# Long wavelength gravity field determination from GOCE using the acceleration approach

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## Introduction

In the GOCE (Gravity field and steady-state Ocean Circulation Explorer) mission two types of techniques are used for the recovery of the gravity field: gradiometry for the medium to short wavelengths and high-low satellite-to-satellite tracking (hl-SST) for the long wavelength features. For the latter, it is necessary to make use of GPS observations due to the limited measurement bandwidth of the gradiometer. We focus on this part in this contribution. Currently, the processing facilities derive the long-wavelength features by using the energy conservation approach. We propose to use the acceleration approach, instead, as earlier studies for CHAMP showed that it offers a superior alternative. The procedure aims at the optimal recovery of a GOCE-only solution which is one of the key objectives within the ESA's Living Planet Programme.

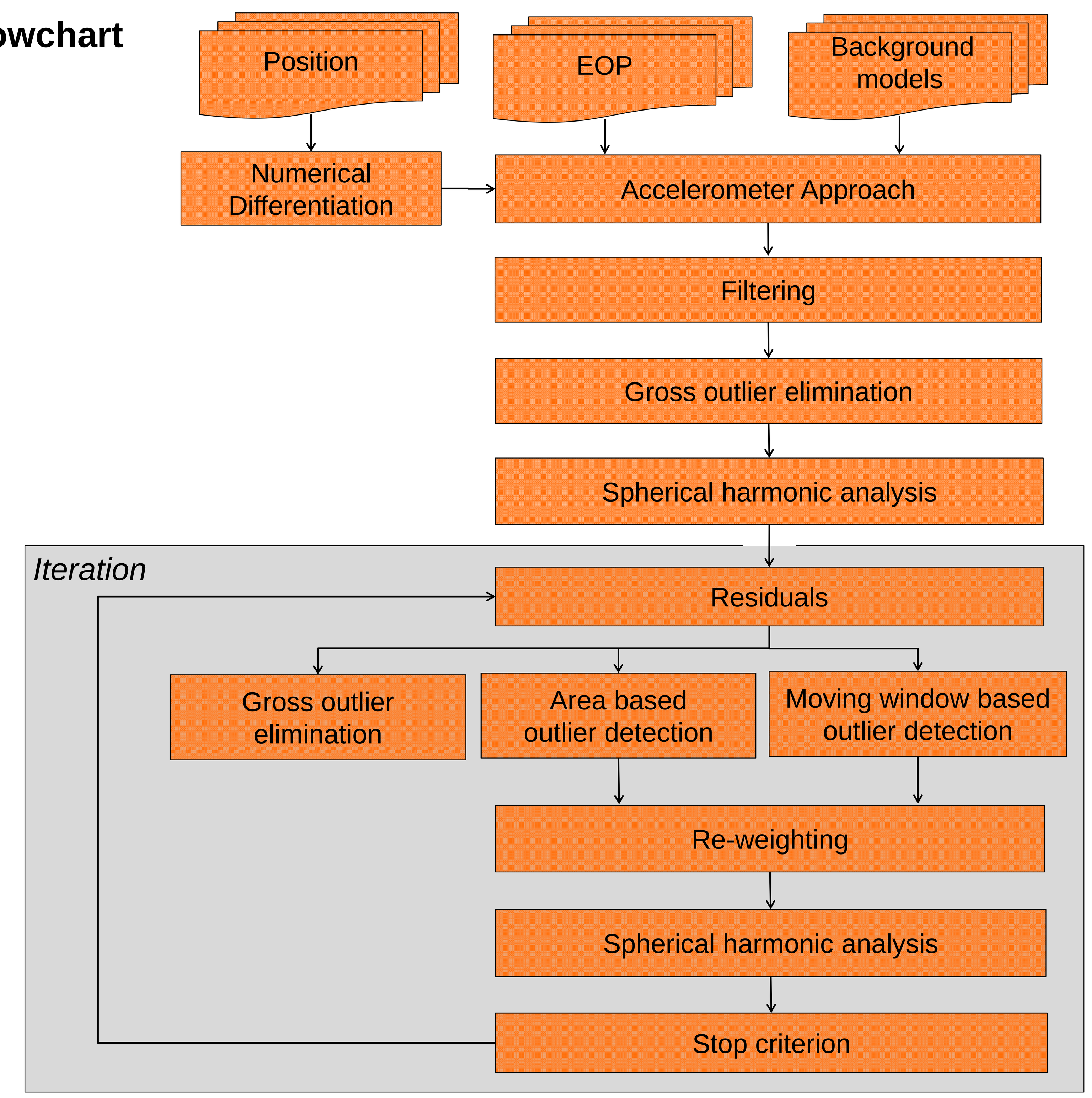
## Methodology

The acceleration approach is based on Newton's equation of motion in the inertial frame. Kinematically derived positions are double differentiated and gravitational and non-gravitational disturbing forces are substracted. Since the gravitational field $\nabla V$ is most conveniently modeled in the local north-oriented frame, rotations need to be applied. The basic equation is then

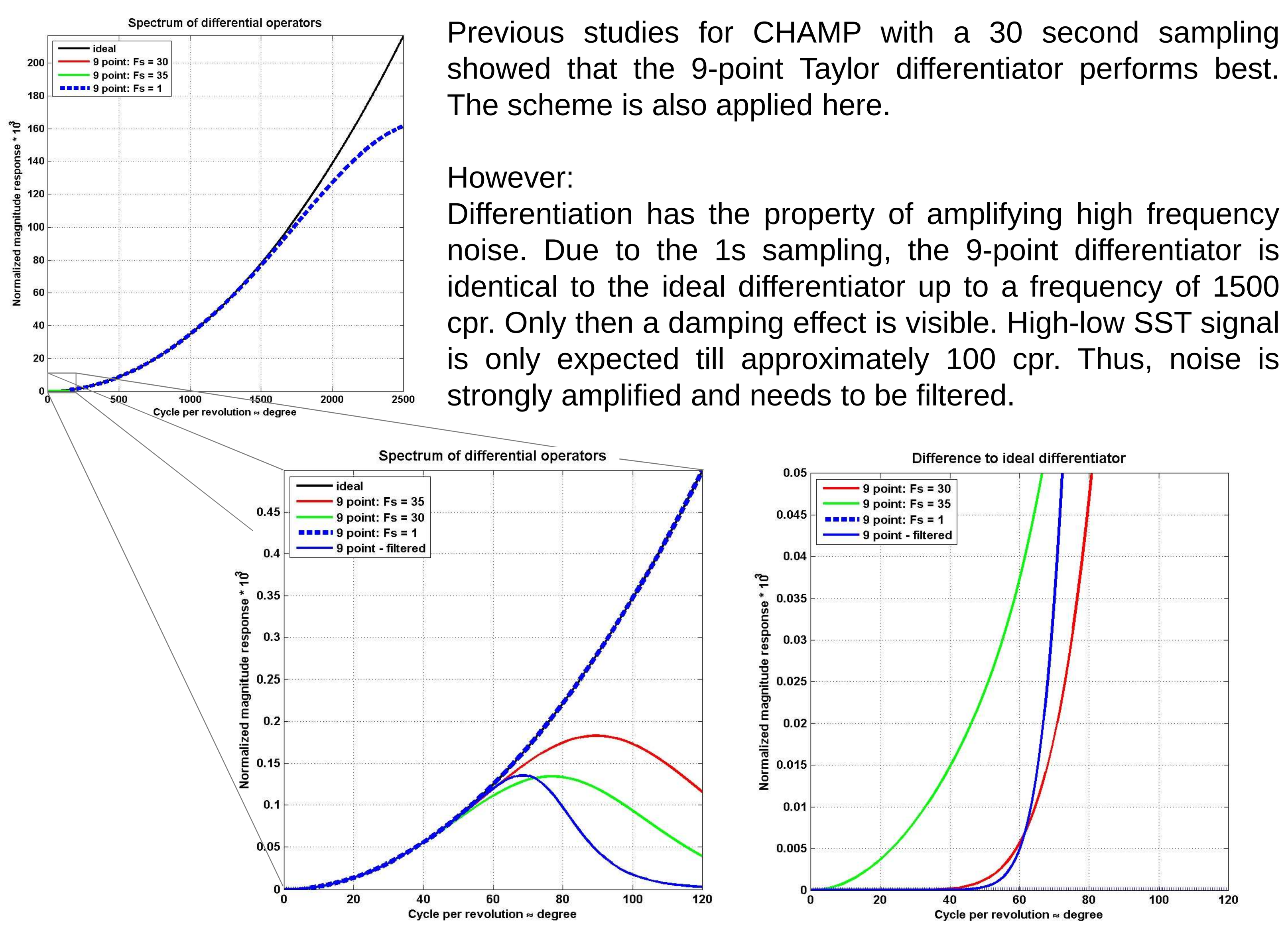
$$\nabla V = R_{\mathrm{L}}^{\mathrm{E}} R_{\mathrm{E}}^{\mathrm{I}} \left( \ddot{\mathbf{x}} - \mathbf{f}^{\mathrm{3rdBody}} - \mathbf{f}^{\mathrm{Tides}} - \mathbf{f}^{\mathrm{Rel}} - \mathbf{f}^{\mathrm{Grav}} \right)$$

with $\ddot{\mathbf{x}}$ = kinematically derived accelerations of the satellite,
$\mathbf{f}^{\mathrm{3rdBody}}$ = direct forces exerted by third bodies like the Sun, Moon and others,
$\mathbf{f}^{\mathrm{Tides}}$ = tidal forces (solid Earth, ocean, solid Earth and ocean pole, atmosphere),
$\mathbf{f}^{\mathrm{Rel}}$ = relativistic corrections,
$\mathbf{f}^{\mathrm{Grav}}$ = time variable gravitational changes (dealiasing products),
$R_{\mathrm{E}}^{\mathrm{I}}$ = rotation matrix from inertial to Earth-fixed frame and
$R_{\mathrm{L}}^{\mathrm{E}}$ = rotation matrix from Earth-fixed to the local north-oriented frame.

## Flowchart




## Numerical Differentiation and Filtering



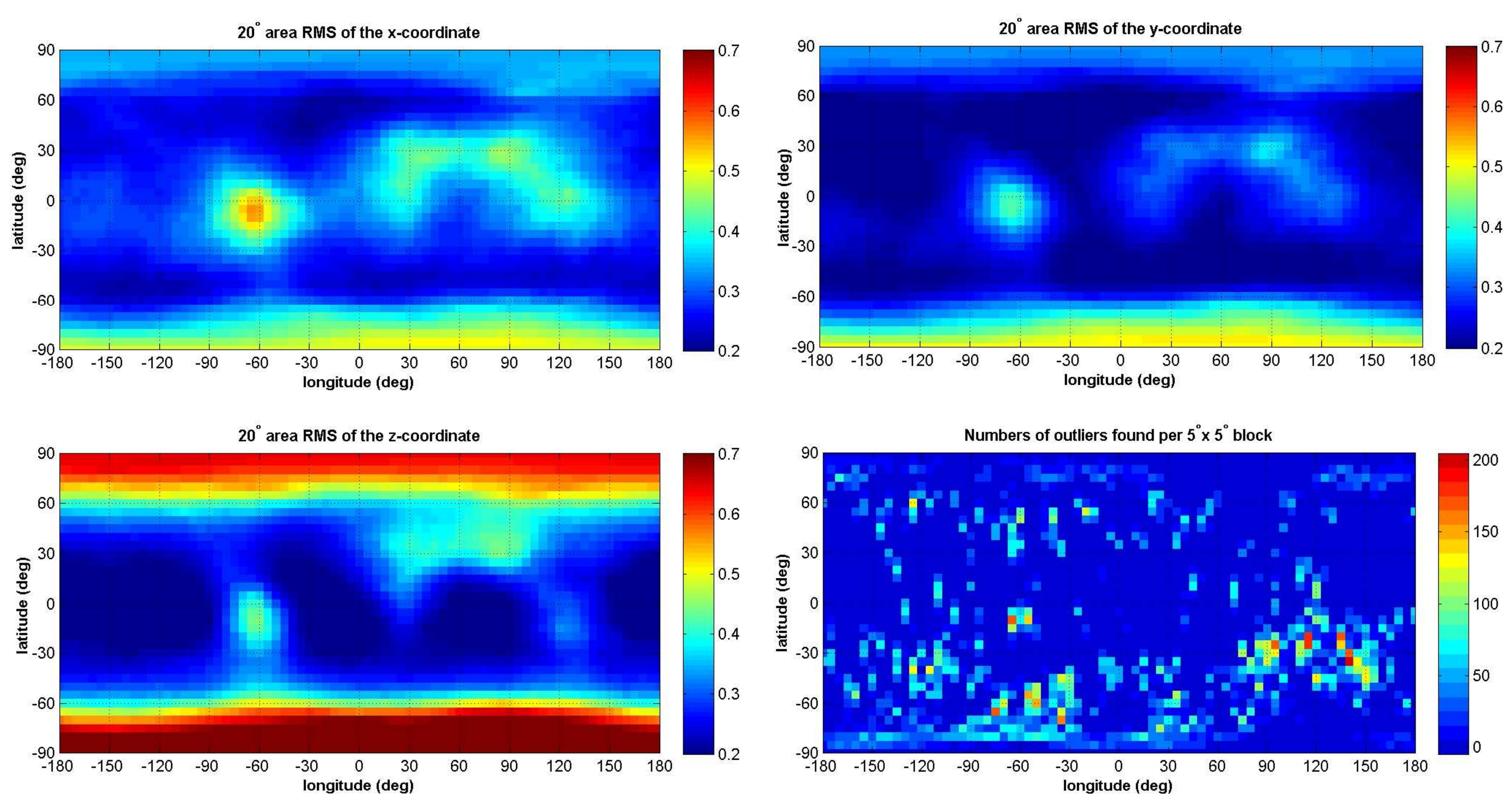

Previous studies for CHAMP with a 30 second sampling showed that the 9-point Taylor differentiator performs best. The scheme is also applied here.

However:
Differentiation has the property of amplifying high frequency noise. Due to the 1s sampling, the 9-point differentiator is identical to the ideal differentiator up to a frequency of 1500 cpr. Only then a damping effect is visible. High-low SST signal is only expected till approximately 100 cpr. Thus, noise is strongly amplified and needs to be filtered.

The sophisticated approach is to design an IIR low-pass filter. This offers high flexibility in the design but needs to account for warmup effects which yields a loss of data. A simpler alternative is to use points every 30 or 35 seconds for differentiation and shift the scheme by 1 second afterwards. The magnitude response above shows that similar filtering is achieved.

## Outlier detection & robust estimation

Investigations show that a simple threshold based outlier detection can only be used for gross outlier determination. Additionally, poor observations are not necessarily indicated by the variance information. Further, the RMS between the solutions of two iteration steps depend on the spatial location and the coordinate.

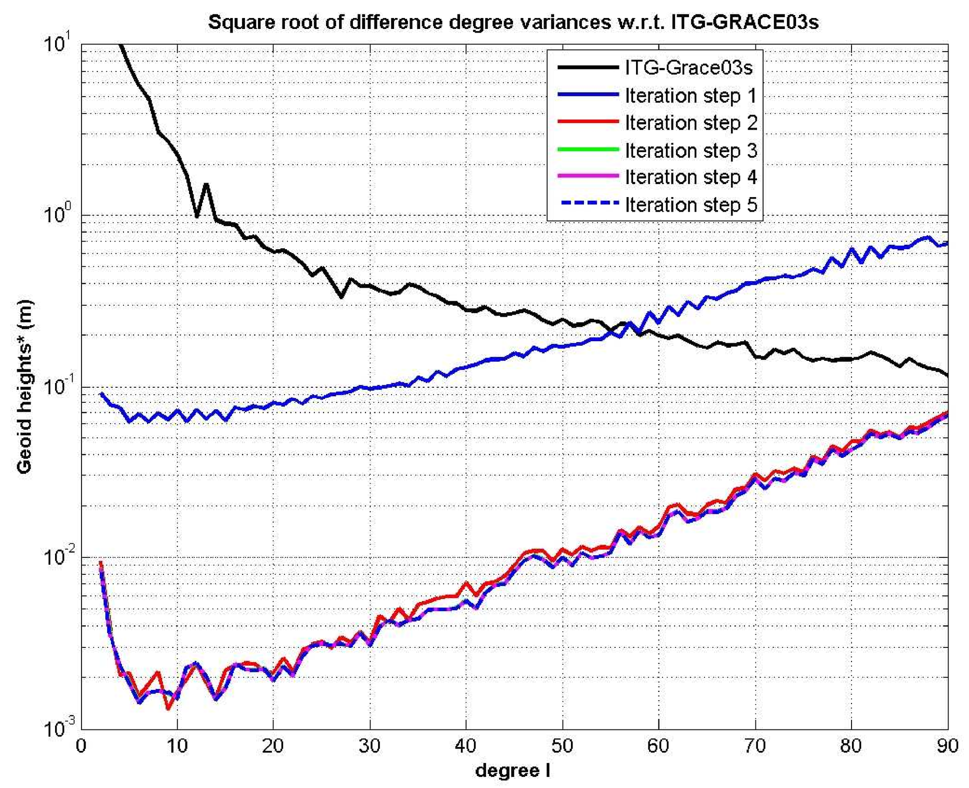



In order to account for the area- and coordinate-dependency, the RMS has been calculated for 5°x 5° blocks using all the available data in a spherical cap of 20 ° around the center of the block. Observations in a specific block deviating more than 3 times from the RMS are downweighted using the difference between the observation and the reconstructed signal of the *i*-th iteration step. The procedure is equivalent to a robust estimation using a Huber estimator.

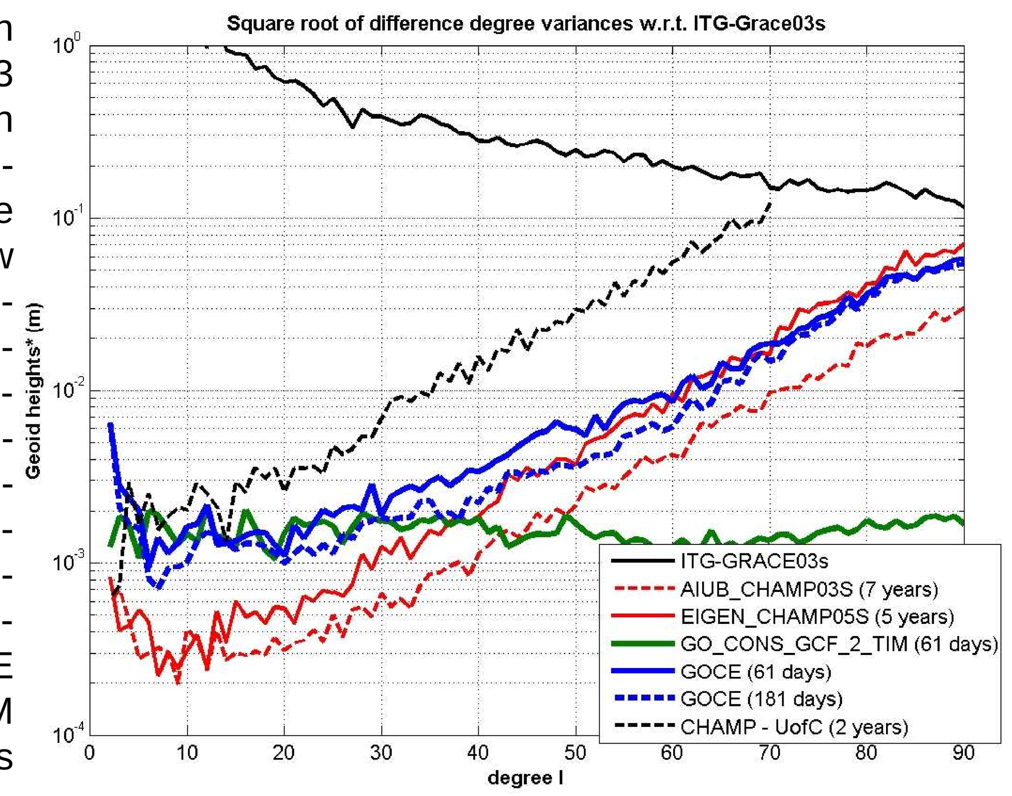
The number of outliers found indicates on the one hand specific arcs, which have been poorly determined, but also shows local concentrations. The reason for the later is not understood, yet.

## Results

The figure on the right shows the results of one month which has been iterated 5 times. The iteration converged after 3 steps. The importance of a proper outlier handling becomes visible between step 1 and 2. In the former case only gross outliers have been removed. In the later, poor observations have been down-weighted additionally.




The lower right figure shows the comparison of the GOCE solutions calculated by the acceleration approach using 61 and 181 days of data, respectively. The solutions outperform the 2-year CHAMP solution UofC for degrees higher than 3 and the 5 year CHAMP solution EIGEN-CHAMP05s for degrees higher than 45 due to the lower orbit of GOCE. In the low degrees the long-term solutions of CHAMP are still significantly better. The GOCE solutions exhibits a loss of accuracy in the low degrees. Possible reasons are neglected covariance information and non-gravitational forces. The comparison to the time-wise GOCE solution GO-CONS-GCF-2-TIM suggest that an improvement is possible using this approach.




## Conclusions

It has been shown that the acceleration approach offers an improvement to the energy balance approach and thus poses an interesting alternative to the current processing strategy. The 61 day solution outperforms already the 2 and 5 year solutions from CHAMP primarily due to the lower orbit of GOCE. Currently, the solutions exhibit problems in the low degree harmonics 2-5. The reason for this is not yet understood. Possible causes are neglected covariance information and non-gravitational forces. Further improvements are also expected by refined filtering strategies and outlier detection. The former is expected to yield an improvement to the high degrees as currently to much signal has been filtered for degrees higher than 50.

## References

GO_CONS_GCF_2_TIM – R. Pail, H. Goiginger, R. Mayrhofer, W.-D. Schuh, J.M. Brockmann, I. Krasbutter, E. Hoeck, T. Fecher, GOCE gravity field model derived from orbit and gradiometry data applying the time-wise method, Proceedings of the ESA Living Planet Symposium, 28 June - 2 July 2010, Bergen, Norway

AIUB-CHAMP03s – L. Prange, A. Jäggi, G. Beutler, U. Meyer, L. Mervart, R. Dach, H. Bock, AIUB-CHAMP03S: A gravity field model from eight years of CHAMP GPS data. paper in preparation

EIGEN-CHAMP05s – F. Flechtner, Ch. Dahle, K.H. Neumayer, R. König and Ch. Förste, The Release 04 CHAMP and GRACE EIGEN Gravity Field Models, In: F. Flechtner, Th. Gruber, A. Güntner, M. Mandea, M. Rothacher, T. Schöne and J. Wickert (Eds.): System Earth via Geodetic-Geophysical Space Techniques, Springer, ISBN 978-3-642-10227-1, DOI 978-3-642-10228-8, pp 41 – 58, 2010

CHAMP – UofC – M. Weigelt, Global and local gravity field recovery from satellite-to-satellite tracking. Ph.D. thesis, University of Calgary, 2007

ITG-Grace03s – T. Mayer-Gürr, "Gravitationsfeldbestimmung aus der Analyse kurzer Bahnbögen am Beispiel der Satellitenmissionen CHAMP und GRACE" Dissertation, University of Bonn, 2006

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