

## Associative formal power series in two indeterminates

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The classical examples of associative formal power series are formal group laws. A one dimensional formal group law over a ring  $\mathbb{A}$  is a formal power series  $F, F \in \mathbb{A}[[X, Y]]$  in two indeterminates of order one, such that

$$F(X, Y) = X + Y + \sum_{\alpha+\beta \geq 2} a_{\alpha, \beta} X^\alpha Y^\beta$$

and the associativity equation

$$F(F(X, Y), Z) = F(X, F(Y, Z))$$

holds, see [2]. One dimensional formal group laws as well as associative polynomials, see [3], and associative rational functions, see [1], are completely described.

For a commutative field  $\mathbb{K}$  we characterize all formal power series  $F \in \mathbb{K}[[X, Y]]$  of arbitrary order which are associative.

- [1] A. Chéritat, *Fractions rationnelles associatives et corps quadratiques*, Revue des Mathématiques de l'Enseignement Supérieur, 109 (1998-1999) 1025–1040.
- [2] M. Hazewinkel, *Formal Groups and Applications*, Acad. Press, New York and London, 1978.
- [3] J.-L. Marichal, P. Mathonet, *A description of  $n$ -ary semigroups polynomial-derived from integral domains*, Semigroup Forum 83(2) (2011) 241–249.