Measurement of the *speed-of-light* perturbation of free-fall absolute gravimeters

Ch Rothleitner[1]‡, TM Niebauer[2] and O Francis[1]
[1]University of Luxembourg
Faculty of Sciences, Technology and Communication, L-1359 Luxembourg
[2]Micro-g LaCoste Inc.
1401 Horizon Avenue, Lafayette, CO 80026, USA
E-mail: christian.rothleitner@ptb.de

Abstract. We report on a direct measurement of the relativistic Doppler shift with a commercial free-fall absolute gravimeter of the type FG5X. The observed Doppler shift, which is commonly called *speed-of-light* perturbation, can be well described by the relativistic Doppler formula, where the constant object velocity is replaced by a time-dependent velocity with constant acceleration. The observed *speed-of-light* perturbation stands in contrast to other publications, which predict a higher frequency shift. It has been be measured with a relative uncertainty of 1.1×10^{-3}.

1. Introduction

Free-fall absolute gravimeters measure the acceleration due to gravity, g, and find applications in geophysics, geodesy, as well as in metrology. The best known of those instruments is the free-fall absolute gravimeter FG5, from Micro-g LaCoste, Lafayette (CO), USA [2]. State-of-the-art free-fall absolute gravimeters, like the FG5, are able to measure g with a precision of about 1 part in 10^9. The traceability to the SI of the gravity measurement is provided by calibrations of the laser frequency and the frequency standard against metrological standards and is verified in international comparisons [1]. The accuracy is limited by both random and systematic errors [2]. Most of those error sources can be investigated independently and their uncertainty can be well assessed. However, other uncertainties are to date estimated in a purely theoretical way. Relativistic effects, for example, are subject to such theoretical estimations. Corrections due to the finite propagation velocity of light can amount to some 10 μGal (1 μGal = 10 nm s^{-2}), corresponding to 1 part in 10^8 of the absolute gravity value. The magnitude of this perturbation has been subject to an open discussion (see [3, 4, 5]). Until recently it was not possible to directly measure this effect, since its signal was well masked by other effects, e.g. seismic noise. In this short communication we report on a direct

‡ Current address: Physikalisch-Technische Bundesanstalt, Bundesallee 100, 38116 Braunschweig, Germany

measurement of the perturbation due to the *speed-of-light* with a commercial FG5X gravimeter from Micro-g LaCoste Inc.

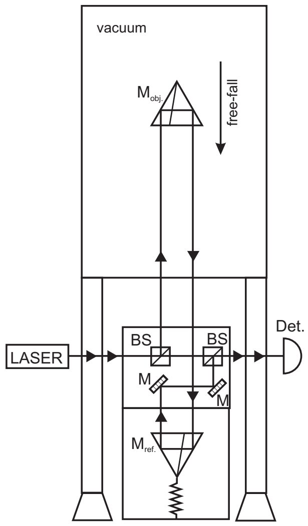

Figure 1. Simplified schematic drawing of a classical free-fall absolute gravimeter. An object mirror M_{obj} is falling freely in a vacuum vessel and is part of a Mach-Zehnder-type laser interferometer. The reference mirror M_{ref} is seismically isolated. After recombining the object beam with the reference beam, which passes straight through the beam splitter BS, a beat signal is formed. The beat signal is detected with a photo detector Det. M denotes a mirror.

2. Theoretical background

In a free-fall absolute gravimeter, like the FG5X, the free-fall trajectory of a test mass is tracked with a laser interferometer, as depicted in figure 1. Due to the increasing speed of the test mass the beat signal of the reflected beam with the reference beam forms a chirped sinusoid covering a frequency range from nearly DC to about 7.7 MHz, for a 30 cm long drop and a laser wavelength of 633 nm (HeNe-laser). In order to get the time-distance data for the test mass trajectory, the times are measured when the signal crosses the mean value of the sinusoid, also called zero-crossings. A full cycle of the fringe signal represents a displacement of the falling body by half the laser wavelength. In a simple treatment, the time dependent free-fall distance is modelled as a parabola with the coefficient of the quadratic term given by half the gravitational value, $g/2$.

The most common way of introducing the speed-of-light correction is through a retarded time given by the difference between the time that the interference is observed and the time in the past when the light actually reflected from the falling object. The new time coordinate is then given by $t \mapsto t + \frac{z}{c}$, where z is the uncorrected path lengths difference in the interferometer. This method of dealing with the speed-of-light has been used as long as free-fall measurements have been made with sufficient accuracy. It was first proposed by AH Cook in 1965 [6] and has been the accepted practice in the gravity community since that time.

Another way of calculating the speed-of-light perturbation is by applying two consecutive relativistic Doppler shifts, as described in [3]. This approach gives the speed-of-light corrected gravity value to be

$$g(T) = g_0 + \frac{1}{c}\left(2v_0 g_0 + 2g_0^2 T\right) \ , \tag{1}$$

where only Doppler terms up to second order have been considered. The constant velocity, which appears in the relativistic Doppler formula, was replaced by a time dependent velocity $v = v_0 + g_0 t$, v_0 representing some initial velocity and g_0 the acceleration due to gravity. T denotes the total drop time of the test mass. The first term of equation (1) arises from first-order terms in $(1/c)$ of the Doppler-shift and describes the trajectory obtained by Newton's second law. The terms in parenthesis are a result of the second-order term of the two consecutive Doppler-shifts and is commonly called *speed-of-light* perturbation.

The derivation of the speed-of-light perturbation, as described in (1), can be made equivalent to this order by introducing a time delay of $t \mapsto t + \frac{2}{3}\frac{z}{c}$. This amounts to 2/3 of the magnitude which is commonly applied. The difference between both predictions provided the impetus for this work, where we apply these corrections to see which one best fits the data.

In order to keep it more general we include an experimental factor a_c, so that the second order term can be written as

$$g_c(T) = a_c \cdot \frac{1}{c}\left(v_0 g_0 + g_0^2 T\right) \ . \tag{2}$$

a_c has to be determined by the experiment then, for which we expect a value of 2 if a derivation via (1) is correct. A derivation via a retarded time, which has remained ubiquitous in the gravity community (see e.g. [2, 7, 8, 6, 9]), would be consistent with $a_c = 3$. Though the difference in g amounts to approximately 4 μGal for a drop length of about 20 cm, as is common in the commercial FG5 gravimeter, a direct measurement of the effect is challenging. The reason is that a true gravity value is unknown. In order to make a decision whether one or the other factor is correct, the drop length has to be varied or the initial velocity of the test mass. This will lead to different perturbations in g. To date the correct factor has not been successfully measured by analyzing the same drop at different drop lengths for the limited signal to noise ratio. A recent improvement in the technology, namely a longer free-fall distance, has now made possible a direct experimental investigation of the speed-of-light on a freely falling mass in the laboratory. The longer drop helps by increasing the time of flight making the speed-of-light delay longer and the Doppler shift larger. It also helps by having a longer time to average other disturbances (mainly residual vibrations of the reference mirror) masking the speed-of-light effect.

3. Experiment

For our experiment a data set of 2400 drops, one drop every five seconds, was taken with our FG5X gravimeter in the Underground Laboratory for Geodynamics, in Walferdange, Luxembourg. Since the gravimeter itself has mass and thus produces a perturbing attraction on the freely falling mirror, the data had to also be corrected for this self attraction [10]. We used the original data from [10], which are based on a careful digitization and analysis of the technical drawings of the FG5X gravimeter. Self-attraction data were thus given for every milimeter covering the whole drop range. By integrating them twice we could directly subtract them from the raw time-position data of each drop. For each drop the model function

$$z = z_0 + v_0 t \left(1 + \frac{1}{6}\gamma t^2\right) + \frac{1}{2} g_0 t^2 \left(1 + \frac{1}{12}\gamma t^2\right) + a\ \cos\left(t 2\pi f_{mod}\right) + b\ \sin\left(t 2\pi f_{mod}\right) \quad (3)$$

was least squares fit to the time-distance data. This model includes the known vertical gravity gradient γ, which is independently measured, and the modulation frequency of the laser f_{mod}, which is about 8.33 kHz. This fit provides us also the velocity v_0 of the test mass. The data were then corrected for tidal influences, ocean loading and direct attraction, polar motion and atmospheric pressure effects. This was necessary, because for each drop length the data of all 2400 drops were averaged. Neglecting time dependent gravity variations would introduce a large scatter which would mask the speed-of-light perturbation. Next the drop length was shortened using the same data set by selectively removing data (fringes) from the least-squares fit (see Fig. 2). This allows us to investigate the dependence of the speed-of-light perturbation as a function of the drop length and initial velocity. We did this first by varying the start fringe of the drop data, i.e. deleting some of the first cycles of the sinusoid, and leaving the last fringe fixed (FF). 401 different drop lengths have thus been investigated–17 cm the shortest and 30 cm the longest drop. Since the start of drop changes when changing the drop length, an additional gravity transfer had to be done by considering the local gravity gradient. This is in addition to the gradient term in equ. 3 and must be done in order to refer all measurements to the same position. A second treatment was done by keeping the initial fringe fixed and varying the last fringe (LF), obtaining the same 401 drop lengths as before. Since all drop lengths start now at the same height no additional gradient correction (gravity transfer) needs to be applied, the gradient term in equ. 3 is sufficient. The reason for this second treatment is, that the magnitude of the speed-of-light correction is different here, since the initial velocities are constant and the drop time varies, while in the first analysis both parameters change. As a consequence, unknown effects, which by coincidence exactly compensate the speed-of-light effect in the first analysis, will not do so in the second analysis. The consistency can thus be verified. In the ideal case, i.e. if all corrections are made correctly, the measured gravity is drop length independent and will have the same mean value for both treatments. It is worth to mention that for both treatments we use the same drop lengths, however, since the initial velocity for the shorter drops in the FF case is higher than in the LF

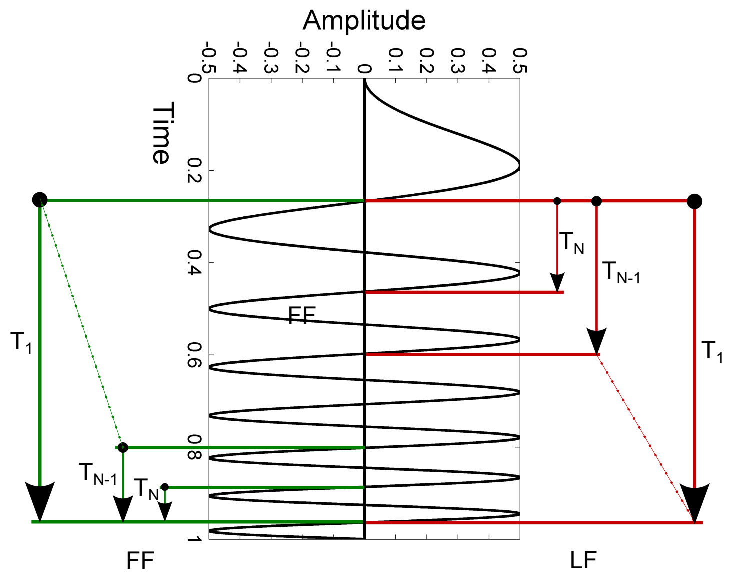

Figure 2. Illustration of how the drop length is changed by selectively removing fringes from the raw fringe signal. This allows to investigate whether the measured gravity is depending on the drop length (or free-fall duration T). FF denotes the case when the first fringe is varied, i.e. fringes from the start are removed. LF refers to when the last fringe is varied, where fringes from the end are removed. Observe that the total drop times T are different for the FF and LF case (except for the longest drop), although the drop length (number of fringes) is the same.

case, total drop times in the FF case are shorter.

4. Results

Fig. 3 shows the results of our experiment, varying the first and the last fringe, respectively. Data have been corrected for time varying signals (tides, ocean loading and direct attraction, polar motion and atmospheric pressure), self attraction of the gravimeter and the local gravity gradient. The same (arbitrary) constant value was subtracted from all drop lengths and for both treatments, in order to emphasize the gravity changes. Blue dots show measurements without any speed-of-light correction for FF, whereas green dots are results from LF without speed-of-light correction. It can be readily seen that the gravity values show a correlation with the total drop time. The maximum difference between the shortest and longest drop data amounts to only 5.1 μGal and 1.54 μGal, for the FF and LF treatment, respectively. To these data we fitted the model equation

$$g_c(T) = a_c \cdot \frac{1}{c}\left[v_0 g_0 + \left(1+\eta_2\frac{1}{5}\right)\frac{1}{2}g_0^2 T\right] , \tag{4}$$

as derived above. The factor $\eta_2 = \frac{5\left(l^2-3\right)l}{7(3l^2-5)}$, with $l = \left(1+\frac{2v_0}{g_0 T}\right)^{-1}$, is introduced in order to take into account that the data are sampled equally spaced in distance (see [3] and

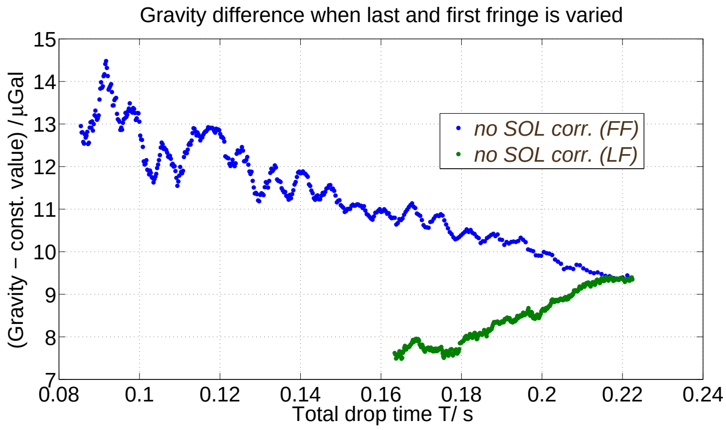

Figure 3. The gravity value as a function of the total drop time T. The same constant value is subtracted from all data. Green dots represent data without speed-of-light (SOL) correction for LF. The blue dots show the data for FF without SOL correction. FF data include shorter total drop times, since the initial velocity is higher for shorter drops, but both treatments, FF and LF, contain the same different drop lengths.

[7]). For g_0 it is of sufficient precision to use the value 9.81 m s^{-2}. Also the degree of the model is of sufficient precision. Higher order terms, e.g. including the gradient, would give negligible contributions as simulations have shown. As a result we obtain $a_{c;FF} = 2.0005(34)$ and $a_{c;LF} = 1.9999(16)$, when the first fringe and the last fringe is varied, respectively. This shows clearly that $a_c = 2$ is more adequate to describe the speed-of-light effect, than $a_c = 3$ does. This can also be seen in figures 4 and 5, where we have plotted the gravity changes after correcting for the speed-of-light (SOL). The model including the factor $a_c = 2$ describes better the perturbation than $a_c = 3$, which leaves still a slope in the residuals.

The speed-of-light effect is of first order in the total drop time, T. Other effects,

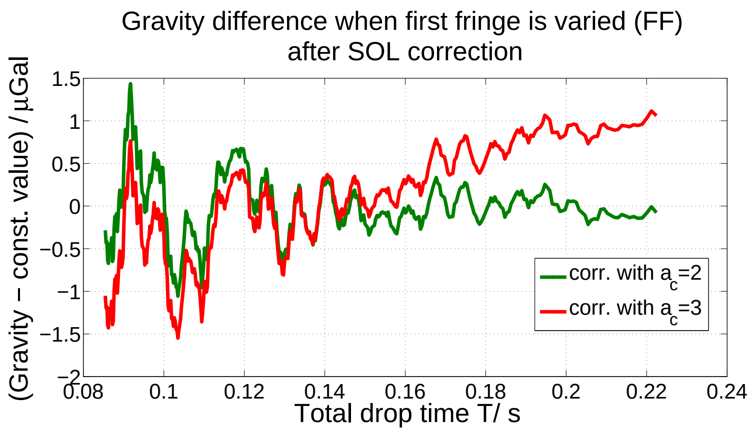

Figure 4. The plot shows the gravity difference when the drop length is changed for the FF case after the SOL correction is applied. The green line uses the factor $a_c = 2$, whereas the red line is obtained after correcting for the SOL using $a_c = 3$. A slope can be observed here.

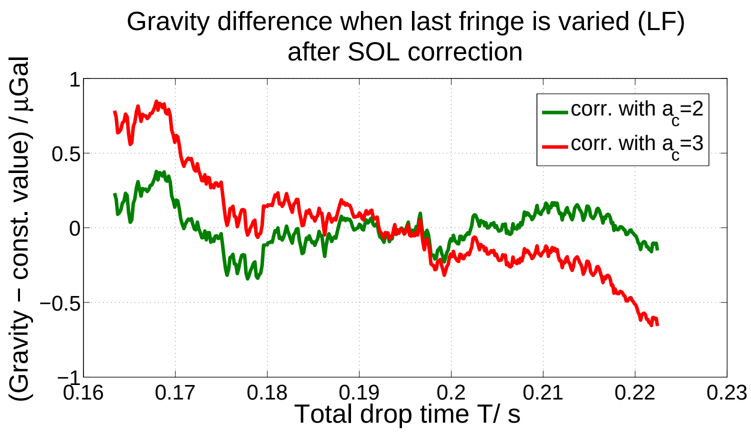

Figure 5. The plot shows the gravity difference when the drop length is changed for the LF case after the SOL correction is applied. The green line uses the factor $a_c = 2$, whereas the red line is obtained after correcting for the SOL using $a_c = 3$. A slope can be observed here.

which are of the same order in T, could give a similar dependence of the gravity as function of drop length and bias the data. Candidates, which could produce such a bias, are residual air friction, corner cube rotation, diffraction correction or frequency dependent phase shifts in the electronics system. Based on the parameters given in [2], and using [11, 7, 12], the uncertainties due to the first three candidates were estimated to be less than a tenth of a microgal. This sounds small, but we are considering here only differential corrections between the longest and the shortest drop. Let us consider the diffraction correction for example. Though the absolute effect amounts to about 1 μGal for a beam waist of 3 mm, the change of this effect, when the drop is increased by 13 cm is much smaller. We are not interested in the absolute perturbation here. The uncertainty due to electronic phase shift is also estimated to be less than a microgal and should average out, since for different drop lengths we have different phases and frequency contributions. In fact, the wave-like shape in the graph, when the first fringe (FF) is varied, is assumed to arise from this effect. Besides it is expected to be asymmetric, i.e. should have different biases to both treatments, since different frequency ranges are covered. Another effect, which could be a reason for the wave-like shape of the plots, can be residual ground vibrations. For an uncertainty estimation we analyzed the stacked residuals over 2400 drops. A power spectral analysis showed harmonics at about 7 Hz with an amplitude of some $2.5{\cdot}10^{-11}$ m. Converted into acceleration it would contribute with roughly 0.46 μGal to the uncertainty. Other harmonics (e.g. 13 Hz, 40 Hz, 117 Hz) had a much smaller contribution, so that the total assessed uncertainty due to residual vibrations is on the order of 0.77 μGal. The uncertainty contribution due the self-attraction is also assumed to be negligible, for we have used the detailed technical drawings, with known material, of the FG5X gravimeter to calculate the self attraction in steps of millimeters along the free-fall trajectory [10]. Finally–and this remains the main contribution to our uncertainty–the gradient γ is measured to better than 2%. It was determined by measuring the gravity at 4 different positions along the drop range by means of a Scintrex Autograv CG5 relative gravimeter. Usually an uncertainty of 1% is assessed to such a measurement. A data analysis using a gradient of $0.98 \times \gamma$ leads to estimates of $a_{c;LF} = 2.0011(17)$ and $a_{c;FF} = 2.0026(37)$. For a gradient of $1.02 \times \gamma$, on the other side, we obtain $a_{c;LF} = 1.9986(18)$ and $a_{c;FF} = 1.9983(35)$. Combining both treatments and taking the estimates for a lower and higher gradient as confidence intervals, we can give the final estimate as $a_c = 2.0002^{+0.0022}_{-0.0019}$. The maximum relative uncertainty amounts thus to 1.1×10^{-3}.

5. Conclusions

We have presented an experiment where we measured 2400 consecutive gravity values at the same location with an absolute free-fall gravimeter FG5X. The average gravity value was obtained by correcting for time-varying signals (tides, ocean loading and direct attraction, polar motion and atmospheric pressure), the measured local gravity gradient and the simulated self-attraction of the FG5X gravimeter. The drops covered a free-fall range of 30 cm. We reduced the drop lengths of the same 2400 drops by chopping fringes from either the start or the end. The same types of corrections (gravity gradient and self-attraction now adjusted to the new time-position data) as in the 30 cm drops were applied again to the shortened drop data and the average gravity value was re-calculated. The data thus obtained showed a dependence of the gravity from the drop length. The observed perturbation can be explained by the speed-of-light effect to the 10^{-3} relative level. The speed-of-light model we used differs from that which is commonly used and was first proposed by Cook [6]. The results support our conclusion that the observed correction is 1/3 less than commonly applied to free-fall absolute gravimeters.

Besides the importance for gravimetry and fundamental physics, this result has implications also for the so-called watt balance experiment. A possible way of a new definition of the SI unit kilogram, which is currently defined by a material artifact, is through the watt balance, first described by Kibble [13] (see also [14] and [15]). This instrument provides a link between electrical and mechanical virtual power. A crucial parameter for the realization of this redefinition is the accurate knowledge of the local acceleration due to gravity. The targeted standard uncertainty for a measurement with such a watt balance lies at 2 parts in 10^8. A bias of 4 μGal can thus not be neglected. Furthermore, interferometers become more and more precise and the requirements for high velocity positioning become tighter. An erroneous speed-of-light correction can thus lead to observable positioning errors (see [16]).

Acknowledgments

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References

[1] O. Francis et al. The European Comparison of Absolute Gravimeters 2011 (ECAG-2011) in Walferdange, Luxembourg: results and recommendations. *Metrologia*, 50(3):257, 2013.

[2] T.M. Niebauer, G.S. Sasagawa, J.E. Faller, and F. Klopping. A new generation of absolute gravimeters. *Metrologia*, 32:159–180, 1995.

[3] Ch. Rothleitner and O. Francis. Second-order doppler-shift corrections in free-fall absolute gravimeters. *Metrologia*, 48(3):187, 2011.

[4] V.D. Nagornyi, Y.M. Zanimonskiy, and Y.Y. Zanimonskiy. Relativity, doppler shifts, and retarded times in deriving the correction for the finite speed of light: a comment on 'second-order doppler-shift corrections in free-fall absolute gravimeters'. *Metrologia*, 2011.

[5] Ch. Rothleitner and O. Francis. Reply to 'comment on second-order doppler-shift corrections in free-fall absolute gravimeters'. *Metrologia*, 48(5):442, 2011.

[6] A.H. Cook. The absolute determination of the acceleration due to gravity. *Metrologia*, 1:84–114, 1965.

[7] L. Robertsson. Absolute gravimetry in a shifted legendre basis. *Metrologia*, 42(5):458, 2005.

[8] V.D. Nagornyi, Y.M. Zanimonskiy, and Y.Y. Zanimonskiy. Correction due to the finite speed of light in absolute gravimeters. *Metrologia*, 48(3):101, 2011.

[9] J.A. Hammond. *A Laser-Interferometer System for the Absolute Determination of the Acceleration of Gravity*. PhD thesis, UNIVERSITY OF COLORADO AT BOULDER., 1970.

[10] T.M. Niebauer, R. Billson, A. Schiel, D. van Westrum, and F. Klopping. The self-attraction correction for the fg5x absolute gravity meter. *Metrologia*, 50(1):1, 2013.

[11] L. Robertsson. On the diffraction correction in absolute gravimetry. *Metrologia*, 44(1):35, 2007.

[12] Ch. Rothleitner and O. Francis. On the influence of the rotation of a corner cube reflector in absolute gravimetry. *Metrologia*, 47(5):567–574, 2010.

[13] B.P. Kibble. A measurement of the gyromagnetic ratio of the proton by the strong field method. In J. H. Sanders and A. H. Wapstra, editors, *Atomic Masses and Fundamental Constants*, volume 5, pages 542–551, New York, 1976. Plenum.

[14] I.A. Robinson. Towards the redefinition of the kilogram: a measurement of the planck constant using the NPL Mark II watt balance. *Metrologia*, 49(1):113, 2012.

[15] M. Stock. Watt balance experiments for the determination of the planck constant and the redefinition of the kilogram. *Metrologia*, 50(1):R1, 2013.

[16] R.K. Heilmann, P.T. Konkola, C.G. Chen, and M.L. Schattenburg. Relativistic corrections in displacement measuring interferometry. *Journal of Vacuum Science & Technology B: Microelectronics and Nanometer Structures*, 18(6):3277–3281, 2000.