From image to analysis: an extended finite element method to simulate the mechanical response of soft tissue

Dr. Jack S. Hale
Prof. Stéphane P. A. Bordas

RUES | RESEARCH UNIT IN ENGINEERING SCIENCES

Overview

- Background and context.
- The problem.
- Traditional pipeline.
- Implicit pipeline.
- The method.
- Progress so far.
- What I won't talk in depth about:
  - Optimal convergence, function spaces, weak forms, imposing Dirichlet boundary conditions, hyperelasticity, mixed formulations, stability…
- But if you have any questions about these topics feel free to ask!

Background

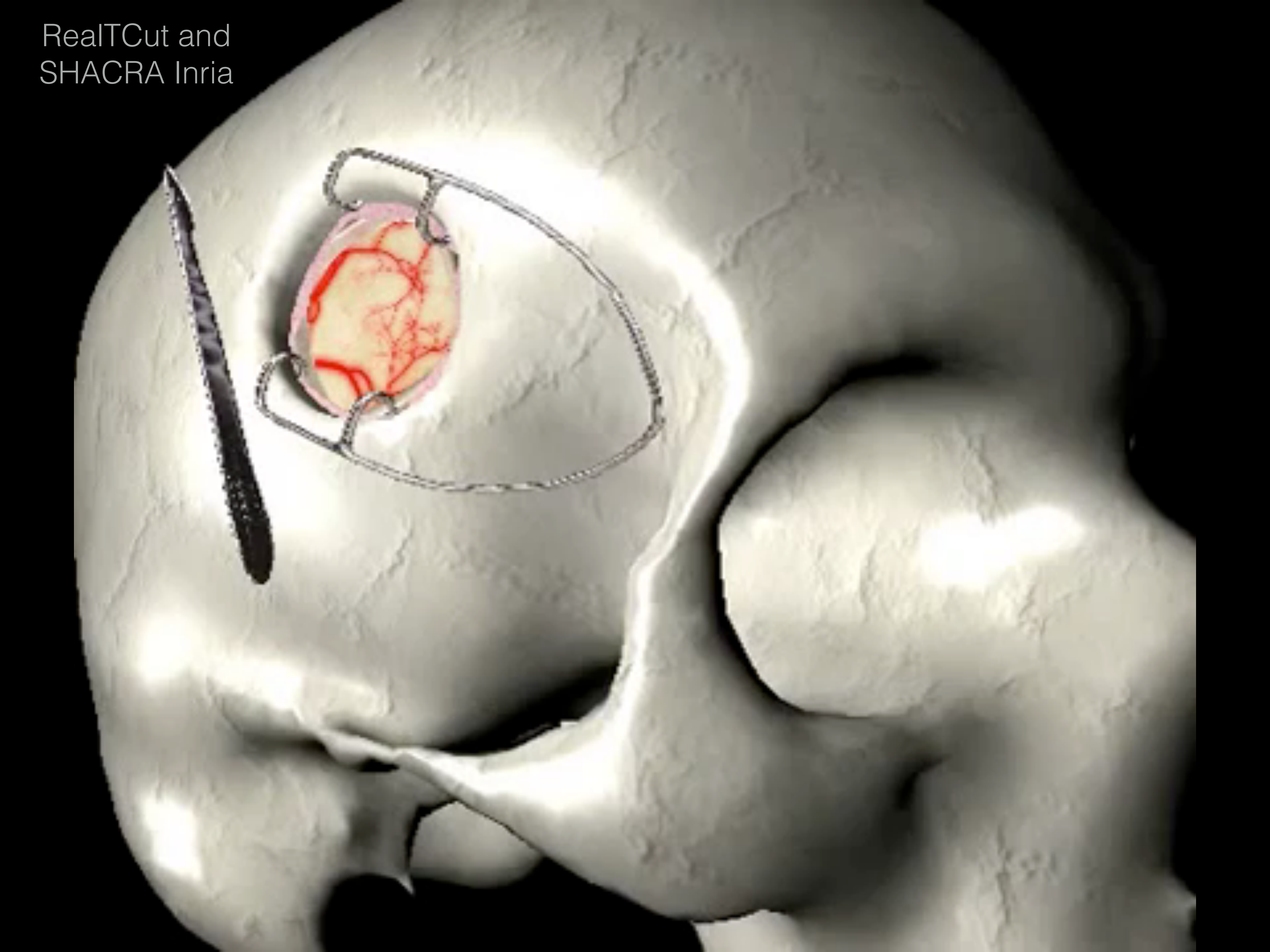
Mechanical

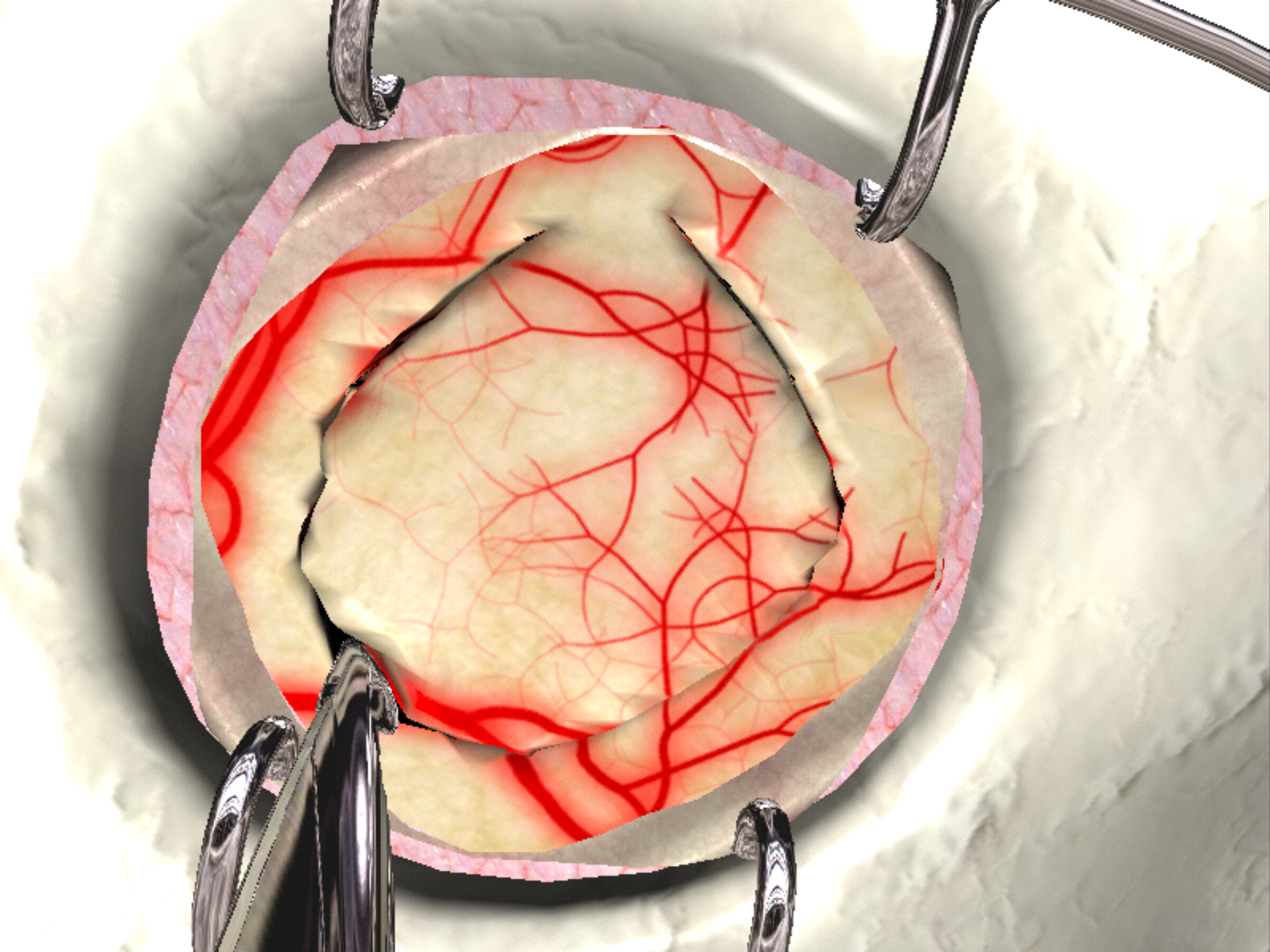
Cadavers of animals and humans

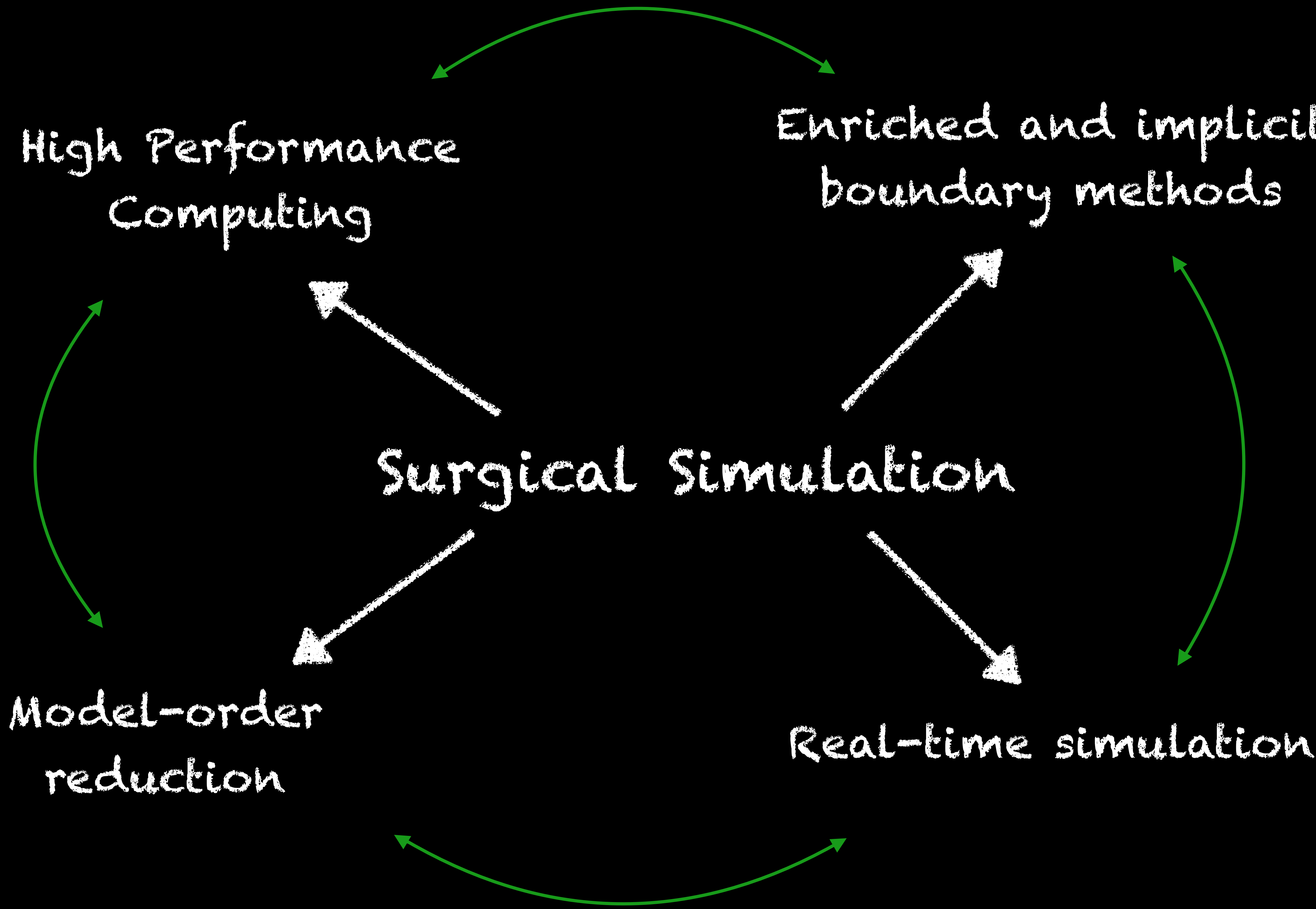
What is a surgical simulator?

Real living and breathing bodies

Computer simulations


RealTCut and
SHACRA Inria



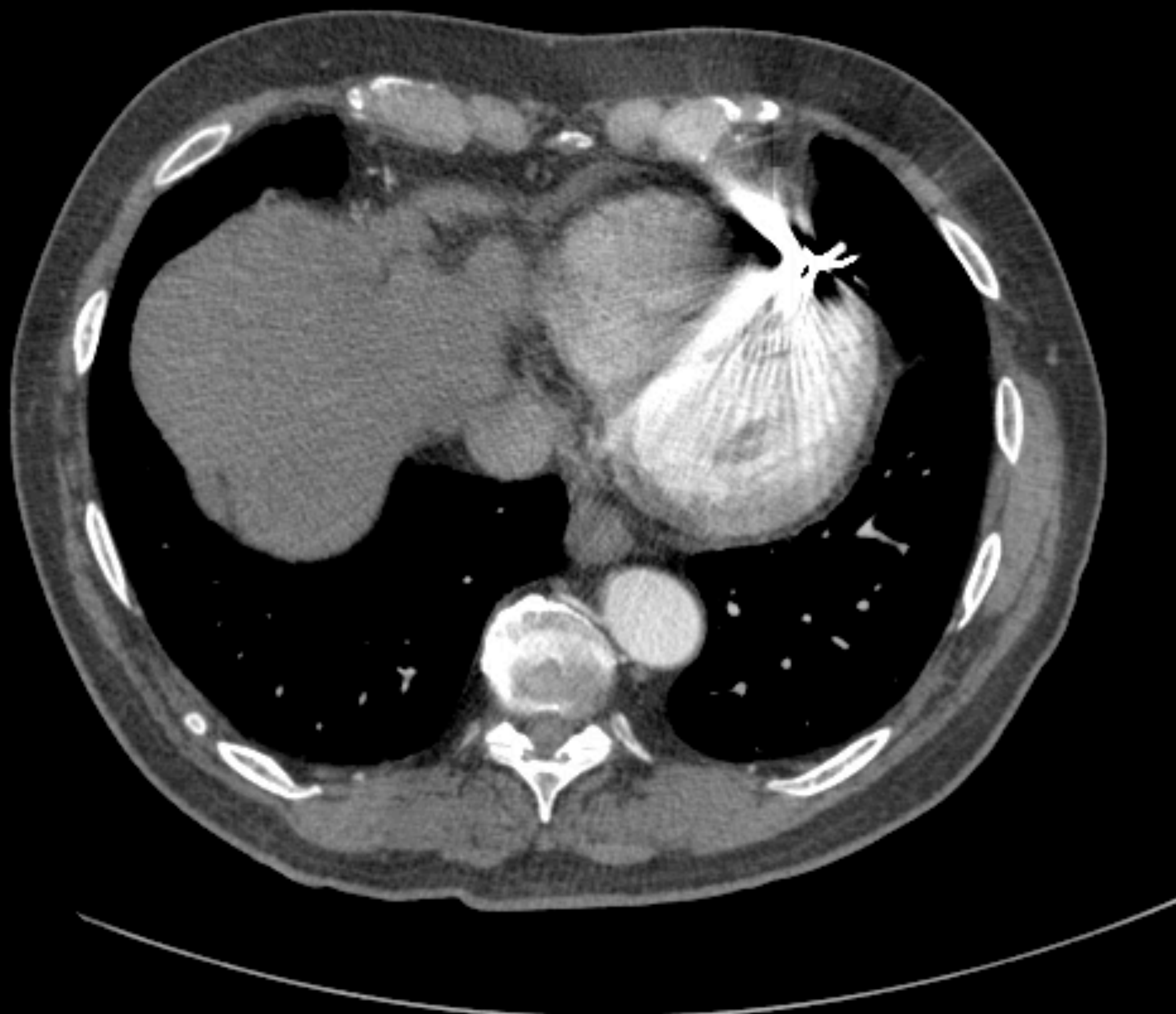

High Performance Computing
Enriched and implicit boundary methods
Surgical Simulation
Model-order reduction
Real-time simulation

The problem

How can we move from an image…

COLONIX, OSIRIX

…or perhaps a series of images…

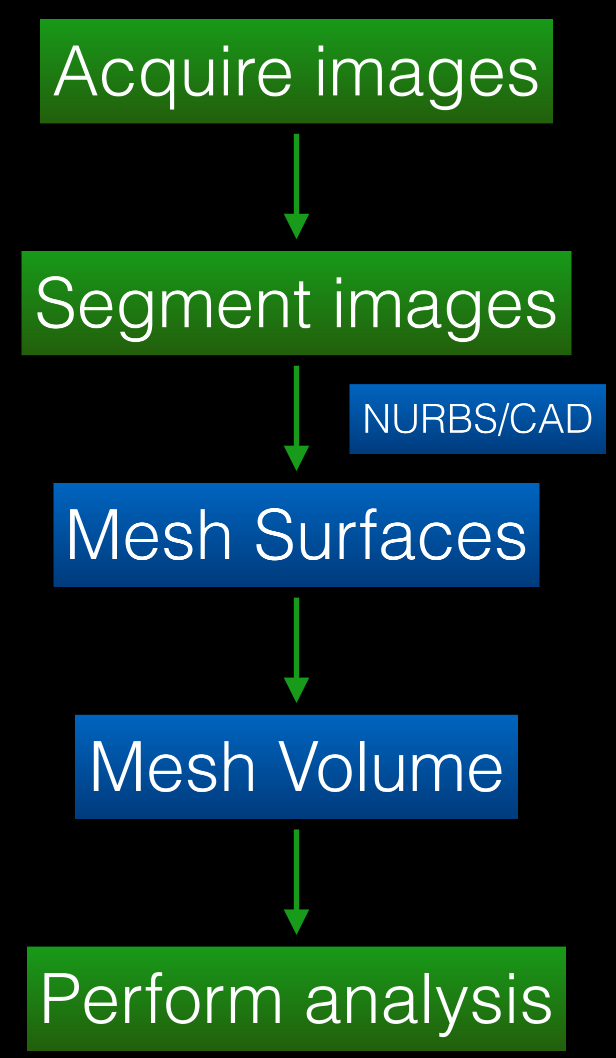


Source: COLONIX, OSIRIX

to a full mechanical analysis?

Pipeline to analysis

Traditional



Each voxel j is a 32-bit floating point measurement

z

Planes (64)

x

Rows (64)

y

Cols (64)

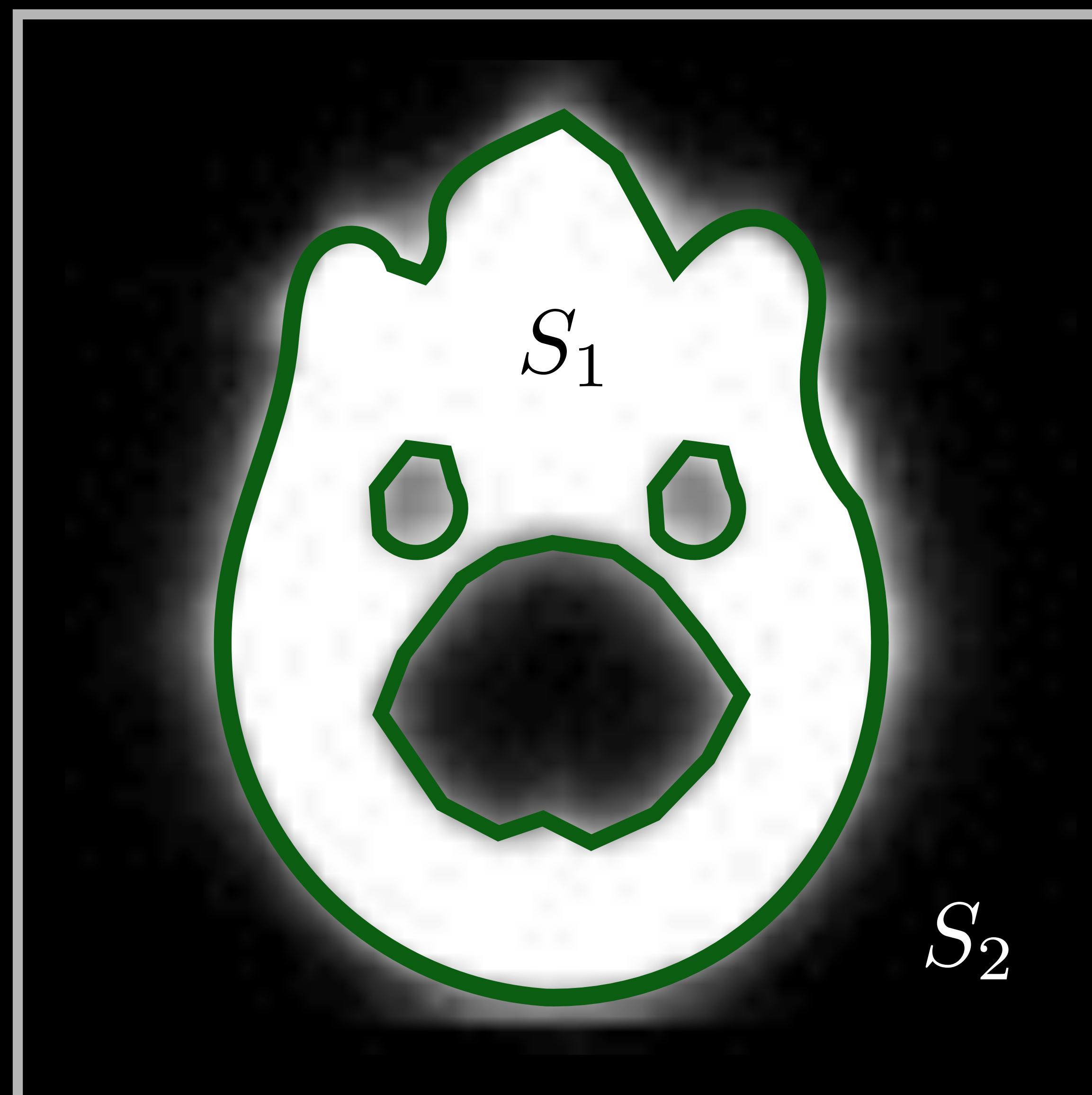
Image: CGAL Project

Soft segmentation

$$0 < m_k(j) < 1 \quad \forall j,\, k$$

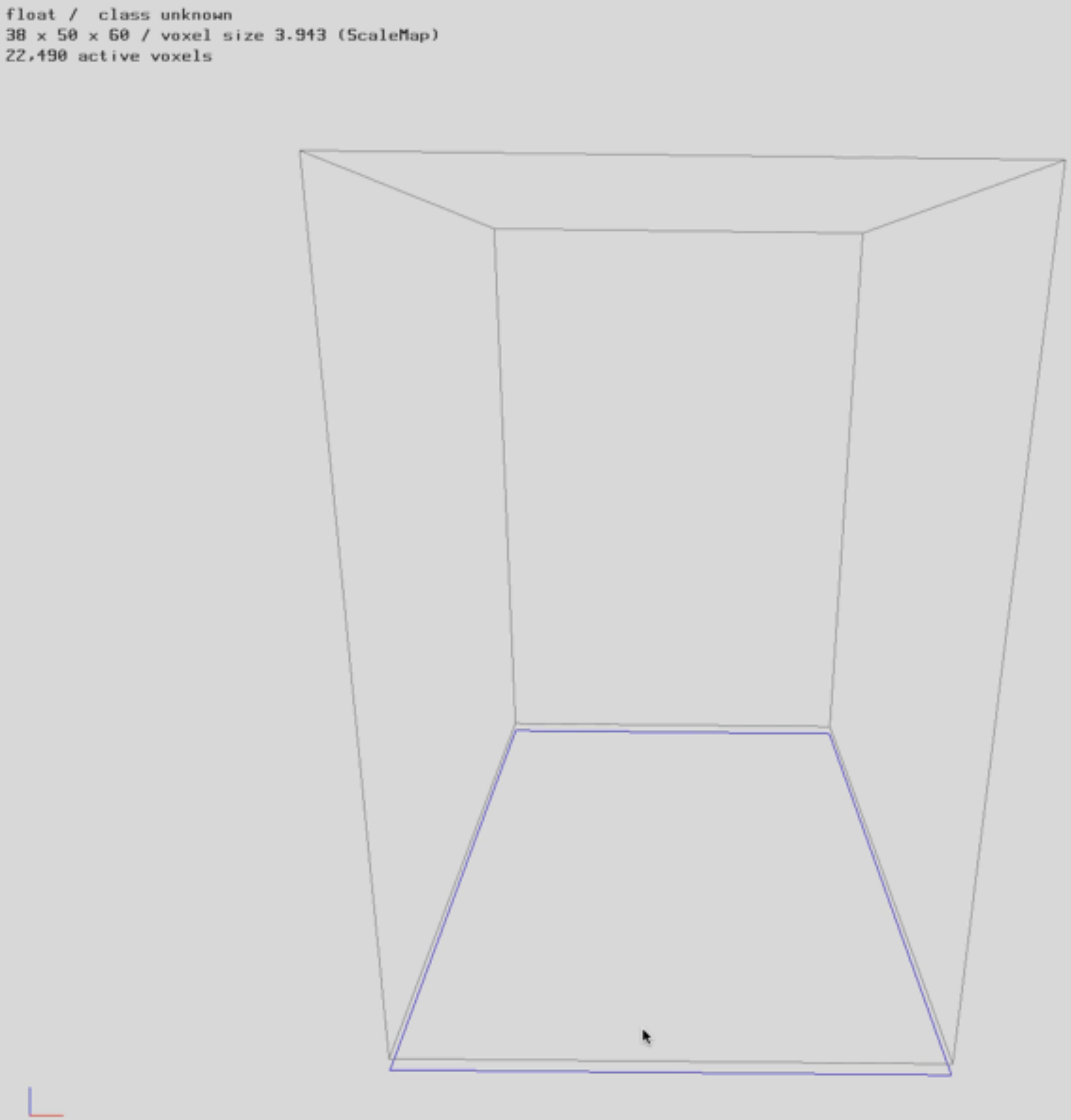
$$\sum_{k=1}^{K} m_k(j) = 1 \quad \forall j$$

Hard segmentation

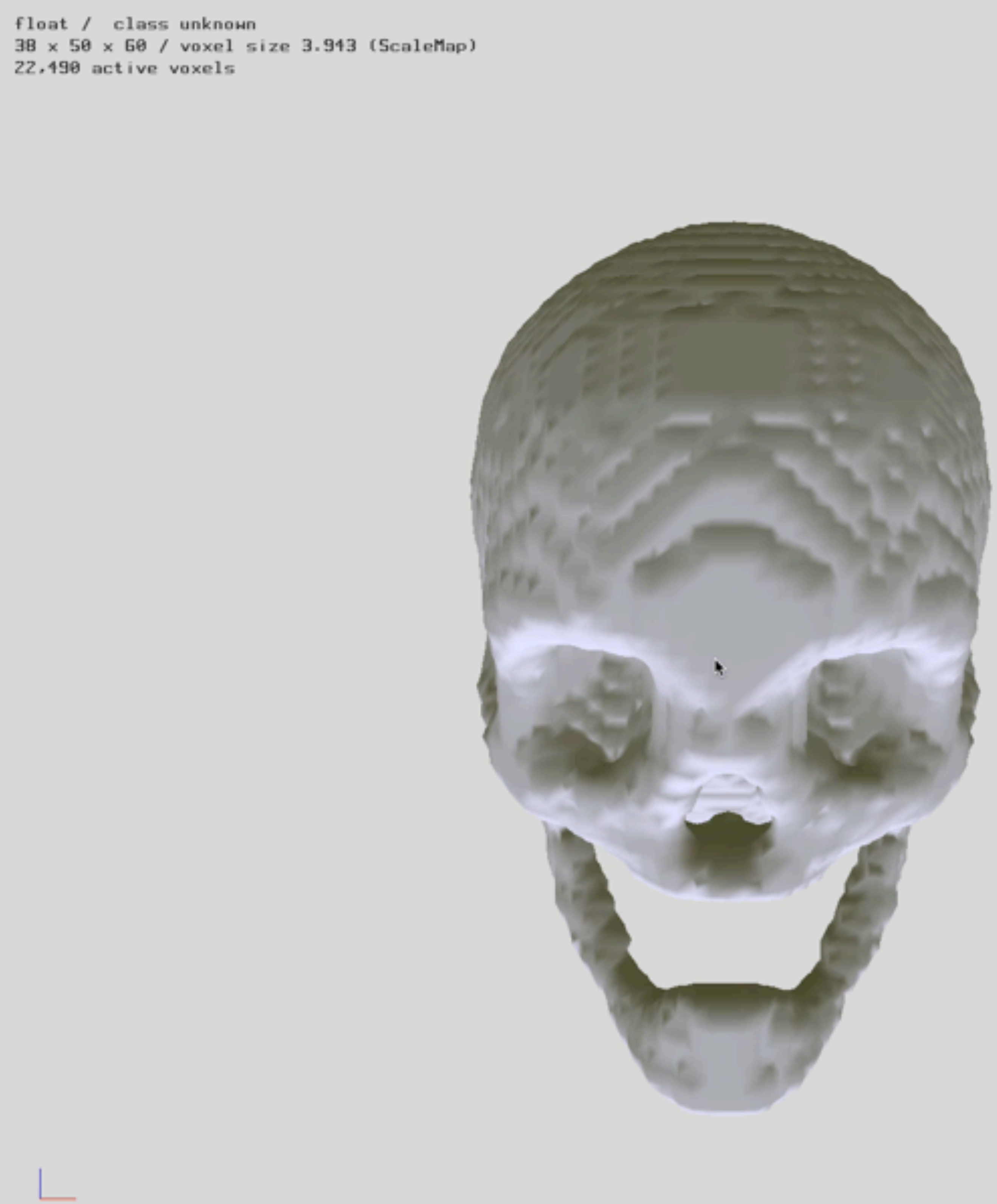


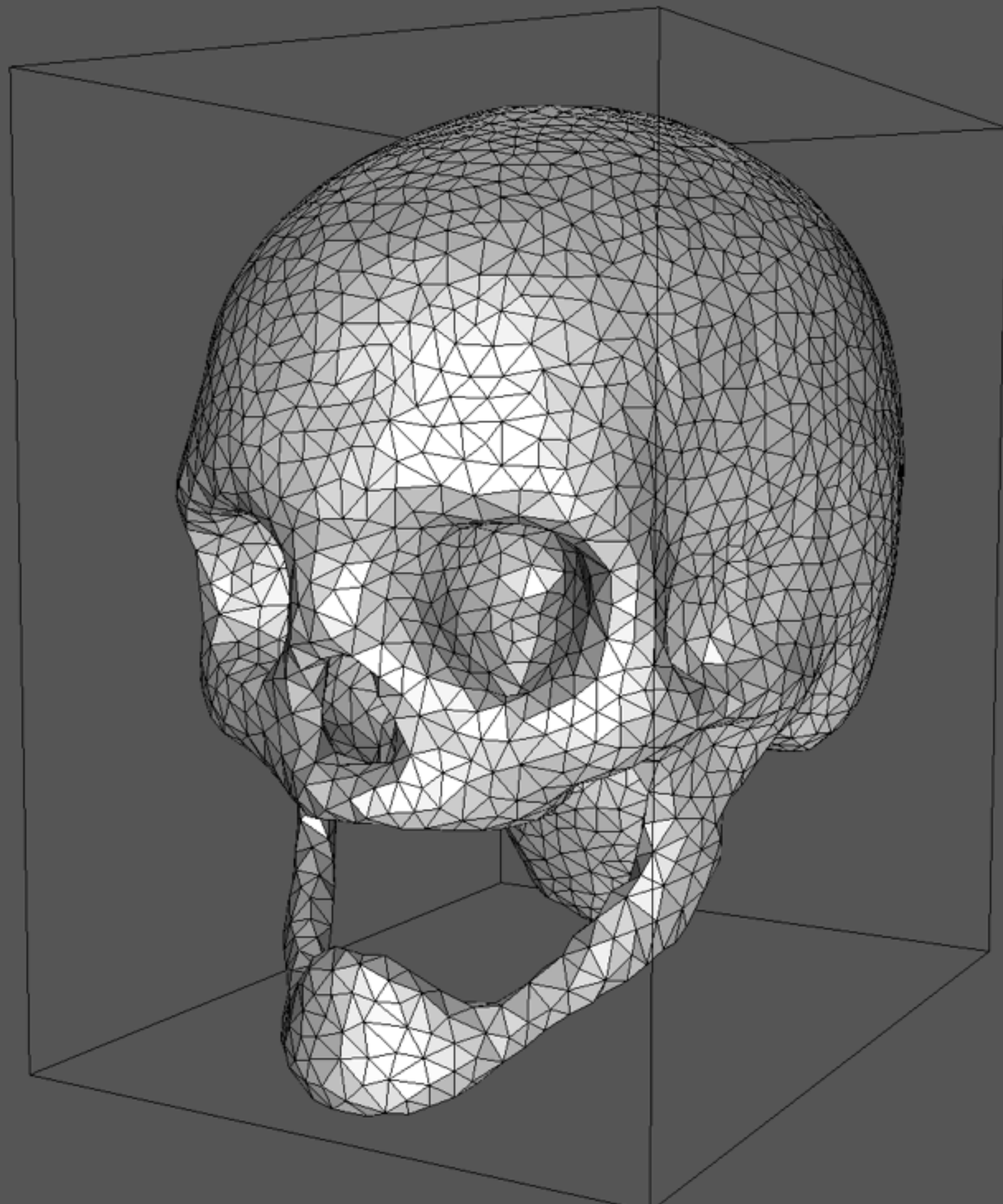
$$\Omega = \bigcup_{k=1}^{K} S_k \qquad S_k \cap S_j = \varnothing \quad \forall k \neq j$$

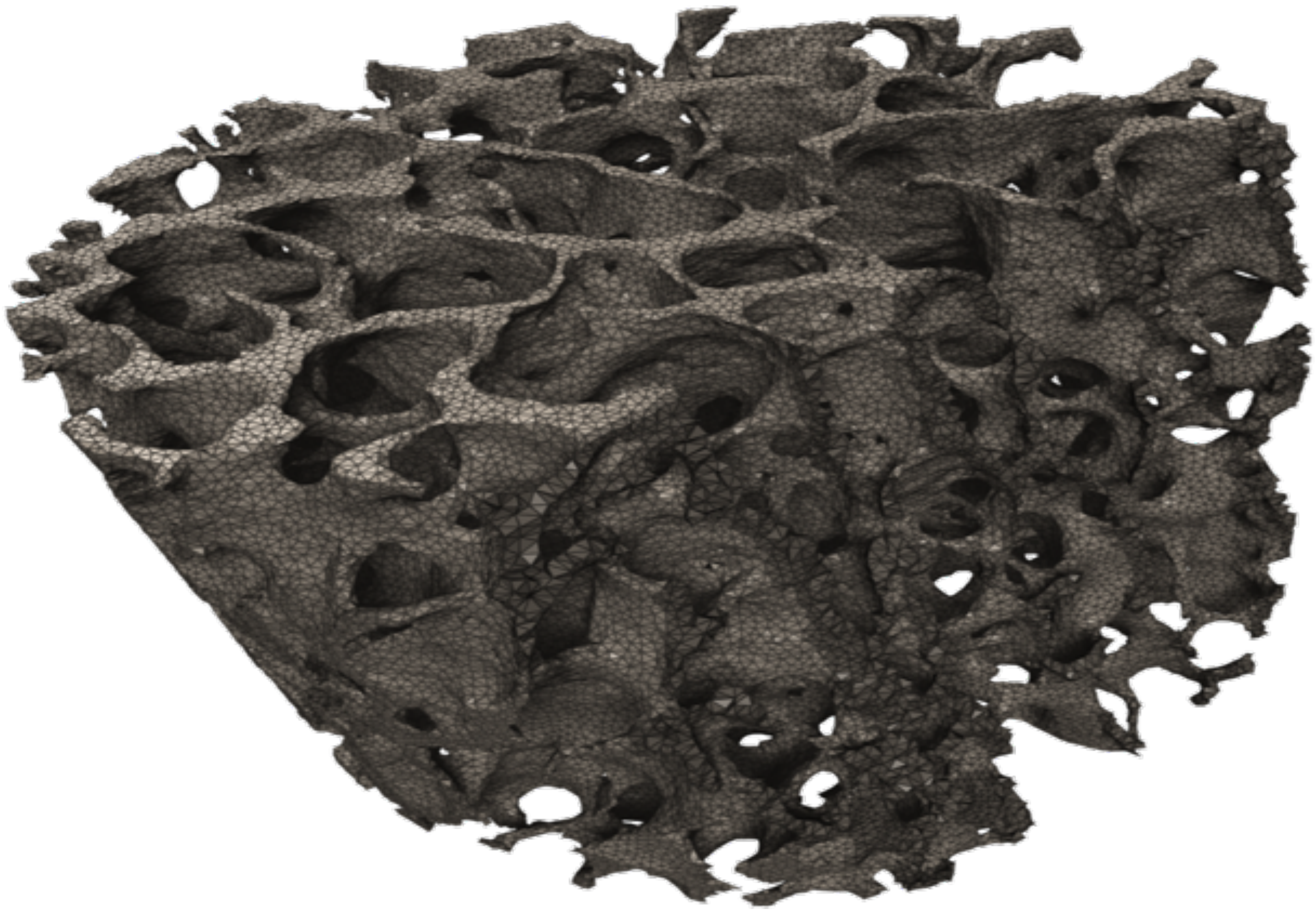
Hard Segmentation at 0.2f



Hard Segmentation at 0.2f with CGAL and OpenVDB

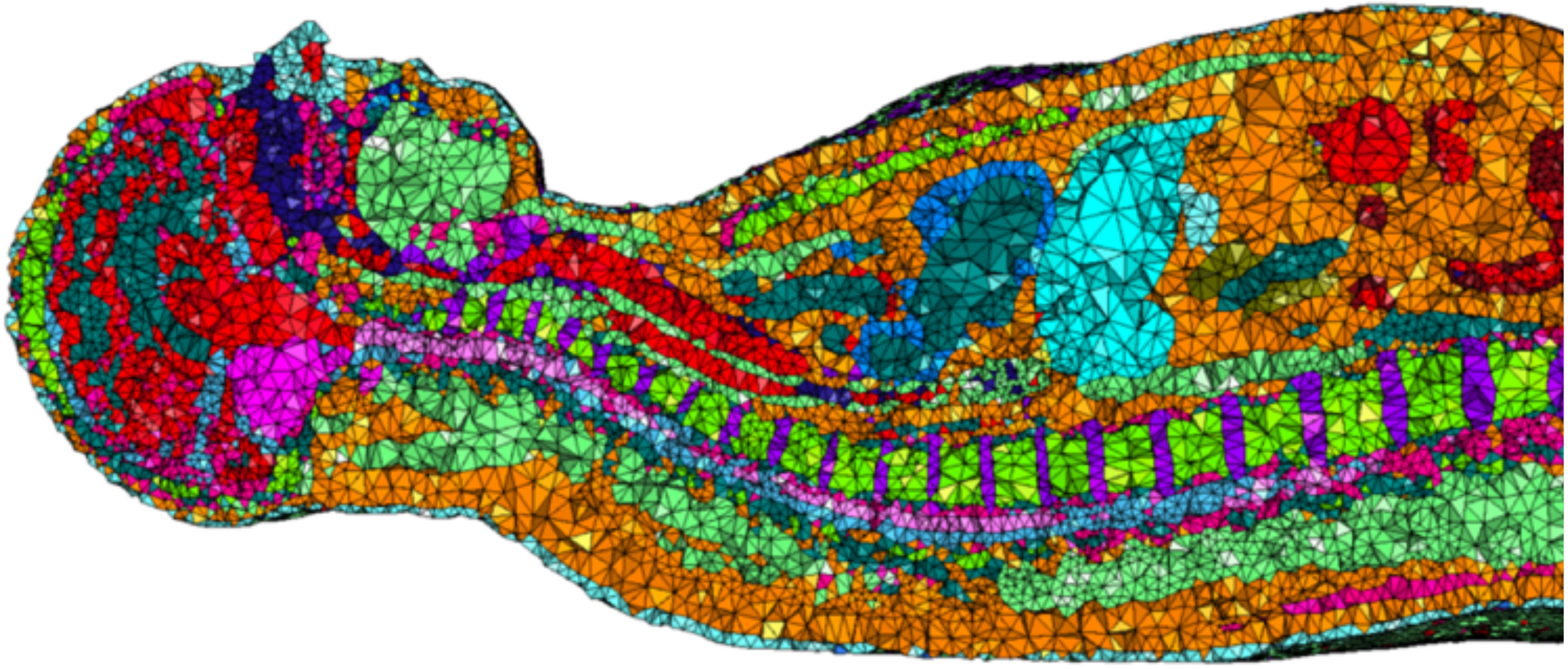






Segmented Bone

INSERM



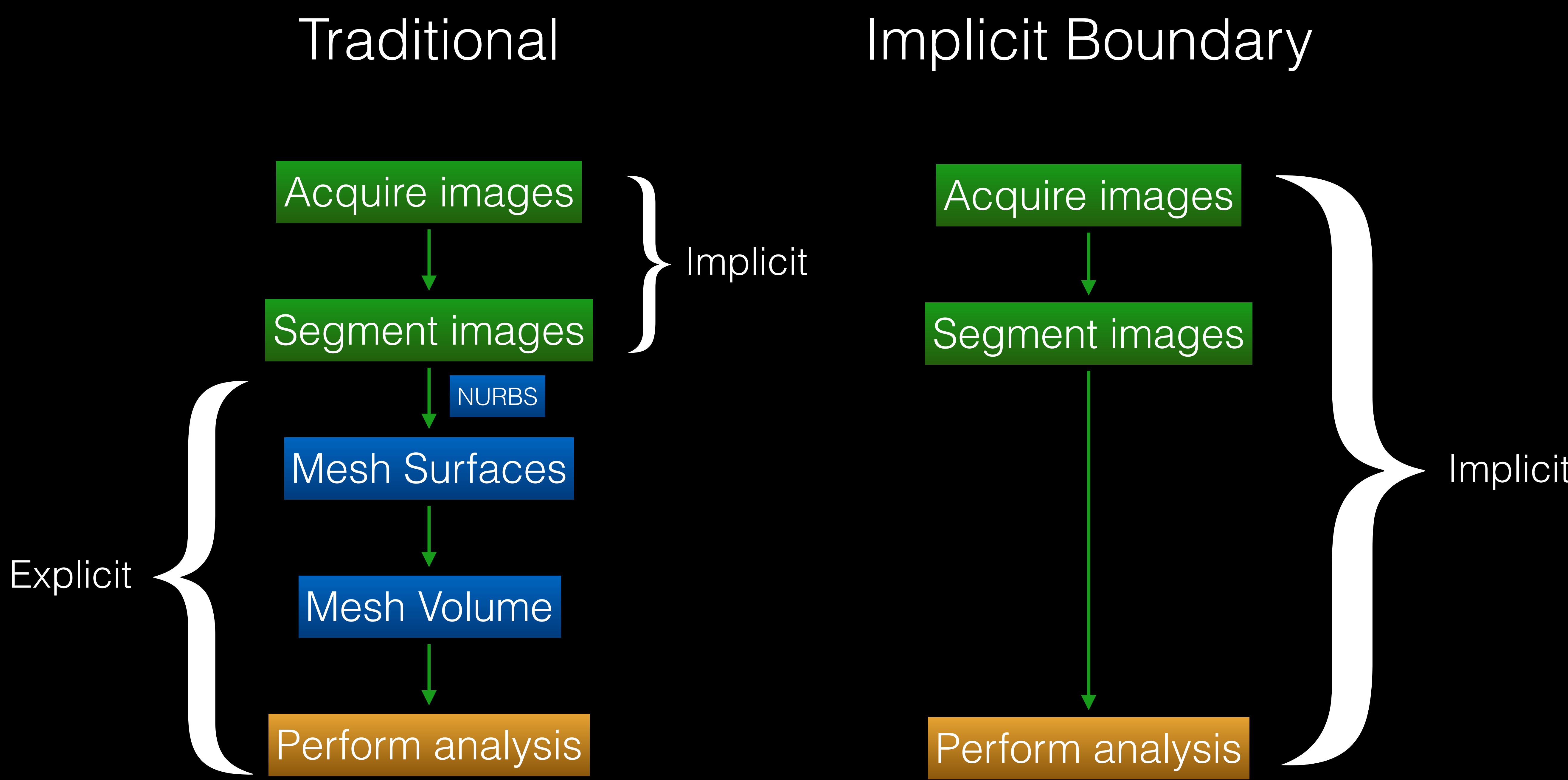
Visible Human

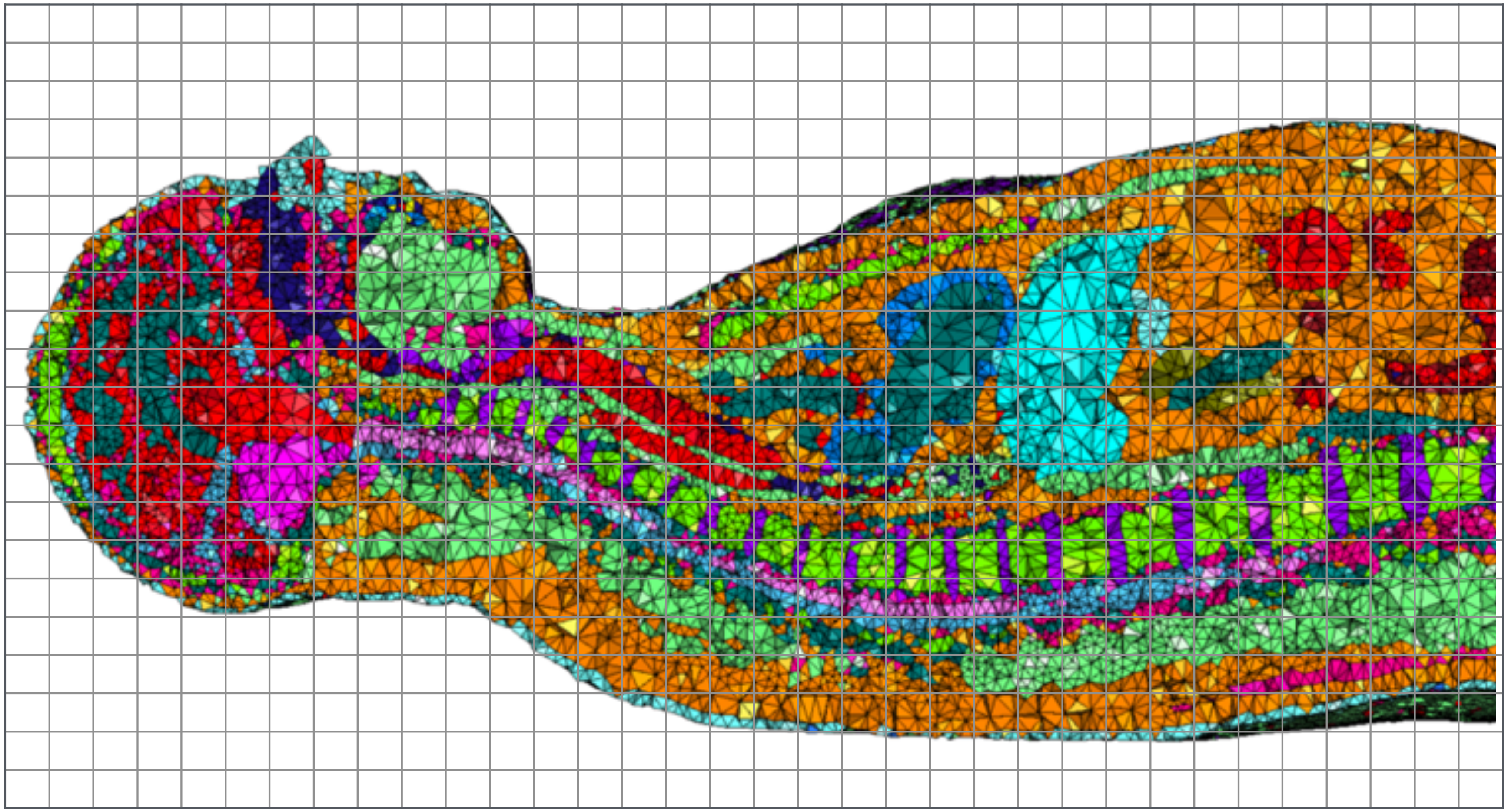
Stephane Lanteri (INRIA) and France Telecom

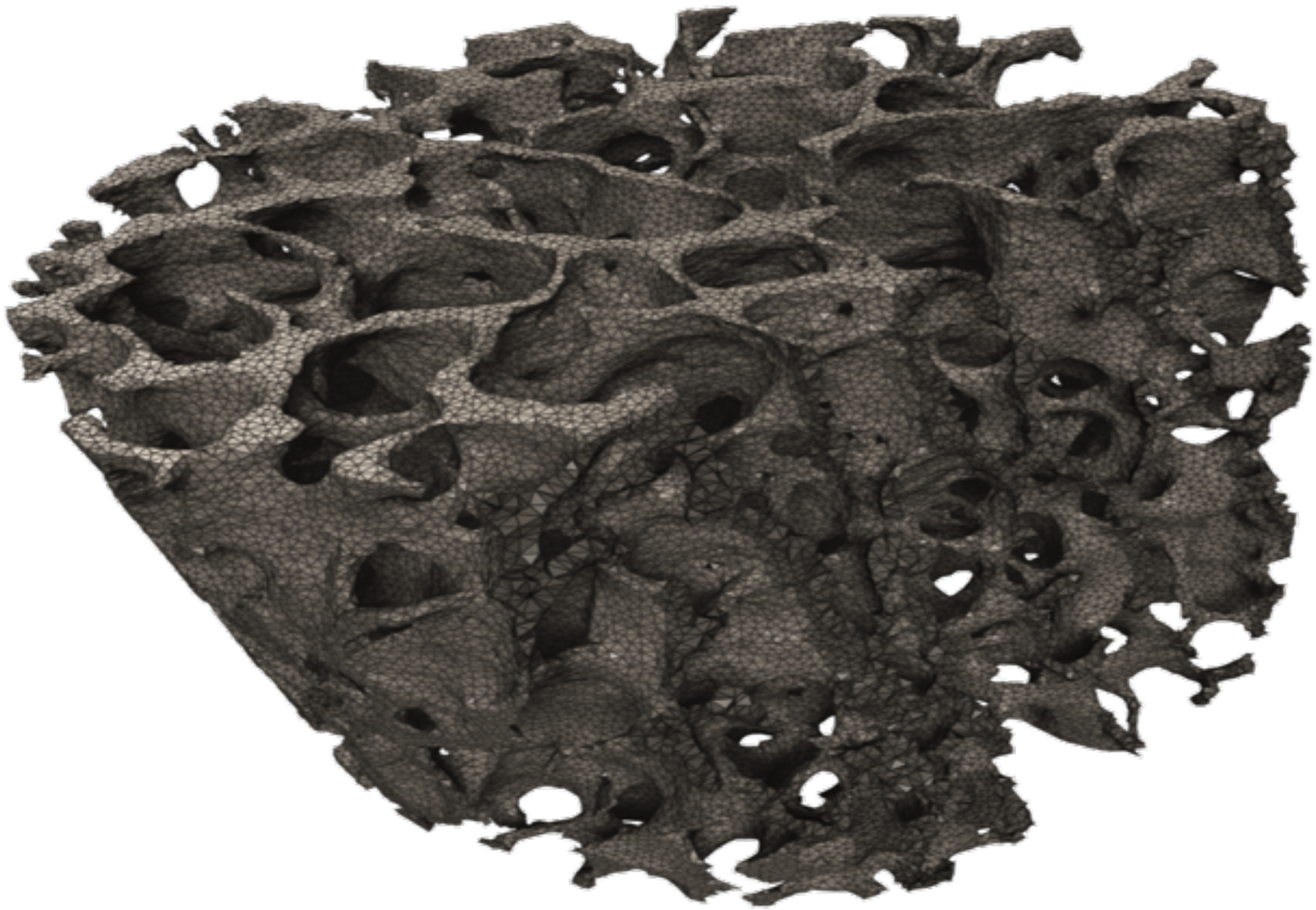
Problems

- **Core problem:** Geometry is tightly coupled with discretisation.
- How will we deal with:
  - Dynamic topology eg. cutting.
  - Clinical environments.
  - Refinement.
  - Complex microstructures.

Pipelines to analysis



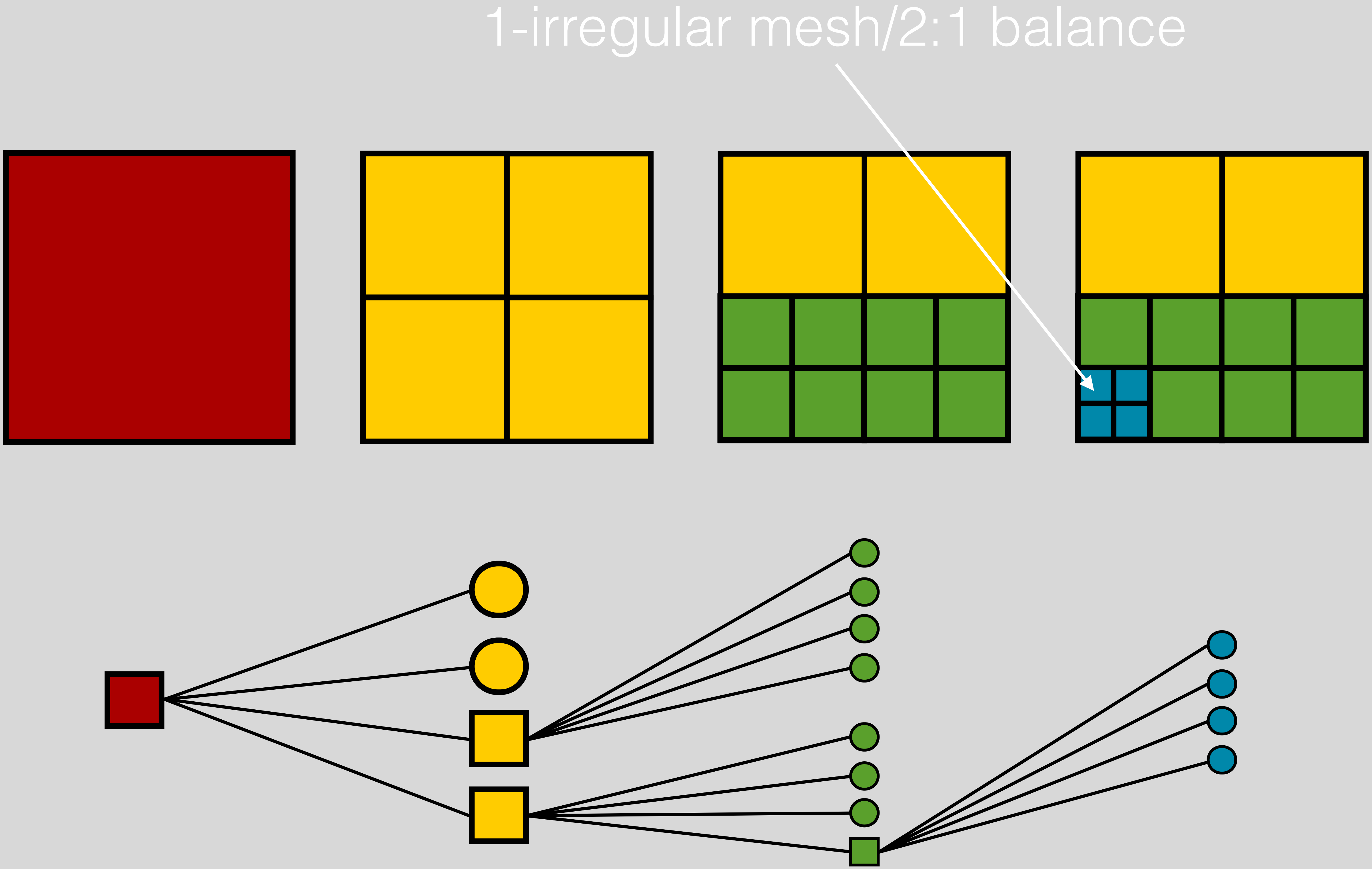




Segmented Bone

INSERM

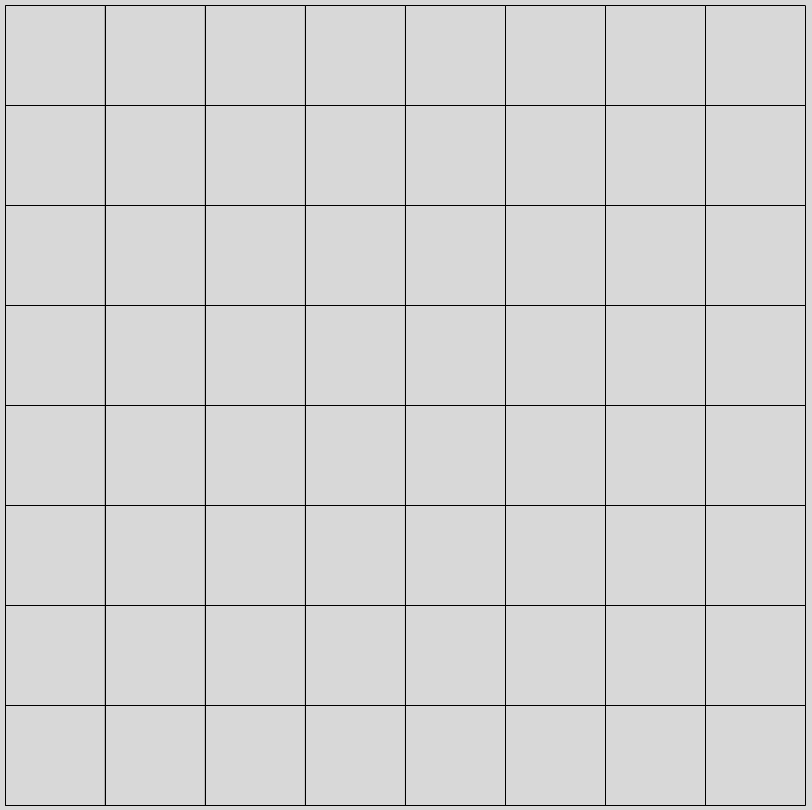
The method



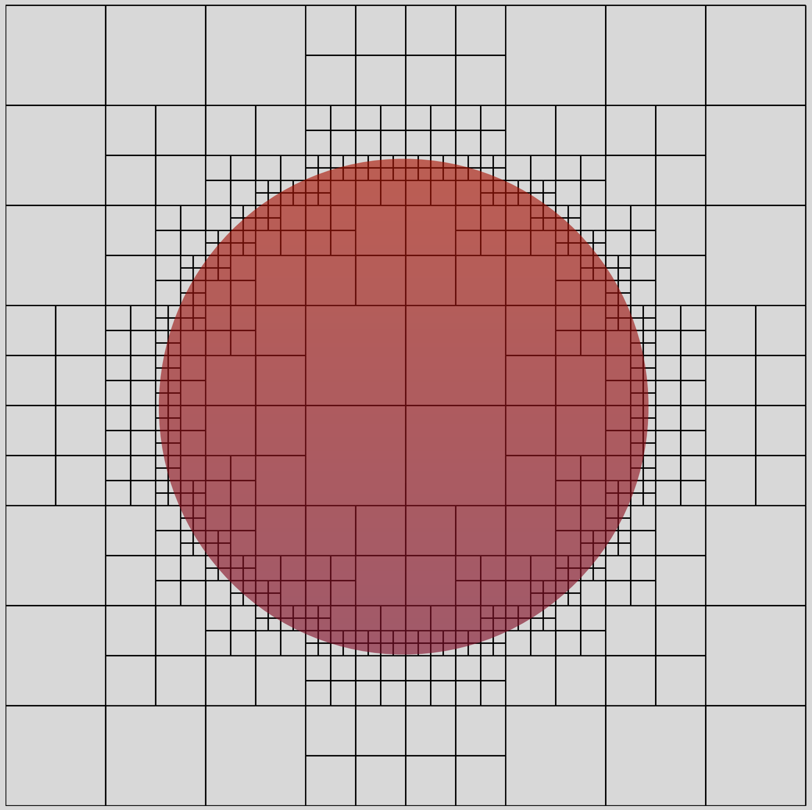
Octree data structure

Nested Octree

Discretisation

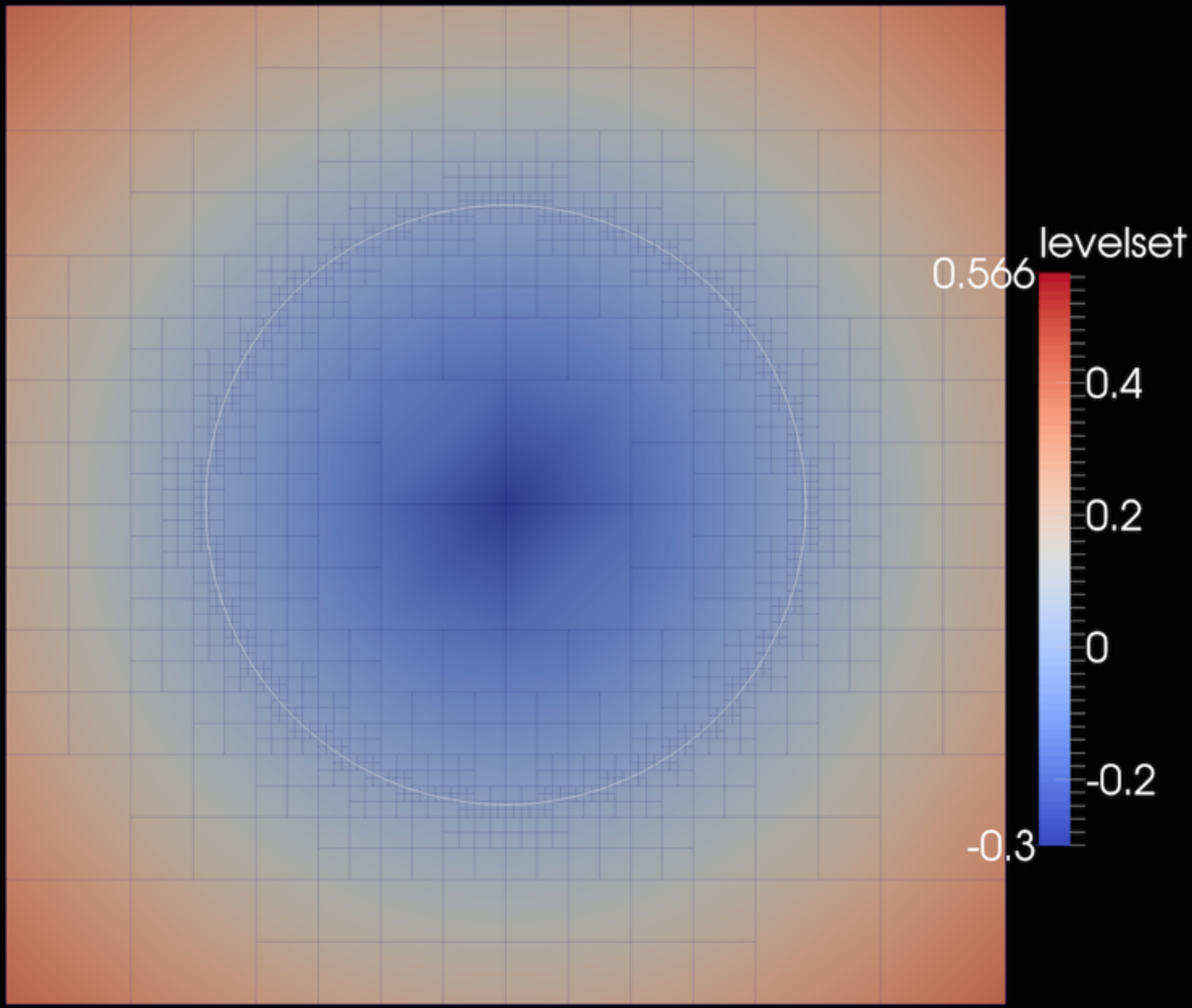


Geometry

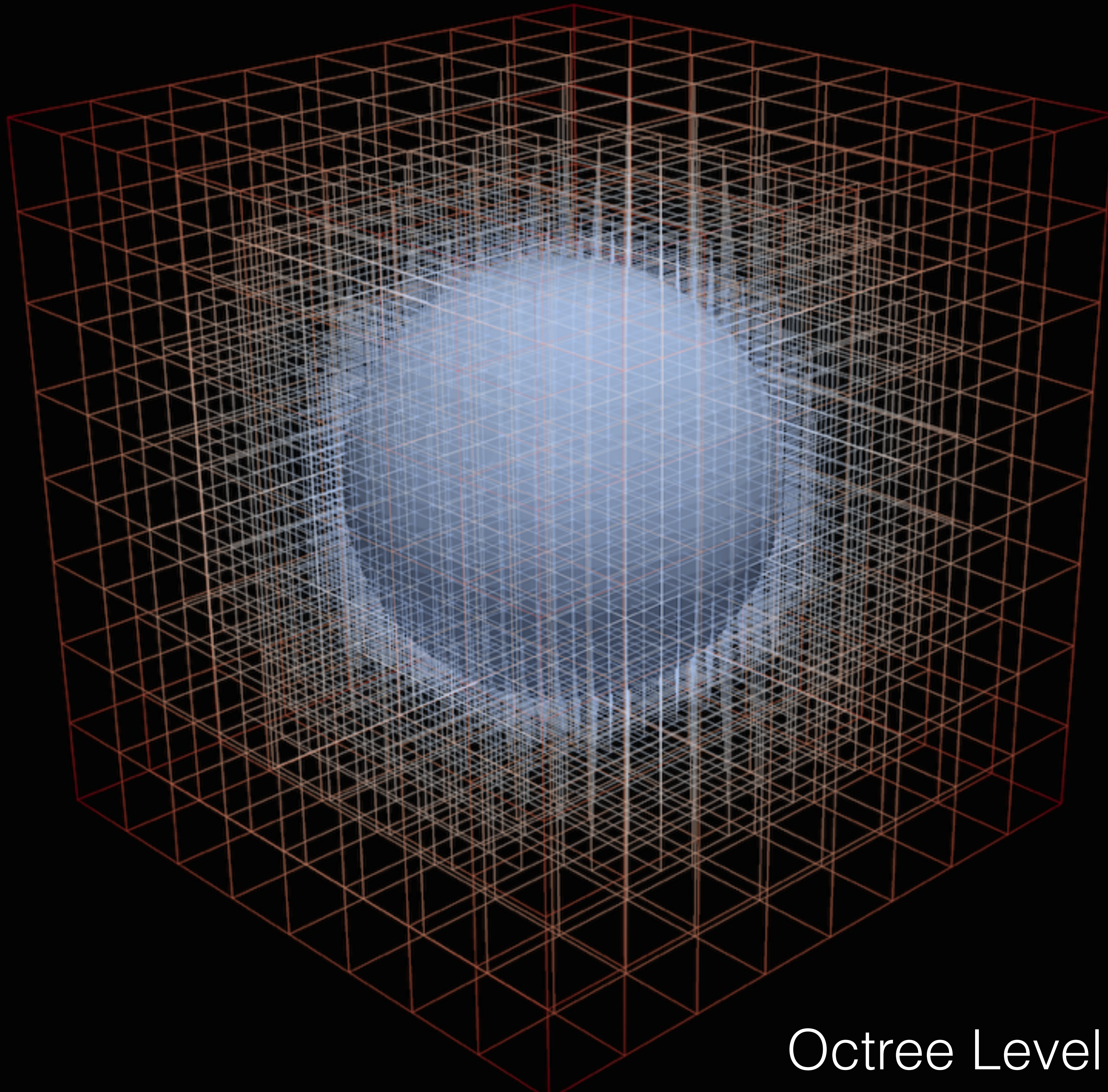


$\mathcal{O}_d$ ⟷ $\mathcal{O}_g$

$\mathcal{M}$



Quadtree Level 7/Level 4

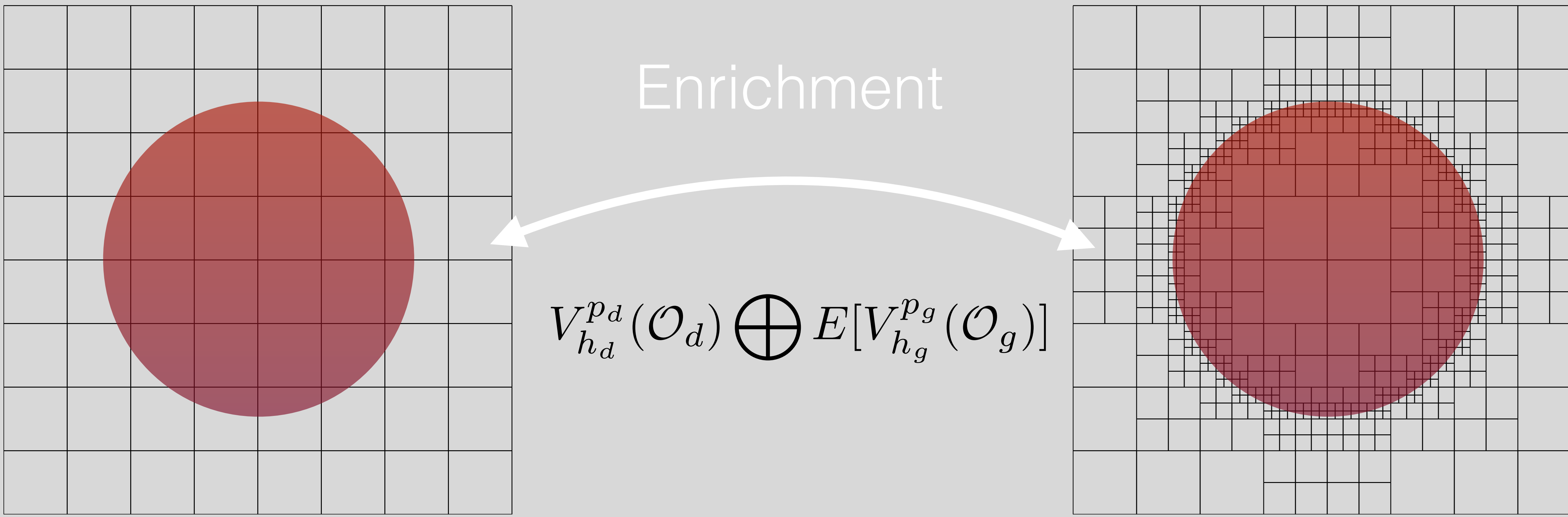


Octree Level 5/Level 3

6a2e86c

Surface

How to transfer geometric information back to the discretisation?



$V_{h_d}^{p_d}(\mathcal{O}_d)$

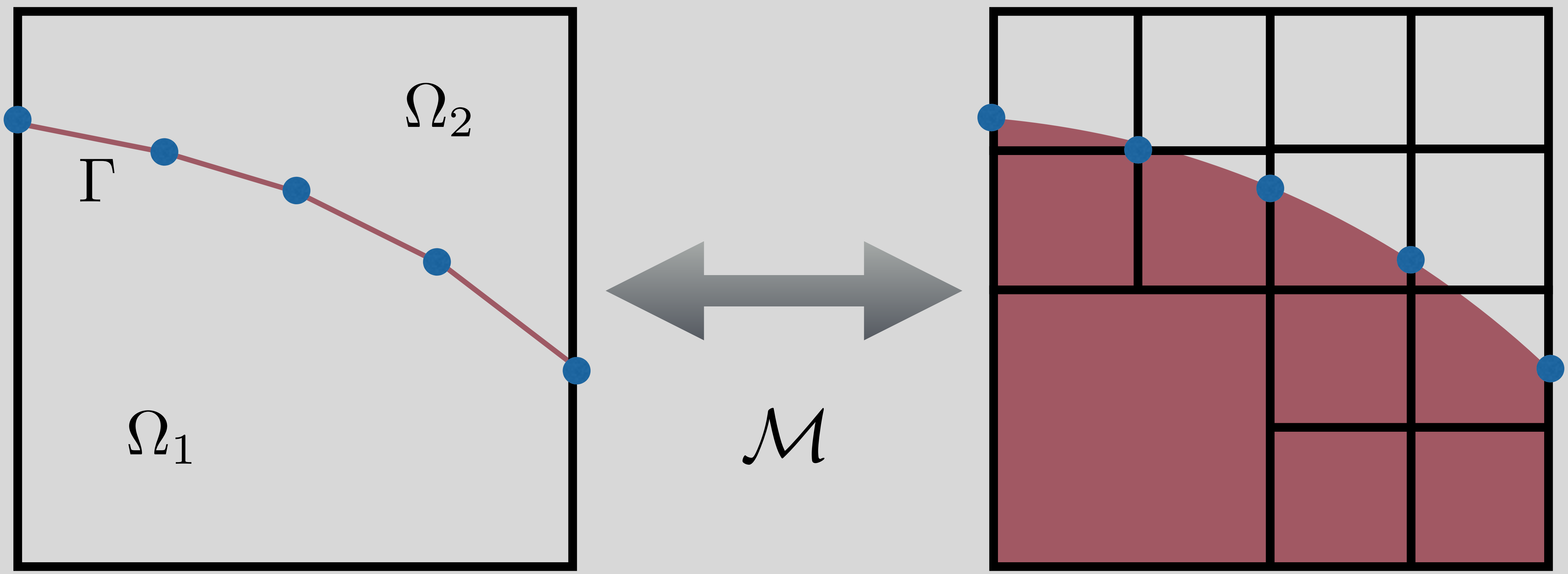
$p_d > p_g$

$h_d > h_g$

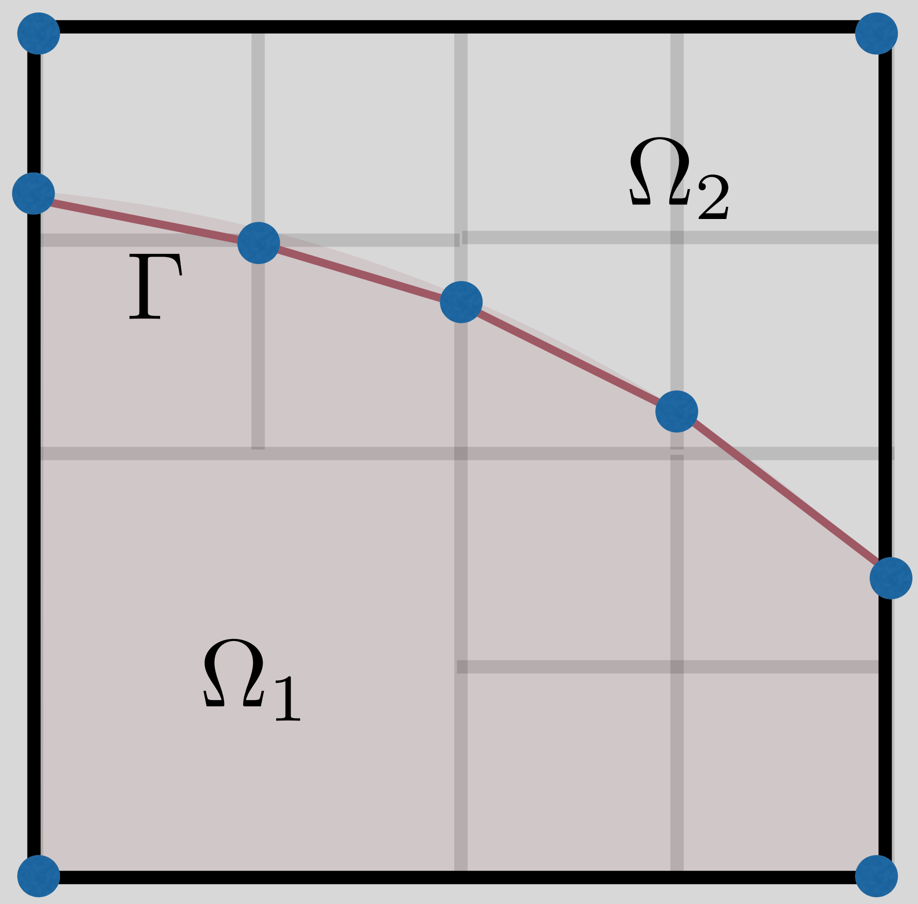
$V_{h_g}^{p_g}(\mathcal{O}_g)$

$p_g = 1$

For each enriched cell in the discretisation…

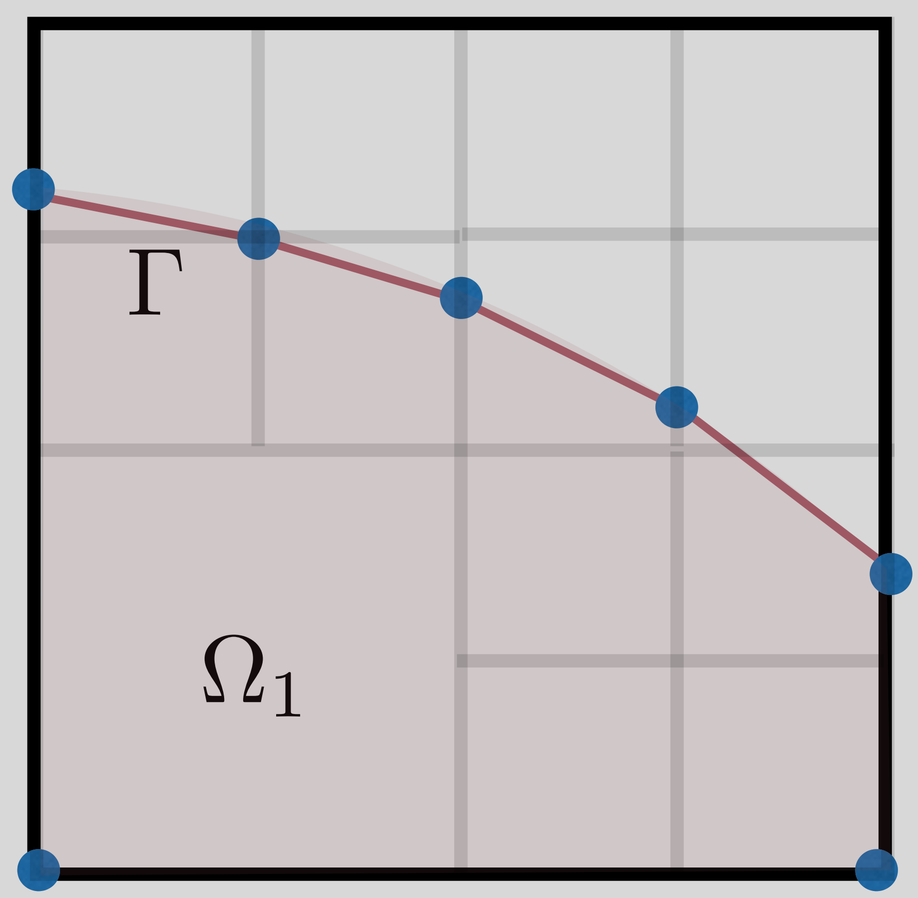


generate local Delaunay triangulation…



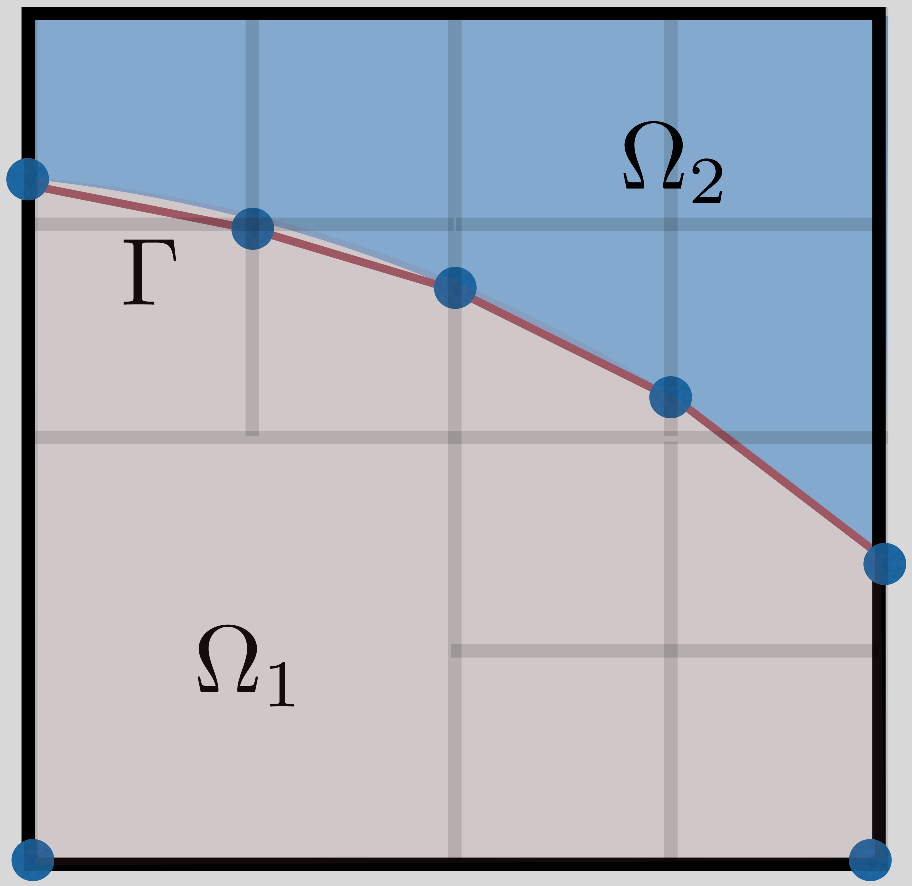
Case 1: boundary

finite cell method, implicit boundary method…



Case 2: inclusion

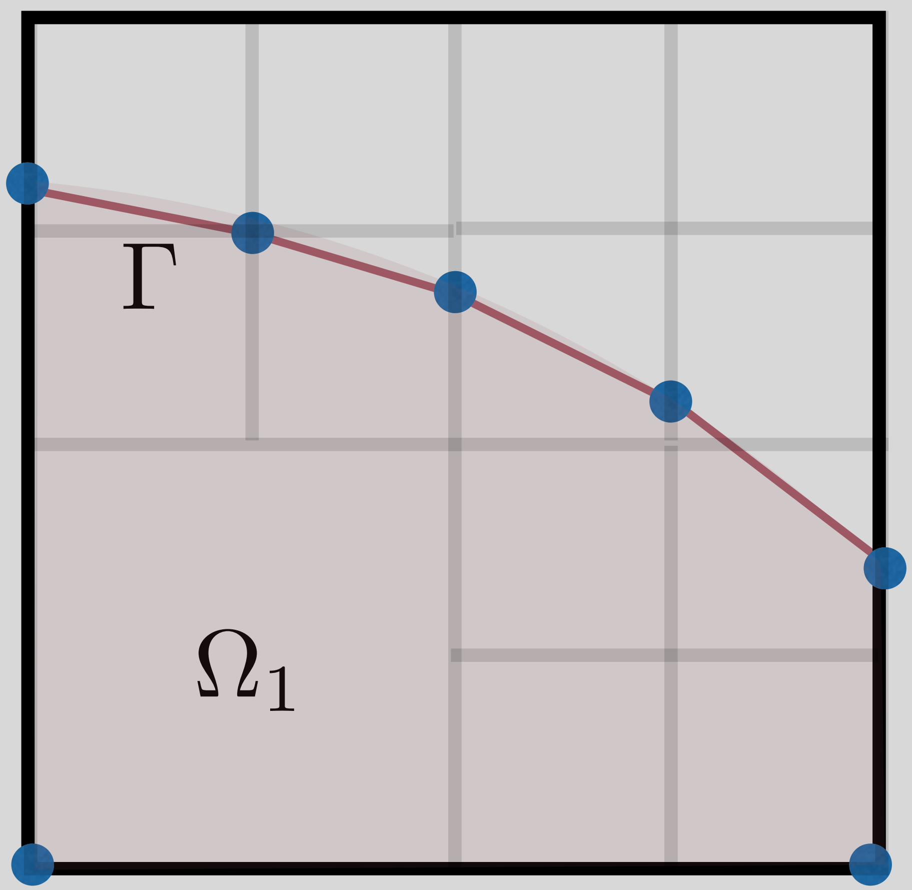
XFEM, PUM…



$$\mathbf{u}_h(\mathbf{x}) = \sum_{i=1}^{N} \mathbf{N}_i u_i + \sum_{i=1}^{N} \mathbf{N}_i \sum_{j=1}^{M} \psi_j(\mathbf{x}) a_i^j$$

Case 3: Dirichlet Boundary

Nitsche's method, Lagrange multipliers…



Present and future.

Progress so far…

- We are developing an add-on toolkit for the deal.ii finite element package specifically designed to ease the development of cartesian grid partition of unity and implicit boundary methods.
- We will release the library under an open-source (probably GPL due to heavy use of CGAL) license once the API has stabilised.
- Lessons learned from in-house codes: OpenXFEM++ (Bordas) and UPC-Implicit (Rodenas Garcia).
- Key lesson: if someone has already coded it, don't code it yourself (CGAL, deal.ii, ITK).

Features

- 2D and 3D problems on the same code-path.
- parallel hybrid MPI/TBB assembly and solution.
- fast and robust computational geometry using CGAL.
  - automatic Delaunay tessellation of integration subdomains.
- completely separate representation of discretisation and geometry via nested octree data structures.
  - constructs to represent soft and hard segmentations of image data.
  - implicit representation via level-sets, inside-outside functions.
- independent hpe-type adaptivity on discretisation and geometry.
  - fast refinement and coarsening operations.

Outlook

- We are developing a cartesian grid implicit boundary/enriched finite element method toolkit within deal.ii.

- By uncoupling discretisation and geometry we will develop methods particularly suited to image-based analysis.

- Many mathematical and software challenges challenges ahead.

Acknowledgements

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