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Unbiased H_∞ filtering for stochastic systems with data packet losses

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Abstract. This paper presents the unbiased H_∞ filter design for stochastic systems with data packet losses. By constructing unbiased filter, the complexity and computational burden of the real-time filtering process are reduced greatly. Delay-dependent sufficient conditions for stochastic system with data packet losses are proposed in terms of linear matrix inequalities (LMIs). Numerical example demonstrates the proposed approaches are effective.

Introduction

Recently, the H_∞ filtering and control problems of stochastic system have gained extensive attentions and achieved comprehensive applications in many fields [1,2,3,4,5,6,7]. But, the order of filter error system in [1,2,3,4,5,6,7] is the twice as great as the original system, in the real time filtering process, which directly result in the complexity and computational burden. In addition, it is well known that data packet dropouts are commonly encountered in control field and it usually cause instability and poor performance of signals. Unfortunately, there are very few corresponding works dealing with the filter design problems for systems with data packet losses. These motivate us to investigate the unbiased filter design problem for stochastic systems with data packet losses.

This paper discusses the H_∞ filtering problem of stochastic system with data packet losses. By constructing an unbiased filter such that the order of filter error dynamic system is the same as the original system, in contrast with the normal H_∞ filtering approaches, which result in the order of filter error dynamic system is the twice as great as the original system. Therefore，the unbiased filter reduce the complexity and computational burden of the real-time filtering process. Based on LMI algorithm, delay-dependent sufficient conditions for stochastic system with data packet dropouts are proposed and the minimum H_∞ performance lever γ is obtained. Numerical simulation example shows the results are effective.

Unbiased H_∞ Filtering for Stochastic Systems Based on Data Packet Losses

consider the following stochastic system

$$\begin{aligned} dx(t) &= [A_0 x(t) + B_0 v(t)]dt + C_0 x(t) d\beta(t) \\ x(t) &= \varphi(t), t \in [-\tau, 0] \\ z(t) &= Lx(t)\ . \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $\varphi(t)$ is a continuous vector-valued initial function, $z(t) \in R^r$ is the state combination to be estimated, $v(t) \in L_F^2([0,\infty); R^n)$ stands for the exogenous disturbance signal. A_0, B_0, C_0 and L are known constant matrices with appropriate dimensions. Where the stochastic variables $\beta(t)$ is zero-mean real scalar Wiener processes.

Due to the data packet dropouts lead to the measurement signal contains the uncertainty. So we construct a new measurement model in the section and the model with uncertainty can be described as

$$y_{pd} = \theta(t)y = \theta(t)A_1x(t) + \theta(t)A_{1d}x(t-h(t)) + \theta(t)B_1v(t). \tag{2}$$

where $h(t) = t - t_k + \tau_{sc} + d(k)h$ and represents there are $d(k)$ packets dropout at time t_k. The delay part $h(t)$ may vary with time t and it is assumed that $0 \le \tau_1 \le h(t) = t - t_k + \tau_{sc} + d(k)h \le h + \tau_{sc} \le \tau_2,\ t \in [t_k, t_{k+1})$ and $\dot{h}(t) \le \mu$. Assuming $\theta(t)$ is a stochastic matrix of the form

$$\theta(t) = diag(\theta_1, \theta_2, \cdots, \theta_m). \tag{3}$$

where $\theta_i(t), (i = 1, \cdots, m)$ taking the values of 0 and 1 is independent stochastic variable one another and satisfies the following probability

$$\text{Prob}\{\theta_i(t) = 0\} = E\{\theta_i(t)\} = \rho_i\ , i = 1, \cdots, m, \text{Prob}\{\theta_i(t) = 1\} = E\{1 - \theta_i(t)\} = 1 - \rho_i. \tag{4}$$

$\theta(t) = 0$ represents the case that data packet losses occur, while $\theta(t) = I$ represents the case that no data packets are dropped, and $\theta_i(t) = 0$ means the ith output signal dropout, $\theta_i(t) = 1$ means the ith output signal is transmitted normally. $\rho_i \in [0,1]$ reflects the occurrence probability of the ith output signal losses. Assuming $\theta_i(t)$, $i = 1, \cdots, m$ are independent stochastic variable one another and satisfy

$$E(\theta_i(t)\theta_j(t)) = E(\theta_i(t))E(\theta_j(t)), i \ne j, j = 1, \cdots, m. \tag{5}$$

We set up the following unbiased filtering system

$$\begin{aligned} d\hat{x}(t) &= [A_f\hat{x}(t) - \theta(t)B_fA_{1d}\hat{x}(t-h(t))]dt + B_fdy(t) + C_0\hat{x}(t)d\beta(t) \\ \hat{x}(0) &= 0 \\ \hat{z}(t) &= C_f\hat{x}(t). \end{aligned} \tag{6}$$

where $\hat{x}(t) \in R^n$ is the filter state, $\hat{z}(t) \in R^r$, the constant matrices A_f, B_f, C_f are filter parameters to be designed. Denote $x_e(t) = x(t) - \hat{x}(t)$ and $z_e(t) = z(t) - \hat{z}(t)$ then, we obtain the following filtering error system:

$$\begin{aligned} dx_e(t) &= [A_fx_e(t) + (A_0 - A_f - \theta(t)B_fA_1)x(t) - \theta(t)B_fA_{1d}x_e(t-h(t) + (B_0 - \theta(t)B_fB_1)v(t)]dt + C_0x_e(t)d\beta(t) \\ z_e(t) &= C_fx_e(t) + (L - C_f)x(t). \end{aligned} \tag{7}$$

Unbiasedness of the filter requires that the estimation error system be independent of the system state x, that is the following conditions are satisfied :

$$A_f = A_0 - \theta(t)B_fA_1,\ C_f = L. \tag{8}$$

then we obtain an unbiased filter and the filtering error system can be described as

$$\begin{aligned} dx_e(t) &= [A_ex_e(t) + A_{ed}x_e(t-h(t) + B_ev(t)]dt + C_0x_e(t)d\beta(t) \\ z_e(t) &= C_fx_e(t). \end{aligned} \tag{9}$$

where

$$A_e = A_0 - \theta(t)B_fA_1, A_{ed} = -\theta(t)B_fA_{1d}, B_e = B_0 - \theta(t)B_fB_1. \tag{10}$$

Based on the above discussion, the problem to be addressed in this paper is stated as follows. Design an unbiased filter of the form (6), such that the filtering error system (9) with $v(t) \equiv 0$ is exponentially stable in mean square, and the H_∞ performance $\int_0^\infty \| z_e(t) \|^2\, dt \le \gamma^2 \int_0^\infty \| v(t) \|^2\, dt$ is satisfied for zero initial conditions and all nonzero $v(t) \in L_F^2((0,\infty), R^n)$ with a prescribed $\gamma > 0$.

Theorem 1: Consider the system (1) and (2). Given scalars $\tau_1, \tau_2 > 0$ and μ for a prescribed $\gamma > 0$, the system (9) with $v(t) = 0$ is exponentially stable in mean square, and the H_∞ performance $\hat{J} < 0$ holds for all nonzero $v(t)$, $v(t) \in L_F^2((0,\infty), R^n)$, if there exist symmetric positive definite matrix $P > 0, Q_r \geq 0, Z > 0, R > 0$, $(r = 1,2,3)$ scalars $h_i > 0$ and appropriately dimensioned matrices S, X, T_i and $Y_i (i = 1,2,3,4,5,6)$ satisfying the following LMI

$$\begin{bmatrix} \Omega & \tilde{Y} & K & \tilde{F}(\rho) \\ * & -(\frac{R}{\tau_2-\tau_1}+Z) & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{11}$$

If a solution of the LMI exists then the filter that guarantees the estimation error level of γ is given by (6) with $A_f = A_0 - S^{-1}X\rho A_1, B_f = S^{-1}X$ and $C_f = L$.

Where

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} \\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} \\ * & * & * & \Omega_{44} & \Omega_{45} & \Omega_{46} \\ * & * & * & * & \Omega_{55} & \Omega_{56} \\ * & * & * & * & * & \Omega_{66} \end{bmatrix}, \tilde{F}(\rho) = \begin{bmatrix} h_1(SB_0 - X\rho B_1) \\ h_2(SB_0 - X\rho B_1) \\ h_3(SB_0 - X\rho B_1) \\ h_4(SB_0 - X\rho B_1) \\ h_5(SB_0 - X\rho B_1) \\ h_6(SB_0 - X\rho B_1) \end{bmatrix}, K = \begin{bmatrix} L^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{12}$$

$$\rho = diag(\rho_1, \rho_2, \cdots, \rho_m)$$

and

$$\Omega_{11} = h_1 SA_0 + h_1 A_0^T S^T - h_1 X\rho A_1 - h_1 A_1^T \rho^T X^T + T_1 C_0 + C_1^T T_1^T + Q_1 + Q_2 + Q_3$$
$$\Omega_{12} = -h_1 X\rho A_{1d} + h_2 A_0^T S^T - h_2 A_1^T \rho^T X^T + C_0^T T_2^T, \Omega_{13} = h_3 A_0^T S^T - h_3 A_1^T \rho^T X^T + C_0^T T_3^T + Y_1$$
$$\Omega_{14} = h_4 A_0^T S^T - h_4 A_1^T \rho^T X^T + C_0^T T_4^T - Y_1, \Omega_{15} = P - h_1 S + h_5 A_0^T S^T - h_5 A_1^T \rho^T X^T + C_0^T T_5^T$$
$$\Omega_{16} = h_6 A_0^T S^T - h_6 A_1^T \rho^T X^T - T_1 + C_0^T T_6^T, \Omega_{22} = -h_2 X\rho A_{1d} - h_2 A_{1d}^T \rho^T X^T - (1-\mu)Q_3$$
$$\Omega_{23} = -h_3 A_{1d}^T \rho^T X^T + Y_2, \Omega_{24} = -h_4 A_{1d}^T \rho^T X^T - Y_2, \Omega_{25} = -h_2 S - h_5 A_{1d}^T \rho^T X^T, \Omega_{26} = -T_2 - h_6 A_{1d}^T \rho^T X^T$$
$$\Omega_{33} = -Q_1 + Y_5 + Y_5^T, \ \Omega_{34} = -Y_5, \Omega_{35} = -h_3 S + Y_3^T, \ \Omega_{36} = -T_3 + Y_4^T,$$
$$\Omega_{44} = -Q_2 - Y_6 - Y_6^T, \ \Omega_{45} = -h_6 S - Y_3^T, \Omega_{46} = -T_6 - Y_4^T, \ \Omega_{55} = -h_5 S - h_5 S^T + (\tau_2 - \tau_1)R$$
$$\Omega_{56} = -h_6 S^T - T_5, \ \Omega_{66} = P - T_6^T - T_6 + (\tau_2 - \tau_1)Z$$

Proof: For convenience, set

$$q(t) = A_e x_e(t) + A_{ed} x_e(t - h(t)) + B_e v(t)$$
$$g(t) = C_0 x_e(t). \tag{13}$$

then system (9) becomes the following descriptor stochastic systems

$$dx_e(t) = q(t)dt + g(t)d\beta(t)$$
$$z_e = C_f x_e(t). \tag{14}$$

Choose a Lyapunov-Krasovskii functional for system (14) to be

$$V(t) = \sum_{i=1}^{5} V_i(t). \tag{15}$$

in which

$$V_1(t)=x_e(t)^T Px_e(t), V_2(t)=\sum_{i=1}^{2}\int_{t-\tau_i}^{t} x_e(s)^T Q_i x_e(s)ds, V_3(t)=\int_{t-h(t)}^{t} x_e(s)^T Q_3 x_e(s)ds$$
$$V_4(t)=\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} q^T(s)Rq(s)dsd\theta, V_5(t)=\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} trace[g^T(s)Z(g(s)]dsd\theta.$$

where P, Q_i, Q_3, Z, R are symmetric positive definite matrices with appropriate dimensions. L be the weak infinitesimal operator of (15), the Newton-Leibniz formula provides

$$x_e(t-\tau_1)-x_e(t-\tau_2)=\int_{t-\tau_2}^{t-\tau_1} q(s)ds+\int_{t-\tau_2}^{t-\tau_1} g(s)d\beta(s). \tag{16}$$

For appropriately dimensioned matrices N_j, T_j and Y_j $(j=1,2,3,4,5,6)$, equations in (13) ensure that

$$\begin{aligned}&2[x_e^T(t)N_1+x_e^T(t-h(t))N_2+x_e^T(t-\tau_1)N_3+x_e^T(t-\tau_2)N_4+q^T(t)N_5+g^T(t)N_6]\\&*\left[A_e x_e(t)+A_{ed}x_e(t-h(t))+B_e v(t)-q(t)\right]\equiv 0.\end{aligned} \tag{17}$$

$$2[x_e^T(t)T_1+x_e^T(t-h(t))T_2+x_e^T(t-\tau_1)T_3+x_e^T(t-\tau_2)T_4+q^T(t)T_5+g^T(t)T_6]*\left[C_0 x_e(t)-g(t)\right]\equiv 0. \tag{18}$$

$$\begin{aligned}&2[x_e^T(t)Y_1+x_e^T(t-h(t))Y_2+x_e^T(t-\tau_1)Y_3+x_e^T(t-\tau_2)Y_4+q^T(t)Y_5+g^T(t)Y_6]\\&*\left[x_e(t-\tau_1)-x_e(t-\tau_2)-\chi-\varsigma\right]\equiv 0.\end{aligned} \tag{19}$$

$$(\tau_2-\tau_1)\Lambda-\int_{t-\tau_2}^{t-\tau_1}\Lambda ds\geq 0. \tag{20}$$

where $\chi^T=\int_{t-h(t)}^{t} q^T(s)ds$, $\varsigma^T=\int_{t-\tau_2}^{t-\tau_1} g^T(s)d\beta(s)$, $\Lambda=\eta^T(t)\tilde{Y}R^{-1}\tilde{Y}^T\eta(t)$. Adding the terms on the left of (17)-(20) to $L_{v=0}V$, by Lemma [8], for any matrix $Z>0$, $-2\eta^T(t)\tilde{Y}\varsigma\leq\eta^T(t)\tilde{Y}Z^{-1}\tilde{Y}^T\eta(t)+\varsigma^T Z\varsigma$.
then $L_{v=0}V$ can be expressed as

$$L_{v=0}V\leq\eta^T(t)\Xi\eta(t)-\int_{t-\tau_2}^{t-\tau_1}\Theta R^{-1}\Theta^T ds-\int_{t-\tau_2}^{t-\tau_1} trace\left[g^T(s)Zg(s)\right]ds+\varsigma^T Z\varsigma. \tag{21}$$

where

$$\Xi=\Omega+\tilde{Y}(\frac{R}{\tau_2-\tau_1}+Z)^{-1}\tilde{Y}^T, \Theta=\eta^T(t)\tilde{Y}+q^T(s)R. \tag{22}$$

$$\eta^T(t)=\begin{bmatrix}x_e(t) & x_e(t-h(t)) & x_e(t-\tau_1) & x_e(t-\tau_2) & q(t) & g(t)\end{bmatrix}^T, \tilde{Y}^T=\begin{bmatrix}Y_1 & Y_2 & Y_3 & Y_4 & Y_5 & Y_6\end{bmatrix}^T. \tag{23}$$

where Ω is defined in (12).

Since $E\left(\varsigma^T Z\varsigma\right)=E\int_{t-h(t)}^{t} trace\left[g^T(s)Zg(s)\right]ds$. It follows that $EL_{v=0}V(t)\leq E\eta^T(t)\Xi\eta(t)$. By [9] a scalar α_0 exists, such that $\lim_{t\to\infty}\sup\frac{1}{t}\log E\left\|x_e(t)\right\|^2\leq-\alpha_0$. Which implies system (9) is exponentially stable in mean square. Next, we prove $\hat{J}<0$ for all nonzero $v(t)\in L_F^2((0,+\infty),R^n)$ with $x_e(t,0)=0$. Note that for any $T>0$

$$\begin{aligned}\hat{J}(T)&=E\int_0^T(\left\|z_e(t)\right\|^2-\gamma^2\left\|v(t)\right\|^2)dt\leq E\int_0^T[\left\|z_e(t)\right\|^2-\gamma^2\left\|v(t)\right\|^2+L_vV(\xi(t),t)]dt\\&=E\int_0^T\begin{bmatrix}\eta(t)\\v(t)\end{bmatrix}^T\Psi\begin{bmatrix}\eta(t)\\v(t)\end{bmatrix}dt-E\int_0^T\int_{t-h(t)}^{t}\Theta R\Theta^T dsdt.\end{aligned} \tag{24}$$

Where

$$\Psi=\begin{bmatrix}\tilde{\Xi} & F\\ * & -\gamma^2 I\end{bmatrix}, \tilde{\Xi}=\Omega+\tilde{Y}^T(\frac{R}{\tau_2-\tau_1}+Z)^{-1}\tilde{Y}^T+K^TK. \tag{25}$$

Therefore, if $\Psi<0$ then $\hat{J}(T)<-\lambda_{\min}(-\Psi)E\int_0^t\left\|v(t)\right\|^2)dt<0.$ for any nonzero $v(t)\in L_F^2((0,+\infty),R^n)$

which yield $\hat{J}(t) < -\lambda_{\min}(-\Psi)E\int_0^t \|v(t)\|^2)dt < 0$. Substituting (10) and (22) into (25), and letting $SB_f = X$ then $\Psi < 0$ is equivalent to (12). From our assumption, $B_f = S^{-1}X$ and a H_∞ filter is constructed as in the form of (6). The proof of Theorem 1 is completed.

Numerical Simulation

Consider a CH-47 double helix helicopter who makes standard 40 kt (1kt=1.85 km/h) for level flight and its linear model is as follows:

$$\dot{x} = A_0 x(t) + B_0 v(t) + C_0 x(t)\dot{\beta}(t)$$
$$z = Lx(t) \tag{26}$$
$$x(t) = \varphi(t), t \in [-\tau, 0].$$

where

$$A_0 = \begin{bmatrix} -9 & 1 \\ -2 & -10 \end{bmatrix}, B_0 = \begin{bmatrix} -0.3 \\ -0.42 \end{bmatrix}, C_0 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, L = \begin{bmatrix} 1 & -1 \end{bmatrix}. \tag{27}$$

state x_1 denotes flight altitude; x_2 means forward velocity; $v(t)$ is an exogenous disturbance signal, $\dot{\beta}(t)$ is a white noise. Assume the measurement output experiences the network transmission delay and results in the data packet losses. The output can be given as

$$y = \theta(t)[A_1 x(t) + A_{1d} x(t - h(t)) + B_1 v(t)]. \tag{28}$$

$$A_1 = \begin{bmatrix} -8 & 1 \\ 0.5 & -15 \end{bmatrix}, A_{1d} = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & -0.4 \end{bmatrix}, B_1 = \begin{bmatrix} 0.4 \\ -0.2 \end{bmatrix}. \tag{29}$$

where stochastic matrix $\theta(t)$ and the probability of output signal losses are as follows, respectively

$$\theta(t) = \begin{bmatrix} \theta_1(t) & 0 \\ 0 & \theta_2(t) \end{bmatrix}, \rho_1 = 0.3, \rho_2 = 0. \tag{30}$$

Let $h_1 = 0.1, h_2 = 0.3, h_3 = 0.1, h_4 = 0.5, h_5 = 0.012, h_6 = 0.0513$. By Theorem1, $\gamma_{\min} = 1.5 \times 10^{-6}$ (for $t_1 = 0, t_2 = 0.1$, $\mu = 0.3$). When we take $\gamma = 0.5$, the corresponding filter matrices are

$$A_f = \begin{bmatrix} \text{-7.1648} & 0.7706 \\ \text{-2.7471} & \text{-9.9066} \end{bmatrix}, B_f = \begin{bmatrix} 0.7647 & 0 \\ \text{-0.3113} & 0 \end{bmatrix}, C_f = \begin{bmatrix} 1 & -1 \end{bmatrix}. \tag{31}$$

Figure 1 gives the time responses of states x_{e1} and x_{e2} with $v(t) = 0$. The trajectories of real states x_1, x_2 and filter states $\hat{x}_1, \hat{x}_2$ for the filter (31) are displayed in Figure 2. The simulation results demonstrate that the prescribed performance requirements on the filtering process are guaranteed by the Theorem 1.

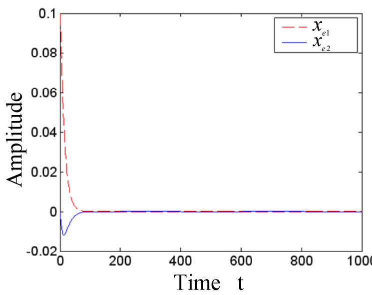
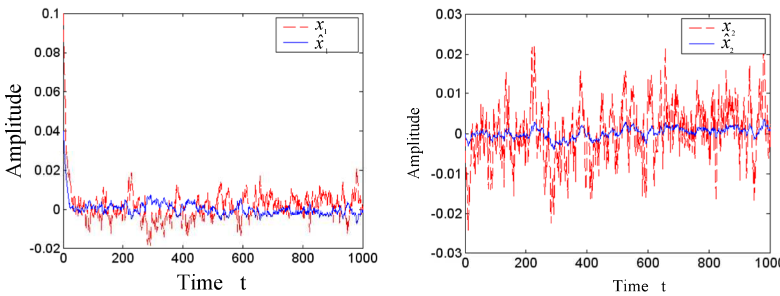


Figure 1. The state responses of the filtering error system



(a) (b)

Figure 2. The time responses of state x and $\hat{x}$ for the filter (31)

Summary

LMI-based technique, delay-dependent sufficient conditions for stochastic system with data packet losses are presented. By constructing unbiased filter, the complexity and computational burden are reduced greatly in the process of seeking the filter parameters. Numerical example has clearly indicated our design is effective.

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