi^T(t)T_1 + \xi^T(t-h(t))T_2 + q^T(t)T_3 + g^T(t)T_4\right] * \left[\tilde{C}\xi(t) + \tilde{C}_d\xi(t-h(t)) - g(t)\right] \equiv 0 \quad (16)$$

$$2\left[\xi^T(t)Y_1 + \xi^T(t-h(t))Y_2 + q^T(t)Y_3 + g^T(t)Y_4\right] * [\xi(t) - \xi(t-h(t)) - \varkappa - \varsigma] \equiv 0 \quad (17)$$

$$\tau\Lambda - \int_{t-h(t)}^{t} \Lambda ds \geq 0 \quad (18)$$

where $\varkappa^T = \int_{t-h(t)}^{t} q^T(s)ds$, $\varsigma^T = \int_{t-h(t)}^{t} g^T(s)d\beta(s)$, $\Lambda = \eta^T(t)\tilde{Y}R^{-1}\tilde{Y}^T\eta(t)$. Adding the terms on the left of (15)-(18) to $L_{v=0}V$, Moreover, by Lemma[8], for any matrix $Z > 0$

$$-2\eta^T(t)\tilde{Y}\varsigma \leq \eta^T(t)\tilde{Y}Z^{-1}\tilde{Y}^T\eta(t) + \varsigma^T Z\varsigma$$

then $L_{v=0}V$ can be expressed as

$$L_{v=0}V \leq \eta^T(t)\Xi\eta(t) - \int_{t-h(t)}^{t} \Theta R^{-1}\Theta^T ds - \int_{t-h(t)}^{t} trace\left[g^T(t)Zg(t)\right]ds + \varsigma^T Z\varsigma$$

where

$$\eta^T(t) = \left[\xi^T(t), \xi^T(t-h(t)), q^T(t), g^T(t)\ \right]$$

$$\begin{aligned}
\Xi &= \Gamma + \tau\tilde{Y}R^{-1}\tilde{Y}^T + \tilde{Y}Z^{-1}\tilde{Y}^T \\
\Theta &= \eta^T(t)\tilde{Y} + q^T(s)R \\
\tilde{Y}^T &= \begin{bmatrix} Y_1^T & Y_1^T & Y_1^T & Y_1^T \end{bmatrix} \quad (19) \\
\Gamma &= \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ * & * & \Gamma_{33} & \Gamma_{34} \\ * & * & * & \Gamma_{44} \end{bmatrix} \quad (20)
\end{aligned}$$

Since

$$E\left(\varsigma^T Z\varsigma\right) = E\int_{t-h(t)}^{t} trace\left[g^T(t)Zg(t)\right]ds$$

It follows that

$$EL_{v=0}V(t) \leq E\eta^T(t)\Xi\eta(t) \quad (21)$$

By Schur's complement, $\Xi < 0$ is equivalent to LMI (10).

Set $\lambda_0 = \lambda_{\min}(-\Xi), \lambda_1 = \lambda_{\min}(P)$,by (21)

$$EL_{v=0}V(t) \leq -\lambda_0 E\eta^T(t)\eta(t) \leq -\lambda_0 Ex^T(t)x(t) \quad (22)$$

From the definitions of $V(t), q(t)$ and $g(t)$, there exist positive scalars $\alpha_{1,}\alpha_2$ such that

$$\lambda_1\|x(t)\|^2 \leq V(t) \leq \alpha_1\|x(t)\|^2 + \alpha_2\int_{t-2\tau}^{t}\|x(s)\|^2 ds \quad (23)$$

Choose a scalar $\alpha_0 > 0$ such that

$$\alpha_0(\alpha_1 + 2\alpha_2\tau e^{2\alpha_0\tau}) \leq \lambda_0 \quad (24)$$

then, by $It\hat{o}$ differential formula, for $t_0 \geq 2\tau$, it has

$$Ee^{\alpha_0 t}V(t) - Ee^{\alpha_0 t_0}V(t_0) = E\int_{t_0}^{t} L_{v=0}(e^{\alpha_0 s}V(s))ds$$

$$\leq E\int_{t_0}^{t} e^{\alpha_0 s}[\alpha_0(\alpha_1\|x(s)\|^2 + \alpha_2\int_{s-2\tau}^{s}\|x(u)\|^2 du) - \lambda_0\|x(s)\|^2]ds$$

By (23) and (24), getting

$$\lim_{t\to\infty}\sup\frac{1}{t}\log E\|x(t)\|^2 \leq -\alpha_0$$

which implies system (13) is exponentially stable in mean square. The proof of Lemma 1 is completed. ■

Theorem 1: Consider the system (1)-(3) with (4) and (5). Given scalars $\tau > 0$ and $\mu < 1$, for a prescribed $\gamma > 0$, the H_∞ performance $\hat{J} < 0$ holds for all nonzero $v(t) \in L_F^2((0,\infty), R^n)$, if there exist symmetric positive definite matrix $P > 0, Q \geq 0, Z > 0, R > 0$, scalars $h_i > 0, e_i > 0$ and appropriately dimensioned matrices S_i ,W_i and Y_i $(i = 1, 2, 3, 4)$ satisfying the following LMI

$$\begin{bmatrix} \Gamma & \tau\tilde{Y} & \tilde{Y} & \tilde{K} & \tilde{F} \\ * & -\tau R & 0 & 0 & 0 \\ * & * & -Z & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (25)$$

If a solution of the LMI exists then the filter that guarantees the estimation error level of γ is given by (6) with

$$A_f = S_4^{-1}X_4, B_f = S_4^{-1}X_5 \ and \ C_f \ .$$

where Γ and $\tilde{Y}$ are defined in (19) and (20), respectively ,

$$\tilde{F} = \begin{bmatrix} N_1\tilde{B} \\ N_2\tilde{B} \\ N_3\tilde{B} \\ N_4\tilde{B} \end{bmatrix}, \tilde{K} = \begin{bmatrix} \tilde{L} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{26}$$

and

$$\Gamma_{ii} = \begin{bmatrix} \Gamma_{ii}^1 & \Gamma_{ii}^2 \\ * & \Gamma_{ii}^4 \end{bmatrix}, \ \Gamma_{ij} = \begin{bmatrix} \Gamma_{ij}^1 & \Gamma_{ij}^2 \\ \Gamma_{ij}^3 & \Gamma_{ij}^4 \end{bmatrix} \tag{27}$$

$$\begin{aligned}
\Gamma_{11}^1 &= h_1S_1A_0 + h_1A_0^TS_1^T + h_1X_2A_1 + h_1A_1^TX_2^T \\
&\quad +e_1W_1C_0 + e_1C_0^TW_1^T + e_1X_3C_1 \\
&\quad +e_1C_1^TX_3^T + Q_1 + Y_{11} + Y_{11}^T \\
\Gamma_{11}^2 &= h_1X_1 + h_1A_0^TS_3^T + h_1A_1^TX_5^T + e_1C_0^TW_3^T \\
&\quad +e_1C_1^TX_6^T + Q_2 + Y_{12} + Y_{13}^T \\
\Gamma_{11}^4 &= h_1X_4 + h_1X_4^T + Q_3 + Y_{14} + Y_{14}^T \\
\Gamma_{12}^1 &= h_1S_1A_{0d} + h_1X_2A_{1d} + h_2A_0^TS_1^T + h_2A_1^TX_2^T \\
&\quad +e_1W_1C_{0d} + e_1X_3C_{1d} + e_2C_0^TW_1^T \\
&\quad +e_2C_1^TX_3^T - Y_{11} + Y_{21}^T \\
\Gamma_{12}^2 &= h_2A_0^TS_3^T + h_2A_1^TX_5^T + e_2C_0^TW_3^T \\
&\quad +e_2C_1^TX_6^T - Y_{12} + Y_{23}^T \\
\Gamma_{12}^3 &= h_1S_3A_{0d} + h_1X_5A_{1d} + h_2X_1^T + e_1W_3C_{0d} \\
&\quad +e_1X_6C_{1d} - Y_{13} + Y_{22}^T \\
\Gamma_{12}^4 &= h_2X_4^T - Y_{14} + Y_{24}^T \\
\Gamma_{13}^1 &= P_1 - h_1S_1 + h_3A_0^TS_1^T + h_3A_1^TX_2^T \\
&\quad +e_3C_0^TW_1^T + e_3C_1^TX_3^T + Y_{31}^T \\
\Gamma_{13}^2 &= P_2 - h_1S_2 + h_3A_0^TS_3^T + h_3A_1^TX_5^T \\
&\quad +e_3C_0^TW_3^T + e_3C_1^TX_6^T + Y_{33}^T
\end{aligned}$$

$$\begin{aligned}
\Gamma_{13}^3 &= P_2^T - h_1S_3 + h_3X_1^T + Y_{32}^T \\
\Gamma_{13}^4 &= P_3 - h_1S_4 + h_3X_4^T + Y_{34}^T \\
\Gamma_{14}^1 &= h_4A_0^TS_1^T + h_4A_1^TX_2^T - e_1W_1 \\
&\quad +e_4C_0^TW_1^T + e_4C_1^TX_3^T + Y_{41}^T \\
\Gamma_{14}^2 &= h_4A_0^TS_3^T + h_4A_1^TX_5^T - e_1W_2 \\
&\quad +e_4C_0^TW_3^T + e_4C_1^TX_6^T + Y_{43}^T \\
\Gamma_{14}^3 &= h_4X_1^T - e_1W_3 + Y_{42}^T \\
\Gamma_{14}^4 &= h_4X_4^T - e_1W_4 + Y_{44}^T \\
\Gamma_{14}^4 &= h_4X_4^T - e_1W_4 + Y_{44}^T \\
\Gamma_{15}^1 &= \tau Y_{11}, \ \Gamma_{15}^2 = \tau Y_{12}, \Gamma_{15}^3 = \tau Y_{13}, \ \Gamma_{15}^4 = \tau Y_{14} \\
\Gamma_{16}^1 &= h_1S_1B_0 + h_1X_2B_1 \\
\Gamma_{16}^2 &= h_1S_3B_0 + h_1X_5B_1 \\
\Gamma_{22}^1 &= h_2S_1A_{0d} + h_2X_2A_{1d} + h_2A_{0d}^TS_1^T + h_2A_{1d}^TX_2^T \\
&\quad +e_2W_1C_{0d} + e_2X_3C_{1d} + e_2C_{0d}^TW_1^T \\
&\quad +e_2C_{1d}^TX_3^T - (1-\mu)Q_1 - Y_{21} - Y_{21}^T \\
\Gamma_{22}^2 &= h_2A_{0d}^TS_3^T + h_2A_{1d}^TX_5^T + e_2C_{0d}^TW_3^T \\
&\quad +e_2C_{1d}^TX_6^T - (1-\mu)Q_2 - Y_{22} - Y_{23}^T \\
\Gamma_{22}^4 &= -(1-\mu)Q_3 - Y_{24} - Y_{24}^T \\
\Gamma_{23}^1 &= -h_2S_1 + h_3A_{0d}^TS_1^T + h_3A_{1d}^TX_2^T \\
&\quad +e_3C_{0d}^TW_1^T + e_3C_{1d}^TX_3^T - Y_{31}^T \\
\Gamma_{23}^2 &= -h_2S_2 + h_3A_{0d}^TS_3^T + h_3A_{1d}^TX_5^T \\
&\quad +e_3C_{0d}^TW_3^T + e_3C_{1d}^TX_6^T - Y_{33}^T \\
\Gamma_{23}^3 &= -h_2S_3 - Y_{32}^T, \ \Gamma_{23}^4 = -h_2S_4 - Y_{34}^T \\
\Gamma_{24}^1 &= h_4A_{0d}^TS_1^T + h_4A_{1d}^TX_2^T + e_4C_{0d}^TW_1^T \\
&\quad +e_4C_{1d}^TX_3^T - e_2W_1 - Y_{41}^T \\
\Gamma_{24}^2 &= h_4A_{0d}^TS_3^T + h_4A_{1d}^TX_5^T + e_4C_{0d}^TW_3^T \\
&\quad +e_4C_{1d}^TX_6^T - e_2W_2 - Y_{43}^T \\
\Gamma_{24}^3 &= -e_2W_3 - Y_{42}^T, \ \Gamma_{24}^4 = -e_2W_4 - Y_{44}^T \\
\Gamma_{25}^1 &= \tau Y_{21}, \Gamma_{25}^2 = \tau Y_{22}, \Gamma_{25}^3 = \tau Y_{23}, \ \Gamma_{25}^4 = \tau Y_{24} \\
\Gamma_{26}^1 &= h_2S_1B_0 + h_2X_2B_1 \\
\Gamma_{26}^2 &= h_2S_3B_0 + h_2X_5B_1 \\
\Gamma_{33}^1 &= -h_3S_1 - h_3S_1^T + \tau R_1 \\
\Gamma_{33}^2 &= -h_3S_2 - h_3S_3^T + \tau R_2 \\
\Gamma_{33}^4 &= -h_3S_4 - h_3S_4^T + \tau R_3 \\
\Gamma_{34}^1 &= -h_4S_1^T - e_3W_1, \ \Gamma_{34}^2 = -h_4S_3^T - e_3W_2 \\
\Gamma_{34}^3 &= -h_4S_2^T - e_3W_3, \ \Gamma_{34}^4 = -h_4S_4^T - e_3W_4
\end{aligned}$$

$$\begin{aligned}
\Gamma_{35}^1 &= \tau Y_{31}, \ \Gamma_{35}^2 = \tau Y_{32}, \ \Gamma_{35}^3 = \tau Y_{33}, \ \Gamma_{35}^4 = \tau Y_{34} \\
\Gamma_{36}^1 &= h_3S_1B_0 + h_3X_2B_1, \Gamma_{36}^2 = h_3S_3B_0 + h_3X_5B_1 \\
\Gamma_{44}^1 &= P_1 - e_4W_1^T - e_4W_1 + \tau Z_1 \\
\Gamma_{44}^2 &= P_2 - e_4W_3^T - e_4W_2 + \tau Z_2
\end{aligned}$$

$$\begin{aligned}
\Gamma_{44}^4 &= P_3 - e_4W_4^T - e_4W_4 + \tau Z_3 \\
\Gamma_{45}^1 &= \tau Y_{41}, \Gamma_{45}^2 = \tau Y_{42}, \Gamma_{45}^3 = \tau Y_{43}, \Gamma_{45}^4 = \tau Y_{44} \\
\Gamma_{46}^1 &= h_4S_1B_0 + h_4X_2B_1, \ \Gamma_{46}^2 = h_4S_3B_0 + h_4X_5B_1
\end{aligned}$$

$$\begin{aligned}
Y_i &= \begin{bmatrix} Y_{i1} & Y_{i2} \\ Y_{i3} & Y_{i4} \end{bmatrix}, i = 1, 2, 3, 4 \\
\tilde{L} &= \begin{bmatrix} L & -C_f \end{bmatrix}
\end{aligned} \tag{28}$$

Proof: First by the proof of the Lemma 1, we can deduce that the augmented system (7) with $v(t) = 0$ to be robust exponential stable in mean square. Second, we prove $\hat{J} < 0$.for all nonzero $v(t) \in L_F^2((0, +\infty), R^n)$ with $\xi(t, 0) = 0$,Note that for any $T > 0$

$$\begin{aligned}\hat{J}(T) &= E\int_0^T (\|\tilde{z}(t)\|^2 - \gamma^2 \|v(t)\|^2)dt \\ &\leq E\int_0^T [\|\tilde{z}(t)\|^2 - \gamma^2 \|v(t)\|^2 + L_v V(\xi(t), t)]dt \\ &= E\int_0^T \begin{bmatrix} \eta(t) \\ v(t) \end{bmatrix}^T \Psi \begin{bmatrix} \eta(t) \\ v(t) \end{bmatrix} dt \\ &\quad -E\int_0^T \int_{t-h(t)}^t \Theta R \Theta^T ds dt\end{aligned}$$

Where $\tilde{F}$ and $\tilde{K}$ are defined in (26), and

$$\begin{aligned}\Psi &= \begin{bmatrix} \tilde{\Xi} & \tilde{F} \\ * & -\gamma^2 I \end{bmatrix} \\ \tilde{\Xi} &= \Gamma + \tau \tilde{Y} R^{-1} \tilde{Y}^T + \tilde{Y} Z^{-1} \tilde{Y}^T + \tilde{K}\tilde{K}^T \quad (29)\end{aligned}$$

Therefore, if $\Psi < 0$,then

$$\begin{aligned}\hat{J}(T) &\leq -\lambda_{\min}(-\Psi) E\int_0^T (\|\xi(t)\|^2 + \|v(t)\|^2)dt \\ &\leq -\lambda_{\min}(-\Psi) E\int_0^T \|v(t)\|^2)dt < 0\end{aligned}$$

for any nonzero $v(t) \in L_F^2((0,+\infty), R^n)$,which yield $\hat{J}(t) < -\lambda_{\min}(-\Psi)E\int_0^t \|v(t)\|^2)dt < 0$. If we take

$$\begin{aligned}N_i &= h_i \begin{bmatrix} S_1 & S_2 \\ S_3 & S_4 \end{bmatrix}, Y_i = \begin{bmatrix} Y_{i1} & Y_{i2} \\ Y_{i3} & Y_{i4} \end{bmatrix} \quad (30) \\ T_i &= e_i \begin{bmatrix} W_1 & W_2 \\ W_3 & W_4 \end{bmatrix}, i = 1,2,3,4 \quad (31) \\ Q &= \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix}, P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} \\ R &= \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix}, Z = \begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix} \quad (32)\end{aligned}$$

Substituting (8) and (32) into (29), and letting $S_2 A_f = X_1$, $S_2 B_f = X_2, W_2 B_f = X_3, S_4 A_f = X_4$, $S_4 B_f = X_5, W_4 B_f = X_6$ then $\Psi < 0$ is equivalent to (25). From our assumption, $A_f = S_4^{-1} X_4$, $B_f = S_4^{-1} X_5$,and an H_∞ filter is constructed as in the form of (6), and the proof of Theorem 1 is completed. ■

III. NUMERICAL SIMULATION

Consider the stochastic time-varying delay system[10] with parameters as follows:

$$\begin{aligned}A_0 &= \begin{bmatrix} 0 & 3 \\ -4 & -5 \end{bmatrix}, B_0 = \begin{bmatrix} -0.4545 \\ 0.9090 \end{bmatrix} \\ A_{0d} &= \begin{bmatrix} -0.1 & 0 \\ 0.2 & -0.2 \end{bmatrix}, A_{1d} = \begin{bmatrix} 0.5 & 0.3 \end{bmatrix} \\ A_1 &= \begin{bmatrix} 1 & 2 \end{bmatrix}, L = \begin{bmatrix} 3 & 4 \end{bmatrix}, B_1 = 1 \\ C_0 &= C_{0d} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, C_1 = C_{1d} = \begin{bmatrix} 1 & 2 \end{bmatrix}\end{aligned}$$

For the simple choice of $h_1 = 0.7, h_2 = 0.41, h_3 = 0.7, h_4 = 0.2, e_1 = 0.5$, $e_2 = 0.3, e_3 = 0.1, e_4 = 0.5, e_5 = 0.01$,

The minimum achievable noise attenuation level obtained in [10] is $\gamma = 0.6074$, However, applying the delay-dependent criterion of Theorem 1, we obtain that the minimum achievable bound on the noise attenuation level is $\gamma_{\min} = 8\times10^{-7}$ (for $\tau = 0.1, \mu = 0.3$), which is far smaller than the result obtained in [10].When we take $\gamma = 0.6074$, the corresponding filter matrices obtained by Theorem 1 and [10] respectively are

$$\begin{aligned}A_f &= \begin{bmatrix} -9.7029 & 1.1090 \\ 0.9684 & -9.4364 \end{bmatrix}, B_f = \begin{bmatrix} -0.1195 \\ -0.1562 \end{bmatrix} \\ C_f &= \begin{bmatrix} 0.0399 & 0.0965 \end{bmatrix} \quad (33)\end{aligned}$$

and

$$\begin{aligned}A_f &= \begin{bmatrix} 0.3065 & 2.5820 \\ -4.4618 & -5.0860 \end{bmatrix}, B_f = \begin{bmatrix} 0.1797 \\ -0.5045 \end{bmatrix} \\ C_f &= \begin{bmatrix} -2.8091 & -3.4756 \end{bmatrix} \quad (34)\end{aligned}$$

Set the initial conditions as $x(0) = [0.1, 0]^T, x_f = [0.04, 0]^T$ respectively. The exogenous disturbance input $v(t)$ is set as random noise and $v(0) = 0.1$. Fig.1 give the time responses of the system states and filter states for the filter (33) with $v(t) = 0$. It shows the filtering error system (7) is exponential stable in mean square with $v(t) = 0$.Fig.2 show the time response of the function $\omega(t) = \int_0^\infty \|\tilde{z}(s)\|^2 ds \big/ \int_0^\infty \|v(s)\|^2 ds$ with $v(t) \neq 0$,It is seen that the maximum value of this function is less than 0.25, which reveals that the H_∞ performance level $\sqrt{0.25} = 0.5$ is much less than the prescribed level 0.6074. Therefore, we can see that the designed H_∞ filter meets the specified requirements. When $v(t) \neq 0$,the time responses of the system states x_1, x_2 and filter state x_{f1}, x_{f2} for the filter (33) and (34) are displayed in Fig.3 and Fig.4 respectively. The simulation results imply that the desired goal is well achieved. Moreover, our method is better than the method in [10].

IV. CONCLUSION

The problem of robust H_∞ filtering for stochastic systems with time-varying delay has been addressed in this paper. LMI-based technique, exponentially stable in mean square linear filter are designed which guarantee L_2 gain to be less than a prescribed level $\gamma > 0$.Delay-dependent sufficient condition for stochastic system is presented, the

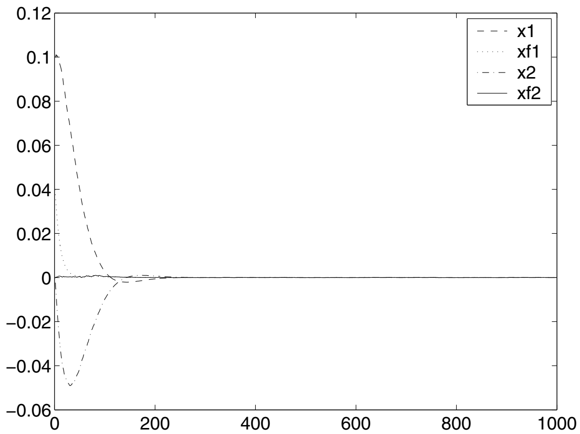

Fig. 1. The time responses of state x and x_f

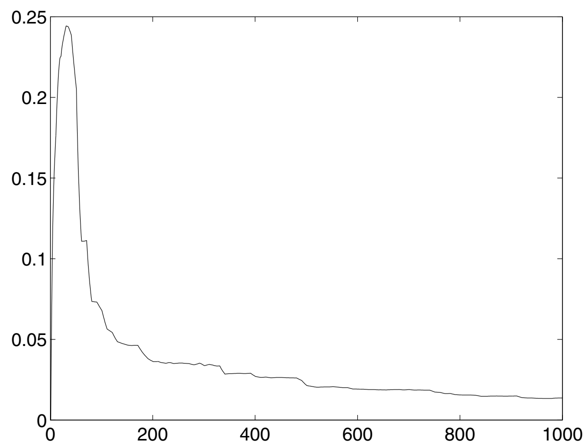

Fig. 2. the time response of the function $\omega(t)$

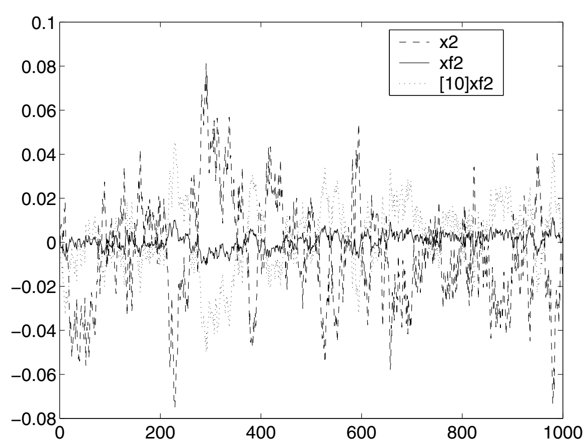

Fig. 3. The time responses of state x_1 and x_{f1}

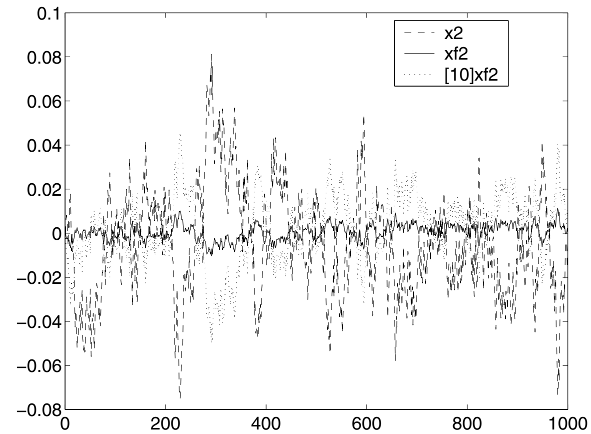

Fig. 4. The time responses of state x_2 and x_{f2}

minimum γ obtained are less conservative than corresponding results in the literature due to applying descriptor model transformation of the system and introducing some free weighting matrices. Numerical example has clearly indicated the less conservatism of our design.

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