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On business cycles of variety and quality

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On business cycles of variety and quality ☆

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Abstract

This paper explores the role played by product variety and quality in a real business cycle model. Firms are heterogeneous in terms of their specific quality as well as productivity levels. Firms which have costly technology enter in a period of high aggregated demand and produce high quality goods. Thus, the average quality level and number of available varieties are procyclical, as in the data. The model can replicate the observed inflationary bias in the conventional Consumer Price Index due to a rise in the number of new product varieties and quality.

Keywords: *entry and exit, firm heterogeneity, the Schumpeterian destruction, product quality, business cycles*
JEL classification: D24, E23, E32, L11, L60

1. Introduction

Measuring fluctuations in the number of product varieties and their quality level is important for a better assessment of the true cost of living. It is well known, however, that conventional consumer price indices (CPI) do not fully appreciate changes in the number

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of product varieties and quality. Many empirical studies have documented the existence of bias in the conventional CPI (Boskin et al. 1996, Hausman 2003, Bils 2001, 2009,, Broda and Weinstein 2004, 2006, 2007, 2011). Boskin et al (1996) report the U.S. CPI had an upward bias of 1.1 percent per year. Of the total bias, 0.6 percent was ascribed to unmeasured quality improvements. Broda and Weinstein (2010) document an 0.8 percent annual upward bias using a dataset that covers around 40 percent of all expenditures on goods corrected at the US household level. They also report that quality and variety bias are procyclical and conventional official business cycle statistics underestimate the variability of major economic variables.

This paper investigates the role played by product variety and quality in a closed-economy real business cycle model. The theoretical model can be considered an extension of Ghironi and Melitz (2005) and Bilbiie et al. (2007), in which I incorporate endogenous-Schumpeterian destruction among variety representing firms. Firms are assumed to be heterogeneous in firm specific quality as well as productivity level. When higher firm specific quality level requires higher firm specific marginal costs as found in Verhoogen (2008) and Kugler and Verhoogen (2012), firms which implement costly technology enter in order to produce high quality goods in the period of relatively high aggregated demand. As a result, the number of producers and average quality level in the economy rise, showing their procyclical patterns. By capturing imperfectly such a rise in the number of new products and quality, the conventional consumer price indices exhibit an upward bias, as documented in Broda and Weinstein (2010).[1] Because of this procyclicality of CPI bias, the variability of major economic variables are underestimated using the empirical-based or statistically relevant CPI. In addition to the dynamic of bias in CPI as documented in Broda and Weinstein (2010), the theoretical model in this paper will replicate the second

[1] Broda and Weinstein (2010) document that turnover of product variety takes place mainly *within* firms. The assumption of one firm for one product variety in the paper is taken for the sake of simplicity rather than reality. Although extending the model to such a multi-product setting as in Bernard et al. (2010) and Bernard et al. (2011) would be a promising direction of future research, the main message of the paper would remain.

moments of major economic variables, including a high turnover in the number of firms.

The paper is also related to the recent literature in international trade on product quality.(Schott (2004),Hummels and Klenow (2005), Hallak (2006), Verhoogen (2008), Khandelwal (2010), Hallak and Schott (2011), Baldwin and Harrigan (2011), Johnson (2012)) The unit prices of exporting goods tend to rise with distance or difficulty in entering the market. This pattern cannot be explained in the model with homogenous quality in which only a subset of efficient firms which can charge a lower price are successful in penetrating foreign markets. With the possibility of quality upgrading, however, the prices of exporting goods can be higher for firms which produce high quality goods. This is because quality-adjusted price matters for consumers. Even with such high prices, if the quality of goods is high, these firms can attract demand, thus overcoming difficult market accessibility. We can draw a complete parallel with the above literature in international trade. While the difficulty of market accessibility sorts firms based on quality level in exporting market, in this paper's model, a relatively high aggregated demand in the boom period lets firms which have costly technology enter and produce high quality goods.[2]

The paper is organized as follows. Section 2 presents the benchmark model. The model is considered an extension of Hamano (2012) with product quality. Section 3, the theoretical model is calibrated with conventional parameters and shock process. In particular, I consider the case of a positive aggregate productivity shock in the following subsection and argue how the quality upgrading and variety and quality bias in CPI appear in impulse response functions. In the next subsection, the second moment of the theoretical model is presented. The last section provided a brief concluding remark.

2. The model

The economy is inhabited by one unit mass of atomic households. New firms or varieties, i.e. extensive margins appear as a result of investment motivated by their

[2] Although there is no quality dimension, such a behavior is consistent with Shleifer (1986), in which firms defer implementation of new efficient technology until the time of boom in order to take advantage of high aggregated demand.

consumption smoothing. Each firm represents one product variety. Firms draw their specific capability level from a distribution upon entry. These capabilities are either transformed into firm specific quality or productivity level following Sutton (1998, 2005). In addition to sunk entry costs, fixed operational costs are required for production from the next period. These costs are paid in terms of effective labor. Firms exit from the market endogenously depending on their profitability as well as exogenously.

2.1. Households

The representative household maximizes the expected discounted sum of utilities, $E_t \sum_{i=t}^{\infty} \beta^{i-t} U_t$, where β (< 1) denotes discount factor. The utility at time t depends on consumption C_t and labor supply L_t as follows

$$U_t = \ln C_t - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}. \tag{1}$$

The parameter χ (> 0) represents degree of non-satisfaction supplying labour and φ stands for the Frisch elasticity of labor supply[3]. With the above specification, the marginal disutility in providing one unit of additional labor is increasing.

Consumption is defined over a continuum of goods Ω. At any given time t, only a subset of goods $\Omega_t \in \Omega$ is available as

$$C_t = V_t \left(\int_{\omega \in \Omega_t} \left(q(\omega) c_t(\omega)\right)^{1-\frac{1}{\sigma}} d\omega \right)^{\frac{1}{1-\frac{1}{\sigma}}}, \tag{2}$$

where $c_t(\omega)$ is individual demand and $q(\omega)$ stands for quality for a variety ω which is invariant across time and $V_t \equiv S_t^{\psi - \frac{1}{\sigma-1}}$ in which S_t denotes the number of available varieties at time t. Following Benassy (1996), ψ represents the marginal utility associated with one additional increase in the number of varieties in the basket. When $\psi = 1/(\sigma - 1)$, the preference coincides with those implied by Dixit and Stiglitz (1977). σ (> 1) stands for the elasticity of substitution among varieties.

[3]With $\varphi = \infty$ the marginal disutility of supplying labor becomes constant, χ. When $\varphi = 0$ the marginal disutility becomes infinite and the labor supply becomes inelastic.

2.1.1. Law of motion and budget constraint

N_t and H_t denote the number of potential producers and new entrants, respectively. "Potential", because only a subset number of S_t firms which has survived the Schumpeterian destruction actually produce as will be detailed in the following sections. New entrants do not produce immediately. They are assumed to need "one time to build" in order to become potential producers in the next period. It is also assumed that δ fraction of potential producers and new entrants exit exogenously in each period. These assumptions imply that the number of potential producers at time t is given by the following law of motion

$$N_t = (1-\delta)\left(N_{t-1} + H_{t-1}\right). \tag{3}$$

And following Caballero and Hammour (2005), I distinguish the number of firms destroyed through the endogenous Schumpeterian destruction from those who destroyed by exogenous exit rate. They are defined respectively as

$$D_t^S \equiv N_t - S_t \quad \text{and} \quad D_t^\delta \equiv \delta\left(S_t + H_t\right). \tag{4}$$

Thus, gross destruction at time t is given by

$$D_t \equiv D_t^S + D_t^\delta. \tag{5}$$

I choose the price of consumption basket P_t as numéraire and let "˜" stand for the "average" level. Using this notation, the period-by-period real budget constraint for the representative household is given by

$$C_t + x_{t+1}v_t\left(N_t + H_t\right) = L_t w_t + x_t N_t\left(v_t + \widetilde{d}_t\right). \tag{6}$$

v_t denotes real share price of a mutual fund among N_t potential producers and H_t new entrants. At time t, the household purchases consumption goods and a share of the mutual fund, x_{t+1}. As revenue, she receives labour income and the returns of the fund on a share previously held, x_t. w_t and $\widetilde{d}_t$ denote real wage and average real dividends among

N_t potential producers, respectively. Accordingly, it is particularly important to assume that the Schumpeterian destruction takes place only after households have completed investing.

2.1.2. First order conditions

The representative household maximizes U_t with respect to C_t, x_{t+1} and L_t under the budget constraint (6) for every period. The first-order condition with respect to labor supply L_t is

$$\chi \left(L_t\right)^{\frac{1}{\psi}} = w_t C_t^{-1}. \tag{7}$$

Taking into account the motion of firms (3), the first-order condition with respect to share holdings x_{t+1} is

$$v_t = \beta \left(1-\delta\right) E_t \left(\frac{C_{t+1}}{C_t}\right)^{-1} \left(v_{t+1} + \widetilde{d}_{t+1}\right). \tag{8}$$

Iterating forward and ruling out the Ponzi schema in the above expression, we have

$$v_t = E_t \sum_{i=t+1}^{\infty} \left[\beta \left(1-\delta\right)\right]^{i-t} \left(\frac{C_i}{C_t}\right)^{-1} \widetilde{d}_i. \tag{9}$$

The current asset price v_t can be expressed as the expected discounted sum of future dividends. It is discounted by the exogenous destruction rate $1-\delta$ as well as discount factor β.

The optimal consumption for a variety ω is given by

$$c_t\left(\omega\right) = \left(V_t q_t(\omega)\right)^{\sigma-1} \left(\frac{p_t\left(\omega\right)}{P_t}\right)^{-\sigma} C_t. \tag{10}$$

where $p_t\left(\omega\right)$ denotes the "physical unit" price of a variety ω. The price index which minimizes nominal expenditures is found to be

$$P_t = \frac{1}{V_t} \left(\int_0^{S_t} \left(\frac{p_t\left(\omega\right)}{q_t(\omega)}\right)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}}. \tag{11}$$

Specifically, $p_t\left(\omega\right)/q_t(\omega)$ is defined as the "quality-adjusted" individual price. The above expression is also defined by a welfare-basis.

2.2. Heterogeneous firms and the Schumpeterian destruction

2.2.1. Entry

Firms are monopolistically competitive and each firm produces one specific product variety. Upon entry, new entrants draw a firm specific "capability" level α which is defined as the product of firm specific quality and productivity level such as $\alpha \equiv q(\alpha) z(\alpha)$. This specification flows Sutton (1998, 2005). Specifically, α is assumed to be transformed into either firm specific quality or productivity through $q(\alpha) = \alpha^{\phi}$ and $z(\alpha) = \alpha^{1-\phi}$. Thus quality is associated with firm specific productivity through

$$q(\alpha) = z(\alpha)^{\frac{\phi}{1-\phi}}$$

In the above expression, the parameter ϕ determines the degree of competition in quality, i.e. the quality ladder. When $\phi > 1$ or $\phi < 0$, we have a negative correlation between firm specific quality and productivity. Firms implement costly firm specific technology to produce high quality goods. Note when $\phi = 0$, all firms have the same quality $q(\alpha) = 1$ independent from their specific capability level. In such a case, the model collapses to the one discussed in Hamano (2012) where only the productivity level matters in sorting firms.[4]

Also, α is assumed to be drawn from a c.d.f, $G(\alpha)$, which is defined as the following Pareto distribution:

$$G(\alpha) = 1 - \left(\frac{\alpha_{\min}}{\alpha}\right)^{k}. \tag{12}$$

where $\alpha_{\min}$ is the minimum productivity level and k $(> \sigma - 1)$ is the parameter which governs the shape of distribution. When k rises, the distribution becomes more skewed towards the minimum level and heterogeneity decreases.

[4] In the same vein, based on Melitz (2003), Mandel (2010) and Kugler and Verhoogen (2012) present a model where firms choose quality level as a result of the optimization problem. As argued in Baldwin and Harrigan (2007), however, the key is to have a positive relation between firm specific marginal costs and its quality level.

Throughout this paper, entry is assumed to be identical. Once entrants draw a productivity level, every firm enters and pays the sunk entry costs which consist of $l_{E,t} \equiv f_{E,t}/Z_t^{\vartheta}$ units of effective labor, where $f_{E,t}$ is exogenous and represents (de)regulation on entry, Z_t stands for productivity of labor and the parameter ϑ governs its spillover to the efficiency of workers for firm setup activity.

In equilibrium, the following free entry condition which equates current share price and cost of entry must hold pining down the number of new entrants H_t:

$$v_t = \frac{w_t f_{E,t}}{Z_t^{\vartheta}}. \tag{13}$$

2.2.2. Production

The production technology of a firm which has drawn a capability level α is summarized by

$$l_t(\alpha) = \frac{y_t(\alpha)}{Z_t z(\alpha)} + \frac{f_t}{Z_t^{\theta}}, \tag{14}$$

where $l_t(\alpha)$ stands for labor demand for production. $y_t(\alpha)$ denotes the scale of production, i.e. intensive margins. In addition to variable costs $y_t(\alpha)/Z_t z(\alpha)$, production requires operational fixed costs which are defined in terms of effective labor f_t/Z_t^{θ}. This fixed cost is assumed to fluctuate along labour productivity Z_t with a degree of spillover θ. f_t is exogenous and represents "subsidies" when it reduces.

Provided (14), operational real profits of the firm are expressed as

$$d_t(\alpha) = \left(\rho_t(\alpha) - \frac{w_t}{Z_t z(\alpha)}\right) y_t(\alpha) - \frac{w_t f_t}{Z_t^{\theta}}, \tag{15}$$

where $\rho_t(\alpha)$ stands for real price measured in terms of consumption basket. Goods market clearing requires that $y_t(\alpha) = c_t(\alpha)$ and taking into account the demand addressed to each firm as (10), the maximization of profits gives a standard pricing in monopolistic competition:

$$\rho_t(\alpha) = \frac{\sigma}{\sigma - 1} \frac{w_t}{Z_t z(\alpha)}, \tag{16}$$

The real price is markup over real marginal costs.

Finally using the above optimal pricing, the profits can be rewritten as

$$d_t(\alpha) = \frac{1}{\sigma} S_t^{\psi(\sigma-1)-1} \left(\frac{\rho_t(\alpha)}{q_t(\alpha)}\right)^{1-\sigma} C_t - \frac{w_t f_t}{Z_t^{\theta}}, \tag{17}$$

Since $\sigma > 1$, lower real quality-adjusted price induces a rise in profits. The term $S_t^{\psi(\sigma-1)-1}$ captures additional impact on profits arising from fluctuations in extensive margins. Its magnitude depends on how much consumers appreciate varieties, the value of ψ.

2.2.3. The cutoff firm and the number of survivors-producers

Provided with a specific capability level α, the firm produces if $d_t(\alpha) > 0$, otherwise it exits. The survival depends on how cheaply the price firms can charge, hence their marginal costs. Inefficient firms which have drawn a lower productivity than a cutoff level ($\alpha \leq \alpha_{s,t}$) exit immediately without producing. Thus, the endogenous Schumpeterian destruction takes place following "strict capability ranking". This is very similar to Caballero and Hammour (1994, 1996, 2005) where the destruction happens through productivity ranking rather capability.

Operational profits become zero for the firm with the cutoff productivity $\alpha_{s,t}$ providing the following zero profit cutoff (ZCP) condition:

$$d_t(\alpha_{s,t}) = \frac{1}{\sigma} S_t^{\psi(\sigma-1)-1} \left(\frac{\rho_t(\alpha_{s,t})}{q_t(\alpha_{s,t})}\right)^{1-\sigma} C_t - \frac{w_t f_t}{Z_t^{\theta}} = 0. \tag{18}$$

Following Melitz (2003) and Ghironi and Melitz (2005), the average firm specific capability of survivors-producers $\widetilde{\alpha}_{s,t}$ with which the heterogeneous capability is perfectly summarized is defined as follows,[5]

[5]The average capability level is defined as a harmonic mean weighted by quality-adjusted output. From the goods market clearing condition, we have

$$\frac{q_t(\alpha_{s,t})\, y_t(\alpha_{s,t})}{q_t(\widetilde{\alpha}_{s,t}) y_t(\widetilde{\alpha}_{s,t})} = \left(\frac{\alpha_{s,t}}{\widetilde{\alpha}_{s,t}}\right)^{\sigma}.$$

Thus $\widetilde{\alpha}_{s,t}$ can be defined as

$$\widetilde{\alpha}_{s,t} \equiv \left[\frac{1}{1-G(\alpha_{s,t})}\int_{\alpha_{s,t}}^{\infty}\alpha^{\sigma-1}dG(\alpha)\right]^{\frac{1}{\sigma-1}} = \alpha_{s,t}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \tag{19}$$

The second identity comes from the use of the Pareto distribution defined previously.

And average real profits among surviving firms are defined as follows

$$\widetilde{d}_{s,t} = \frac{1}{\sigma}S_t^{\psi(\sigma-1)-1}\left(\frac{\widetilde{\rho}_{s,t}}{\widetilde{q}_{s,t}}\right)^{1-\sigma}C_t - \frac{w_t f_t}{Z_t^{\theta}},$$

where $\widetilde{\rho}_{s,t} \equiv \rho_t\left(\widetilde{\alpha}_{s,t}\right)$ and $\widetilde{q}_{s,t} \equiv q_{s,t}(\widetilde{\alpha}_{s,t})$. By optimal pricing, we have

$$\widetilde{\rho}_{s,t} = \frac{\sigma}{\sigma-1}\frac{w_t}{Z_t z\left(\widetilde{\alpha}_{s,t}\right)}. \tag{20}$$

where $\widetilde{z}_{s,t} \equiv z\left(\widetilde{\alpha}_{s,t}\right)$. At the same time, by definition of the price index (11), the average quality-adjusted price is expressed as a function of the number of varieties as $\widetilde{\rho}_{s,t}/\widetilde{q}_{s,t} = S_t^{\psi}$. Using this latter expression, the average profits are rewritten as

$$\widetilde{d}_{s,t} = \frac{1}{\sigma}\frac{C_t}{S_t} - \frac{w_t f_t}{Z_t^{\theta}}, \tag{21}$$

Finally, using (18), (21) and (19) the ZCP is rewritten as

$$\frac{1}{\sigma}\frac{C_t}{S_t} = \frac{k}{k-(\sigma-1)}\frac{w_t f_t}{Z_t^{\theta}}. \tag{22}$$

Also using the average firm productivity and with the Pareto density function, the Schumpeterian surviving rate is given by

$$\frac{S_t}{N_t} = \alpha_{\min}^{k}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{k}{\sigma-1}}\widetilde{\alpha}_{s,t}^{-k}. \tag{23}$$

In the end, average operational profits among potential producers are given by

$$\widetilde{d}_t = \frac{S_t}{N_t}\widetilde{d}_{s,t}. \tag{24}$$

$$\widetilde{\alpha}_{s,t}^{-1} \equiv \frac{1}{1-G(\alpha_{s,t})}\int_{\alpha_{s,t}}^{\infty}\alpha^{-1}\frac{q_t\left(\alpha_{s,t}\right)y_t\left(\alpha_{s,t}\right)}{q_t(\widetilde{\alpha}_{s,t})y_t(\widetilde{\alpha}_{s,t})}dG(\alpha).$$

2.3. Labor market clearing

The labor market should be clear in general equilibrium. L_t units of endogenously supplied labor are employed in the production of intensive margins as well as the creation of extensive margins. This implies

$$L_t = S_t \widetilde{l}_{s,t} + H_t l_{E,t}. \tag{25}$$

where $\widetilde{l}_{s,t} \equiv l_t \left(\widetilde{\alpha}_{s,t}\right)$The above condition can be further developed as follows[6]

$$L_t = S_t \left[(\sigma - 1) \frac{\widetilde{d}_{s,t}}{w_t} + \sigma \frac{f_t}{Z_t^{\theta}} \right] + H_t \frac{v_t}{w_t}. \tag{26}$$

This condition is equivalent to the aggregated identity obtained by summing up budget constraints among households: $Y_t \equiv C_t + v_t H_t = L_t w_t + S_t \widetilde{d}_{s,t}$, whereby Y_t stands for real GDP measured in welfare-basis from expenditures and income.

Finally the model consists of 13 equations and 13 endogenously determined variables among which the number of potential producers, N_t, behaves like a state variable. Table 1 summarizes the benchmark model.

3. Calibration

3.1. Choice of parameters' values, the non-stochastic steady state and productivity process

The models are calibrated by the following value parameters in Table 2. The calibration is performed on annual basis. The value of discount factor (β) and the Frisch elasticity of labor supply (φ) are set to 0.96 and 2, respectively. These values are well in the range used in the real business cycle literature.

The elasticity of substitution among varieties (σ) is set to 3.8 following Ghironi and Melitz (2005), who choose it based on empirical findings of Bernard et al. (2003) about U.S. manufacturing plants and macro trade data. Bernard et al. (2003) also document that

[6] Note that

$$\widetilde{d}_{s,t} = \frac{\widetilde{\rho}_{s,t}}{\sigma} \widetilde{y}_{s.t} - \frac{w_t f_t}{A_t},$$

where $\widetilde{y}_{s,t}$ is average intensive margins.

Table 1: Summary of the benchmark model

Average pricing	$\widetilde{\rho}_{s,t} = \frac{\sigma}{\sigma-1} \frac{w_t}{Z_t \widetilde{z}_{s,t}}$
Variety effect	$\widetilde{\rho}_{s,t} / \widetilde{q}_{s,t} = S_t^{\psi}$
Average survivors' profits	$\widetilde{d}_{s,t} = \frac{1}{\sigma} \frac{C_t}{S_t} - \frac{w_t f_t}{Z_t^{\theta}}$
Average profits	$\widetilde{d}_t = \frac{S_t}{N_t} \widetilde{d}_{s,t}$
Free entry condition	$v_t = \frac{w_t f_{E,t}}{Z_t^{\theta}}$
Motion of firms	$N_{t+1} = (1-\delta)(N_t + H_t)$
Euler equation	$v_t = \beta (1-\delta) E_t \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(v_{t+1} + \widetilde{d}_{t+1} \right)$
Optimal labor supply	$\chi (L_t)^{\frac{1}{\psi}} = w_t C_t^{-1}$
ZCP	$\frac{1}{\sigma} \frac{C_t}{S_t} = \frac{k}{k-(\sigma-1)} \frac{w_t f_t}{Z_t^{\theta}}$
Schumpeterian surviving rate	$\frac{S_t}{N_t} = \alpha_{\min}^{k} \left[\frac{k}{k-(\sigma-1)} \right]^{\frac{k}{\sigma-1}} \widetilde{\alpha}_{s,t}^{-k}$
Labor market clearing	$L_t = S_t \left[(\sigma-1) \frac{\widetilde{d}_{s,t}}{w_t} + \sigma \frac{f_t}{Z_t^{\theta}} \right] + H_t \frac{v_t}{w_t}$
Average quality	$\widetilde{q}_{s,t} = \widetilde{\alpha}_{s,t}^{\phi}$
Average productivity	$\widetilde{z}_{s,t} = \widetilde{\alpha}_{s,t}^{1-\phi}$

Table 2: Parametrization for the benchmark economy

β	discount factor	0.96
φ	Frisch elasticity of labor supply	2
σ	elasticity of substitution among varieties	3.8
ψ	love for variety	$1/(\sigma-1)$
δ	death shock	0.01
ϕ	quality parameter	-1.10

standard deviation of log U.S. plant sales is 1.67. The corresponding standard deviation in the theoretical model is given by $1/(k-\sigma+1)$, according to which the value of k is provided. I set ψ, the love for variety, at $1/(\sigma-1)$ according to the Dixit-Stiglitz preference. The destruction rate (δ) is set to 0.01 implying that one percent of potential producers exit exogenously per year on average. ϕ is set to -1.10 with which quality-new variety CPI bias amounts to 0.8 percent at the steady state. This is the average annual bias documented in Broda and Weinstein (2010) using a dataset that covers around 40 percent of all expenditures on goods in the CPI for US households. For the purposes of comparison, I also consider an economy without fluctuations in quality by setting $\phi = 0$.

Variables in the non-stochastic steady state are expressed without a time index. I assume that $A = f_E = 1$. The minimum level productivity $z_{\min}$ is set to unity without loss of generality. The average annual destruction rate of U.S. manufacturing establishment is 9.3 percent, from the data. In the benchmark model, I set f (the steady state value of subsides) to 0.073 so that the Schumpeterian endogenous destruction rate, $1 - S/N$, matches to 8.3 percent provided the one percent exogenous destruction. For the extended version of the model, f is set to 0.079 according to the above logic. The parameter value χ is determined such that the steady state labor supply becomes the unity. See the appendix for details about the steady state.

The productivity process is estimated using a Solow residual such as $\ln Z_t = \ln Y_t - 0.64 \ln L_t$ where Y_t and L_t represent time series of U.S. real GDP and hours worked for the period of 1977 to 2009. The $\ln Z_t$ is assumed to follow the following AR(1) process: $\ln Z_t = a + \rho \ln Z_{t-1} + \epsilon_t$. The estimation by OLS provides the value of AR(1) coefficient ρ as 0.98 and the standard deviation of the shock ϵ_t as 0.019. Because I do not have the data about sunk entry and fixed operational costs, the spillover coefficients of productivity on these costs are arbitrarily chosen as $\vartheta = 0.25$ and $\theta = 0.50$.

3.2. IRFs following a positive productivity shock

3.2.1. Quality upgrading

Figure 1 provides the impulse response functions following a one percent rise in productivity for the benchmark economy (the solid lines) and the homogenous-quality economy (the dashed lines). In these figures, vertical axes measure percent deviations from the steady state values and horizontal axes represent years. Figure 1 highlights four important variables which characterize the benchmark economy with heterogenous quality. Following a positive shock, the number of new entrants rises and less capable firms enter the market. As a result, the number of producers S_t rises and the average capability cutoff $\widetilde{\alpha}_{s,t}$ decreases on impact. These less capable firms use costly firm specific technology (a further decrease in $\widetilde{z}_{s,t}$ in the benchmark economy) in order to produce high quality goods (a rise in $\widetilde{q}_{s,t}$). While the average quality-adjusted price rises in exactly the same manner in both economies, the average real price $\widetilde{\rho}_{s,t}$ rises further in the benchmark economy due to quality upgrading.

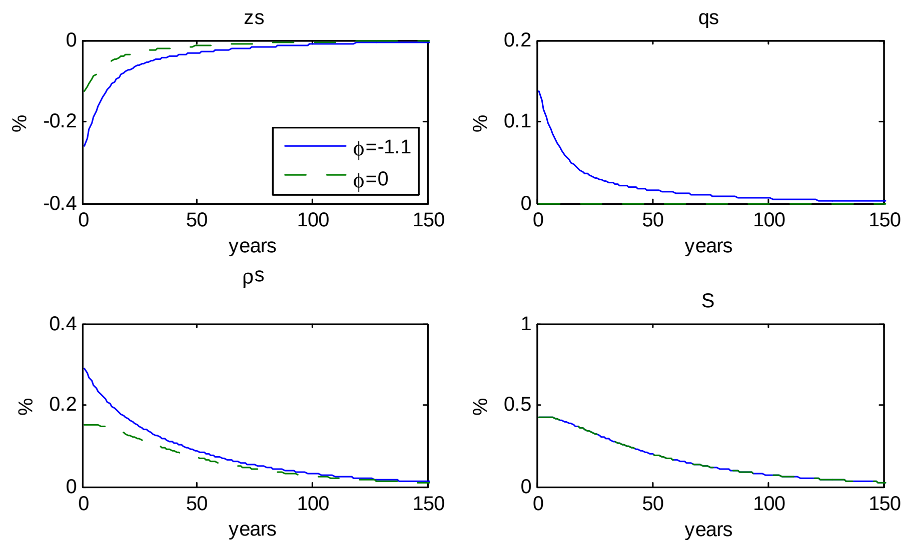

Figure 1: IRFs following a positive productivity shock ($\phi = -1.1$ and $\phi = 0$)

3.2.2. Variety and quality bias

Capturing fluctuations in the number of varieties and quality in price indices is important since only such welfare-based price indices can provide the true cost of living. As argued in Bils (2009), Bils and Klenow (2001) and Broda and Weinstein (2004, 2007), however, this is not the case in practice. Let assume that, for simplicity, such empirical-based CPIs completely fail to capture fluctuations in variety and quality. Specifically, by letting hat stand for log deviations, we have the following relation about bias:

$$\text{Total bias} \equiv \underbrace{\widehat{P}_{e,t}}_{\text{Empirical-based CPI}} - \widehat{P}_t = \underbrace{\psi \widehat{S}_t}_{\text{Variety bias}} + \underbrace{\widehat{\widetilde{q}}_{s,t}}_{\text{Quality bias}},$$

where, by definition, $P_{e,t}$ is the average individual price $\widetilde{p}_{s,t}$ in the model. There is deflation in the welfare-based CPI, P_t, with a rise in the number of varieties and average quality. The empirical-based CPI, $P_{e,t}$, tends to have a positive bias by imperfectly capturing a rise in S_t and $\widetilde{q}_{s,t}$. Figure 2 shows the decomposition of these two biases following a positive productivity shock in the benchmark economy. By imperfectly measuring a rise in the number of varieties and quality, inflationary variety and quality bias appear in the empirical-based CPI (the dotted and circled lines). There is a positive bias following a positive shock.

What is the consequence of such a bias? Any real variable X_t deflated with the welfare-based CPI, P_t, is transformed to those $X_{R,t}$ deflated with the empirical-based CPI, $P_{e,t}$, by $X_{R,t} \equiv P_t X_t / P_{e,t}$. Empirical-based or statistically relevant consumption, $C_{R,t}$, is defined in the above manner.[7] By omitting a positive welfare impact arising from quality upgrading and a rise in the number of varieties, a rise in consumption following a positive shock can be understated in the benchmark economy (the solid line in the lower panel $C_{R,t}$ in Figure 2). Put differently, due to the procyclicality of bias, consumption can be more volatile than that we observe in official statistical reports as argued in Broda and Weinstein (2010).

[7] Empirical-based GDP, $Y_{R,t}$ and investment, $I_{R,t} \equiv v_{R,t} H_t$ in the theoretical model are also defined in the same manner.

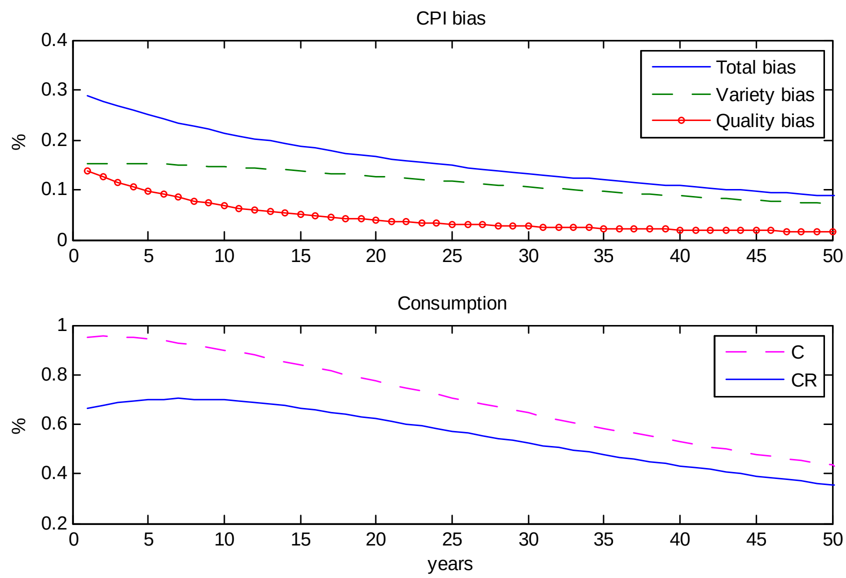

Figure 2: CPI bias and biased consumption fluctuations following a positive productivity shock

3.3. Second moments of the theoretical models

Table 3: Second moments

		Y_R	C_R	I_R	L	H	D	$Bias$
St.div (%)	U.S. Data	1.33	0.73	5.25	0.38	4.62	5.22	0.40*
	Benchmark	1.00	0.77	5.19	0.19	4.62	5.10	0.34
	Homogenous quality	1.17	0.94	5.35	0.19	4.62	5.10	0.18
Relative to Y_R	U.S. Data	1	0.55	3.94	0.28	3.47	3.92	0.30*
	Benchmark	1	0.77	5.18	0.19	4.62	5.10	0.34
	Homogenous quality	1	0.80	4.59	0.17	3.96	4.37	0.15
Corr(Y_R. X)	U.S. Data	1	0.78	0.95	0.67	0.43	-0.15	+*
	Benchmark	1	1.00	1.00	0.99	1.00	-1.00	1.00
	Homogenous quality	1	1.00	1.00	0.99	1.00	-1.00	1.00

Table 3 provides second moments of principal variables for the U.S. data, benchmark and homogenous-quality economies.[8] See the appendix for more details about the U.S. data. The data values with asterisk are taken from Broda and Weinstein (2010).

Removing (unobservable) fluctuations in quality as well as variety, standard deviations of the empirical-based GDP, consumption and investments become lower in the benchmark economy than those in the homogenous-quality model. The standard deviation of the total bias (defined previously) amounts to 0.34 for the benchmark and 0.15 for the homogenous-quality model relative to GDP. The standard deviation of the bias in the benchmark model, 0.34, is roughly in line with the estimate of Broda and Weinstein (2010) who report 0.4 % bias from 2000 I to 2003 IV over four quarters. They also document a strong procyclicality of bias. The model is in accord with this finding also; the correlation with output and total bias amounts to 1.00 in the benchmark model.

As argued in Hamano (2012), due to the endogenous-Schumpeterian destruction, the model can replicate high volatility in destruction margins D (5.10 for both models) as observed in the US data (5.22). On the other hand, firm entry is procyclical while exit is slightly countercyclical. Contemporaneous correlation of H and D with GDP are 0.43 and -0.15, respectively in the US data. Lee and Mukoyama (2008) report the same pattern about U.S. manufacturing firms. Also Broda and Weinstein (2010) document that product creation is highly procyclical while product destruction is countercyclical but with a less important magnitude. Both benchmark and homogenous-quality models can replicate such a procyclical pattern for entry and a countercyclical pattern for destruction, however, the correlations are much higher than those in the data. This discrepancy would be mainly due to the non-existence of adjusting costs in entry and exit in the theoretical models.[9]

[8] All series are detrended by HP filter. The smoothing parameter is set to 6.25. Second moments of the theoretical models are computed by the frequency domain techniques proposed by Uhlig (1998). For the U.S. series, see the appendix.

[9] This is also related to the transmission of aggregated productivity shock on entry and fixed operational costs. Indeed, the standard deviation and correlation for destruction margins are very sensitive to cyclical properties of fixed operational cost $w_t f_t / A_t^{\theta}$ and entry cost $w_t f_{E,t} / A_t^{\vartheta}$. See Hamano (2012) for a detailed discussion.

4. Conclusion

This paper analyzed the role played by product variety and quality in a real business cycle model. Variety representing firms are heterogeneous in terms of their specific quality as well as productivity level. Firms which have costly technology enter during a period of high aggregated demand and produce high quality goods. Thus, quality level and the number of available varieties are procyclical as in the data. The model can replicate the observed inflationary bias in the conventional CPI due to a rise in the number of new product varieties and quality as well as second moments of major economic variables.

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Appendix A. Data

The U.S. data about establishment entry and exit and job creation and destruction in the manufacturing sectors are taken from the Business Dynamics Statistics (BDS) of the U.S. Census Bureau. The series of U.S. real GDP, consumption (private plus government expenditures), investment (fixed capital formation), labour (hours worked) are taken from the OECD data base.

Appendix B. Steady state

I start by arguing the steady state of the benchmark model. The Euler equation (8) gives

$$\frac{1}{\beta} = (1-\delta)\left(1 + \frac{\widetilde{d}}{v}\right). \tag{B.1}$$

Using (21), the ZCP (22) can be transformed as

$$\frac{\widetilde{d}_s}{w} = \frac{\sigma - 1}{k - (\sigma - 1)}. \tag{B.2}$$

We have $\widetilde{d} = S\widetilde{d}_s/N$ from (24) and $v = w$ from the free entry condition (13). Using these relations, (B.1) can be expressed as

$$\frac{1}{\beta} = (1-\delta)\left(1 + \frac{S}{N}\frac{\sigma-1}{k-(\sigma-1)}f\right). \tag{B.3}$$

The above equation provides the steady state Schumpeterian destruction rate, S/N, provided the value of f.

I set the value of χ so that the steady state labor supply equals unity. Note also from the law of motion (3), we have $H = \delta N/(1-\delta)$. Plugging these relations in the labor market clearing condition (26), we get

$$\frac{1}{N} = (\sigma-1)\frac{S}{N}\frac{\sigma-1}{k-(\sigma-1)}f + \sigma\frac{S}{N}f + \frac{\delta}{1-\delta} \tag{B.4}$$

Provided the value of S/N the above equation yields the unique solution for N. Knowing S, the steady state values of other variables are easily found.