Gravity field recovery in high latitude areas

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Geodätische Woche 2007 - Geodetic Week 2007

Session 1

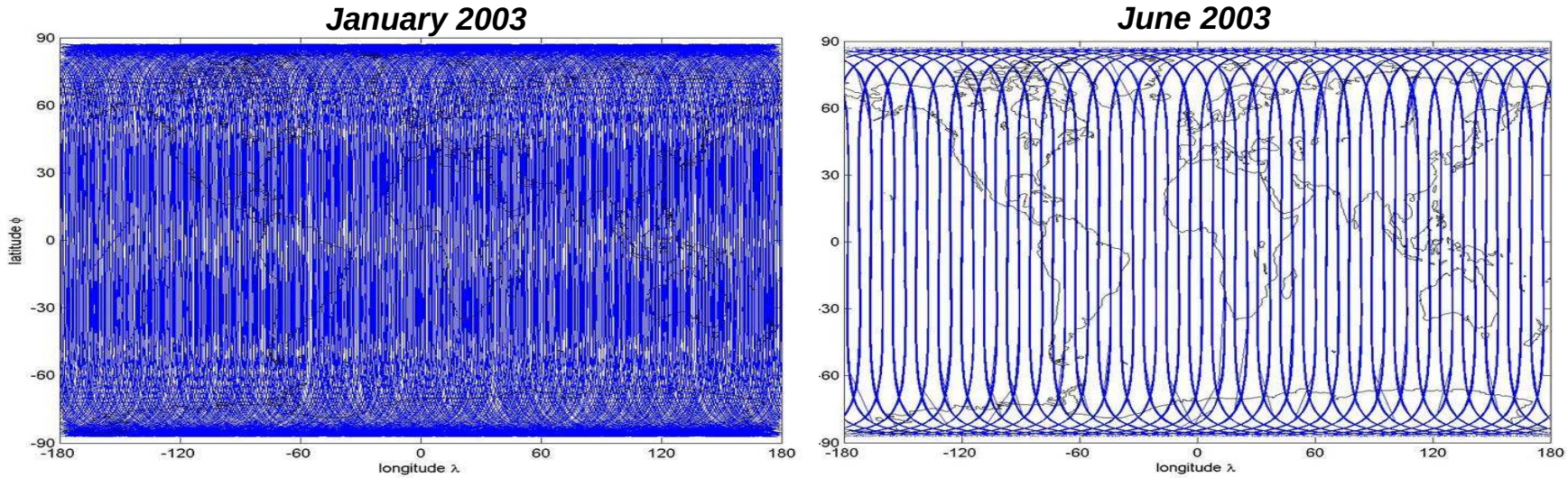
25th September, 2007

Outline

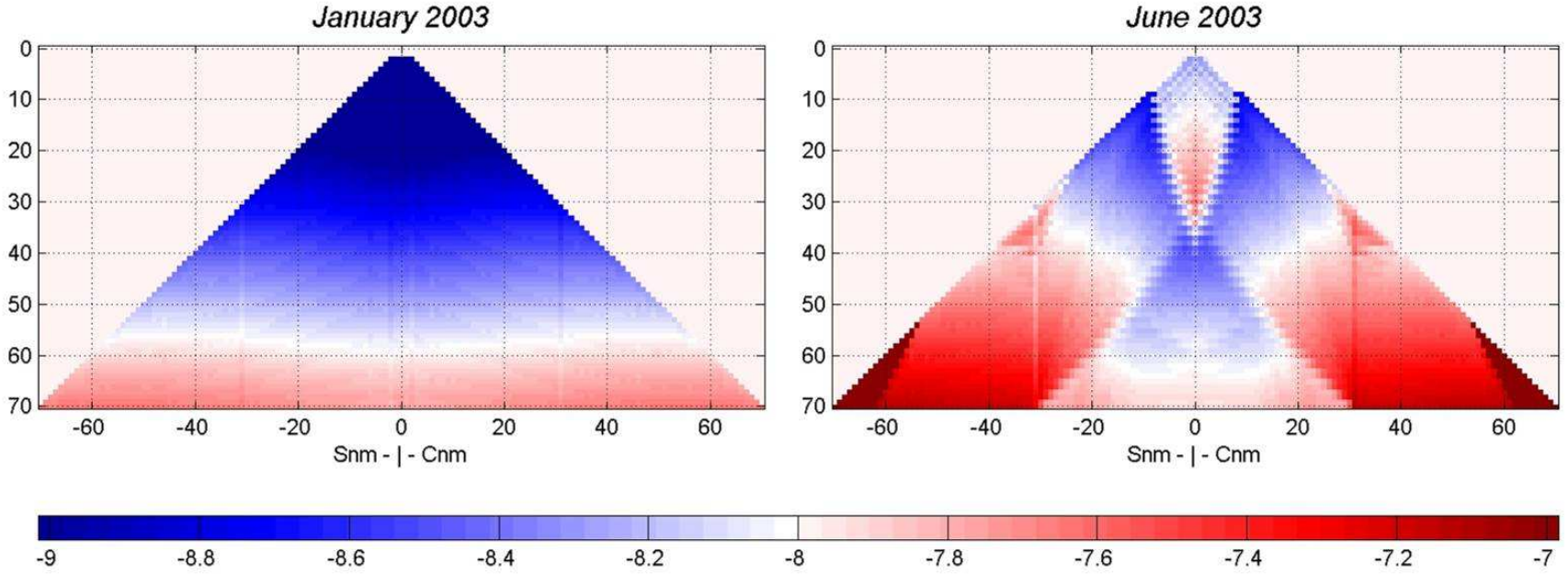
- Motivation for local gravity field recovery
- Slepian functions - an empirical base
  - Approach
  - Properties
- Test Scenario: CHAMP in high latitude areas

Motivation – Influence of the ground track

Ground track:

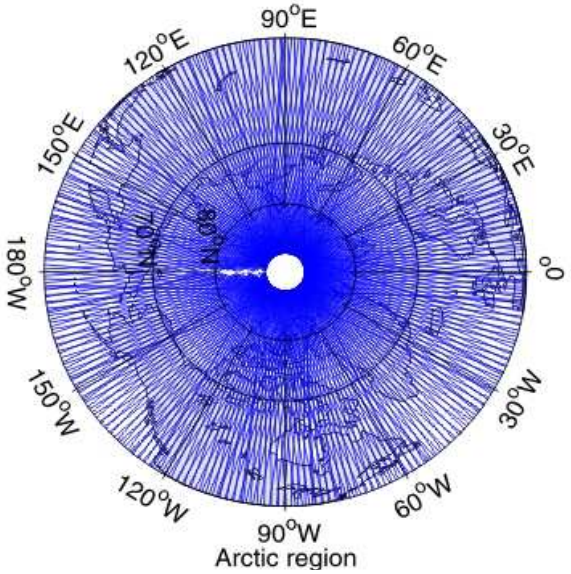


Error spectrum:

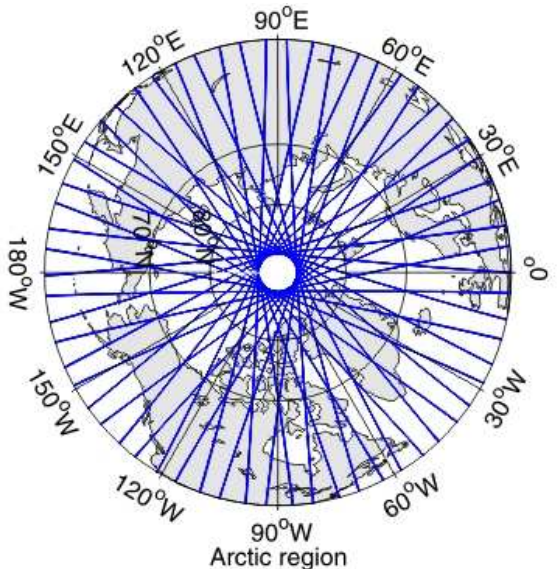


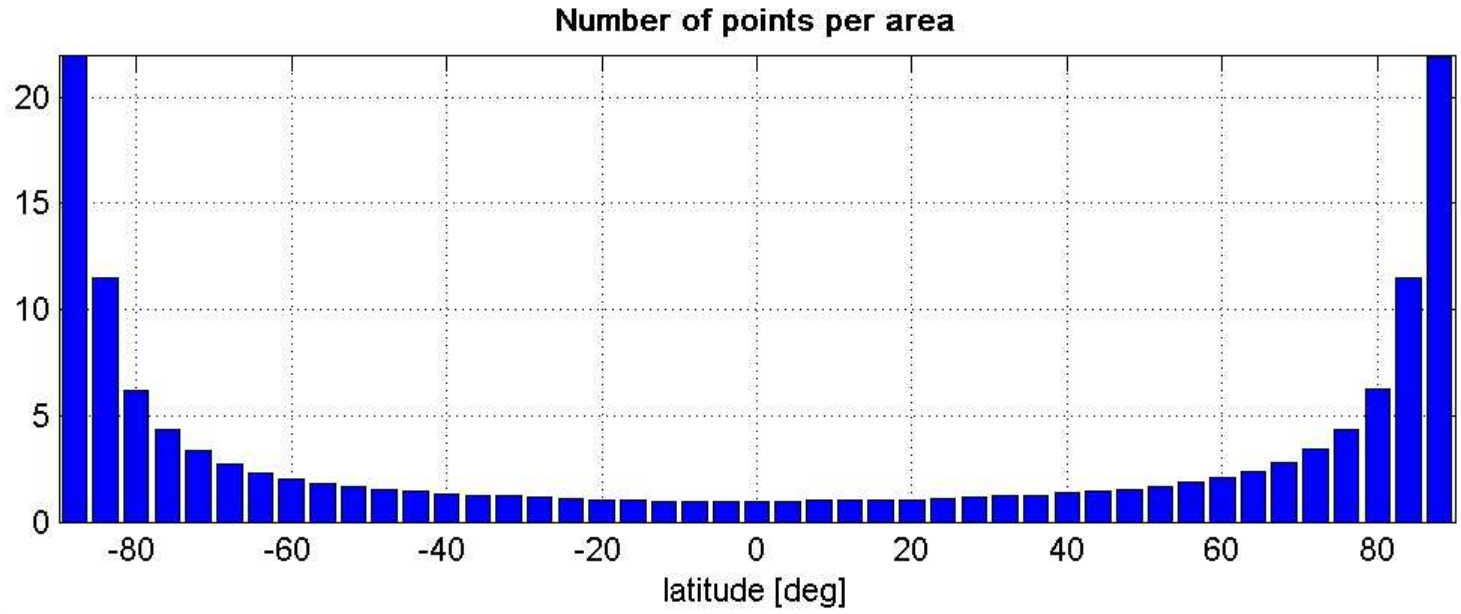
Motivation – Data density

January 2003



June 2003





Slepian functions

- Spatial concentration of bandlimited functions (Simons et al. 2005):
  - given:
$$S = \sum_{l=0}^{L} \sum_{m=-l}^{l} g_{lm} Y^{lm}$$
  - approach:
$$\lambda = \frac{\int_R S^2 \, d\Omega}{\int_\Omega S^2 \, d\Omega} = \text{maximum}$$
  - yields an algebraic eigenvalue problem:
$$\lambda \vec{g} = D\vec{g} \quad \text{with} \quad D = \int_R Y_{lm} Y_{nk} d\Omega$$
$$\rightarrow D = G\Lambda G^{-1}$$

Potential and Orthogonality

- Slepian representation of the potential:

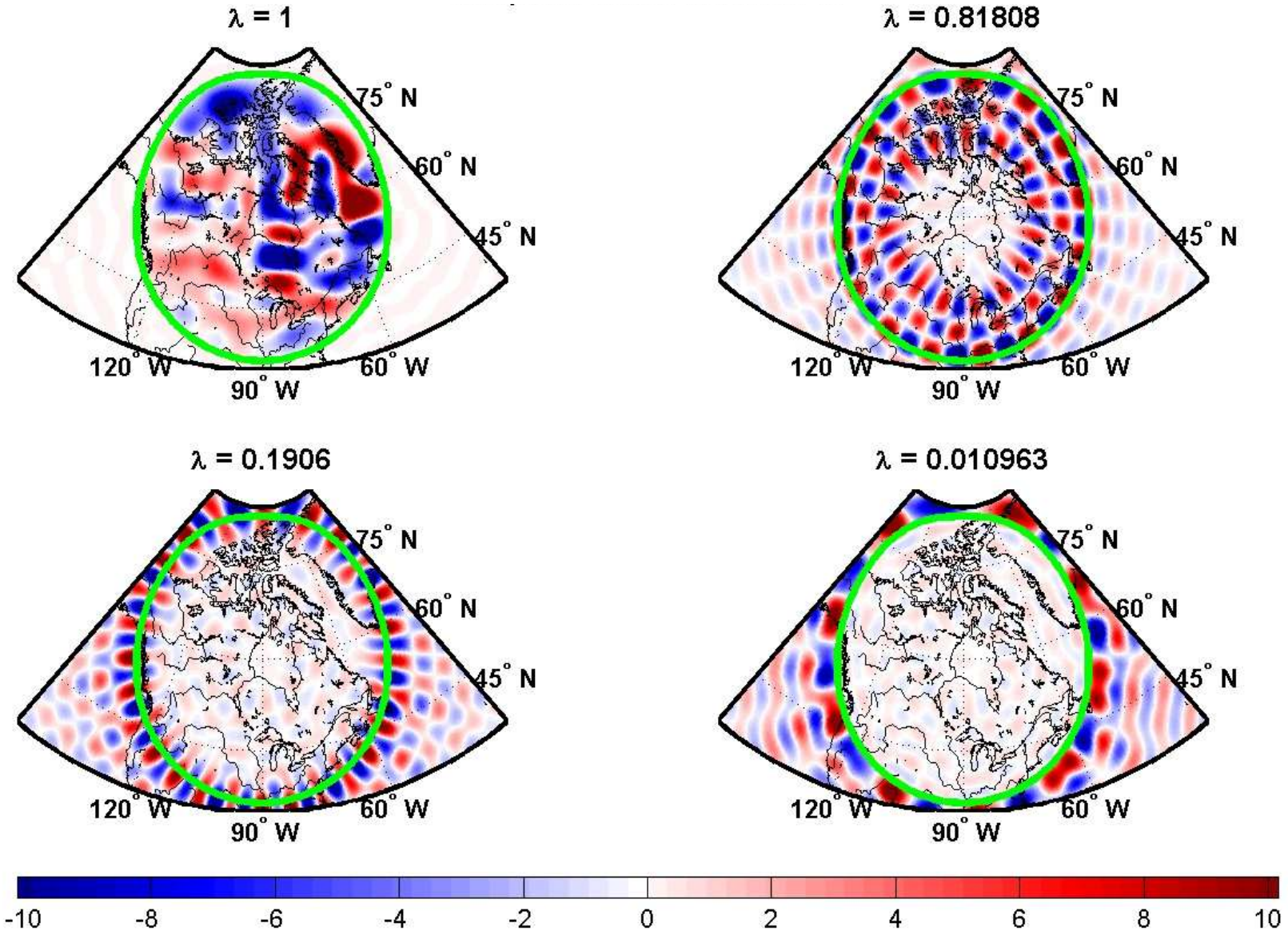
$$T = \frac{GM}{R} \sum_{i=1}^{(L+1)^2} \beta_i S_i = \frac{GM}{R} \sum_{i=1}^{(L+1)^2} \beta_i \sum_{l=0}^{L} \sum_{m=-l}^{l} \left(\frac{R}{r}\right)^{l+1} g_{lm}^i Y^{lm}$$

- double orthogonality (Gilbert & Slepian 1977)

$$\int_\Omega g_{lm}^i \, g_{lm}^j d\Omega = \delta_{ij} \qquad \int_R g_{lm}^i g_{lm}^j d\Omega = \lambda_{ij} \delta_{ij}$$

Localization property

- indication of spatial concentration by eigenvalues

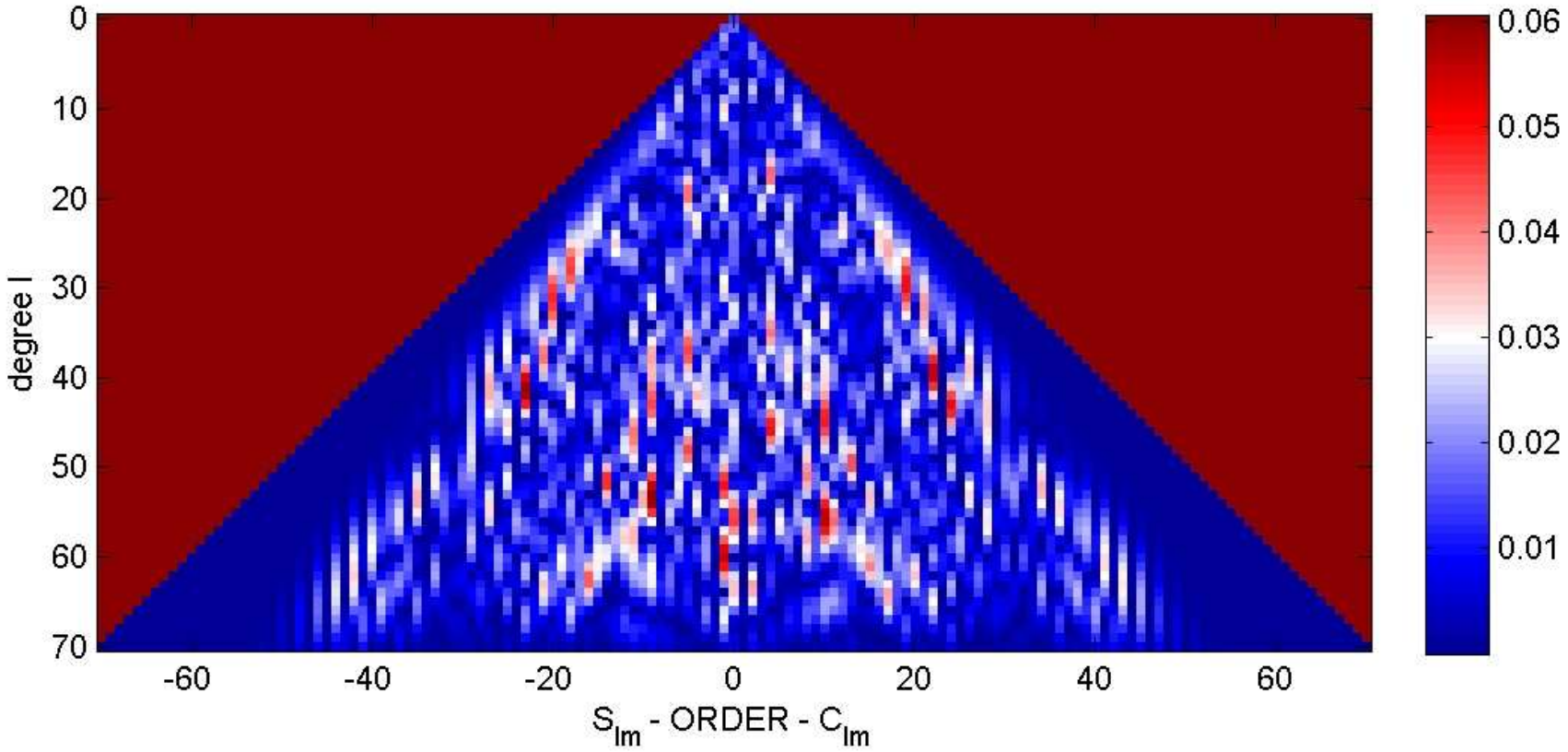


Spherical harmonics ↔ Slepian coefficients

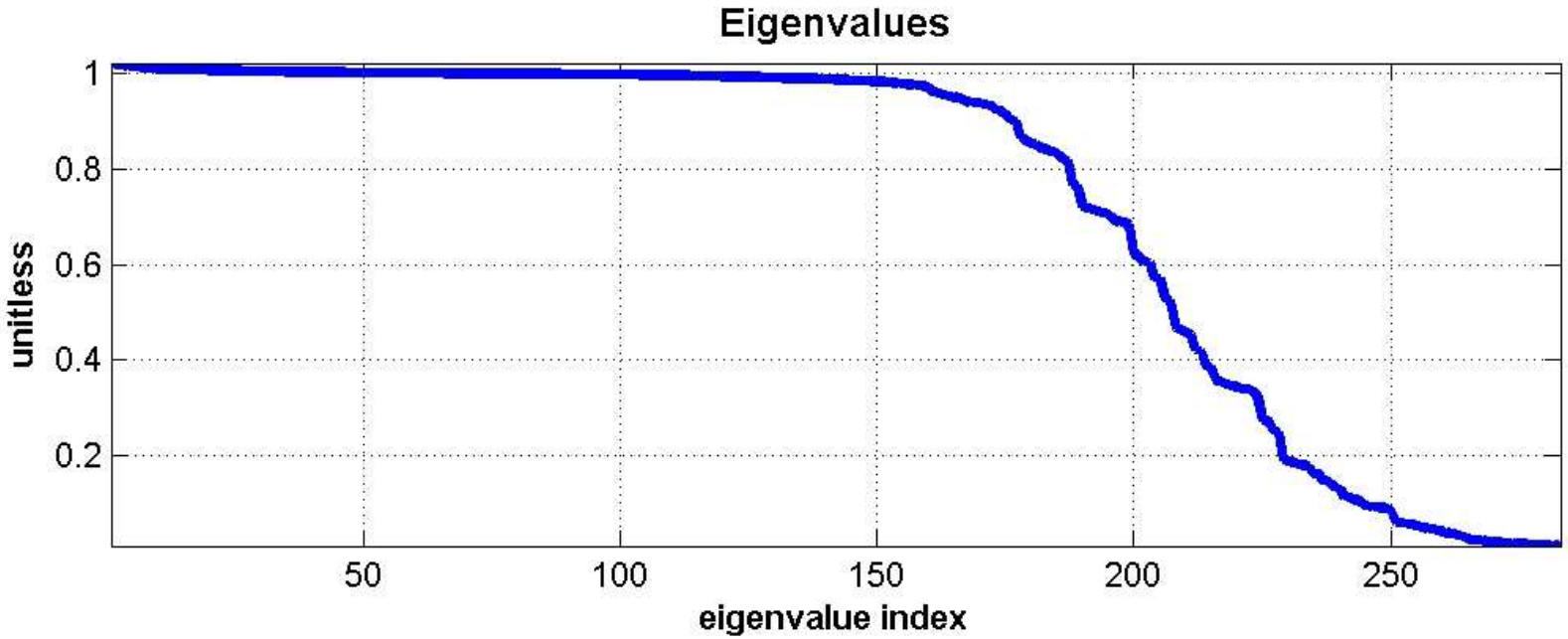
- transformation:

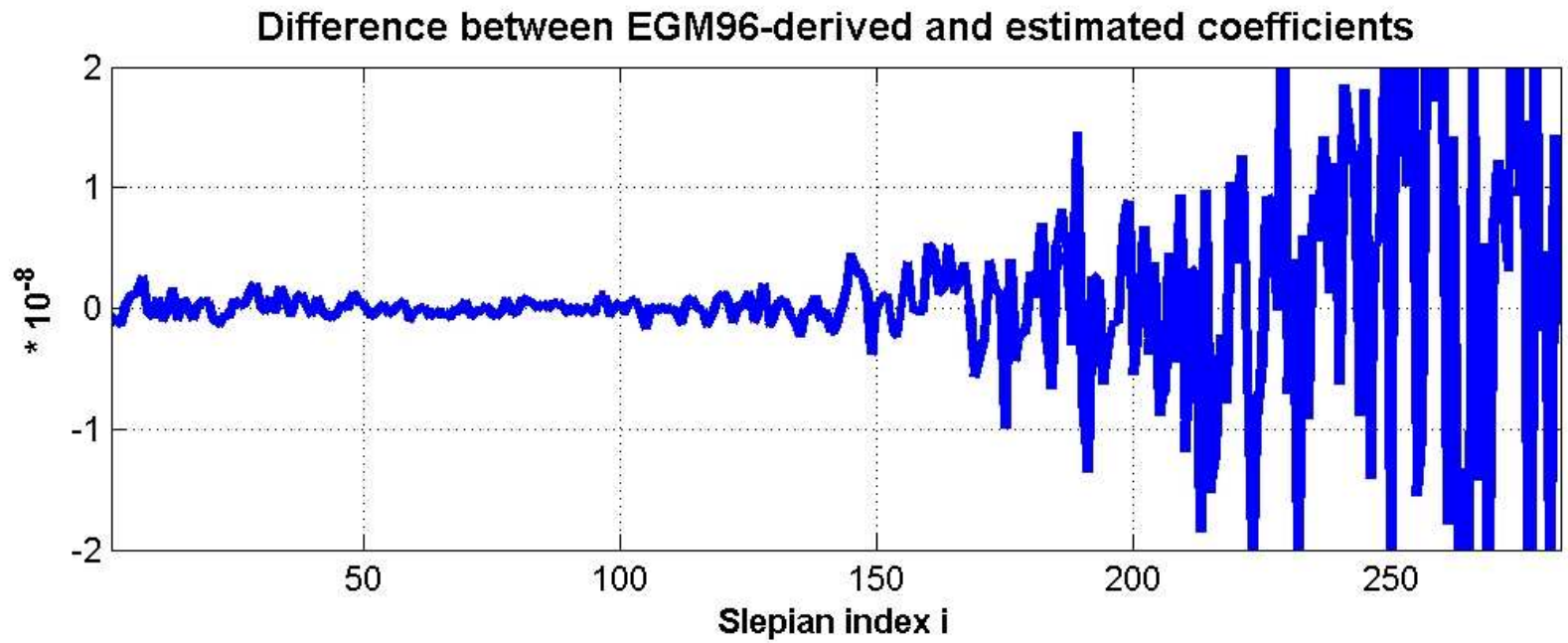
$$\beta_i = \sum_{l=0}^{L} \sum_{m=-l}^{l} K_{lm} g_i^{lm} \qquad K_{lm} = \sum_{i=1}^{(L+1)^2} \beta_i \, g_{lm}^i$$

- influence of one Slepian coefficient, e.g., g_{lm}^3 :



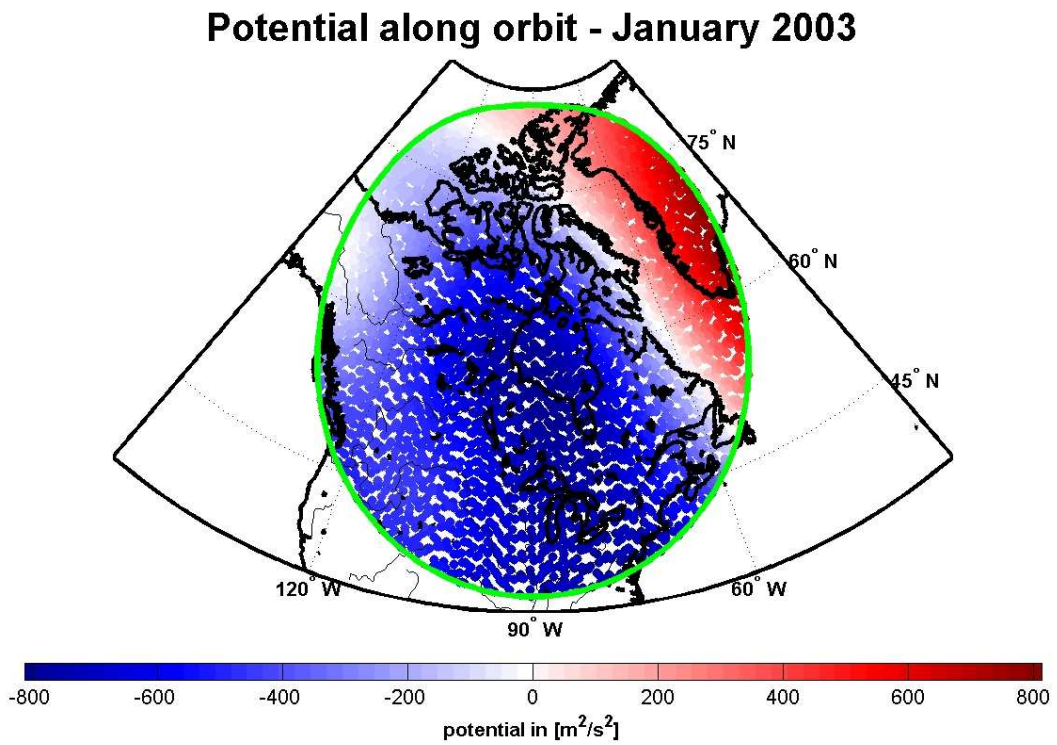
Edge effect

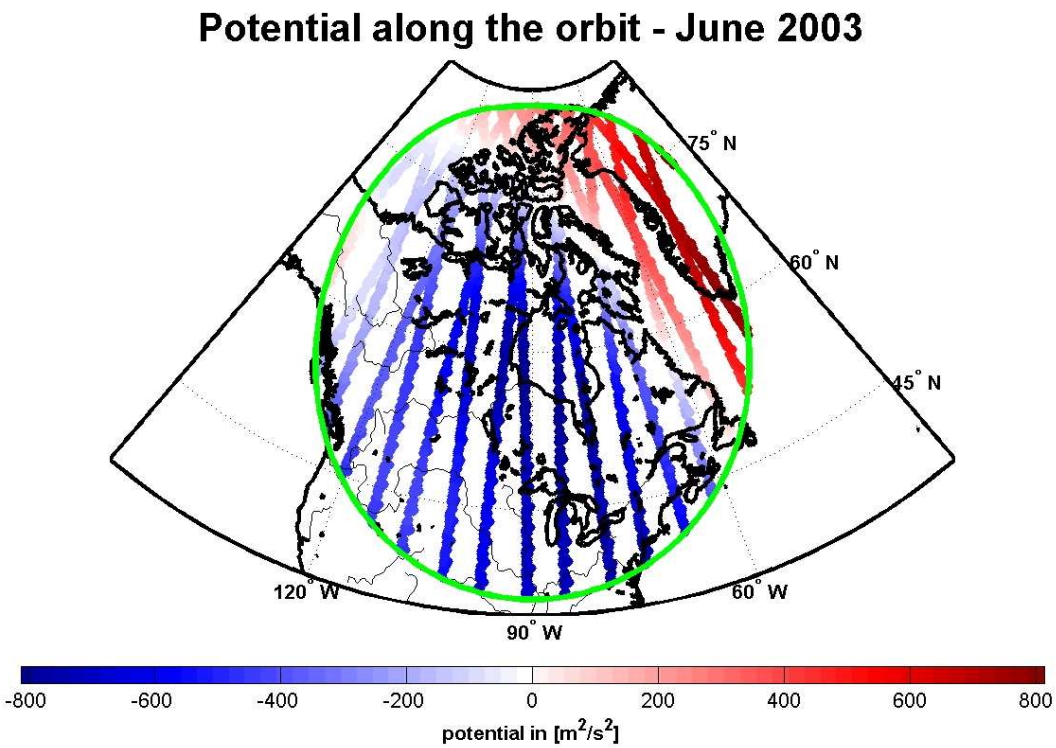

Eigenvalues
unitless
1
0.8
0.6
0.4
0.2
50
100
150
200
250
eigenvalue index


Difference between EGM96-derived and estimated coefficients
* 10^-8
2
1
0
-1
-2
50
100
150
200
250
Slepian index i

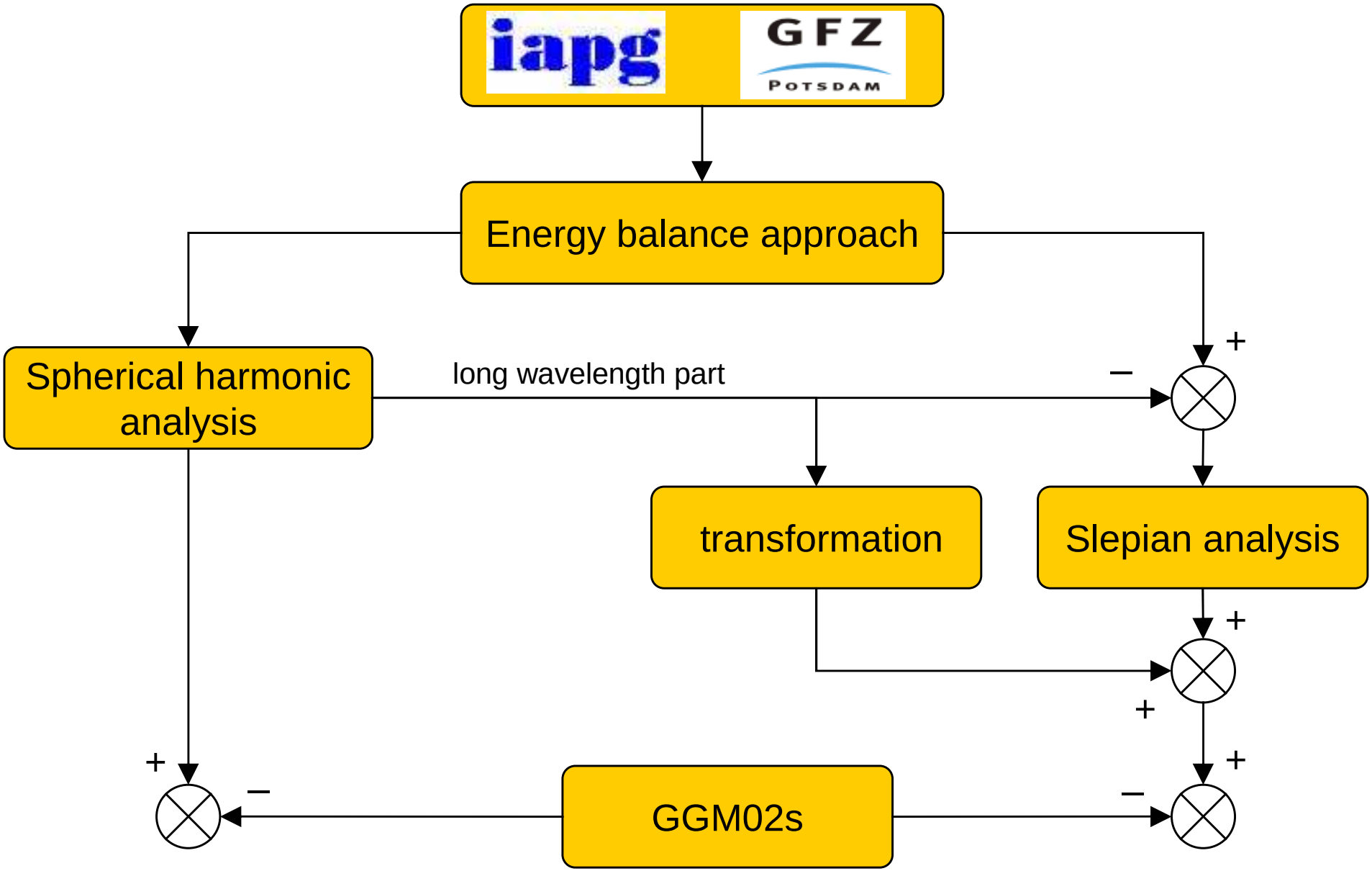
Test scenario

- monthly CHAMP solutions in high-latitude areas
- residual signal: $T(\lambda, \phi, r) - T^{SH}_{Lmax}(\lambda, \phi, r)$
- GGM02s as reference

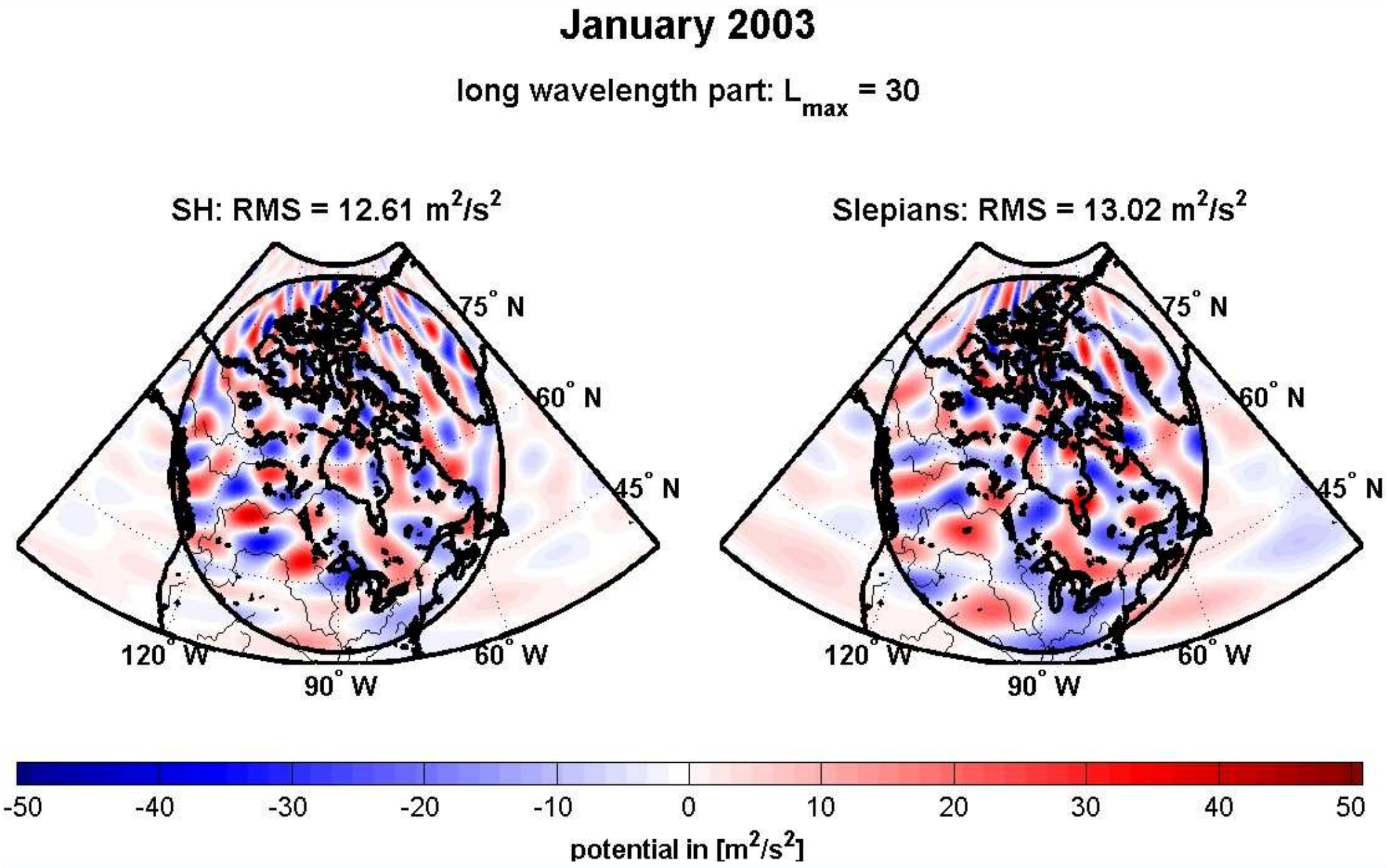




Processing scheme



Spherical harmonic vs. Slepian functions w.r.t. GGM02s



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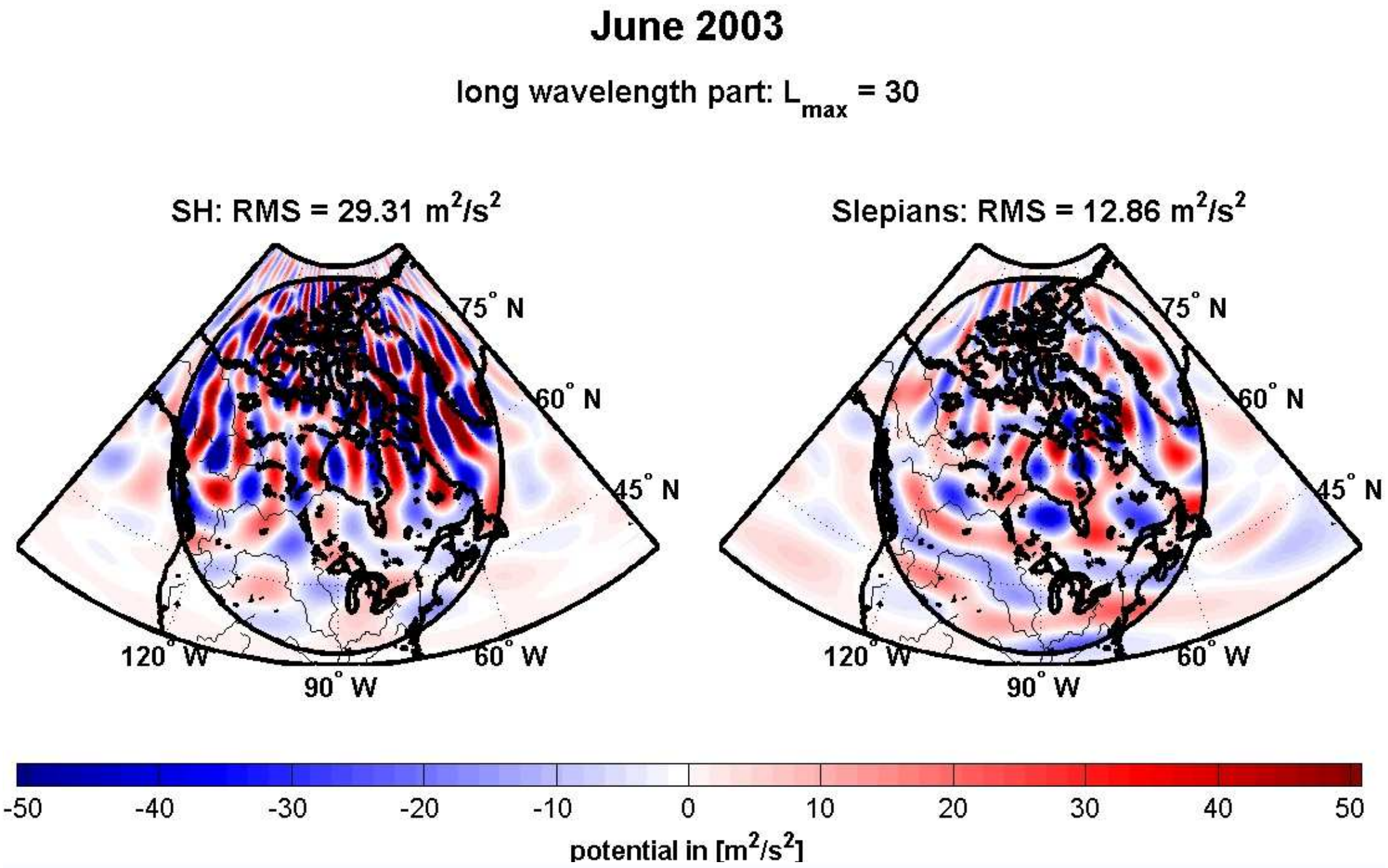

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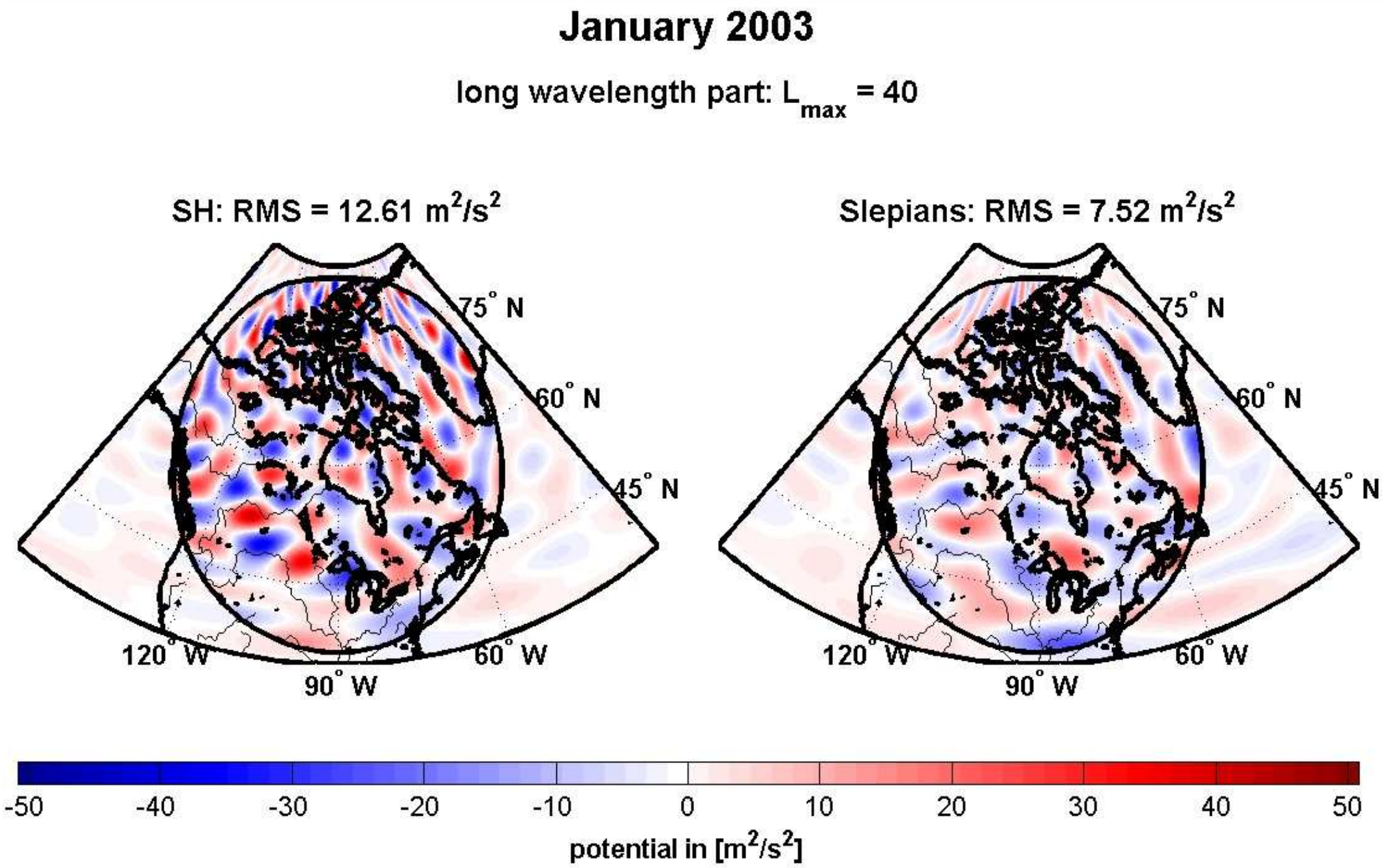
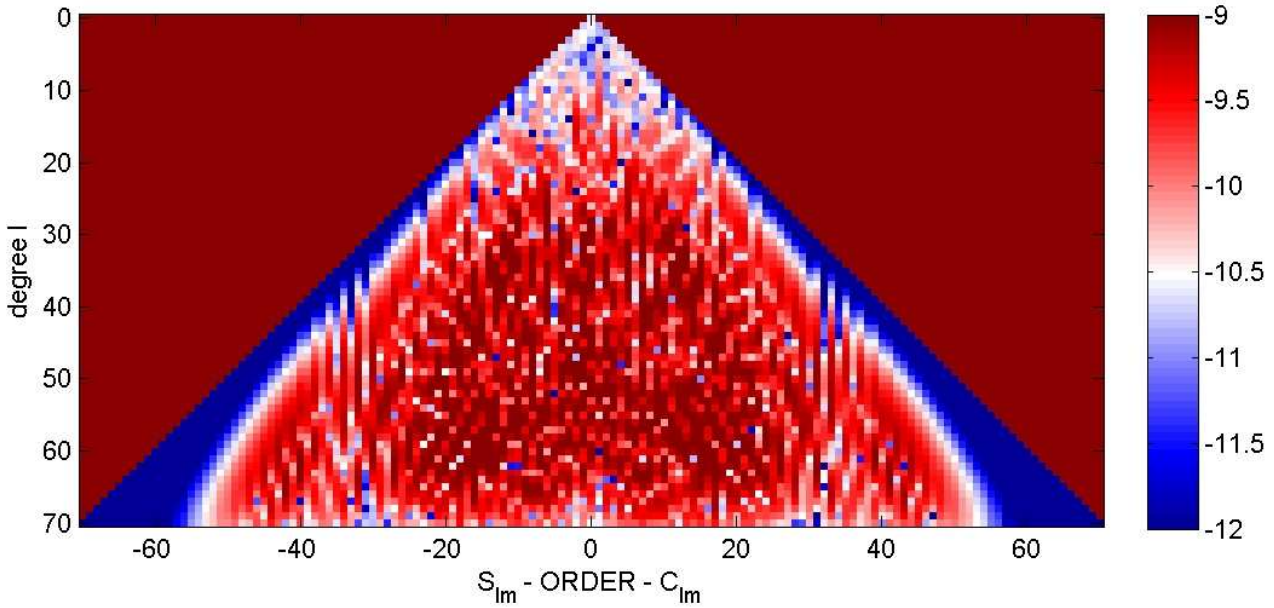
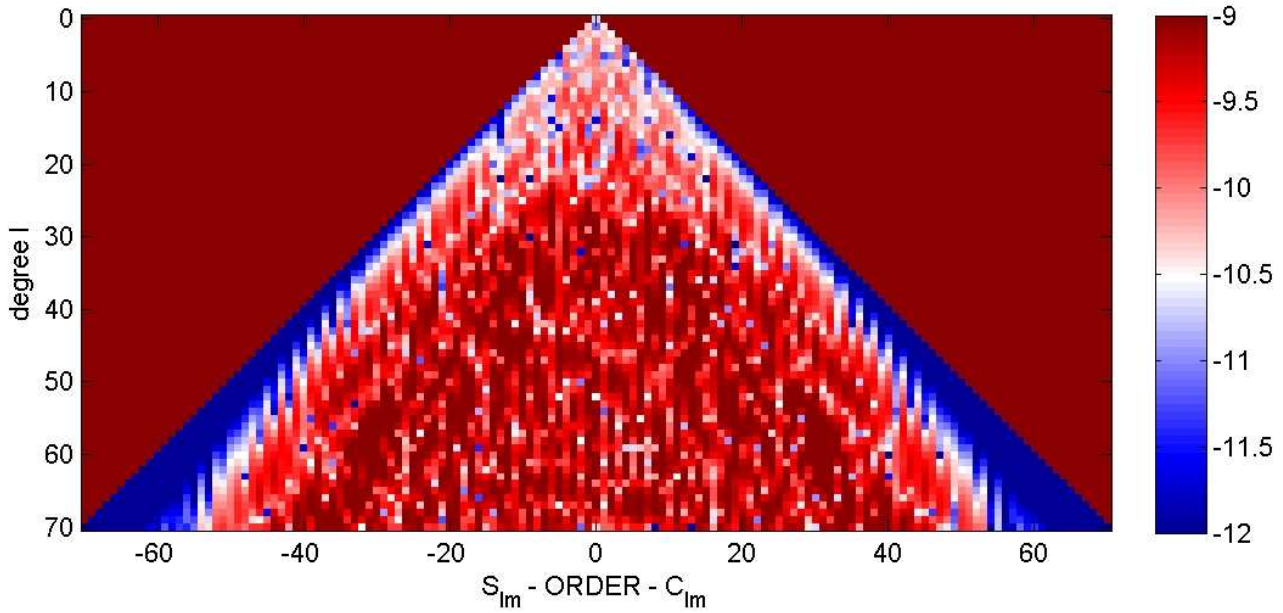


Affected coefficients

January 2003



June 2003



Conclusions

- Slepian functions as a empiricial base function
  - maximum spatial concentration of a bandlimited function
  - double orthogonality
  - spatial concentration indicated by eigenvalue
- improvement in high latitude areas
- dependency on a priori information (edge effect)
- high computational effort

THANK YOU

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