Independence - revision and defaults *

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Abstract

We investigate different aspects of independence here, in the context of theory revision, generalizing slightly work by Chopra, Parikh, and Rodrigues, and in the context of preferential reasoning.

Contents

1 Introduction

We give some results on

(1) Theory revision:

Parikh and co-authors (see [CP00]), and, independently, Rodrigues (see [Rod97]), have investigated a notion of logical independence, based on the sharing of essential propositional variables. We do a semantical analogue here. What Parikh et al. call splitting on the logical level, we call factorization (on the semantical level).

A comparison of the work of Parikh and Rodrigues can be found in [Mak09].

Note that many of our results are valid for arbitrary products, not only for classical model sets.

We go very slightly beyond Parikh's work, as a matter of fact our generalization is alreday contained in the Axiom P2g, due to K.Georgatos (see [CP00]):

$(P2g)$ If T is split between $\mathcal{L}_1$ and $\mathcal{L}_2$, α, β are in $\mathcal{L}_1$ and $\mathcal{L}_2$ respectively, then $T * \alpha * b = T * \beta * \alpha = T * (\alpha \wedge \beta)$.

On the other hand, we stay below Parikh's work and do *not* investigate partial overlap (see $\mathcal{B}-$structures model in [CP00]).

We claim no originality of the basic ideas, just our proofs and perhaps an example might be new - but they are always elementary and very easy.

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(2) Preferential reasoning:

We shortly discuss preferential structures which have properties of defaults in the fact that they permit to treat sub-ideal information. Usually, we have only the ideal case, where all "normal" information holds, and the classical case. (Reiter) defaults, but also e.g., inheritance systems, permit to satisfy only some, but not necessarily all, default rules, and are thus more flexible. We show how to construct preferential structures with the same properties.

1.1 The situation in the case of theory revision

We work here with arbitrary, non-empty products. Intuitively, $\mathcal{Y}$ is the set of models for the propositional variable set U. We assume the Axiom of Choice.

Definition 1.1

Let U be an index set, $\mathcal{Y} = \Pi\{Y_k : k \in U\}$, let all $Y_k \neq \emptyset$, and $\mathcal{X} \subseteq \mathcal{Y}$. Thus, $\sigma \in \mathcal{X}$ is a function from U to $\bigcup\{Y_k : k \in U\}$ s.t. $\sigma(k) \in Y_k$. We then note $X_k := \{y \in Y_k : \exists\sigma \in \mathcal{X}.\sigma(k) = y\}$.

If $U' \subseteq U$, then $\sigma\lceil U'$ will be the restriction of σ to U', and $\mathcal{X}\lceil U' := \{\sigma\lceil U' : \sigma \in \mathcal{X}\}$.

If $\mathcal{A} := \{A_i : i \in I\}$ is a partition of U, $U' \subseteq U$, then $\mathcal{A}\lceil U' := \{A_i \cap U' \neq \emptyset : i \in I\}$.

Let $\mathcal{A} := \{A_i : i \in I\}$, $\mathcal{B} := \{B_j : j \in J\}$ both be partitions of U, then $\mathcal{A}$ is called a refinement of $\mathcal{B}$ iff for all $i \in I$ there is $j \in J$ s.t. $A_i \subseteq B_j$.

A partition $\mathcal{A}$ of U will be called a factorization of $\mathcal{X}$ iff $\mathcal{X} = \{\sigma \in \mathcal{Y} : \forall i \in I(\sigma\lceil A_i \in \mathcal{X}\lceil A_i)\}$, we will also sometimes say for clarity that $\mathcal{A}$ is a partition of $\mathcal{X}$ over U.

We will adhere to above notations throughout these pages.

If $\mathcal{X}$ is as above, $U' \subseteq U$, and $\sigma \in \mathcal{X}\lceil U'$, then there is obviously some (usually not unique) $\tau \in \mathcal{X}$ s.t. $\tau\lceil U' = \sigma$. This trivial fact will be used repeatedly in the following pages. We will denote by σ^+ some such τ - context will tell which are the U' and U. (To be more definite, we may take the first such τ in some arbitrary enumeration of $\mathcal{X}$.)

Given a propositional language $\mathcal{L}$, $v(\mathcal{L})$ will be the set of its propositional variables, and $v(\phi)$ the set of variables occuring in ϕ. A model set C is called definable iff there is a theory T s.t. $C = M(T)$ - the set of models of T.

1.2 Organization

We treat here the following:

(1) We give a purely algebraic description of factorization.

This is the algebraic analogue of work by Parikh and co-authors, and we claim almost no originality, perhaps with the exception of an example and the remark on language independence.

(2) We generalize slightly the Parikh approach so it can be described as commuting with decomposition into sublanguages and addition.

We show by a trivial argument that this corresponds to a generalized Hamming distance between models.

(3) We go beyond Rational Monotony and show how to construct a preferential structure from a set of (normal) defaults. Thus, we give an independent semantics to normal defaults, translating their usual treatment into a homogenous construction of the preferential structure.

In the general case, this gives nothing new, as any preferential structure can be constructed this way. (We consider the one-copy case only.) Most of the time, it will result in a special structure which automatically takes into account the specificity criterion to resolve conflicts. The essential idea is to take a modified Hamming distance on the set of satisfied defaults, modified as we do not count the defaults, but look at them as sets, together with the subset relation.

We also show that our approach can be seen as a revision of the ideal, perhaps non-existant case, or as an approach to this ideal case as the limit. Of course, when the ideal case is consistent, then this will be our result.

(4) Independence in the case of TR is treated by looking at "independent" parts "independently", and later summing up. In the case of defaults, we treat the defaults independently, just as in the Reiter approach, but also "inside" the model sets, we treat subsets just as we the sets themselves, resulting in a partial kind of rankedness (by default).

(5) We conclude by giving a simple informal argument why the TR situation is more complicated than the default situation.

2 Factorisation

Fact 2.1

If $\mathcal{A}$, $\mathcal{B}$ are two partitions of U, $\mathcal{A}$ a factorization of $\mathcal{X}$, and $\mathcal{A}$ a refinement of $\mathcal{B}$, then $\mathcal{B}$ is also a factorization of $\mathcal{X}$.

Proof

Trivial by definition. $\square$

Fact 2.2

Let $\mathcal{A}$ be a factorization of $\mathcal{X}$ over U, $U' \subseteq U$. Then $\mathcal{A}\lceil U'$ is a factorization of $\mathcal{X}\lceil U'$ over U'.

Proof

If $A_i \cap U' \neq \emptyset$, let $\sigma'_i \in \mathcal{X}\lceil(A_i \cap U')$. Let then $\sigma_i := \sigma_i'^{+}\lceil A_i$. If $A_i \cap U' = \emptyset$, let $\sigma_i := \tau\lceil A_i$ for any $\tau \in \mathcal{X}$. Then $\sigma := \bigcup\{\sigma_i : i \in I\} \in \mathcal{X}$ by hypothesis, so $\sigma\lceil U' \in \mathcal{X}\lceil U'$, and $\sigma\lceil(A_i \cap U'\} = \sigma'_i$. $\Box$

Fact 2.3

If $\{A, A'\}$ is a factorization of $\mathcal{X}$ over U, $\mathcal{A}$ a factorization of $\mathcal{X}\lceil A$ over A, $\mathcal{A}'$ a factorization of $\mathcal{X}\lceil A'$ over A', then $\mathcal{A} \cup \mathcal{A}'$ is a factorization of $\mathcal{X}$ over U.

Proof

Trivial $\Box$.

Fact 2.4

If $\mathcal{A}$, $\mathcal{B}$ are two factorizations of $\mathcal{X}$, then there is a common refining factorization.

Proof

Let σ s.t. $\forall i \in I \forall j \in J(\sigma\lceil(A_i \cap B_j) \in \mathcal{X}\lceil(A_i \cap B_j))$, show $\sigma \in \mathcal{X}$. Fix $i \in I$. By Fact 2.2 (page 2), $\mathcal{B}\lceil A_i$ is a factorization of $\mathcal{X}\lceil A_i$, so $\cup\{\sigma\lceil(A_i \cap B_j) : j \in J,\ A_i \cap B_j \neq \emptyset\} = \sigma\lceil A_i \in \mathcal{X}\lceil A_i$. As $\mathcal{A}$ is a factorization of $\mathcal{X}$, $\sigma \in \mathcal{X}$. $\Box$

This does not generalize to infinitely many factorizations:

Example 2.1

Take as index set $\omega + 1$, all $Y_k := \{0, 1\}$. Take $\mathcal{X} := \{\sigma : \sigma\lceil\omega$ arbitrary, and $\sigma(\omega) := 0$ iff $\sigma\lceil\omega$ is finally constant $\}$. Consider the partitions $\mathcal{A}_n := \{n, (\omega + 1) - n\}$, they are all fatorizations of $\mathcal{X}$, as it suffices to know the sequence from $n + 1$ on to know its value on ω. A common refinement $\mathcal{A}$ will have some $A \in \mathcal{A}$ s.t. $\omega \in A$. Suppose there is some $n \in \omega \cap A$, then $A \not\subseteq n + 1$, $A \not\subseteq (\omega + 1) - (n + 1)$, this is impossible, so $A = \{\omega\}$. If $\mathcal{A}$ were a factorization of $\mathcal{X}$, so would be $\{\omega, \{\omega\}\}$ by Fact 2.1 (page 2), but $\mathcal{X}$ does not factor into $\mathcal{X}\lceil\omega$ and $\mathcal{X}\lceil\{\omega\}$.

Comment 2.1

Above set $\mathcal{X}$ is not definable as a model set of a corresponding language $\mathcal{L}$: If ϕ is not a tautology, there is a model m s.t. $m \models \neg\phi$. ϕ is finite, let its variables be among $p_1, \ldots, p_n$ and perhaps p_ω. If p_ω is not among its variables, it is trivially also false in some m' in $\mathcal{X}$. If it is, then modify m accordingly beyond n. Thus, exactly all tautologies are true in $\mathcal{X}$, but $\mathcal{X} \neq \mathcal{Y} =$ the set of all $\mathcal{L}-$models.

We have, however:

Fact 2.5

Let $\mathcal{X} = \bigcap\{\mathcal{X}_m : m \in M\}$ and $\mathcal{X}, \mathcal{X}_m \subseteq \mathcal{Y}$ for all $m \in M$.

Let $\mathcal{A}$ be a partition of U, and a factorization of all $\mathcal{X}_m$.

Then $\mathcal{A}$ is also a factorization of $\mathcal{X}$.

Proof

Let σ s.t. $\forall i \in I\ \sigma\lceil A_i \in \mathcal{X}\lceil A_i$.

But $\mathcal{X}\lceil A_i = (\bigcap\{\mathcal{X}_m : m \in M\})\lceil A_i \subseteq \bigcap\{\mathcal{X}_m\lceil A_i : m \in M\}$: Let $\tau \in \mathcal{X}\lceil A_i$, so by $\mathcal{X} = \bigcap\{\mathcal{X}_m : m \in M\}$ $\tau^+ \in \mathcal{X}_m$ for all $m \in M$, so $\tau \in \mathcal{X}_m\lceil A_i$ for all $m \in M$.

Thus, $\forall i \in I, \forall m \in M : \sigma\lceil A_i \in \mathcal{X}_m\lceil A_i$, so $\forall m \in M. \sigma \in \mathcal{X}_m$ by prerequisite, so $\sigma \in \mathcal{X}$. $\Box$

Fact 2.6

Let $A \cup A'$ be a partition of U, and for all $\sigma \in \mathcal{X}\lceil A$ and all $\tau : A' \to \bigcup\{X_k : k \in A'\}$ with $\tau(k) \in X_k$ $\sigma \cup \tau \in \mathcal{X}$. Then

(1) $A \cup A'$ is a factorization of $\mathcal{X}$ over U.

(2) Any partition $\mathcal{A}' = \{A'_k : k \in I'\}$ of A' is a factorization of $\mathcal{X}\lceil A'$ over A'.

(3) If $\mathcal{A}$ is a factorization of $\mathcal{X}\lceil A$ over A, and $\mathcal{A}'$ a partition of A', then $\mathcal{A} \cup \mathcal{A}'$ is a factorization of $\mathcal{X}$.

Proof

(1) and (2) are trivial, (3) follows from (1), (2), and Fact 2.3 (page 3). □

Corollary 2.7

Let $U = v(\mathcal{L})$ for some language $\mathcal{L}$. Let $\mathcal{X}$ be definable, and $\{\mathcal{A}_m : m \in M\}$ be a set of factorizations of $\mathcal{X}$ over U. Then $\mathcal{A} := \cup\{\mathcal{A}_m : m \in M\}$ is also a factorization of $\mathcal{X}$.

Proof

Let $\mathcal{X} = M(T)$. Consider $\phi \in T$. $v(\phi)$ is finite, consider $\mathcal{X}\lceil v(\phi)$. There are only finitely many different ways $v(\phi)$ is partitioned by the $\mathcal{A}_m$, let them all be among $\mathcal{A}_{m_0}, \ldots, \mathcal{A}_{m_p}$. $M(\phi)\lceil v(\phi)$ might not be factorized by all $\mathcal{A}_{m_0}\lceil v(\phi), \ldots, \mathcal{A}_{m_p}\lceil v(\phi)$, but $M(T)\lceil v(\phi)$ is by Fact 2.2 (page 2). By Fact 2.4 (page 3), $\mathcal{A}\lceil v(\phi)$ is a factorization of $M(T)\lceil v(\phi)$.

Consider now $\mathcal{X}_\phi := (M(T)\lceil v(\phi)) \times \Pi\{(0,1) : k \in v(\mathcal{L}) - v(\phi)\}$.

By Fact 2.6 (page 3), (1) $\{v(\phi), v(\mathcal{L}) - v(\phi)\}$ is a factorization of $\mathcal{X}_\phi$ over $v(\mathcal{L})$.

By Fact 2.6 (page 3), (2) $\mathcal{A}\lceil(v(\mathcal{L}) - v(\phi))$ is a factorization of $\mathcal{X}_\phi\lceil(v(\mathcal{L}) - v(\phi))$ over $v(\mathcal{L}) - v(\phi)$.

By Fact 2.6 (page 3), (3) $\mathcal{A}$ is a factorization of $\mathcal{X}_\phi$ over $v(\mathcal{L})$.

$M(T) = \bigcap\{(M(T)\lceil v(\phi)) \times \Pi\{(0,1) : k \in v(\mathcal{L}) - v(\phi)\} \colon \phi \in T\}$, so by Fact 2.5 (page 3), $\mathcal{A}$ is a factorization of $M(T)$.

□

Comment 2.2

Obviously, it is unimportant here that we have only 2 truth values, the proof would just as well work with any, even an infinite, number of truth values. What we really need is the fact that a formula affects only finitely many propositional variables, and the rest are free.

Unfortunately, the manner of coding can determine if there is a factorization, as can be seen by the following example:

Example 2.2

(1) p= "*blue*", q= "*round*", q'= "*blue iff round*".
Then
$p \wedge q = blue\ and\ round$, $\neg p \wedge \neg q = \neg blue\ and\ \neg round$
$p \wedge q' = blue\ and\ round$, $\neg p \wedge q' = \neg blue\ and\ \neg round$
Thus, both code the same (meta-) situation, the first cannot be factorized, the second can.
Our example (first presented in [Sch07a]) is discussed in more detail in [Mak09], see Section 5. there.

(2) More generally, we can code e.g. the non-factorising situation $\{p \wedge q \wedge r, \neg p \wedge \neg q \wedge \neg r\}$ also using $q' = p \leftrightarrow q$, $r' = p \leftrightarrow r$, and have then the factorising situation $\{p \wedge q \wedge r, \neg p \wedge q' \wedge r'\}$.

(3) The following situation cannot be made factorising: $\{p \wedge q,\ p \wedge \neg q,\ \neg p \wedge \neg q\}$. Suppose there were some such solution. Then we need some p' and q', and all 4 possibilities $\{p' \wedge q',\ p' \wedge \neg q',\ \neg p' \wedge q',\ \neg p' \wedge \neg q'\}$. If we do not admit impossible situations (i.e. one of the 4 possibilities is a contradictory coding), then 2 possibilities have to contain the same situation, e.g. $p \wedge q$. But they are mutually exclusive (as they are negations), so this is impossible.

□

Remark 2.8

As we worked with abstract sequences, which need not be models, we can apply our results and the ideas behind them (essentially due to Parikh/Rodrigues) e.g., to

- Update: if a set of sequences (where the points are now models, and not true/false) factorizes, then we can update the components (i.e. look for the locally "best" subsequences), and then compose them to the globally "best" sequences.
- Utility streams: if a set of utility streams factorizes, we can do the same, commutativity and associativity of addition will guarantee the desired result.
- Preferential reasoning: Again, we factorize, and choose the locally best which we compose to the globally best.

The idea is always the same: If a set factorizes, choose locally, and compose to the global choice - provided this is the desired result!

3 Factorisation and Hamming distance

Definition 3.1

Given $x, y \in \Sigma$, a set of sequences over an index set I, the Hamming distance comes in two flavours:

$d_s(x, y) := \{i \in I : x(i) \neq y(i)\}$, the set variant,

$d_c(x, y) := card(d_s(x, y))$, the counting variant.

We define $d_s(x, y) \leq d_s(x', y')$ iff $d_s(x, y) \subseteq d_s(x', y')$,

thus, $s-$distances are not always comparabel.

We can also give different importance to different i in the counting variant, so e.g., $d_c(\langle x, x' \rangle, \langle y, y' \rangle)$ might be 1 if $x \neq y$ and $x' = y'$, but 2 if $x = y$ and $x' \neq y'$.

Fact 3.1

d_c has the normal addition, set union takes the role of addition for d_s, $\emptyset$ takes the role of 0 for d_s, both are distances in the following sense:

(1) $d(x, y) = 0$ iff $x = y$,

(2) $d(x, y) = d(y, x)$,

(3) the triangle inequality holds, for the set variant in the form $d_s(x, z) \subseteq d_s(x, y) \cup d_s(y, z)$.

Proof

(3) If $i \notin d_s(x, y) \cup d_s(y, z)$, then $x(i) = y(i) = z(i)$, so $x(i) = z(i)$ and $i \notin d_s(x, z)$.

The others are trivial.

□

Both Hamming distances cooperate well with factorization, as we will see now. This is not surprising, as Hamming distances work componentwise.

Definition 3.2

We say that a revision function $*$ factorizes iff for all T and ϕ and joint factorisations, which we write for simplicity (and immediately for models) $M(T) = M(T)\lceil\mathcal{L}_1\times, \ldots, \times M(T)\lceil\mathcal{L}_n$, $M(\phi) = M(\phi)\lceil\mathcal{L}_1\times, \ldots, \times M(\phi)\lceil\mathcal{L}_n$, $M(T * \phi) = M(T) \mid M(\phi) = ((M(T)\lceil\mathcal{L}_1) \mid (M(\phi)\lceil\mathcal{L}_1))\times, \ldots, \times((M(T)\lceil\mathcal{L}_n) \mid (M(\phi)\lceil\mathcal{L}_n))$.

To simplify notation, we will speak about $\Sigma \mid T$, $\Sigma_i \mid T_i$, $\sigma_1 \times \ldots \sigma_n$, etc.

The advantage is that, when factorisation is possible, we can work with smaller theories, formulas, and languages, and then do a trivial composition operation by considering the product.

Fact 3.2

If $*$ is defined by the counting or the set variant of the Hamming distance, then $*$ factorises.

Proof

We do the proof for the set variant, the counting variant proof is similar.

Let a factorisation as in the definition be given, and suppose $\tau \in T$ has minimal distance from Σ, i.e. $\tau \in \Sigma \mid T$. We show that each τ_i has minimal distance from Σ_i. If not, there is τ_i' closer to Σ_i, but then τ', which is like τ, only τ_i is replaced by τ_i' is also in T, by factorisation. By definition of the Hamming distance, τ' is closer to Σ than τ is, *contradiction*. Thus $\tau \in (\Sigma_1 \mid T_1) \times \ldots \times (\Sigma_n \mid T_n)$. Conversely, let all $\tau_i \in (\Sigma_i \mid T_i)$, we have to show that $\tau := \tau_1 \times \ldots \times \tau_n \in \Sigma \mid T$. By factorisation, $\tau \in T$. If there were a closer $\tau' \in T$, then at least one of the components τ_i' would be closer than τ_i, *contradiction.*

□

The authors do not know if all factorising distance defined revisions can be defined by one of the above Hamming distances.

4 Preferential modelling of defaults

Reiter defaults have the advantage to give results also for non-ideal cases. If, by default, α and α' hold, but α is inconsistent with the current situation, then α' will still "fire". Preferential structures say nothing about non-ideal cases. We construct special preferential structures which have the same behaviour as Reiter defaults. In addition, specificity will be used to solve conflicts.

The idea is simple.

For simplicity, we admit direct contradictions: $\phi \mid\sim \psi$ and $\phi \mid\sim \neg\psi$. This is done only to make the representation proof simple. One can do without, but pays with more complexity (see below). We also use structures with one copy of each model only.

Definition 4.1

(1) We call the default $\phi \mid\sim \psi$ more specific than the default $\phi' \mid\sim \psi'$ iff $\phi \vdash \phi'$ - $\vdash$ is classical consequence.

(2) We say that the default $\phi \mid\sim \psi$ separates m and m' iff $m, m' \models \phi$, but only one of m, m' satisfies ψ.

Consider two models, m, m'. Take the most specific defaults which separate them. For each such default $\phi \mid\sim \psi$, if $m \models \psi$, $m' \not\models \psi$, then set $m \prec m'$. We might introduce cycles of length 2 here.

Remark 4.1

(1) The construction has a flavour of rankedness, as, if possible, we make each "good" element smaller than each "less good" element. If e.g. $\phi \mid\sim \psi$ is the default, $m, m', n, n' \models \phi$, and $m, n \models \psi$, but $m', n' \not\models \psi$, then $m \prec m'$, $n \prec m'$, $m \prec n'$, $n \prec n'$.

(2) We may create indirectly loops, as we may have $m \prec m' \prec m''$, and also see $m'' \prec m$.

(3) Let $m \in X$ be minimal in our construction. Then there is no default $\phi \mid\sim \psi$ s.t. $m \models \phi \wedge \neg\psi$, and there is $m' \in X$, $m' \models \phi \wedge \psi$. Thus, minimal elements are "as good as possible", i.e. there is no better one in X. Of course, there might be a default $\phi \mid\sim \psi$ with $m \models \phi \wedge \neg\psi$, but "better" elements are outside X. Thus, minimal elements are an approximation of the ideal case. We can also consider minimal elements as a revision of the ideal case by X, in the sense that we cannot get closer to the ideal within X.

Conversely, given *any* preferential structure, we take any two models m, m', if $m \prec m'$ (which we see by $m' \notin \mu(\{m, m'\})$), we add the default $Th\{m, m'\} \mid\sim Th\{m\}$. By the basic law of 1-copy preferential structures, we create the structure again. (Note that $\{m, m'\}$ is the most specific set containing both.) Thus, our approach cannot result in new structural rules for preferential structures, like smoothness, rankedness, etc.

5 Remarks on independence

The idea of independence was realized for defaults by trying to satisfy them independently, so if one fails, the others still have a chance. This is a simple idea.

The case of theory revision is more complicated, as we have no predefined structure. In particular, the starting theory T can be just a "blob" which makes independence difficult to realize. Moreover, we may try to revise just once, so no multiple default satisfaction or so is needed. Perhaps the Parikh idea, and its refinement through (modified) Hamming distances is all one can achieve.

It is evident how to treat our form of independent revision with IBRS, it can just be written as a diagram.

Perhaps the best way to write defaults as a diagram, is the trivial one: $\alpha \mid\sim \beta$ will be written $\alpha \Rightarrow \beta$, and the treatment is in the evaluation of the diagram - as outlined above.

5.1 Epistemic states and independence

It is probably adequate to say that [AGM85] consider the revision function $*$ an epistemic state (depending on K), as revealed by the notion of epistemic entrenchment, and its equivalence to a revision function. In [LMS01], the global distance can probably be seen as (fixed, global) epistemic state. Essentially the critique of such too rigid, fixed, epistemic states resulted in dynamic states of [DP94] and [DP97]. The approach in [Spo88] incorporated already a dynamic approach. An excellent short overview of such dynamic revision approaches can be found in [Ker99].

In preferential structures, we might see the relation choosing the normal situations as (again fixed) epistemic state. In counterfactual conditionals, the distance can again be seen as the underlying epistemic state.

The present authors see the higher order arrows of reactive structures (see e.g., [GS08b]) as expressing epistemic states or changes of epistemic states. This will be explored in future research by the present authors.

But, we can also see the approaches discussed in this article as an epistemic state, which can perhaps be resumed as: "divide and conquer".

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