FACULTY OF LAW, ECONOMICS AND FINANCE

CREA Discussion Paper 2013-01

Center for Research in Economic Analysis
University of Luxembourg

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Masashige Hamano, CREA, University of Luxembourg

January, 2012

For editorial correspondence, please contact: crea@uni.lu
University of Luxembourg
Faculty of Law, Economics and Finance
162A, avenue de la Faïencerie
L-1511 Luxembourg

UNIVERSITÉ DU LUXEMBOURG

The consumption-real exchange rate anomaly with extensive margins[☆]

Masashige Hamano[a,*]

[a] *University of Luxembourg-CREA*

This draft: January 2013

Abstract

This paper investigates a consumption-real exchange rate anomaly from the open macroeconomics literature known as the Backus-Smith puzzle . We both analytically and quantitatively examine how an expansion of trade along extensive margins can contribute to the puzzle's resolution. Our argument is based on 1) a wealth effect due to changes in the number of product varieties, 2) statistical inefficiency in measuring the number of product varieties, and 3) market incompleteness. Contrary to complete asset markets which, in general, feature overly strong risk sharing properties, changes in the number of product varieties under incomplete markets may produce a wealth effect under high trade elasticity. Since statistical agencies systematically fail to capture the welfare impact arising from that changes, data-consistent terms of trade and real exchange rates tend to appreciate due to this positive wealth effect. This provides a realistic correlation between data-consistent real exchange rates and consumption.

☆I would like to thank the editor and two anonymous referees for their valuable comments. I gratefully acknowledge Philippe Martin for his advice and encouragement. I would like to also thank Nicolas Coeurdacier, Hideaki Hirata, Jan Kranich, Eiji Ogawa, Eiji Okano and Robert Kollmann for providing comments and insightful discussions. I thank all seminar participants at the Universities Rennes1, Paris1, Hitotsubashi, the 13th T2M conference, the 14th SMYE, the 25th EEA meeting and the 13th macroeconomic conference in Tokyo. The paper is based on the second chapter of my Ph.D. dissertation. The present project is supported by the National Research Fund, Luxembourg, and cofounded under the Marie Curie Actions of the European Commission (FP7-COFUND) and by the grant F2R-CRE-PUL-10EGQH at the University of Luxembourg. All remaining errors are my own.

*Corresponding author: Faculty of Law, Economics and Finance, University of Luxembourg, 162a, avenue de la Faïencerie, L-1511 Luxembourg. +352 4666446655

Email address: `masashige.hamano@uni.lu` (Masashige Hamano)

Keywords: terms of trade, the Backus-Smith puzzle, firm entry
JEL classification: F12, F41, F43

Perfect international risk sharing under complete asset markets predicts that consumption in one country rises when its prices become relatively cheap. Such a pattern of consumption growth and real exchange rate fluctuations, however, is strongly rejected by data. Table 1 reports correlations between relative consumption and real exchange rates (defined as the price of a foreign consumption basket of goods in terms of the domestic basket) among industrialized countries. The correlations are close to zero or even negative indicating a pattern contradictory to the prediction of a model of complete asset markets. Indeed, households in one country consume *more* when the consumer price index in that country *increases* relative to other countries. This pattern of low risk sharing is known as the Backus-Smith (BS) puzzle (Backus and Smith, 1993; Kollmann, 1995). It is even more surprising given the rise in international capital flow and the progressive integration of world financial markets.[1]

This paper presents a possible resolution of the puzzle based on three elements: 1) a wealth effect due to changes in the number of product varieties; 2) statistical inefficiency in measuring the number of product varieties; and 3) market incompleteness.

A higher number of product varieties creates a wealth effect which further appreciates relative wages. The higher the elasticity of substitution between domestic and imported goods, the stronger the wealth effect due to an expansion of trade along extensive margins will be. Since consumers appreciate product variety, a greater number of product varieties in their consumption basket results in a *welfare-based depreciation* of real exchange rates.

As argued in Broda and Weinstein (2004, 2006), and as an important number of empirical research shows, statistical agencies systematically fail to capture such welfare-

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[1] The lack of international risk sharing can be stated in other forms. Because of strong risk sharing properties, the theoretical model typically features a higher cross country correlation in consumption than output, which is, in general, also not observed in data. See Obstfeld and Rogoff (2000) for other related puzzles in open macroeconomics.

Table 1: KBS correlation

Country	U.S.	ROW	Country	U.S.	ROW
Austria	-0.11	0.05	Italy	-0.28	-0.52
Belgium/Luxembourg	-0.16	0.50	Japan	0.05	0.25
Canada	-0.52	-0.31	Netherlands	-0.45	-0.20
Denmark	-0.14	-0.10	Portugal	-0.61	-0.77
Finland	-0.30	-0.49	Spain	-0.63	-0.64
France	-0.20	0.43	Sweden	-0.56	-0.40
Germany	-0.51	-0.27	U.K.	-0.51	-0.21
Greece	-0.45	-0.35	U.S.	N/A	-0.71
Ireland	-0.39	0.72	Median	-0.42	-0.27

Source: Corsetti et al. (2008a)

relevant fluctuations in the number of product varieties.[2] The BS puzzle is no exception. What we refer to as a puzzle is the anomaly in observed or data-consistent consumption and real exchange rates which only imperfectly measure changes in the number of product varieties. Because of this fundamental discrepancy, *the empirical-based (data-consistent)* real exchange rate, ignoring the real depreciation provided by a higher number product varieties, tends to *appreciate* only according to the above mentioned wealth effect due to an expansion of trade along extensive margins, thus mimicking a realistic BS correlation. A corollary is that the *welfare-based* BS correlation cannot be fully tested unless we know the exact variations in the number of product varieties.

In this paper, we also explore the role played by financial market incompleteness. Even under complete markets, the introduction of trade along extensive margins can replicate a realistic BS correlation. Such a realistic correlation, however, requires unrealistic dynamics in particular, for relative consumption. Due to perfect risk sharing, the terms

[2] Broda and Weinstein (2004, 2006) point out that large welfare gains stem from the increased number of imported varieties. They also note that the data-consistent import price indices have an inflation bias. For the US, this can be as much as 1.2% per year. In Broda and Weinstein (2007), they report an upward bias in the US's CPI: around 0.8% per year from 1994 to 2003.

of trade tend to remain *depreciated* under complete markets even with a positive wealth effect induced by a higher number of product varieties. We show that, complete markets can replicate a realistic BS correlation, only when foreign consumption *rises* relative to domestic consumption via a terms of trade depreciation. There is a detailed discussion about complete markets in Appendix A. A caveat is that only incomplete markets in which trade along extensive margins is allowed can reproduce a realistic BS correlation in a plausible way.

This paper is related to the recent literature which attempts to solve the puzzle using an open macroeconomic model. Under incomplete asset markets, Corsetti et al. (2008a) (henceforth CDL) argue that the puzzle is attenuated due to a wealth effect induced by either low trade elasticity or a combination of high trade elasticity and a highly persistent productivity shock. When trade elasticity is low, the income effect is stronger than the substitution effect. Wealthy domestic households due to an appreciation of the terms of trade consume vast majority of supplied domestic goods, following a positive productivity shock. When a shock is persistent and cross-border borrowing and lending is allowed using state non-contingent bonds, an expected rise in future wealth raises current demand for domestic goods in excess of supply. This creates a short-run terms of trade appreciation under high trade elasticity. The resolution of the puzzle relying on this latter mechanism is also explored in Opazo (2006) and Nam and Wang (2010).

As explained previously, the wealth effect, which is one of the key elements in replicating a realistic BS correlation, is here induced by changes in the number of product varieties under a high trade elasticity.[3] Ghironi and Melitz (2005) and Devereux and Hnatkovska (2012) calibrate a model with endogenous tradability and show, quantita-

[3]Other than the wealth effect induced by either mechanism, there are several competing or complementary arguments. Under incomplete markets with internationally held bonds, Benigno and Thoenissen (2008) quantitatively show that a standard international real business cycle model which includes a non-traded sector can successfully provide a realistic BS correlation through the well-known Harrod-Balassa-Samuelson effect. On the other hand, Kollmann (2009) and Devereux et al. (2009) provide another mechanism relying on the hand-to-mouth behavior of subset households.

tively, a realistic correlation. In this paper, the analysis will go in great detail, including analytical investigation, on the above three key elements.

The paper is organized as follows. The following section presents a static general equilibrium model in the spirit of Corsetti et al. (2007). The implication of trade along extensive margins for the BS puzzle is discussed analytically in Section 2. In Section 3, dynamics are introduced following Ghironi and Melitz (2005), with some extensions. And the BS puzzle is explored quantitatively using the standard calibration method in Section 4. The final section offers brief concluding remarks.

1. The model

We build a simple static general equilibrium model. There are two countries, Home and Foreign, each of which is populated by a unit mass of atomic households (Foreign variables are denoted with asterisks). Firms are monopolistically competitive and their number is determined endogenously in each country. Each firm represents one product variety. In this model, there is no international borrowing and lending. Therefore, trade is balanced.

1.1. Households

The Home representative household inelastically supplies one unit of labor. Utility is defined as

$$U = \frac{C^{1-\gamma}}{1-\gamma},$$

where C is consumption and γ (≥ 1) denotes relative risk aversion. Specifically, C is a bundle of domestic and imported goods as follows

$$C = \left[\alpha^{\frac{1}{\omega}} C_H^{1-\frac{1}{\omega}} + (1-\alpha)^{\frac{1}{\omega}} C_F^{1-\frac{1}{\omega}}\right]^{\frac{1}{1-\frac{1}{\omega}}},$$

where α ($\geq 1/2$) captures home bias in consumption. ω (> 0) denotes the elasticity of substitution between locally produced (C_H) and imported goods (C_F). They are defined over a continuum of goods Ω as

$$C_H = V_H \left(\int_{h\in\Omega} c(h)^{1-\frac{1}{\sigma}} \, dh \right)^{\frac{1}{1-\frac{1}{\sigma}}}, \qquad C_F = V_F^* \left(\int_{f\in\Omega} c(f)^{1-\frac{1}{\sigma}} \, df \right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

where $V_H \equiv N^{\psi-\frac{1}{\sigma-1}}$ and $V_F^* \equiv N^{*\psi-\frac{1}{\sigma-1}}$ in which N and N^* stand for the number of domestic and imported varieties. $c(h)$ and $c(f)$ are the consumption of individual domestic and imported goods indexed by h and f, respectively.

In the above expressions, σ (> 1) denotes the elasticity of substitution among varieties. We conventionally assume $\sigma \geq \omega$. ψ (≥ 0) determines the marginal utility stemming from one additional rise in the number of varieties. This specification follows Benassy (1996), who argues the distinction of firms' markup from the preference for variety. Specifically, preferences follow Dixit and Stiglitz (1977); that is, they are denoted $\psi = \frac{1}{\sigma-1}$. When $\psi = 0$, there is no utility gain in consuming a higher number of varieties.

Consumer price index P, which minimizes nominal spending, is

$$P = \left[\alpha P_H^{1-\omega} + (1-\alpha) P_F^{1-\omega} \right]^{\frac{1}{1-\omega}}.$$

In the above expression, P_H and P_F denote the price of C_H and C_F, respectively. They are determined by

$$P_H = \frac{1}{V_H} \left(\int_{h\in\Omega} p(h)^{1-\sigma} \, dh \right)^{\frac{1}{1-\sigma}}, \qquad P_F = \frac{1}{V_F} \left(\int_{f\in\Omega} p(f)^{1-\sigma} \, df \right)^{\frac{1}{1-\sigma}}, \tag{1}$$

where $p(h)$ and $p(f)$ represent the price of an individual product variety in Home. Observe that P, P_H and P_F are defined on a *welfare basis*: they decrease (increase) with a rise (decrease) in the number of varieties, given a preference for variety, $\psi > 0$.

Finally, optimal consumption demand functions are as follows

$$C_H = \alpha \left(\frac{P_H}{P} \right)^{-\omega} C, \qquad C_F = (1-\alpha) \left(\frac{P_F}{P} \right)^{-\omega} C,$$

$$c(h) = V_H^{\sigma-1} \left(\frac{p(h)}{P_H} \right)^{-\sigma} C_H, \qquad c(f) = V_F^{\sigma-1} \left(\frac{p(f)}{P_F} \right)^{-\sigma} C_F.$$

The welfare-based consumer price index P is set as a numéraire in Home, and define real prices as $\rho_H \equiv \frac{P_H}{P}$, $\rho_F \equiv \frac{P_F}{P}$, $\rho(h) \equiv \frac{p(h)}{P}$, and $\rho(f) \equiv \frac{p(f)}{P}$.

Similar expressions hold in Foreign.

1.2. Firms

Firms must pay sunk entry costs f_E upon entry. The latter is defined in terms of effective labor as

$$f_E = z_E l_{EM}(h), \tag{2}$$

where z_E denotes the level of labor productivity in firm setup and $l_{EM}(h)$ represents the amount of labor demanded by an entering firm. In the above expression, an increase (decrease) in f_E is interpreted as an exogenous increase in entry (de)regulation.

Once entered, the firm produces an amount of output $y(h)$ according to the following production function

$$y(h) = zl(h),$$

where z denotes the level of labor productivity in production and $l(h)$ represents the amount of labor demanded.

We now specify a firm's pricing behavior. Using the production function, operational real profits (dividends) are expressed by

$$d(h) = \left(\rho(h) - \frac{w}{z}\right) y(h), \tag{3}$$

where w denotes real wages in terms of consumption.

The market clearing condition implies $y(h) = c(h) + c^*(h)$. Combined with the optimal demand noted previously, $y(h)$ can be rewritten as

$$y(h) = N^{\psi(\sigma-1)-1} \rho(h)^{-\sigma} \rho_H^{\sigma-\omega} \left[\alpha C + (1-\alpha) Q^{\omega} C^*\right],$$

where Q denotes the real exchange rate; and $Q \equiv P^*/P$.

A firm maximizes operational profits (3) given the above expression of $y(h)$. This yields

$$\rho(h) = \frac{\sigma}{\sigma - 1}\frac{w}{z}. \tag{4}$$

The real domestic price of an individual product variety $\rho(h)$ is set to equal real marginal costs over markup. We denote the exported goods price by $\rho^*(h) = Q^{-1}\rho(h)$, which is denominated in Foreign consumption units.

Since firms are symmetric in equilibrium, hereafter we can denote without loss of generality the equilibrium price as $\rho_h \equiv \rho(h)$. The same type of notation holds for other variables, including those in Foreign.

Finally, using the optimal pricing (4) and the fact that $\rho_H = N^{-\psi}\rho_h$ from (1), real dividends can be rewritten as

$$d_h = \frac{1}{\sigma}\rho_h^{1-\omega}N^{\psi(\omega-1)-1}\left[\alpha C + (1-\alpha) Q^{\omega}C^*\right]. \tag{5}$$

The above expression highlights how dividends change with individual price fluctuations depending on whether local and imported goods are complements ($\omega < 1$) or substitutes ($\omega > 1$). The term $N^{\psi(\omega-1)-1}$ in the above expression captures an additional competing effect arising from the number of domestic firms. In particular, when $\omega = \sigma$ and $\psi = \frac{1}{\sigma-1}$, this term is equal to unity.

1.3. Equilibrium

In this subsection, we fully characterize the general equilibrium by considering free entry, the labor market clearing condition and the balanced trade condition.

Each firm's dividends must be equal to its entry costs in equilibrium, giving the following free entry condition:[4]

$$d_h = \frac{f_E w}{z_E}. \tag{6}$$

[4] In this static version of the model, there is no investment choice by households. This is a distinct feature from a full dynamic model that we discuss in the following section.

One unit of supplied labor is demanded for goods production and firm creation by N number of firms. Therefore, in equilibrium, we have the following labor market clearing condition: $1 = Nl_h + Nl_{EM,h}$. Note that $y_h = (\sigma - 1)\frac{d_h}{w}z$ by (3) and (4), and $l_{EM} = \frac{d_h}{w}$ by (2) and (6). Using these expressions, the above condition can be rewritten as[5]

$$1 = \sigma \frac{Nd_h}{w}.$$

Similar expressions hold in Foreign.

Finally, balanced trade implies that the exported value is equal to the imported value, as $N\rho_h c_h^* = QN^*\rho_f^* c_f$. Using the optimal demands found previously, the balanced trade condition can be rewritten as

$$Q^{2\omega-1}N^{\psi(\omega-1)}\rho_h^{1-\omega}C^* = N^{*\psi(\omega-1)}\rho_f^{*1-\omega}C. \tag{7}$$

In what follows, we solve the linearized version of the model and explore the role played by extensive margins in the BS puzzle.

2. The BS puzzle with extensive margins

2.1. Product varieties, relative wages and the terms of trade

We express percentage deviations of variables from their steady state level using sans-serif fonts. Relative wages and the relative number of varieties are defined as $\mathsf{w}^R \equiv \mathsf{w}-(\mathsf{Q}+\mathsf{w}^*)$ and $\mathsf{N}^R \equiv \mathsf{N}-\mathsf{N}^*$. By linearizing, the model is reduced to a system of two equations, the labor market clearing and free entry conditions, and two unknown variables, w^R and N^R. Without loss of generality, we assume that no regulation shocks take place, as $\mathsf{f}_E = \mathsf{f}_E^* = 0$. Therefore, by solving the system, w^R and N^R are expressed as a function of two relative exogenous shocks, $\mathsf{z}^R \equiv \mathsf{z}-\mathsf{z}^*$ and $\mathsf{z}_E^R \equiv \mathsf{z}_E - \mathsf{z}_E^*$:[6]

[5] It is easily shown that the labor market clearing condition is identical to the aggregate identity. This can be obtained from aggregating budget constraints across households, i.e. $C = w$.

[6] The linearized version of the labor market clearing condition is $\mathsf{w}^R = \mathsf{N}^R + \mathsf{d}^R$. That of the free entry condition is $\mathsf{d}^R = \mathsf{w}^R - \mathsf{z}_E^R$. Using the balanced trade condition, we can write dividends as

$$\mathsf{d}^R = -2\alpha(\omega-1)\left(\mathsf{w}^R - \mathsf{z}^R\right) + \left[2\alpha\psi(\omega-1) - 1\right]\mathsf{N}^R.$$

$$\mathsf{N}^R = \mathsf{z}_E^R,$$

$$\mathsf{w}^R = \frac{2\alpha(\omega-1)}{1+2\alpha(\omega-1)}\left(\mathsf{z}^R + \psi \mathsf{z}_E^R\right). \tag{8}$$

As we can see, the number of varieties N^R only changes one-for-one with the labor productivity shock on entry costs, z_E^R. Relative wages, w^R, instead change following both shocks, z^R and z_E^R. Since a firm setup shock z_E^R induces fluctuations in the number of product varieties, z_E^R can impact w^R. Following positive marginal costs or entry cost shocks ($\mathsf{z}^R > 0$ or $\mathsf{z}_E^R > 0$), the relative wages appreciate for Home when $\omega < 1 - \frac{1}{2\alpha}$ or $\omega > 1$. Observe that the appreciation is higher, the higher the value of ψ under a positive firm setup shock z_E^R.

Specificity of the model can be highlighted by looking for fluctuations in the terms of trade. These are defined as the relative price of Foreign goods in terms of Home goods: $TOT \equiv p_f^*/p_h$. Provided the above expression of relative wages (8), we have

$$\mathsf{TOT} = \frac{1}{1+2\alpha(\omega-1)}\mathsf{z}^R - \psi\frac{2\alpha(\omega-1)}{1+2\alpha(\omega-1)}\mathsf{z}_E^R. \tag{9}$$

The first term is the same one found in CDL under an endowment economy. CDL point out a possibility of terms of trade appreciation ($\mathsf{TOT} < 0$) following a positive endowment shock (here, the equivalence is a positive productivity shock on marginal costs, $\mathsf{z}^R > 0$). The terms of trade appreciate when trade elasticity is *low:* $0 < \omega < 1 - \frac{1}{2\alpha}$. Why do they appreciate despite a more efficient technology? Note that this is exactly the same range of ω in which a positive wealth effect is generated when relative wages appreciate. Such an appreciation in wages is relatively stronger than efficiency gains due to a positive shock, thus leading to a rise in the marginal costs of production. Put differently, as explained in CDL, under such a low elasticity, the income effect is stronger than the substitution effect. Such a strong wealth effects can drive aggregate demand for domestic goods above

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Plugging this expression into the above two equations, the model is reduced to a system of two equations and two unknowns.

supply, appreciating the terms of trade in spite of a positive productivity shock. With $\omega > 1 - \frac{1}{2\alpha}$, however, the terms of trade depreciate since the wealth effect is weaker under such a high range of trade elasticity.

Unlike in CDL, the second term is at work in (9). Under a shock of positive entry costs ($\mathsf{z}_E^R > 0$), this term can prevent the terms of trade from depreciating. Specifically, when $\omega > 1$ under $\psi > 0$, the second term adds an appreciation and this is again exactly the same range where relative wages appreciate because of a higher number of product varieties (as we see in (8)). When the elasticity of substitution is high, new product varieties are further demanded and wages further appreciate due to a strong substitution effect. Such a wealth effect resulting from an expansion of trade along extensive margins can work to counteract the first term depreciation due to a more efficient production technology ($\mathsf{z}^R > 0$). This mechanism is, in essence, the point raised by Krugman (1989). He argues that countries' terms of trade can appreciate by providing a higher number of product varieties during their economic growth. This is not the case if these countries continue to provide a larger quantity of the same set of goods, i.e. an expansion of trade along intensive margins.[7]

The left-hand side panel in Figure 1 provides a numerical example of the terms of trade variations for different values of ω. The figure shows two cases, one with love for variety (solid line) and one without (dotted line). The love for variety is set to the standard Dixit-Stiglitz preference obtained with $\sigma = 6$, and home bias in consumption α is set to 0.72 following CDL. In calibrating (9), two shocks; z^R and z_E^R, are assumed to be perfectly correlated ($\mathsf{z}^R = \mathsf{z}_E^R$), for simplicity. As has been discussed, the terms of trade appreciate when the trade elasticity is very low. When the value of ω exceeds the threshold where the relative strength between the income and substitution effect changes, the terms of trade depreciate for the case where there is love for variety and the case where isn't. However,

[7] For instance, Corsetti et al. (2006) and Kollmann (2008), report with VAR estimates the terms of trade appreciation in several industrialized countries. Using a micro-founded estimation, Hummels and Klenow (2005) and Galstyan and Lane (2008) document the terms of trade appreciation due to the higher number of exported varieties.

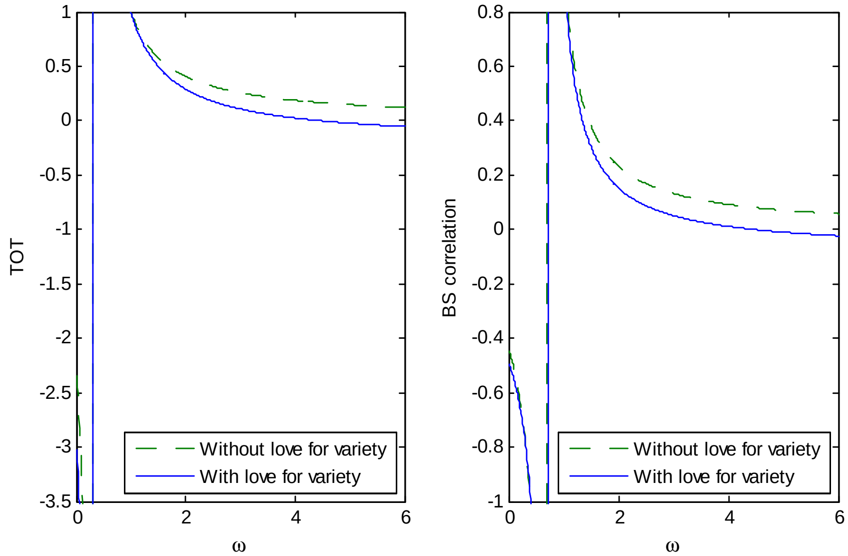

Figure 1: Terms of trade fluctuations and the BS correlation under balanced trade. The figure is calibrated with various values of trade elasticity ω under Dixit-Stiglitz preferences with $\sigma = 6$, $\alpha = 0.72$ and perfect correlation between shocks on marginal and entry costs.

the terms of trade depreciate by a smaller magnitude under preferences for variety, and finally they start to appreciate as ω rises in the figure.

2.2. The welfare vs. empirical-based measure

In addition to the wealth effect induced by trade along extensive margins, another key element in solving the BS puzzle is the fact that we never perfectly observe *welfare-based* fluctuations in real exchange rates and consumption. As Broda and Weinstein (2006, 2007) have pointed out, contrary to the welfare-based measure, *the empirical-based (data-consistent)* price indices capture changes in extensive margins only in a very limited manner. To clarify this point, let us assume for simplicity that price indices do not reflect any fluctuations in the number of product varieties. Denoting empirical fluctuations of price indices using tilda, the welfare-based real exchange rate fluctuations are broken down into two parts:

$$\mathsf{Q} = \widetilde{\mathsf{Q}} + \psi \left(2\alpha - 1\right) \mathsf{N}^{R}, \tag{10}$$

where

$$\widetilde{\mathsf{Q}} \equiv \widetilde{P}^* - \widetilde{P} = (2\alpha - 1)\,\mathsf{TOT}.$$

Expression (10) demonstrates that the welfare-based real exchange rate Q depreciates for a relatively higher number of product varieties ($\mathsf{N}^R > 0$) under home bias ($\alpha > 1/2$) and love for variety ($\psi > 0$). The empirical-based real exchange rate $\widetilde{\mathsf{Q}}$ fluctuates only with the conventional terms of trade, TOT.

Consumption is also poorly measured using these price indices. Specifically, statistical agencies divide total nominal consumption expenditure, PC, by $\widetilde{P}$ to deduce empirical-based consumption $\widetilde{C}$. As a result, we have a relation between C and $\widetilde{C}$ as follows

$$\widetilde{\mathsf{C}} \equiv \mathsf{P} + \mathsf{C} - \widetilde{\mathsf{P}} = \mathsf{C} - \psi\left[\alpha \mathsf{N} + (1-\alpha)\,\mathsf{N}^*\right]. \tag{11}$$

The empirical-based fluctuations in consumption are defined as the true welfare-based fluctuation minus fluctuations along the extensive margins weighed by home bias and love for variety. A similar decomposition holds for $\widetilde{\mathsf{C}}^*$.

The above distinction between the welfare and empirical basis is crucial in considering the BS puzzle, since what we refer to as a puzzle is the empirically observed relationship between real exchange rates and consumption.

2.3. The BS puzzle with extensive margins

Now we are equipped with all key elements to analyze the BS puzzle with extensive margins. Using the balanced trade condition (7), we can find a relation between the real exchange rate and relative consumption on a *welfare basis* as follows

$$\mathsf{Q} = \frac{2\alpha - 1}{2\alpha\omega - 1}\left(\mathsf{C} - \mathsf{C}^*\right). \tag{12}$$

The above expression is exactly identical to the one found in CDL, excepting only that the real exchange rate and consumption include fluctuations in the number of product varieties.[8]

[8] It is worth noting that when $\alpha = 1/2$ and $\omega = 1$, the complete market allocation is achieved without any financial assets. This is the case discussed in Cole and Obstfeld (1991). The terms of

It is worth emphasizing again that (12) holds on a welfare basis, however the BS puzzle is a puzzle about the data-consistent real exchange rate and consumption. Noting that $\mathsf{C} = \mathsf{w}$, $\mathsf{C}^* = \mathsf{w}^*$ and using the definition of empirically measured consumption (11), its Foreign counterpart and (10), we have $\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^* = \mathsf{w}^R + \widetilde{\mathsf{Q}}$ and $\widetilde{\mathsf{Q}} = (2\alpha - 1)\,\mathsf{TOT}$. Finally, plugging the solution of w^R and TOT found previously in these expressions, we have

$$\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^* = \frac{2\alpha\omega - 1}{1 + 2\alpha(\omega - 1)}\mathsf{z}^R + \psi\frac{4\alpha(1-\alpha)(\omega - 1)}{1 + 2\alpha(\omega - 1)}\mathsf{z}_E^R,$$

$$\widetilde{\mathsf{Q}} = \frac{2\alpha - 1}{1 + 2\alpha(\omega - 1)}\mathsf{z}^R - \psi\frac{2\alpha(2\alpha - 1)(\omega - 1)}{1 + 2\alpha(\omega - 1)}\mathsf{z}_E^R.$$

The first terms in both expressions, induced by a shock on marginal costs of production z^R, are the terms found in CDL. Assuming that $\psi = 0$, a negative BS correlation between $\widetilde{\mathsf{Q}}$ and $\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^*$ appears for a low range of trade elasticity as $\omega < \frac{1}{2\alpha}$. Remembering that $\widetilde{\mathsf{Q}} = (2\alpha - 1)\,\mathsf{TOT}$, the negative correlation is due to the terms of trade appreciation induced by a wealth effect from low trade elasticity. When trade elasticity is relatively high, as when $\omega > \frac{1}{2\alpha}$, however, the BS correlation remains positive.

Nevertheless, a negative BS correlation is still possible for a high enough range of trade elasticity ω under a preference for variety ($\psi > 0$). This is due to the second terms in both expressions being driven by a shock on firm setup costs z_E^R. As discussed previously, the terms of trade and the empirical-based real exchange rate $\widetilde{\mathsf{Q}}$ appreciate due to a wealth effect generated by a higher number of product varieties. The higher the value of ω and ψ are, then the stronger the wealth effect from extensive margins. Thus it approximates empirical-based BS correlation.[9]

trade fluctuations perfectly insure consumption risk such that the level of consumption in both countries remains unchanged.

[9] In addition to low trade elasticity, CDL mention a possibility of short-run terms of trade appreciation due to an anticipated future wealth effect in a bond economy. A negative BS correlation takes place when there is a combination of a highly persistent productivity shock and high trade elasticity (sufficiently larger than unity). Such a mechanism is also present in this paper's model. However, when the persistence of a shock rises, new product varieties which appear only gradually over time reduce the current wealth effect.

The right-hand side panel in Figure 1 gives a numerical example of the BS correlation under the same parameters and a perfect correlation between two types of shocks, as in the left-hand side panel. The BS correlation is negative or close to zero for a high enough range of trade elasticity under a preference for variety (solid line in Figure 1).[10]

3. Quantitative investigation

In Sections 3 and 4, we will investigate whether the intuition described in the previous sections quantitatively holds in a fully-specified dynamic model. We present a two-country DSGE model in which the number of firms is endogenously determined (following Ghironi and Melitz, 2005).

All variables defined previously take time index t in a dynamic model. Furthermore, investment takes place in the form of new firm creation as a result of households' consumption smoothing. As in CDL, we assume incomplete financial markets. Specifically, there exists international borrowing and lending by state non-contingent bonds. Other than these modifications, two more realistic extensions are added compared to the static model: endogenous labor supply, and entry costs which are paid in both capital goods as well as labor. Only these modified points are discussed below.

Since the latter mechanism counteracts the anticipated wealth effect discussed in CDL, here the higher persistence of the productivity shock does not improve the BS correlation. See a detailed discussion in Appendix B.

[10] Under the perfect correlation between two types of shocks, we have

$$\widetilde{\mathsf{Q}} = \frac{(2\alpha - 1)\left[1 - 2\psi\alpha\left(\omega - 1\right)\right]}{2\alpha\left(\omega - 1\right)\left[1 + 2\psi\left(1 - \alpha\right)\right] + 2\alpha - 1}\left(\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^{*}\right).$$

In the above expression the BS correlation becomes negative, not only for a low range of elasticity of substitution ($0 < \omega < 1 - \frac{2\alpha-1}{2\alpha[1+2\psi(1-\alpha)]}$), but also for a high range of elasticity ($1 + \frac{1}{2\psi\alpha} < \omega$). Note also that without a love for variety ($\psi = 0$), the expression collapses to the one discussed in CDL with an endowment economy.

3.1. Households

Utility of the Home representative household at time t is given by

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}},$$

where χ (> 0) captures the degree of non-satisfaction in supplying labor service, L_t, and φ is the Frisch elasticity of labor supply.[11] With this specification, the marginal disutility in providing one additional labor is increasing.

The budget constraint at time t is given by

$$\begin{aligned} B_{t+1} + Q_t B_{*,t+1} + \frac{\vartheta}{2} B_{t+1}^2 + \frac{\vartheta}{2} Q_t B_{*,t+1}^2 + s_{h,t+1} \left(N_t + N_{E,t}\right) x_{h,t} + C_t \\ = \left(1 + r_t\right) B_t + Q_t \left(1 + r_t^*\right) B_{*,t} + s_{h,t} N_t \left(d_{h,t} + x_{h,t}\right) + T_t^f + w_t L_t, \quad (13) \end{aligned}$$

where B_{t+1} and $B_{*,t+1}$ denote holdings of Home and Foreign bonds at t into the next period, respectively. We assume that Home and Foreign bonds provide a risk-free real return, r_t and r_t^*. Given these bond-holding terms in the budget constraint, indeterminacy of the equilibrium portfolio position and non-stationarity arise when using a linear approximation. We overcome such a problem by introducing quadratic adjusting costs of bond holdings, ϑ. This addition guarantees a locally unique symmetric steady state with zero bond holdings and stationarity. T_t^f is a free rebate of adjusting costs; it is exogenous for households.[12]

[11] With $\varphi = \infty$, the marginal disutility in supplying one additional unit of labor becomes constant, χ. When $\varphi = 0$, marginal disutility becomes infinite and the labor supply becomes inelastic.

[12] This specification of the quadratic adjustment costs of bond holdings is the same one argued in Ghironi and Melitz (2005). Another possible remedy is to assume an endogenous discount factor, as in CDL. As argued in Schmitt-Grohe and Uribe (2003), these two methods are quantitatively equivalent. Recent literature has instead focused on a solution which relies on higher order approximation of Euler equations about asset holdings. See Devereux and Sutherland (2008) and Tille and van Wincoop (2008) for such a method. We do not incorporate these in order to keep the model and its solution procedure as simple as possible. See also Hamano (2012) for the portfolio choice with a model where trade along extensive margins as well as intensive margins are allowed using the Devereux-Sutherland-Tille-van Wincoop method.

Contrary to internationally-held bonds, domestic firms (N_t number of incumbents and $N_{E,t}$ number of entrants in Home) are held only by domestic households who purchase a share of mutual funds, $s_{h,t+1}$ at t. Its real price is given by $x_{h,t}$.

Similar expressions hold in Foreign.

3.2. Firms

Assume that new entrants need capital goods as well as labor in order to set up their production unit. One firm/variety creation requires a number of firms setting up goods, f_E. Production technology is now defined by a Cobb-Douglas function with capital K_t and labor $l_{EM,t}$ as inputs:

$$f_E = \left(\frac{z_t l_{EM,t}}{\theta}\right)^{\theta} \left(\frac{K_t}{1-\theta}\right)^{1-\theta},$$

where θ $(1-\theta)$ is the share of labor (capital) in total costs. For simplicity, assume that capital goods K_t have the same composition as consumption goods C_t.

Production takes place only one period after entry. The production technology is identical to the previous static model. However, because capital goods are also required for entry, demand for each firm producing a specific product variety now includes capital goods demand:

$$y_{h,t} = c_{h,t} + c^*_{h,t} + N_{E,t} k_{h,t} + N^*_{E,t} k^*_{h,t},$$

where $k_{h,t}$ $(k^*_{h,t})$ denotes the capital demand from Home (Foreign) new entrants. The expression of dividends also changes:

$$d_{h,t} = \frac{1}{\sigma} \rho_{h,t}^{1-\omega} N_t^{\psi(\omega-1)-1} \left[\alpha M_t + (1-\alpha) Q_t^{\omega} M_t^*\right], \tag{14}$$

where M_t and M^*_t are defined as

$$M_t \equiv C_t + N_{E,t} K_t, \qquad M^*_t \equiv C^*_t + N^*_{E,t} K^*_t.$$

Note that, using factor demand functions for entry costs, when $\theta = 1$ (implying that only labor is used as input), the expression of dividends (14) is similar to that obtained under the static model.[13]

Finally, we define the motion of firms following Ghironi and Melitz (2005) as

$$N_{t+1} = (1 - \delta)\left(N_t + N_{E,t}\right),$$

where δ denotes a "death shock". This takes place at the very end of each period after entry has been completed.

Similar expressions hold in Foreign.

3.3. Equilibrium

A household maximizes the expected value of its lifetime utility $E_0 \sum_{s=0}^{\infty} \beta^s U_s$ subject to the budget constraint (13) with respect to $s_{h,t+1}$, B_{t+1}, $B_{*,t+1}$ and L_t, which provides the following first order conditions.

$$x_{h,t} = \beta (1 - \delta) E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(x_{h,t+1} + d_{h,t+1}\right),$$

$$C_t^{-\gamma} \left(1 + \vartheta B_{t+1}\right) = \beta \left(1 + r_{t+1}\right) E_t C_{t+1}^{-\gamma},$$

$$C_t^{-\gamma} \left(1 + \vartheta B_{*,t+1}\right) = \beta \left(1 + r_{t+1}^{*}\right) E_t \frac{Q_{t+1}}{Q_t} C_{t+1}^{-\gamma},$$

$$\chi \left(L_t\right)^{\frac{1}{\psi}} = w_t C_t^{-\gamma}.$$

[13]Entry costs minimization by firms yields the following factor demands:

$$l_{EM,t} = \frac{\theta}{w_t} \mu_t f_E, \qquad K_t = (1 - \theta)\, \mu_t f_E,$$

where $\mu_t \equiv \left(\frac{w_t}{z_{E,t}}\right)^{\theta}$ denotes real costs of firm creation.

As before, we characterize the general equilibrium by a free entry condition, labor market clearing condition and, since trade is no longer balanced, net foreign asset dynamics.

Because of free entry, real share price is equal to real entry costs in equilibrium

$$x_{h,t} = f_E \left(\frac{w_t}{z_{E,t}} \right)^{\theta}. \tag{15}$$

Since labor is allocated by N_t number of incumbents and firm creation by $N_{E,t}$ number of new entrants, the labor market clearing condition gives $L_t = N_t l_t + N_{E,t} l_{EM,t}$. Noting $y_{h,t} = (\sigma - 1) \frac{d_{h,t}}{w_t} z_t$ and $l_{EM,t} = \theta \frac{x_{h,t}}{w_t}$, this condition can be rewritten as

$$L_t = (\sigma - 1) \frac{N_t d_{h,t}}{w_t} + \theta \frac{N_{E,t} x_{h,t}}{w_t}.$$

Similar expressions hold in Foreign.

And due to internationally held non-contingent bonds, we have the following net foreign asset dynamics[14]

$$\begin{aligned} B_{t+1} + Q_t B_{*,t+1} = (1 + r_t) B_t + Q_t (1 + r_t^*) B_{*,t} & \\ + \frac{1}{2} \left[L_t w_t + N_t d_{h,t} - Q_t \left(L_t^* w_t^* + N_t^* d_{f,t}^* \right) \right] & \\ - \frac{1}{2} \left[N_{E,t} x_t + C_t - Q_t \left(N_{E,t}^* x_t^* + C_t^* \right) \right]. & \end{aligned} \tag{16}$$

Finally, bond markets clear in equilibrium via

[14] Aggregation implies the following net foreign assets accumulation for each country:

$$B_{t+1} + Q_t B_{*,t+1} = (1 + r_t) B_t + Q_t (1 + r_t^*) B_{*,t} + L_t w_t + N_t d_{h,t} - N_{E,t} x_t - C_t,$$

$$\frac{B_{t+1}^*}{Q_t} + B_{*,t+1}^* = \frac{(1 + r_t) B_t^*}{Q_t} + (1 + r_t^*) B_{*,t}^* + L_t^* w_t^* + N_t^* d_{f,t}^* - N_{E,t}^* x_t^* - C_t^*.$$

The above two equations (eliminating the bond-holding position by Foreign using bond market clearings (17)) yield (16).

$$B_{t+1} + B_{t+1}^{*} = 0, \qquad B_{*,t+1} + B_{*,t+1}^{*} = 0. \tag{17}$$

The dynamic model contains 33 equations and 33 variables among which eight are endogenous state variables ($N_t, N_t^*, B_t, B_t^*, B_{*.t}, B_{*.t}^*, r_t$ and r_t^*) and four are exogenous shocks (z_t, z_t^*, $z_{E,t}$ and $z_{E,t}^*$). Table 2 summarizes the system. In the next section, we calibrate the linearized version of the model and quantitatively explore the mechanism which can generate a realistic BS correlation with extensive margins.[15]

4. Calibration

The dynamic model is calibrated with parameters in Table 3. The value of constant risk aversion (γ), the discount factor (β), and the Frisch elasticity of labor supply (φ) are taken from Bilbiie et al. (2007). The value of the death shock (δ) is selected such that it matches the US's annual job destruction rate as in Ghironi and Melitz (2005). The adjusting costs of bond holdings (ϑ) is also taken from Ghironi and Melitz (2005). The value of home bias in consumption (α) is taken from CDL. The share of labor (θ) in entry costs is 0.64, as in Heathcote and Perri (2002). This is a standard value in a model with a Cobb-Douglas production function that includes capital and labor.

The elasticities of substitution among varieties (σ) and between Home and Foreign goods (ω) are set to six, as in Rotemberg and Woodford (1992). Although the value of trade elasticity is well in the range of micro-founded estimates, it may be considered too high compared to the open macroeconomics literature. These values typically range from 0.5 to 2. For the purposes of comparison, we will also consider a lower value of elasticity, $\omega = 2$, following Benigno and Thoenissen (2008).[16] The value of love for variety (ψ) is

[15] Since we have zero bond holdings at the steady state, percent deviations of bond positions are defined relative to the steady state consumption C . We choose χ such that the steady state labor supply becomes unity ($L = 1$). Details about the steady state are available upon request.

[16] With his micro-founded estimates, for instance, Romalis (2007) provides elasticities which range from 4 to 13. Recently Imbs and Mejean (2009) question the conventional estimation procedure in the open macroeconomics literature. They note a downward bias when heterogeneity among sectors is not considered.

Table 2: The model

Price indices	$\alpha\rho_{H,t}^{1-\omega}+(1-\alpha)\,\rho_{F,t}^{1-\omega}=1$
	$\rho_{H,t}=N_t^{-\psi}\rho_{h,t}, \qquad \rho_{F,t}=N_t^{*-\psi}\rho_{f,t}$
	$\alpha\rho_{F,t}^{*1-\omega}+(1-\alpha)\,\rho_{H,t}^{*1-\omega}=1$
	$\rho_{F,t}^{*}=N_t^{*-\psi}\rho_{f,t}^{*}, \qquad \rho_{H,t}^{*}=N_t^{-\psi}\rho_{h,t}^{*}$
Pricing	$\rho_{h,t}=\frac{\sigma}{\sigma-1}\frac{w_t}{z_t}, \quad \rho_{h,t}^{*}=Q_t^{-1}\rho_{h,t}$
	$\rho_{f,t}^{*}=\frac{\sigma}{\sigma-1}\frac{w_t^{*}}{z_t^{*}}, \quad \rho_{f,t}=Q_t\rho_{f,t}^{*}$
Profits	$d_{h,t}=\frac{1}{\sigma}N_t^{\psi(\omega-1)-1}\rho_{h,t}^{1-\omega}\left[\alpha M_t+(1-\alpha)\,Q_t^{\omega}M_t^{*}\right]$
	$d_{f,t}^{*}=\frac{1}{\sigma}N_t^{*\psi(\omega-1)-1}\rho_{f,t}^{*1-\omega}\left[\alpha M_t^{*}+(1-\alpha)\,Q_t^{-\omega}M_t\right]$
Definition of M	$M_t=C_t+(1-\theta)\,N_{E,t}x_{h,t}$
	$M_t^{*}=C_t^{*}+(1-\theta)\,N_{E,t}^{*}x_{f,t}^{*}$
Free entry	$x_{h,t}=f_E\left(\frac{w_t}{z_{E,t}}\right)^{\theta}, \quad x_{f,t}^{*}=f_E^{*}\left(\frac{w_t^{*}}{z_{E,t}^{*}}\right)^{\theta}$
Optimal labor supply	$\chi\,(L_t)^{\frac{1}{\psi}}=w_tC_t^{-\gamma}, \quad \chi\,(L_t^{*})^{\frac{1}{\psi}}=w_t^{*}C_t^{*-\gamma}$
Labor Market clearing	$L_t=(\sigma-1)\,\frac{N_td_{h,t}}{w_t}+\theta\frac{N_{E,t}x_{h,t}}{w_t^{*}}$
	$L_t^{*}=(\sigma-1)\,\frac{N_t^{*}d_{f,t}^{*}}{w_t^{*}}+\theta\frac{N_{E,t}^{*}x_{f,t}^{*}}{w_t^{*}}$
Number of firms	$N_t=(1-\delta)\,(N_{t-1}+N_{E,t-1})$
	$N_t^{*}=(1-\delta)\left(N_{t-1}^{*}+N_{E,t-1}^{*}\right)$
Euler equation (shares)	$x_{h,t}=\beta\,(1-\delta)\,E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(x_{h,t+1}+d_{h,t+1})$
	$x_{f,t}^{*}=\beta\,(1-\delta)\,E_t\left(\frac{C_{t+1}^{*}}{C_t^{*}}\right)^{-\gamma}\left(x_{f,t+1}^{*}+d_{f,t+1}^{*}\right)$
Euler equation (bonds)	$C_t^{-\gamma}\,(1+\vartheta B_{t+1})=\beta\,(1+r_{t+1})\,E_tC_{t+1}^{-\gamma}$
	$C_t^{-\gamma}\,(1+\vartheta B_{*,t+1})=\beta\left(1+r_{t+1}^{*}\right)E_t\frac{Q_{t+1}}{Q_t}C_{t+1}^{-\gamma}$
	$C_t^{*-\gamma}\left(1+\vartheta B_{*,t+1}^{*}\right)=\beta\left(1+r_{t+1}^{*}\right)E_tC_{t+1}^{*-\gamma}$
	$C_t^{*-\gamma}\left(1+\vartheta B_{t+1}^{*}\right)=\beta\,(1+r_{t+1})\,E_t\frac{Q_t}{Q_{t+1}}C_{t+1}^{*-\gamma}$
Net foreign Asset	$B_{t+1}-(1+r_t)\,B_t+Q_t\left[B_{*,t+1}-(1+r_t^{*})\,B_{*,t}\right]$
	$=\frac{1}{2}\left[L_tw_t+N_td_{h,t}-Q_t\left(L_t^{*}w_t^{*}+N_t^{*}d_{f,t}^{*}\right)\right]$
	$-\frac{1}{2}\left[N_{E,t}x_t+C_t-Q_t\left(N_{E,t}^{*}x_t^{*}+C_t^{*}\right)\right]$
Bond market clearing	$B_{t+1}+B_{t+1}^{*}=0, \quad B_{*,t+1}+B_{*,t+1}^{*}=0.$

Table 3: Baseline parametrization

γ	constant risk aversion	2
β	discount factor	0.99
φ	Frisch elasticity of labor supply	2
σ	elasticity of substitution among varieties	6
ω	between Home and Foreign goods	6
α	home bias in consumption	0.72
δ	death shock	0.025
ϑ	bond holding adjusting costs	0.0025
θ	share of labor in entry costs	0.64
ψ	love for variety	Dixit-Stiglitz

set to the value implied by standard Dixit-Stiglitz preferences.[17]

For simplicity, assume that productivity shocks on marginal costs of production z_t and firm creation $z_{E,t}$ are perfectly correlated: $z_t = z_{E,t}$ and $z_t^* = z_{E,t}^*$. These processes are selected from Backus et al. (1992), such that $\mathsf{Z}_{t+1} = \Omega \mathsf{Z}_t + \xi_t$, where $\mathsf{Z}_t = \begin{bmatrix} z_t, & z_t^* \end{bmatrix}'$, $\xi_t = \begin{bmatrix} \xi_t, & \xi_t^* \end{bmatrix}'$ and the correlation of shocks and error terms are given by

$$\Omega = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}, \text{ and } V(\xi) = \begin{bmatrix} 0.73 & 0.19 \\ 0.19 & 0.73 \end{bmatrix}.$$

4.1. Impulse response functions

Figure 2 shows impulse response functions for the real exchange rate, terms of trade and relative consumption following a one percent rise in productivity at Home. The figure's upper panel shows percent deviations of these variables from their steady state values measured on an empirical basis. Below are those on a welfare basis (as defined in section 2.2.).

The intuition and mechanism of the BS correlation with extensive margins, as it was analytically explored using the static model, is now best captured in the impulse response

[17] Ardelean (2006) estimates this value to be 42% of that implied by Dixit-Stiglitz preferences.

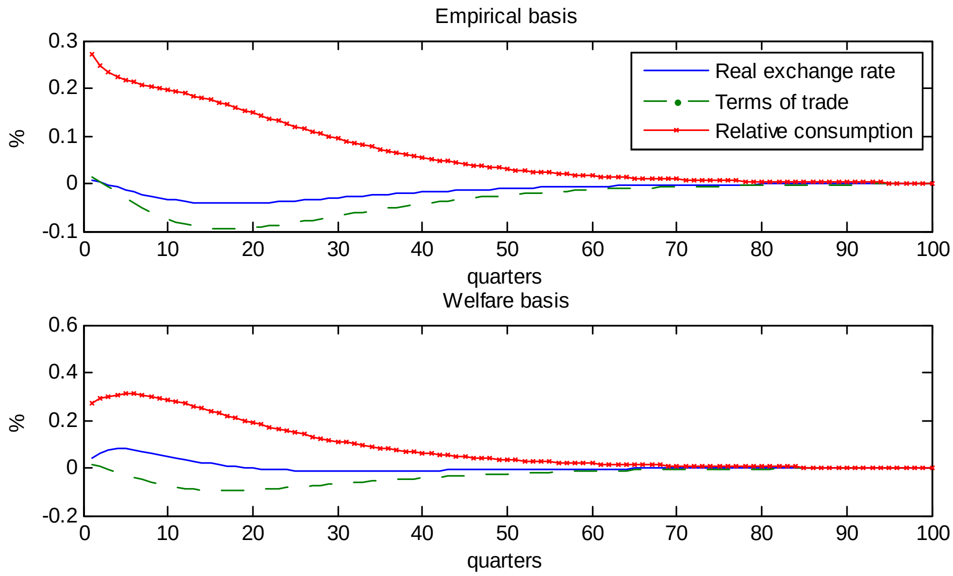

Figure 2: Impulse response functions under incomplete markets with baseline parameters. The empirically-based real exchange rate, terms of trade and relative consumption are shown in the upper panel. Those in welfare based measure in the lower panel.

functions of the dynamic model. With internationally-held state non-contingent bonds, the expected growth rate of real exchange rate is equal to the expected growth rate in relative consumption:

$$E_t\left(\mathsf{Q}_{t+1}-\mathsf{Q}_t\right) \approx \gamma E_t\left[\left(\mathsf{C}_{t+1}-\mathsf{C}_t\right)-\left(\mathsf{C}_{t+1}^{*}-\mathsf{C}_t^{*}\right)\right],$$

where we abstract away from negligible fluctuations in bond holdings arising from quadratic adjustment costs. Home and Foreign households stabilize their welfare-based consumption using non-contingent bonds only in the aftermath of a shock, not ex anté as in the case under complete asset markets. As a result, the tight link between the welfare-based real exchange rate and relative consumption that we observe under complete asset markets is broken.

In the benchmark calibration, market incompleteness alone is not sufficient to replicate a realistic BS correlation. As we can see in the figure, the BS correlation remain positive in welfare-based measures (the crossed and solid lines in the lower panel). The correlation is

0.85 with benchmark parameters. The empirical-based (data-consistent) BS correlation is instead 0.18. How can such a realistic correlation be possible? Following a positive productivity shock, new entry takes place in Home. The relative number of varieties increases steadily, eventually turning into a hump-shaped pattern. Over time, relative wages appreciate strongly due to an expansion of trade along extensive margins and the terms of trade tend to appreciate as well (the dotted lines in the upper and lower panels). Reflecting such a strong appreciation in the terms of trade, the empirical-based real exchange rate appreciates too (the solid line on the upper panel). This approximates a realistic BS correlation. The welfare-based real exchange rate, on the other hand, remains depreciated (the solid line in the lower panel). This reflects a higher number of varieties consumed with home bias.

4.2. Characteristics of the theoretical model

Table 4 reports second moments for the dynamic model. In the table, G7 data comes from Coeurdacier et al. (2010) except for the BS correlation (-0.27) (the median value among OECD countries relative to the rest of the world), drawn from CDL. Theoretical variables which correspond to the data are empirically measured as previously. The theoretical counterpart of GDP, $\widetilde{Y}_t$, investment, $\widetilde{I}_t$, and net exports, $\widetilde{TB}_t$, are defined as follows: $\widetilde{Y}_t \equiv w_t L_t + N_t \widetilde{d}_{h,t}$, $\widetilde{I}_t \equiv N_E \widetilde{x}_h$ and $\widetilde{TB}_t \equiv \left(\widetilde{X}_t - \widetilde{IM}_t\right)/\widetilde{Y}_t$ where $\widetilde{X}_t$ and $\widetilde{IM}_t$ denote empirical-based exports and imports. For the purposes of comparison, second moments obtained under a lower trade elasticity ($\omega = 2$) and complete asset markets are reported.[18]

In the benchmark calibration under incomplete asset markets and high trade elasticity ($\omega = 6$), a highly volatile investment (13.52) and its strongly negative cross-country correlation (-0.89) appear compared to the G7 data. Since investment and firm creation require labor, employment and output are also negatively correlated across countries (-0.93 and -0.49, respectively, in the theoretical model) while they are positive in the G7 data.

[18]Second moments of the theoretical model are computed using the frequency domain technique presented in Uhlig (1998) for HP filtered series. The smoothing parameter is set to 1600.

The high trade elasticity in the benchmark calibration, however, provides a realistic BS correlation, -that is, it is close to zero (0.18). With an alternative low elasticity ($\omega = 2$), investment volatility declines (from 13.52 to 11.44) and investment becomes less correlated internationally (-0.84, compared to -0.89, in the benchmark calibration) while a puzzling BS correlation remains (0.95).

Second moments under complete markets are quite similar to those under incomplete markets. This result is reminiscent of Heathcote and Perri (2002), who noted that a model with balanced trade featuring less risk sharing shows a better match with the data. Although they are similar, under complete markets, investment volatility rises (16.85) and its cross country correlation becomes more negative (-0.93) compared to the benchmark calibration. Since consumption is perfectly insured under complete markets, increased firm entry is observed for more profitable locations. This is why we observe a higher volatility and stronger negative cross-country correlation of investment under complete markets in our dynamic model.

One important observation is that the empirical-based (data-consistent) BS correlation becomes negative (-0.26) under complete markets. The reason is quite different from the one discussed for incomplete markets. Under complete markets, there is an uncommonly strong positive transmission via the terms of trade and the *depreciation* in the empirical-based real exchange rate following a positive shock. As a result, Foreign households consume *more* than Home in empirical-based measure under complete markets. Such a strong transmission, which even goes so far as to reverse the consumption pattern across countries, is unrealistic.[19] Hence, a realistic BS correlation which appears under complete markets cannot be considered plausible. A more detailed discussion about the model under complete markets can be found in Appendix A.

In addition to the cross-country correlation due to strong risk sharing properties, the theoretical model shares other principle characteristics of the standard two-country model argued in Heathcote and Perri (2002) and Chari et al. (2002). These include aspects such

[19] For instance, as Corsetti et al. (2008b) and others have shown, using a VAR model, that consumption increases in a country receiving a positive shock relative to the rest of the world.

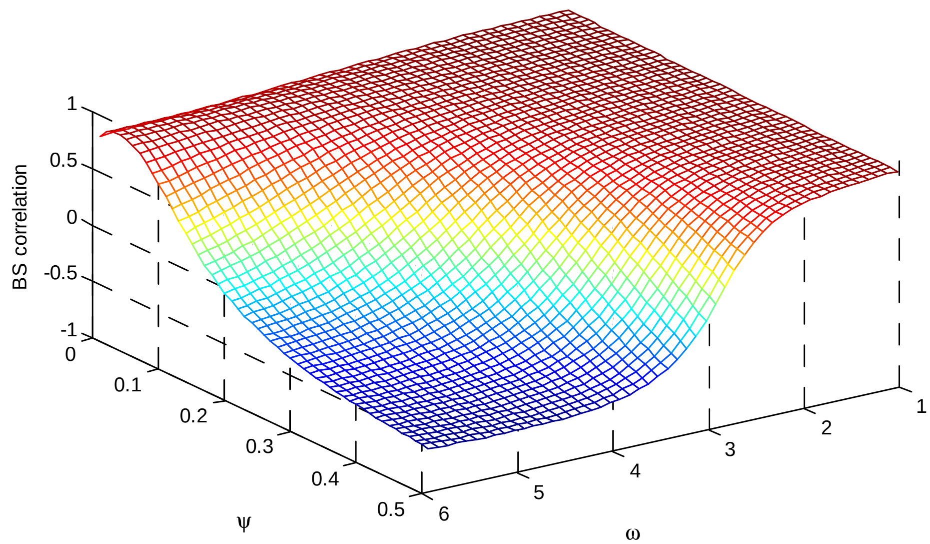

Figure 3: Sensitivity analysis under incomplete financial markets. The BS correlations are computed against various values of trade elasticity, ω, and love for variety, ψ.

as lower volatility in the real exchange rate compared to the data. In summary, allowing trade along extensive margins with high trade elasticity brings the observed BS correlation into a realistic range. On the other hand, other puzzles in the theoretical model remain, without significant quantitative improvement.

4.3. Sensitivity analysis

Figure 3 examines the BS correlation with different values for the elasticity of substitution (ω) and love for variety (ψ). The correlation becomes steadily weaker and finally falls negative as elasticity or the love for variety increases. Again, analytical investigation of a wealth effect due to changes in the number of product varieties sheds light on such a pattern. The interaction between trade elasticity and love for variety strengthens the wealth effect, and this is important in replicating a realistic BS correlation under incomplete markets.

Table 4: Second moments

% std.dev. relative to $\widetilde{Y}$	$\widetilde{Y}$	$\widetilde{C}$	$\widetilde{I}_t$	L	$\widetilde{TB}/\widetilde{Y}$	$\widetilde{Q}$
G7 data	1.87	0.76	8.26	1.31	1.07	1.38
	(1.00)	(0.41)	(4.42)	(0.70)		(0.74)
Benchmark	1.47	0.47	13.52	0.84	0.42	0.04
	(1.00)	(0.32)	(9.19)	(0.57)		(0.03)
$\omega = 2$	1.33	0.47	11.44	0.73	0.16	0.16
	(1.00)	(0.35)	(8.58)	(0.54)		(0.12)
Complete Market	1.76	0.46	16.85	1.20	0.67	0.04
	(1.00)	(0.26)	(9.60)	(0.68)		(0.02)
$\omega = 2$	1.40	0.46	12.12	0.84	0.15	0.22
	(1.00)	(0.33)	(8.63)	(0.60)		(0.16)

Correlation with output	$\widetilde{C}$	$\widetilde{I}_t$	L	$\widetilde{TB}/\widetilde{Y}$	$\widetilde{Q}$
G7 data	0.38	0.71	0.61	-0.07	-0.22
Benchmark	0.65	0.91	0.79	-0.28	0.56
$\omega = 2$	0.68	0.95	0.74	-0.66	0.71
Complete Market	0.41	0.89	0.87	-0.05	0.89
$\omega = 2$	0.60	0.95	0.78	-0.43	0.70

Cross-country correlation	$\widetilde{Y}$	$\widetilde{C}$	$\widetilde{I}_t$	L	$\widetilde{Q}, \frac{\widetilde{C}}{\widetilde{C}^*}$	$Q, \frac{C}{C^*}$
G7 data	0.17	0.58	-0.37	0.18	-0.27^*	
Benchmark	-0.49	0.89	-0.89	-0.93	0.18	0.84
$\omega = 2$	-0.38	0.92	-0.84	-0.90	0.95	0.97
Complete Market	-0.64	0.98	-0.93	-0.96	-0.26	1.00
$\omega = 2$	-0.45	0.98	-0.86	-0.93	0.80	1.00

Data with * from CDL. Others from Coeurdacier et al. (2010).

5. Conclusion

This paper revisits the consumption-real exchange rate anomaly known as the Backus-Smith puzzle. We examine how extensive margins can contribute to the resolution of the puzzle. This paper's argument is based on 1) a wealth effect due to changes in the number of product varieties; 2) statistical inefficiency in measuring the number of product varieties; and 3) market incompleteness.

Contrary to complete asset markets which, in general, feature uncommonly strong risk sharing properties, changes in the number of product varieties under incomplete markets may produce a wealth effect under high trade elasticity.

An important caveat throughout the analysis is that risk sharing must fundamentally include a "variety effect" (Hamano, 2012) as long as consumers display a preference for product varieties. Without considering such a dimension, observed price data might provide a distorted image about the reality of risk sharing across countries. Terms of trade or real exchange rate appreciation may not necessarily indicate a world divergence in wealth once we consider (unobservable) trade flow along extensive margins. Indeed, in the calibrated model under incomplete markets, the *welfare-based* BS correlation remains highly positive. This suggests the existence of a high degree of international risk sharing, while the *empirical-based (data-consistent)* BS correlation in the theoretical model mimics the realistic one.

Undoubtedly one important challenge for future research is in measuring how important such a variety effect can be for international risk sharing. Furthermore, quantifying the contribution of each complementary mechanism in the resolution of the BS puzzle is another fertile avenue for research.

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Appendix A. Complete markets

Appendix A.1. The static model

Under complete asset markets, marginal utility stemming from additional nominal wealth is the same across countries:

$$Q = \left(\frac{C^*}{C}\right)^{-\gamma}. \tag{A.1}$$

Using the above perfect risk sharing condition, we can write relative dividends as $\mathsf{d}^R = -(\lambda - 1)\left(\mathsf{w}^R - \mathsf{z}^R\right) + \left[\psi(\lambda - 1) - 1\right]\mathsf{N}^R$. Plugging this expression into the labor market clearing condition and the free entry condition, we have solutions for w^R and N^R:

$$\mathsf{w}^R = \frac{\lambda - 1}{\lambda}\left(\mathsf{z}^R + \psi \mathsf{z}_E^R\right), \tag{A.2}$$

$$\mathsf{N}^R = \mathsf{z}_E^R, \tag{A.3}$$

where

$$\lambda \equiv \omega\left[1 - (2\alpha - 1)^2\right] + (2\alpha - 1)^2 \frac{1}{\gamma}.$$

In the above expressions, λ roughly represents the elasticity of substitution between local and imported goods. Extensive margins N^R change one by one with an investment shock z_E^R as in the case under balanced trade. Under $\psi > 0$, relative wages w^R increase (decrease) with a positive productivity shock on marginal costs z^R and entry costs z_E^R, given $\lambda > 1$ ($\lambda < 1$).

Provided the above expression of w^R, fluctuations in the terms of trade are given by

$$\mathsf{TOT} = \frac{1}{\lambda}\mathsf{z}^R - \frac{\psi(\lambda-1)}{\lambda}\mathsf{z}_E^R. \tag{A.4}$$

In contrast to the terms of trade fluctuations under incomplete markets, following a positive productivity shock on marginal costs z^R, they *never appreciate* from the first term. Following a positive shock on firm entry costs z_E^R, the second term adds a terms of trade appreciation when $(\lambda > 1)$ given love for variety. Because of the first term, however, the terms of trade are less likely to appreciate under a positive shock such that $\mathsf{z}^R = \mathsf{z}_E^R > 0$.

Appendix A.1.1. The BS puzzle with extensive margins under complete markets

Rewriting the perfect risk sharing condition (A.1) in empirical-based measure and taking its first-order deviations, we have $\widetilde{\mathsf{Q}} = \gamma\left(\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^*\right) + (\gamma-1)\psi(2\alpha-1)\mathsf{N}^R$. Plugging the solution of w^R (A.2), N^R (A.3) and TOT (A.4) in this expression, the BS correlation with extensive margins under complete markets is

$$\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^* = \frac{2\alpha-1}{\lambda\gamma}\mathsf{z}^R - \psi\frac{(2\alpha-1)(\lambda\gamma-1)}{\lambda\gamma}\mathsf{z}_E^R, \tag{A.5}$$

$$\widetilde{\mathsf{Q}} = \frac{2\alpha-1}{\lambda}\mathsf{z}^R - \psi\frac{(2\alpha-1)(\lambda-1)}{\lambda}\mathsf{z}_E^R. \tag{A.6}$$

As in the case of incomplete markets, the BS correlation remains positive when there are only the first terms induced by z^R in the above expressions. Because of the second terms, however, it is possible even under complete markets to generate a realistic BS correlation. The second term in (A.5), which is negative, is larger in absolute value than that in (A.6), which is also negative. The first term in (A.5), which is positive, is smaller in absolute value than that in (A.6), which is also positive. Hence, the BS correlation becomes negative only when $\widetilde{\mathsf{Q}} > 0$ at the same time as $\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^* < 0$, following a positive shock such that $\mathsf{z}^R = \mathsf{z}_E^R > 0$. Intuitively, this is when there is a relatively strong positive transmission via the terms of trade depreciation ($\widetilde{\mathsf{Q}} > 0$), to the extent that the relative empirical-based consumption decreases ($\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^* < 0$). Such a pattern, however, is odd. In the next subsection, we quantitatively investigate this pattern using a dynamic model.

Appendix A.2. The dynamic model

Under complete markets, the dynamic model becomes simpler. We discuss only modified points compared to the dynamic model in the paper. The real budget constraint for the Home representative household now contains state-contingent securities in the place of non-contingent bonds:

$$\begin{aligned} C_t + s_{h,t+1} x_{h,t} \left(N_t + N_{E,t}\right) \\ + \sum_{s_{t+1}} b_{t+1}\left(S_{t+1}\right) q_t\left(S_{t+1} \mid S_t\right) + Q_t \sum_{s_{t+1}} b^*_{t+1}\left(S_{t+1}\right) q^*_t\left(S_{t+1} \mid S_t\right) \\ = w_t L_t + s_{h.t} N_t \left(x_{h,t} + d_{h,t}\right) + b_t\left(S_t\right) + Q_t b^*_t\left(S_t\right). \end{aligned}$$

The state-contingent securities give one unit of Home (Foreign) goods in the next period. The expression $b_{t+1}(S_{t+1})$ ($b^*_{t+1}(S_{t+1})$) captures holdings of such assets into the next period indexed by the future state of nature S_{t+1}. The expression $q_t(S_{t+1} \mid S_t)$ ($q^*_t(S_{t+1} \mid S_t)$) denotes their real price, which is conditional on the current state of nature S_t.

First order conditions about these state-contingent securities yield the well-known perfect risk sharing condition (A.1) with time indices. Under complete markets, Euler equations about non-contingent bonds, bond market clearing conditions and the evolution of net foreign assets are no longer needed. Other first order conditions remain as in Table 2. Finally, the model contains 27 equations and 27 variables, of which two are endogenous state variables (N_t and N^*_t) and four are exogenous shocks (z_t, z^*_t, $z_{E,t}$ and $z^*_{E,t}$).

Appendix A.3. Calibration

Figure A.4 shows impulse response functions with the same baseline parameters and shocks as in the paper. Following a rise in Home labor productivity, the terms of trade depreciate under complete markets (the dotted line in the upper and lower panels). This is because a positive wealth effect due to a rise in the number of product varieties which appreciates the terms of trade under incomplete markets is relatively weak. On the other hand, the welfare-based real exchange rate depreciates (the solid line in the lower panel) because of this rise in the number of product varieties consumed with home bias.

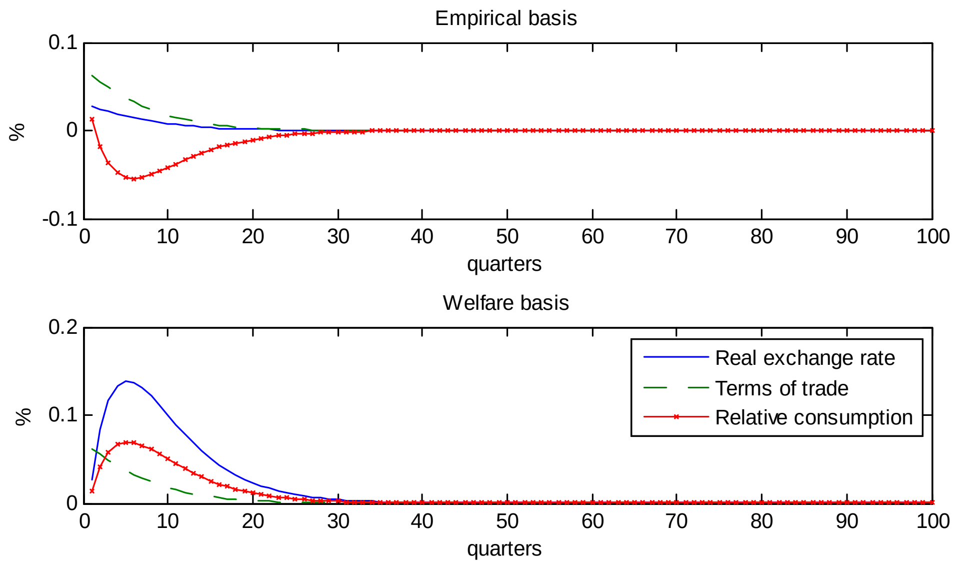

Figure A.4: Impulse response functions under complete markets with baseline parameters. The empirically-based real exchange rate, terms of trade and relative consumption are shown in the upper panel. Those in welfare based measure in the lower panel.

Eliminating fluctuations in the number of product varieties, and combined with a positive transmission to Foreign due to the terms of trade depreciation, makes the empirical-based relative consumption fall (the crossed line in the upper panel). In other words, households in Home appear to consume less in the empirical-based measure while they consume more in the welfare-based measure in order to achieve perfect risk sharing. Although a realistic BS correlation is possible under complete markets, it must be based on such an unrealistic data-consistent consumption pattern.

Figure A.5 show the results of a sensitivity analysis. Under complete markets, as trade elasticity ω and love for variety ψ increase, the BS correlation first declines and then increases. The analytical solution again sheds light on such a non-linearity. As evident in (A.4), when the elasticity of substitution or love for variety approach a sufficiently high value, the terms of trade change their direction, from depreciation to appreciation. Since variations in the empirical-based relative consumption remain negative along such a rise in elasticity or love for variety, the non-linear pattern of the BS correlation appears.

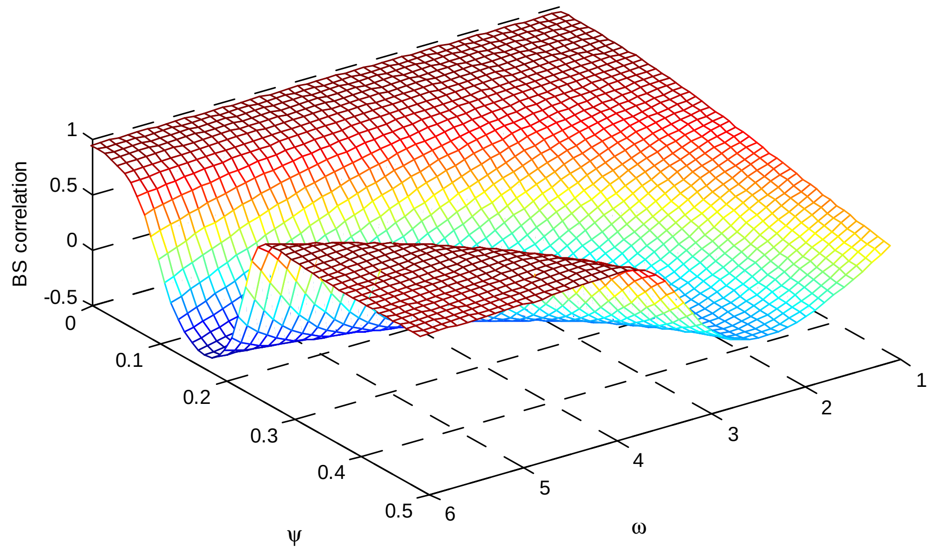

Figure A.5: Sensitivity analysis under complete financial markets. The BS correlations are computed against various values of trade elasticity, ω, and love for variety, ψ.

Appendix B. Shock persistence under a bond economy

CDL argue a possibility that the terms of trade appreciate due to a wealth effect induced by anticipated future output gains. Letting agents smooth consumption with internationally held bonds, the terms of trade appreciate temporally in anticipation of future output gains. This replicates a realistic BS correlation. The key is a highly persistent productivity shock which is combined with high trade elasticity. Such a mechanism is also present in this paper's model under a bond economy.

As in CDL, we assume log utility and no time preference when analyzing international trade in bonds. Letting $\bar{\mathsf{z}}^R$ and $\bar{\mathsf{z}}_E^R$ denote long-run deviations of shocks, we have

$$\mathsf{w}^R = \frac{4\alpha(1-\alpha)(\omega-1)}{1+4\alpha(1-\alpha)(\omega-1)}\left[\left(\mathsf{z}^R-\bar{\mathsf{z}}^R\right)+\psi\left(\mathsf{z}_E^R-\bar{\mathsf{z}}_E^R\right)\right] + \frac{2\alpha(\omega-1)}{1+2\alpha(\omega-1)}\left(\bar{\mathsf{z}}^R+\psi\bar{\mathsf{z}}_E^R\right). \quad \text{(B.1)}$$

The first line in the above expression captures the short-run impact of both marginal and entry cost shocks on relative wages. The second line is the long-run impact of these shocks, which is isomorphic to the one discussed in the paper. Since $\mathsf{z}^R - \bar{\mathsf{z}}^R < 0$ and $\mathsf{z}_E^R - \bar{\mathsf{z}}_E^R < 0$, relative wages (current wealth) unambiguously *depreciate* in the short-run when $\omega > 1$. This is due to decreased production along both the intensive and extensive margins, as compared to their long-run level.

Using the above expression of w^R, we have

$$\begin{aligned}\mathsf{TOT} =& \frac{1}{1+4\alpha(1-\alpha)(\omega-1)}\left(\mathsf{z}^R - \bar{\mathsf{z}}^R\right) - \psi\frac{4\alpha(1-\alpha)(\omega-1)}{1+4\alpha(1-\alpha)(\omega-1)}\left(\mathsf{z}_E^R - \bar{\mathsf{z}}_E^R\right) \\ &+ \frac{1}{1+2\alpha(\omega-1)}\bar{\mathsf{z}}^R - \psi\frac{2\alpha(\omega-1)}{1+2\alpha(\omega-1)}\bar{\mathsf{z}}_E^R.\end{aligned}$$

Again, the first line captures the short-run impact of shocks and the second line their long-run impact. Specifically, the first term in the short-run impact is the one argued in CDL. This term is unambiguously negative when $\omega > 1$. As explained in CDL, in anticipation of future productivity gains, households raise demand above supply in the short-run creating a short-run terms of trade appreciation despite a reduction in current wage income. The second term with changes in product varieties, however, counteracts. This term is unambiguously positive when $\omega > 1$. Intuitively, although households raise demand in the short-run in anticipation of future gains along their intensive and extensive margins, a hump-shaped pattern of rise in the number of product varieties, which appears as a result of high shock persistence, means, at the same time, a reduction of short-run wage income (see (B.1)). This reduced short-run wealth effect dampens the CDL's short-run wealth effect in anticipation of future productivity. Therefore, in our model, raising the shock persistence does not improve the BS correlation. Note that our expression of TOT collapses to the one argued in CDL when $\psi = 0$.

The same result can be obtained with a sensitivity analysis against shock persistence in the fully-specified dynamic model. Figure B.6 provides the BS correlation with various values of shock persistence (from 0.5 to 0.99). The correlation is insensitive.

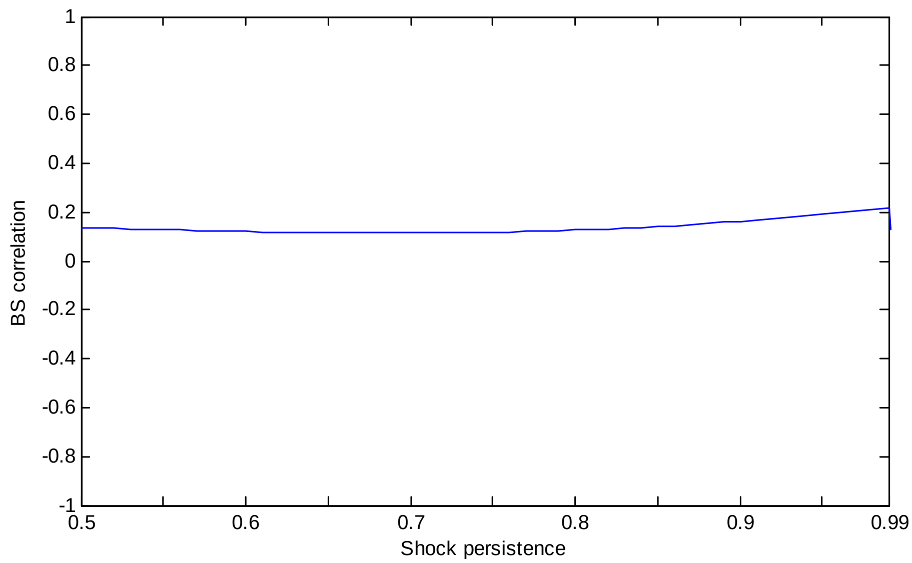

Figure B.6: Sensitivity analysis under incomplete financial markets. The BS correlations are computed against various values of shock persistence with other baseline parameters.