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Nonlinear Protocols for Output Performance Value Consensus of Multi-Agent Systems

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Abstract: This paper investigates output performance value consensus issue, which is a new necessarily considered issue for cooperative and coordinate control of multi-agent systems with fixed topology. Based on Lyapunov theory, this paper presents nonlinear protocols for output performance value consensus of systems and proves the asymptotic consensus is reachable. Simulation example shows the nonlinear protocols are effective for output performance value consensus of multi-agent systems.

Key Words: Nonlinear Consensus Protocol, Output Performance Value Consensus, Multi-Agent Sysyems

1 Introduction

Recent years, distributed coordination of multi-agent systems has attracted considerable attention. This is mainly due to extensive applications of multi-agent systems in many areas including multiorganization scheduling and coordination for maintenance networks [1], coordination techniques for complex environments [2], distributed resource allocation [3], distributed automatic target recognition [4], cooperative control of unmanned air vehicles (UAV's) [5], autonomous underwater vehicles (AUV's) [6]-[7], congestion control in communication networks [8], swarms of autonomous vehicles or robots [9]-[11], autonomous formation flight [12]-[13]. In many cases the aim is to control a group of agents connected through a communication network to reach an agreement on certain quantities of interest. This problem is usually called the consensus problem. Many results have been obtained on this problem [16]-[20]. For example, Lin et al. [16] studied three formation strategies for groups of mobile autonomous agents. Li and Guan [17] gave the nonlinear consensus analysis of multi-agents based on center manifold reduction. Moreau [18] used a set-valued Lyapunov approach to study consensus problems with unidirectional time-dependent communication links. Moreover, by a Lyapunov-based approach, Olfati-Saber and Murray [19] solved the average-consensus problem for network of agents with switching topology and time-delays. P. Lin et al. [20] investigated average consensus in networks of multi-agents with both switching topology and coupling time-delay. Above literatures mentioned are all concerned the internal information state consensus of multi-agent systems, however, the problem of output performance value consensus is a new necessarily considered issue for cooperative and coordinate control of multi-agent systems.

This paper present the output performance value consensus issue of the multi-agent systems for continuous-time case. By exchanging the performance value with others, agents try to work cooperatively in order to reach consensus on their output performance value. In practice, we sometimes only require a group of agents to achieve an output performance value consensus regardless their internal information state. The output performance value consensus mentioned in this paper is significant in some areas, especially, in the biology and chemic areas where only require certain life-form group or chemical reactor to reach an output agreement but not require the agreement about their internal information state. So, it is significant for the discussion about output performance value consensus. The contribution on this aspect is not sufficient in the existing literatures.Naoki et el. [21] propose the nonlinear output consensus issue of the multi-agent systems for discrete-time case, however, the issue for continuous-time case has not yet been considered in the existing literature. Further, the nonlinear output consensus protocol provide a generalization to linear consensus protocols and it may be used to enhance the performance of the dynamic consensus algorithm or satisfy other constraints.These motivate us to investigate the nonlinear output consensus protocol of multi-agent systems. Simulation results show the nonlinear output feedback control strategies is effective for the output performance value consensus of the agents.

2 Output performance consensus protocol

Suppose that the network system under consideration consists of n agents. Each agent is regarded as a node in a directed graph G. Each edge $(j,i) \in E(G)$ corresponds to an available information link from agent j to agent i. Moreover, each agent updates its current state based upon the information received from its neighbors. Let x_i be the state of the ith agent.

We propose an output performance function consensus protocol for locally communication multi-agent systems. Each agent has multi-agent performance value based on its internal information state. By exchanging the performance value with others, agents try to work cooperatively in order to reach consensus on their output performance value.

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Consider the following dynamic directed network of multi-agent systems with nonlinear output function:

$$\dot{x}_i(t) = u_i(t), \forall i \in \Gamma, j \in N_i \tag{1}$$

$$y_i = \phi(x_i) \tag{2}$$

with initial condition $x_i(s) = x_i(0)$, $s \in (-\infty, 0]$, where u_i is the control protocol, $y_i \in R$ be the output state of the ith agent ($i \in \Gamma$). The output performance value corresponding to the internal information state x_i is characterized by the performance function $\phi : R \to R$. We assume that the output performance function $\phi(\cdot)$ is continuous and locally lipschitz and satisfies $(x-y)[xg(x) - yg(y)] > 0, \forall x \neq y$, where the map $g := \dot{\phi} \circ \phi^{-1} : R \to R$.

Our objective is to find appropriate nonlinear protocol to suppress disturbances of agents and make all agents reach output performance value agreement.

We design a nonlinear output feedback control protocol as the following form

$$u_i = \sum_{j \in N_i} (y_j - y_i) \tag{3}$$

Substituting (3) into (1), then the dynamic systems can be rewrite as

$$\dot{x}_i(t) = \sum_{j \in N_i} [\phi(x_j) - \phi(x_i)], \forall i \in \Gamma, j \in N_i \tag{4}$$

$$y_i = \phi(x_i) \tag{5}$$

Equation (4) indicates that each agent updates its internal information state to narrow the performance gap according to the communication among agents. Equation (5) shows that each agent evaluates the performance value based on its internal information state. If Eq. (6) holds for any initial information state $x_i(0) \in D^n$ and for any $i, j \in \Gamma$, we say a group of agents achieve globally output performance consensus.

$$|\phi(x_i) - \phi(x_j)| \to 0, ast \to \infty \tag{6}$$

Denote D^n ($D^n \subseteq R$) is the domains of definition of function $\phi(\cdot)$, i.e. $x_i \in D^n, i = 1, \cdots, n$.

Now we give the following theorem.

Theorem. *Consider a dynamic bidirected network of multi-agent systems (1)-(2), assume the topology graph $G(\Gamma, E)$ is connected, then given multi-agent feedback protocol (3), the agents can achieve globally consensus about their output performance value.*

Proof. Assume the trajectory of Eq.(5) can be written as

$$y_i(t) = a + \delta_i(t) \tag{7}$$

where a is a real constant values, $\delta_i(t)$ can be called the disagreement state. From Eq.(7), we have

$$\dot{y}_i \;=\; \dot{\phi}(x_i)\dot{x}_i = \sum_{j \in N_i} \dot{\phi}[\phi^{-1}(y_i)](y_j - y_i)$$

$$= \sum_{j \in N_i} g(y_i)(y_j - y_i) \tag{8}$$

Notice that $\dot{y}_i(t) = \dot{\delta}_i(t)$, thus δ_i satisfies the following disagreement dynamics equation

$$\dot{\delta}_i(t) = \sum_{j \in N_i} g(a + \delta_i)(\delta_j - \delta_i), \forall i \in \Gamma, j \in N_i \tag{9}$$

Defining the Lyapunov function

$$V(t) = \sum_{i=1}^{n} (\delta_i)^2 \tag{10}$$

then we get

$$\begin{aligned}\dot{V}(t) &= 2\sum_{i=1}^{n} \sum_{j \in N_i} \delta_i g(a + \delta_i)(\delta_j - \delta_i) \\ &= \sum_{(i,j) \in E} \{\delta_i g(a + \delta_i)(\delta_j - \delta_i) \\ &\quad + \delta_j g(a + \delta_j)(\delta_i - \delta_j)\} \\ &= -\sum_{(i,j) \in E} (\delta_j - \delta_i)[\delta_j g(a + \delta_j) \\ &\quad - \delta_i g(a + \delta_i)] \leq 0\end{aligned}$$

so the zero solution of Eq. (9) is globally asymptotically stable, i.e. $y_i(t) \to a$ as $t \to \infty$.

Remark1. Theorem 1 shows that a group of agents can always achieve an output performance value consensus as long as we give multi-agent feedback as (3), whether their internal information state can reach consensus or not.

Remark2. A group of agents can sure reach an output performance value consensus if they can achieve a consensus about their internal information state.

Remark3. The output performance value consensus mentioned in this paper is significant in some areas, especially, in the biology and chemic areas where only require certain life-form group or chemical reactor to reach an output agreement but not require the agreement about their internal information state.

For example, we provide a numerical example with a group of 5 agents. Assume the dynamic bidirected network systems with output performance function $y_i = \sin(x_i), i = 1, 2, 3, 4, 5$, the initial state is $x(0) = [5.7; -3; 0.1; 2, -4.5]^T$. Information topologies among agents are depicted in Fig.2. We give the group agents the following output feedback control protocol

$$u_i = \sum_{j \in N_i} (y_j - y_i) \tag{11}$$

Fig.3. shows the change of output states y_i and internal information states x_i ($i = 1, 2, 3, 4, 5$). We can see that a group of agents reached the global consensus about their output performance values, though the internal information states of agents can't achieve consensus.

However, if we restrict $\sin(x_i)$ is monotone increasing, i.e. $x_i \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ($i = 1, 2, 3, 4, 5$), the initial state is $x(0) = [1; 0.4; -0.8; 0.2, -0.1]^T \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Fig.4. shows the group of agents can achieve a consensus not only about their output performance value but also about their internal information state, and they reach an average-consensus.

Remark4. We only discuss the case that output state vector y_i and internal information state x_i are 1-dimension and

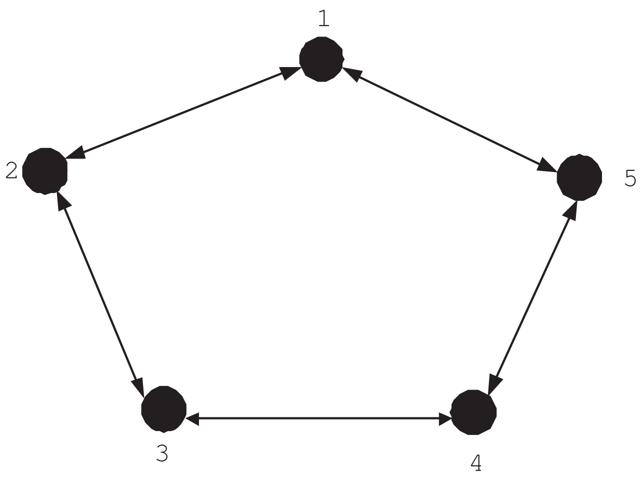

Fig. 1: bidirected graph used for output performance value consensus

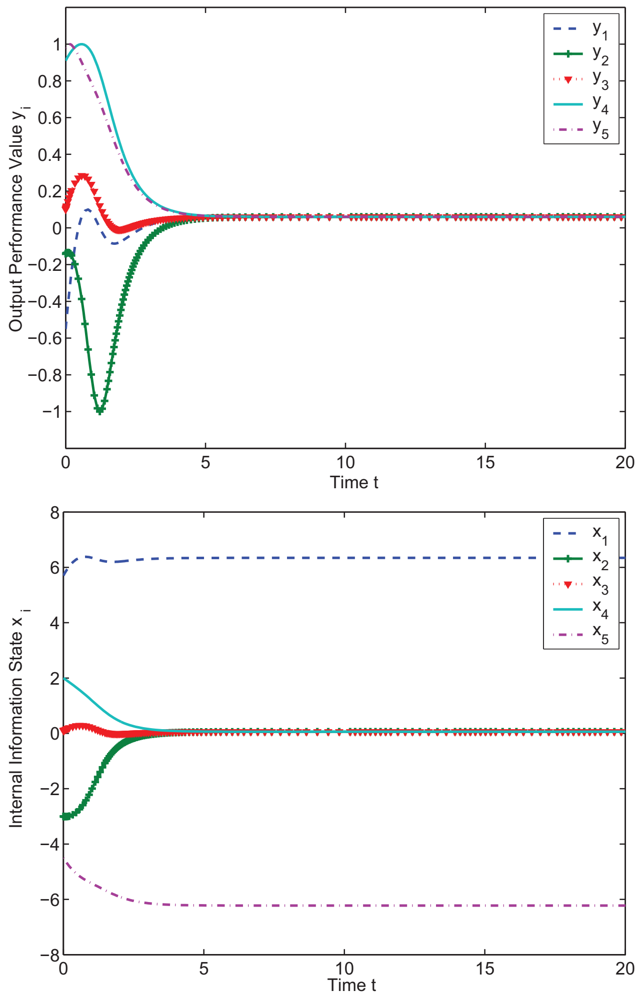

Fig. 2: The state trajectories of output performance value y_i and internal information state x_i (i=1,2,3,4,5)

their dimension is identical, i.e. $y_i \in R, x_i \in R$. However, as $y_i \in R^n, x_i \in R^m$ and $n \neq m$, we don't discuss that case. It deserves further research.

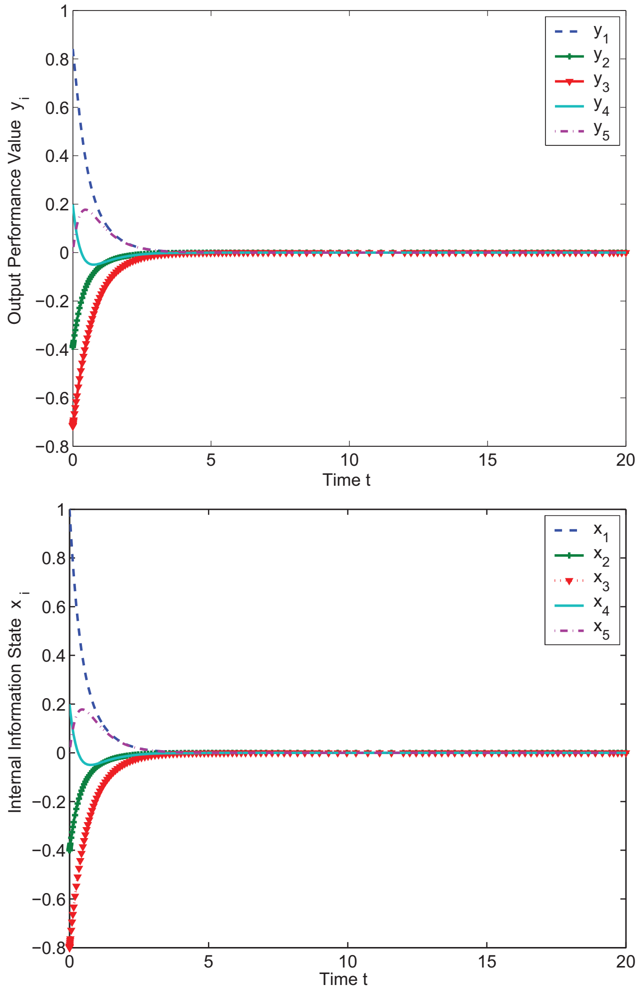

Fig. 3: The state trajectories of output performance value y_i and internal information state x_i (i=1,2,3,4,5)

3 Conclusion

This paper proposes output performance value consensus issue for cooperative and coordinate control of multi-agent systems. By output feedback control protocol(3), we prove that a group of agents can always achieve an output performance value consensus,whether their internal information state can reach consensus or not.We perform simulation studies concerning a group of 5 agents with the fixed communication network topology structure. Simulation results show these nonlinear protocols are effective for output performance value consensus of multi-agent systems.

These results can be applied to many other fields including synchronization, flocking, swarming, distributed decision making and so on. In addition, the paper doesn't consider the influence of time-delay and noise which are unavoidable in the cooperative and coordinate control of multi-agent systems. It deserves further research.

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