"To be or not to be"≠ "to kill or not to kill" a logic on action negation

Xin Sun

Individual and Collective Reasoning Group, University of Luxembourg
xin.sun@uni.lu

Abstract. This paper defines a new action negation operator such that it is more dilemma-free than the existed treatment. A dynamic deontic logic is built on top of this new theory of action. Such new logic satisfies the free choice axiom and avoids all the implausible results often arise with the validation of free choice axiom.

Key words: action negation, dynamic deontic logic

1 Introduction

Research on deontic logic can be divided into two main groups: the ought-to-be group and the ought-to-do group. The ought-to-do group originates from the famous Finnish philosopher von Wright's pioneering paper [11]. Belong to this branch there are dynamic deontic logic [7, 10], and deontic action logic [8, 3, 9].

One issue of dynamic deontic logic is to characterize the negation of action. In the dynamic logic literature [6], the negation of action is usually interpreted as set theoretical complement with respect to the universal relation. [1] and [2] point out that such treatment is not the best choice when dynamic logic is applied to deontic setting. Several new versions of action negation are defined in [1, 2].

In this paper, we will define another action negation operator which is intuitively natural and technically dilemma-free. In Section 2 we recall dynamic logic. In section 3 we define a new operator for action negation therefore give arise to a new dynamic logic. In Section 4 we apply the new logic to the deontic setting. We conclude this paper in Section 5.

2 Dynamic logic

In this section we recall the definitions of dynamic logic. Let $\mathbb{P}$ be a countable set of propositional letters and $\mathbb{A}$ a countable set of symbols of action generators. The language of dynamic logic can be defined by the following BNF:

Definition 1 (Language of dynamic logic). *For $a \in \mathbb{A}$ and $p \in \mathbb{P}$,*

- $\alpha := a|\alpha \cup \alpha|\alpha \cap \alpha|\alpha;\alpha|\alpha^*|\overline{\alpha}$
- $\phi := p|\top|\neg\phi|\phi \wedge \phi|[\alpha]\phi$

Here symbols of the form α are action terms and ϕ are formulas. Formulas are interpreted by the relational model, which can be defined as follows.

Definition 2 (Relational model). *A relational model $\mathbb{M} = (S, R^{\mathbb{A}}, V)$ is a triple:*

- *S is a nonempty set of possible states.*
- *$R^{\mathbb{A}} : \mathbb{A} \to 2^{S \times S}$ is an action interpretation function, assigning a binary relation over $S \times S$ to each action generator $a \in \mathbb{A}$.*
- *V is the valuation function for propositional letters.*

The action interpretation function $R^{\mathbb{A}}$ can be extend to a new function R to interpret arbitrary actions as follows:

- $R(a) = R^{\mathbb{A}}(a)$ for $a \in \mathbb{A}$.
- $R(\alpha \cup \beta) = R(\alpha) \cup R(\beta)$
- $R(\alpha \cap \beta) = R(\alpha) \cap R(\beta)$
- $R(\alpha;\beta) = R(\alpha) \circ R(\beta)$
- $R(\alpha^*) = (R(\alpha))^*$

Here $\circ$ is the composition operator for relations and * is the reflexive transitive closure operator of relations. We leave the case for $R(\overline{\alpha})$ to the next section because that is the theme of this paper. With the function R in hand, we can define the semantics for formulas of dynamic logic use relational model as following:

Definition 3 (Semantics of dynamic logic). *Let $M =< W, R^{\mathfrak{A}}, V >$ be a relational model. Let $w \in W$.*

- *$M, w \models p$ iff $w \in V(p)$*
- *$M, w \models \neg\phi$ iff not $M, w \models \phi$*
- *$M, w \models \phi \wedge \phi$ iff $M, w \models \phi$ and $M, w \models \phi$*
- *$M, w \models [\alpha]\phi$ iff for all v, if $(w, v) \in R(\alpha)$ then $M, v \models \phi$*

3 A New Treatment of Action

In this section we first review the known treatment for dynamic logic on action negation, then we define a new alternative.

3.1 Action negation in the literature

The traditional interpretation of action negation [6] is to let $R(\overline{\alpha}) = W \times W - R(\alpha)$, *i.e.* the set theoretical complement with respect to the universal relation. Jan Broersen [1] and [2] argues that the universal relation is not the ideal background for complement when dynamic logic is applied to normative reasoning, or deontic logic. Instead we should restrict the universal relation such that those worlds which are unreachable by any action are out of concern. With this intuition Jan Broersen suggest we replace the universal relation $W \times W$ in the interpretation of $R(\overline{\alpha})$ by relations like $\bigcup_{\alpha \in \mathbb{A}} R(\alpha)$, $(\bigcup_{\alpha \in \mathbb{A}} R(\alpha))^+$, $(\bigcup_{\alpha \in \mathbb{A}} R(\alpha))^*$ etc.

Jan Broersen's approach is more natural than its traditional counterpart in the deontic setting. But there is a shortcoming both of them can not overcome. For an illustration, first note that for any two action α and β, the action $\alpha \cup \overline{\alpha}$ and $\beta \cup \overline{\beta}$ are identical in both traditional and Broersen's approach. Now suppose Hamlet receives the following authorization: "you are permitted either to be or not to be" and James Bond receives the following authorization:"you are permitted either to kill or not". Intuitively, the first permission offers Hamlet a free choice between to live and to dead and the second offers 007 the license to kill or not. These two permissions convey very different information and should be distinguished. But they are identical in both the traditional and Jan Broersen's approach. Therefore those two approach both lead us to a dilemma.

A dilemma needs a solution. In the following section we will develop a new interpretation of action negation such that the above dilemma is solved.

3.2 A new approach of action negation

We first make a classification about actions. Since actions are interpreted by relations and the simplest relation is a set contains one ordered pair of states, we can naturally call an action α particle if $R(\alpha)$ contains exactly one ordered pair. For a particle action α, we call the first component of $R(\alpha)$ the pre-condition of α. Formally, if $R(\alpha) = (s_1, s_2)$, then $pre(\alpha) = \{s_1\}$. And we call the second component of $R(\alpha)$ the post-condition of α, formally $post(\alpha) = \{s_2\}$. Intuitively, a particle action is a deterministic change from one state to another.

Based on particle action, we build atomic action as a union of particle actions which share the same pre-condition. For instance, for two particle actions α_1 and α_2 with $R(\alpha_1) = \{(s_1, s_2)\}$, $R(\alpha_2) = \{(s_1, s_3)\}$, the action α_3 such that $R(\alpha_3) = \{(s_1, s_2), (s_1, s_3)\}$ is an atomic action.

For an atomic action α, its pre-condition is the same as its consisted particle actions. The post condition of α is the union of the post conditions of its consisted particle actions. Therefore $post(\alpha_3) = \{s_2, s_3\}$. Intuitively, an atomic action is a nondeterministic change from a specific state to other states. For example, if we let s_1 represents "China", s_2 represents "USA", and s_3 represents "Canada", then α_3 means "go to north America from China".

A normal action is a union of simple actions which possibly bears different pre-conditions. For example, let s_1 be "Italy" and s_2 be "Luxembourg", then the action α with $R(\alpha) = \{(s_1, s_2), (s_2, s_1)\}$ is a normal action which can be read as "go to Luxembourg from Italy, or go to Italy from Luxembourg". For a normal action α, we defined its pre-condition as the union of the pre-conditions of its consisted simple actions. Formally, $pre(\alpha) = \{s \in S | (s, t) \in R(\alpha)\}$.

Now we have a classification of actions and the pre/post condition of action has been defined. It is the time to grasp what the negation of an action is. For an atomic action α, say $R(\alpha) = \{(s, s), (s, t)\}$ and $W = \{s, t, u\}$, we tend to define $R(\overline{\alpha}) = \{(s, u)\}$. The intuition is, we understand α as a non-deterministic movement from s to either t or s itself, then the negation of α can be understood as "go to those states other than α goes". More formally, we define $R(\overline{\alpha}) = pre(\alpha) \times (W - post(\alpha))$ for a simple action α.

For a normal action, we can calculate $R(\overline{\alpha})$ via following steps:

1. We decompose α to atomic actions $\alpha_1, \dots, \alpha_n$ such that $R(\alpha) = R(\alpha_1) \cup \dots \cup R(\alpha_n)$ and for every i, j, $pre(\alpha_i) \neq pre(\alpha_j)$. It can be verified such decomposition is unique and each α_i is a maximal sub-atomic-action of α in the sense that for every atomic action β, if $R(\beta) \subseteq R(\alpha)$ then there exist a unique a_i in the decomposition such that $R(\beta) \subseteq R(\alpha_i)$
2. For each $i \in \{1, \dots, n\}$, we calculate $R(\overline{\alpha_i})$. Since α_i is an atomic action, we have $R(\overline{\alpha_i}) = pre(\alpha_i) \times (W - post(\alpha_i))$.
3. We take the union of these $R(\overline{\alpha_i})$ to form $R(\overline{\alpha}) = R(\overline{\alpha_1}) \cup \dots \cup R(\overline{\alpha_n})$.

Equivalent to the procedure above, we can define the negation of action in a more concise manner as follows:

Definition 4. $R(\overline{\alpha}) = Pre(\alpha) \times S - R(\alpha)$.

It is not hard to verify that for two action α and β, as long as $pre(\alpha) \neq pre(\beta)$, we have $R(\alpha \cup \overline{\alpha}) \neq R(\beta \cup \overline{\beta})$. Hence the dilemma from subsection 3.1 is solved.

4 From action to deontic logic

There are several possible approaches to develop deontic logic based on the logic of action above. One is as in [4], for every action we choose a subset of its post-condition to be the ideal outcomes, then use those ideal outcomes to form a neighborhood, served as the source of normativity. A second approach is to build deontic logic via deontic-dynamic reduction as in [2]. In this section we focus on the second approach.

The methodology is to introduce 'normative constant' for obligation, permission and prohibition respectively. Let c_F be the constant of a prohibition, c_O for obligation and c_P be for permission. Intuitively, $V(c_F)$ is the set of states which are morally forbidden and $V(c_O)$ the set of morally required states. V_P is the morally permissive states. For each valuation V of a relational model $M =< W, R^{\mathfrak{A}}, V >$, $V(c) \subseteq W$, for $c \in \{c_F, c_O, c_P\}$. We moreover require $V(c_F) \cap V(c_P) = \emptyset$ and $V(c_O) \subseteq V(c_P)$. For those states which are not in $V(c_F) \cup V(c_P)$, we consider them as morally neutral.

We define normative operators based on normative constant as follows:

- $P(\alpha) := [\alpha]c_P$
- $F(\alpha) := [\alpha]c_F$
- $O(\alpha) := [\overline{\alpha}]\neg c_O$

According to the above definition, an action is permitted iff the post-condition of its execution will always belong to the morally permissive states. An action is forbidden iff all the post-condition of its execution will lead to morally forbidden states. An action is obligatory iff as long as we execute its negation, the outcome will not be morally required.

It can be verified that the above deontic operators satisfies the following logical properties:

- $\models P(\alpha \cup \beta) \rightarrow P(\alpha) \wedge P(\beta)$
- $\not\models O(\alpha) \leftrightarrow \neg P(\overline{\alpha})$
- $\not\models O(\alpha) \rightarrow O(\alpha \cup \beta)$
- $\not\models P(\alpha) \rightarrow P((\alpha \cap \beta) \cup (a \cap \overline{\beta}))$

Those properties are essential to verify the free choice axiom meanwhile block all the potential implausible results arises with the validation of free choice axiom [4],[5].

5 Conclusion

This paper defines a new action negation operator such that it is more dilemma-free than the existed treatment of the action negation. A dynamic deontic logic is build on top of this new logic. Such new logic satisfies the free choice axiom and avoids all the implausible results often arise with the validation of free choice axiom.

References

1. J. Broersen. Relativized action negation for dynamic logics. In P. Balbiani, N-Y. Suzuki, F. Wolter, and M. Zakharyaschev, editors, *Advances in Modal Logic*, volume 4, pages 51–70, 2003.
2. J. Broersen. Action negation and alternative reductions dynamic deontic logics. *Journal of applied logic*, 2004.
3. P. Castro and T. Maibaum. Deontic action logic, atomic boolean algebras and fault-tolerance. *Journal of Applied Logic*, 2009.
4. D. Gabbay, X. Sun, and L. Gammaitoni. The paradoxes of permission, an action based solution. *to appear in Journal of Applied Logic*, 2013.
5. S. Hansson. The varieties of permissons. In *Handbook of deontic logic and normative systems*. College Publication, 2013.
6. D. Harel, D. Kozen, and J. Tiuryn. *Dynamic logic*. The MIT Press, 2000.
7. J. J. Meyer. A different approach to deontic logic: deontic logic viewed as a variant of dynamic logic. *Notre Dame Journal of Formal Logic*, pages 109–136, 1988.
8. K. Segerberg. A deontic logic of action. *Studia Logica*, 1982.
9. R. Trypuz and R. Kulicki. A systematics of deontic action logics based on boolean algebra. *Logic and Logical Philosophy*, 2009.
10. R. van der Meyden. The dynamic logic of permission. *Journal of Logic and Computation*, 6:465–479, 1996.
11. G. H. von Wright. Deontic logic. *Mind*, pages 1–15, 1951.