17th U.S. National Congress on Theoretical and Applied Mechanics
Michigan State University, 15-20 June 2014

Abstract: Z-99-99

A model order reduction approach to construct efficient and reliable virtual charts in computational homogenisation.

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Introduction Computational homogenisation is a widely spread technique to calculate the overall properties of a composite material from the knowledge of the constitutive laws of its microscopic constituents [1, 2]. Indeed, it relies on fewer assumptions than analytical or semi-analytical homogenisation approaches and can be used to coarse-grain a large range of micro-mechanical models. However, this accuracy comes at large computational costs, which prevents computational homogenisation from being used routinely in optimisation, even in the context of linear elastic materials. Indeed, a unit cell problem has to be solved for each microscopic distribution of interest in order to obtain the corresponding homogenised material constants. In the context of nonlinear, time-dependant problem, the computational effort becomes even greater as computational homogenisation requires solving for the time-evolution of the microstructure at every point of the macroscopic domain.

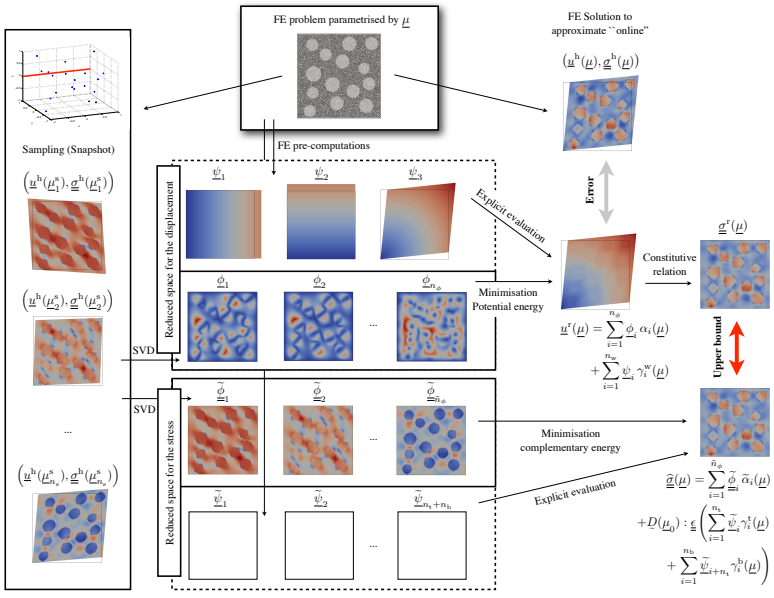

Figure 1. Schematic representation of the error bounding method for the Galerkin-POD.

In this paper, we propose to address these two issues within the unified framework of projection-based model order reduction (see for instance [3, 4, 5, 6]). The smoothness of the solution of the unit cell

problem with respect to parameter or time variations is used to create a reduced order model with very few degrees of freedom, hence reducing the computational burden by orders of magnitude.

Methdology and results We first consider a unit cell problem corresponding to a material composed of several linear elastic phases. We aim at obtaining the homogenised properties for any value of the elastic contrasts between those phases. In order to do so, an offline/online strategy is deployed, whereby an offline snapshot technique is used to construct a reduced space for the solution of the parametrised unit cell problem, and an online Galerkin-type formulation allows us to determine, approximately and at cheap costs, the solution corresponding to the desired set of elastic contrasts.

The novelty of the work lies in the method that we propose to certify this virtual chart. In order to obtain guaranteed error bounds on the homogenised properties, we employ the concept of error in the constitutive law, whereby an auxiliary reduced order model is constructed for the gradient of the solution of the parametrised unit cell. The comparison of the solutions delivered by the two surrogates can be used to bound the error in the homogenised properties (figure 1). An important result is that the bounds can be sharpened at will, by simply increasing the order of the auxiliary surrogate.

In the second part of the talk, we will extend the proposed approach to nonlinear materials, and in particular to unit cells undergoing damage. In this case, we first approximate our unit cell problem using the Empirical Interpolation Method [7], so that the nonlinear terms that appear in our balance equations become affinely dependent on the parameters (here time only). In this case, the accuracy of the virtual chart is not guaranteed. However, we propose to test it heuristically, using a cross-validation methodology. We show that a significant speed-up is obtain compared to a direct solution of the nonlinear unit cell problem.

Acknowledgments

The authors acknowledge the financial support of EPSRC under grant EP/J01947X/1: Towards rationalised computational expense for simulating fracture over multiple scales(RationalMSFrac) and the support of the European Research Council under Starting Independent Research Grant agreement No. 279578: Towards real time multiscale simulation of cutting in non-linear materials with applications to surgical simulation and computer guided surgery (RealTCut).

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