The Decade of the Geopotential – techniques to observe the gravity field from space

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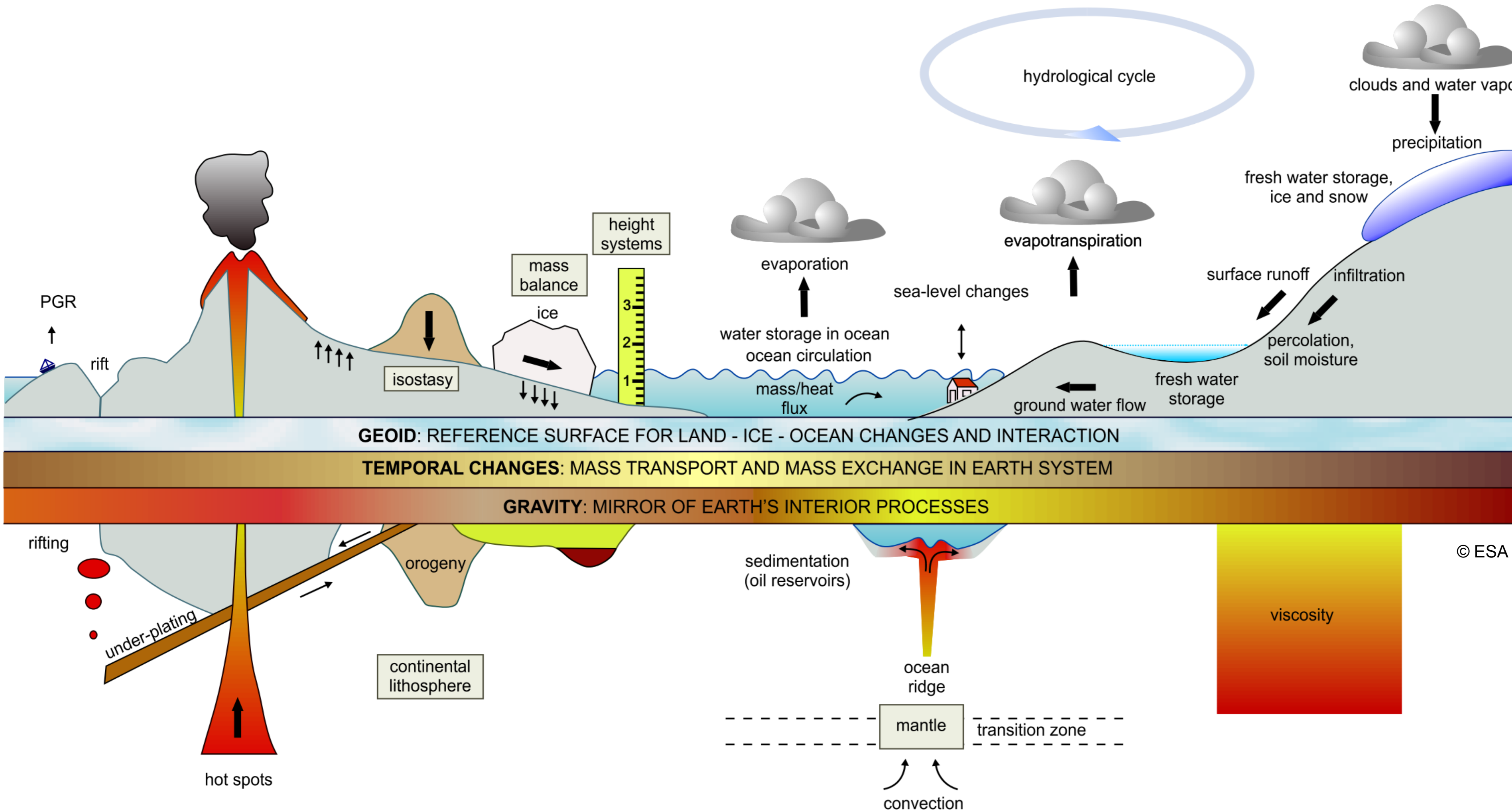
Geophysical implications of the gravity field



Gravity field modelling

$$V = \frac{GM}{R}\sum_{l=0}^{\infty}\left(\frac{R}{r}\right)^{l+1}\sum_{m=0}^{l}\bar{P}_{lm}\left(\sin\phi\right)\left(\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\right)$$

with	GM	gravitational constant times mass of the Earth
	R	radius of the Earth
	r, ϕ, λ	spherical coordinates of the calculation point
	$\bar{P}_{lm}$	Legendre function
	l, m	degree, order
	$\bar{C}_{lm}, \bar{S}_{lm}$	(unknown) spherical harmonic coefficients

Gravity field modelling

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with

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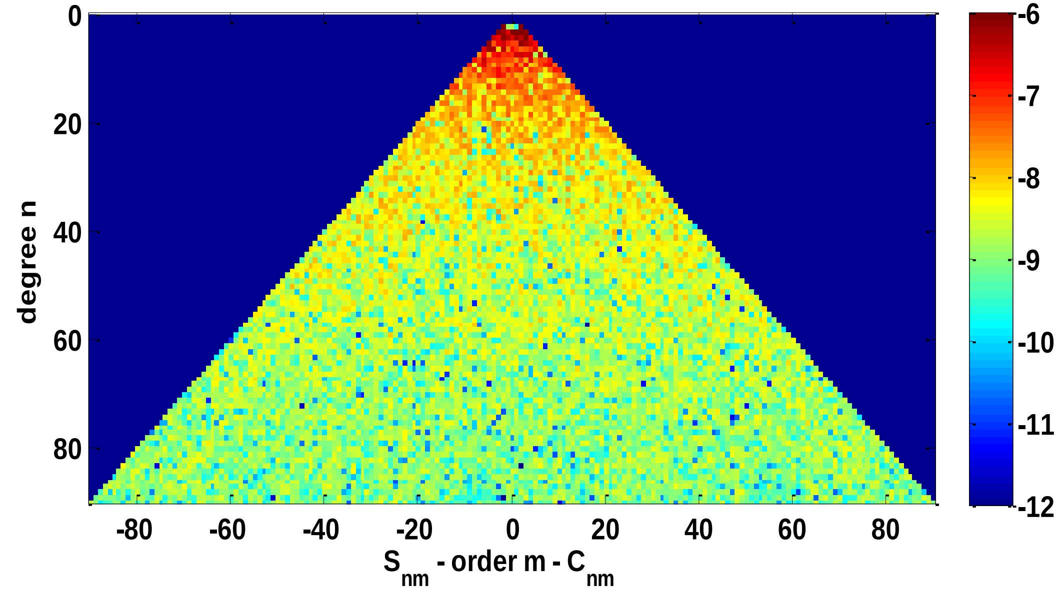
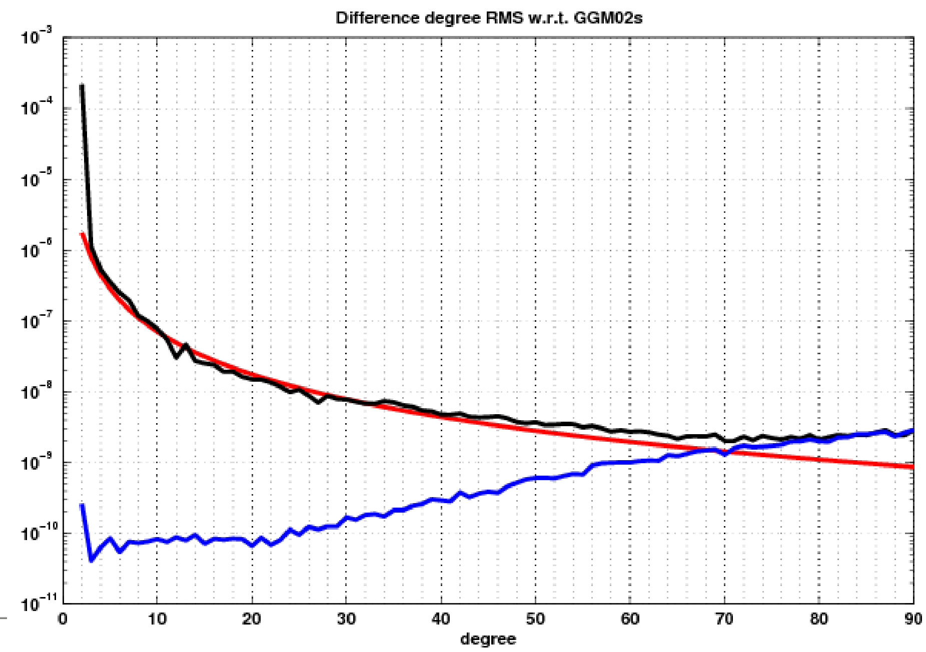
Spectral representation

Spherical harmonic representation



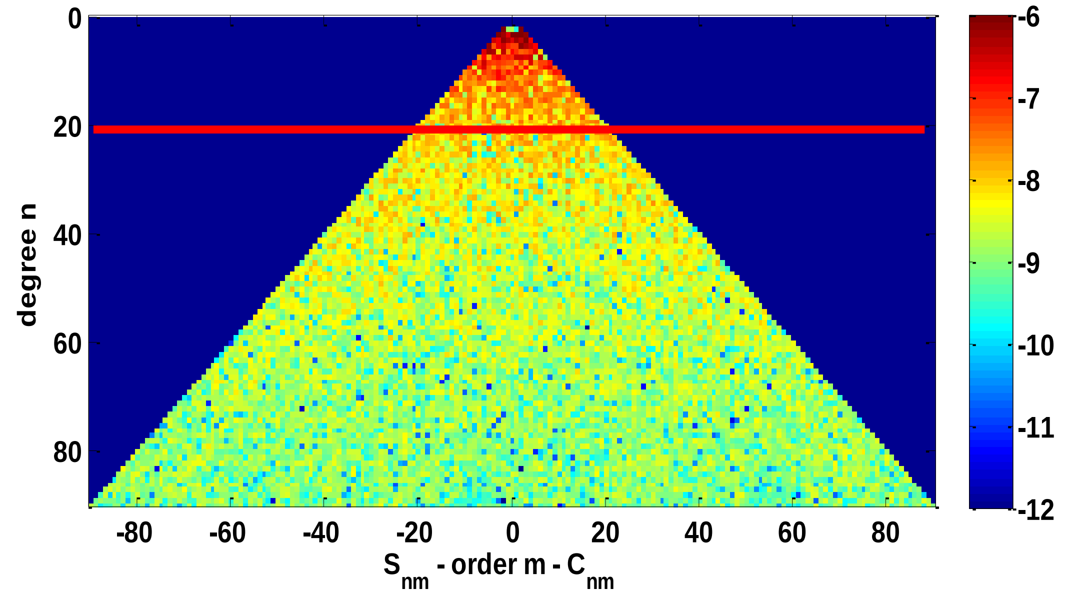
degree-RMS

$$c_n = \sqrt{\frac{1}{n}\sum_{m=0}^{n}\bar{C}_{\mathrm{nm}}^2 + \bar{S}_{\mathrm{nm}}^2}$$



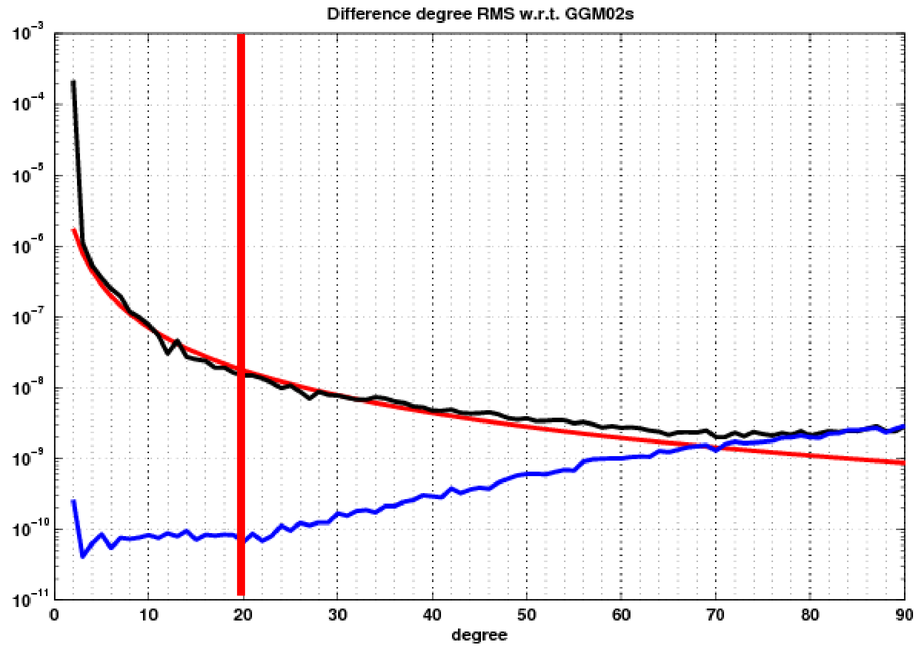
Spectral representation

Spherical harmonic representation



degree-RMS

$$c_n = \sqrt{\frac{1}{n}\sum_{m=0}^{n} \bar{C}^2_{\mathrm{nm}} + \bar{S}^2_{\mathrm{nm}}}$$

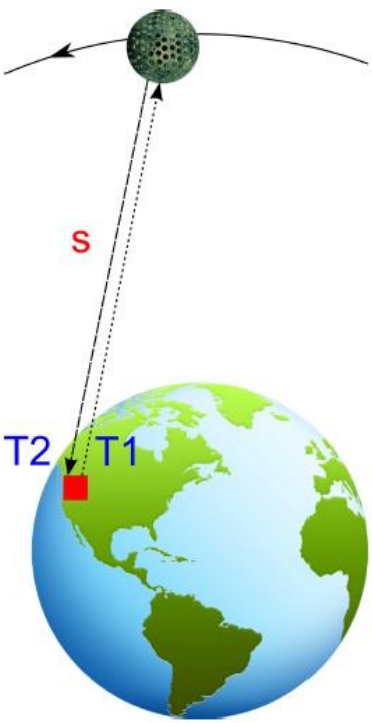


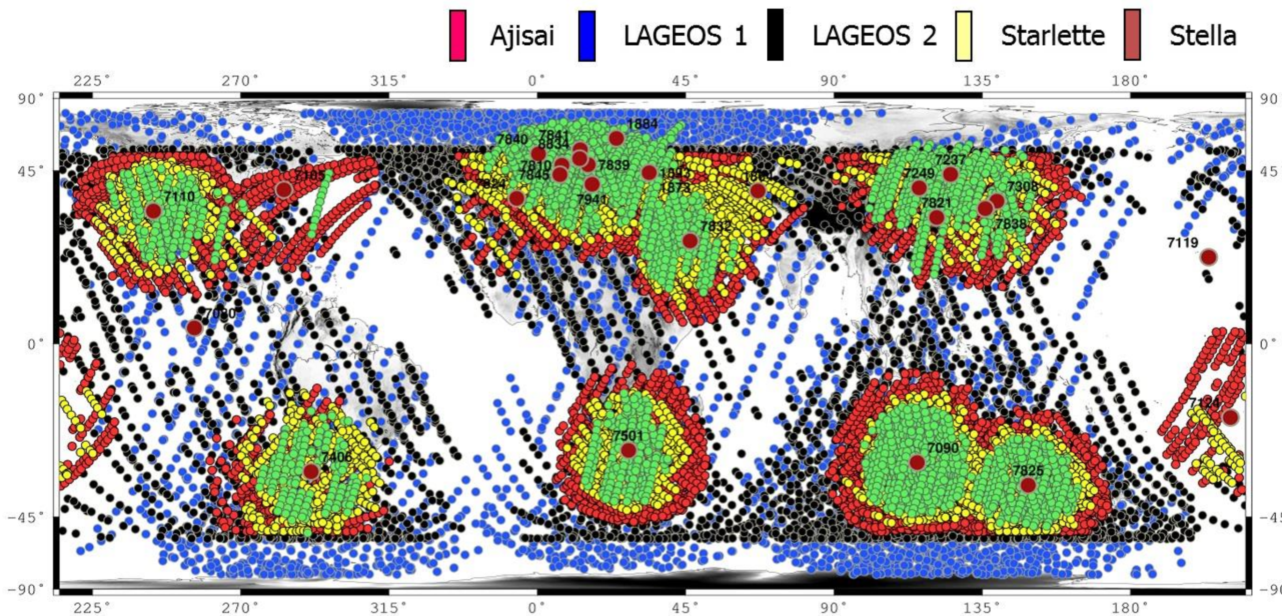
Pre CHAMP era

Space geodetic observation techniques

Optical:

SLR:

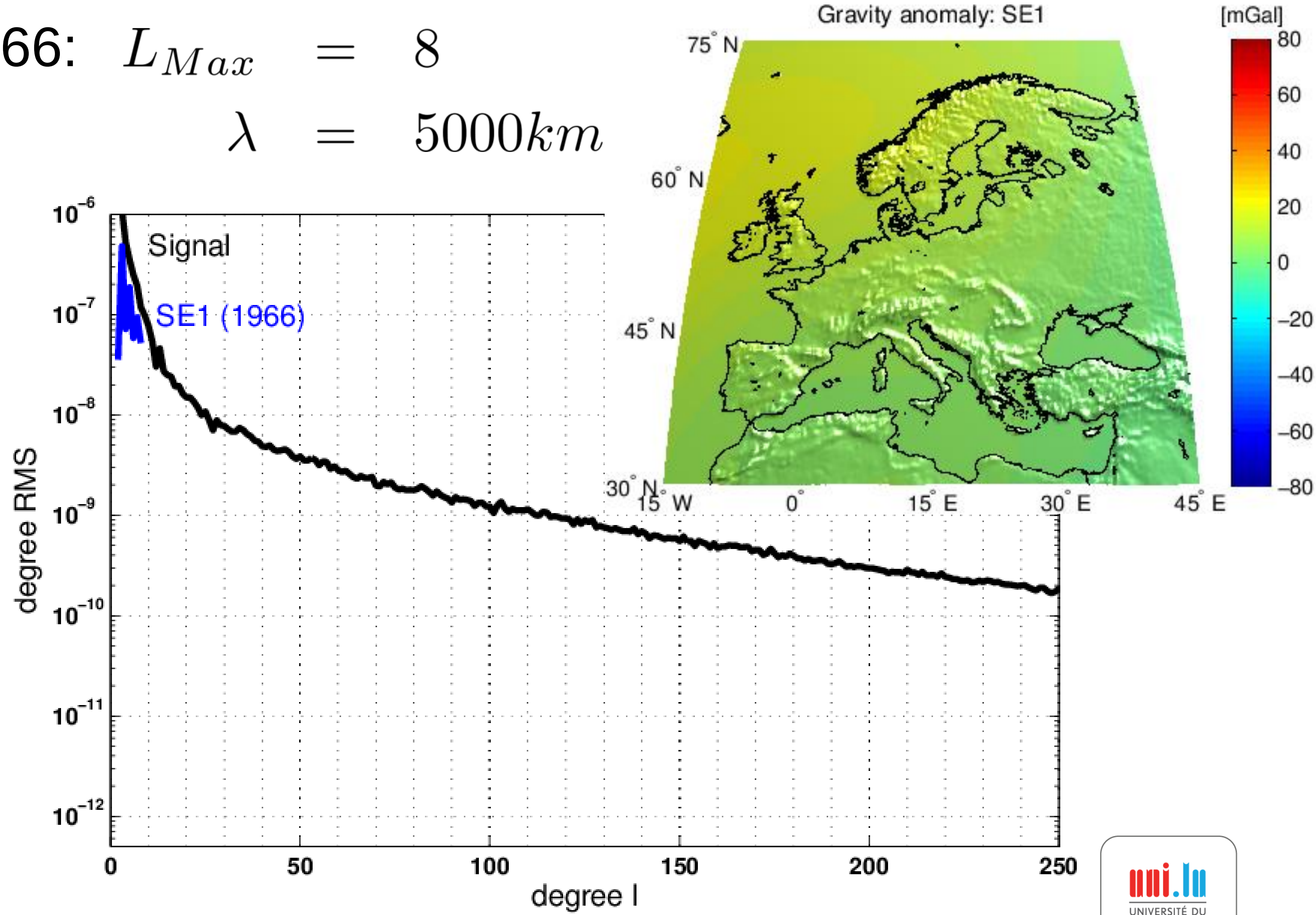




courtesy Oliver Baur

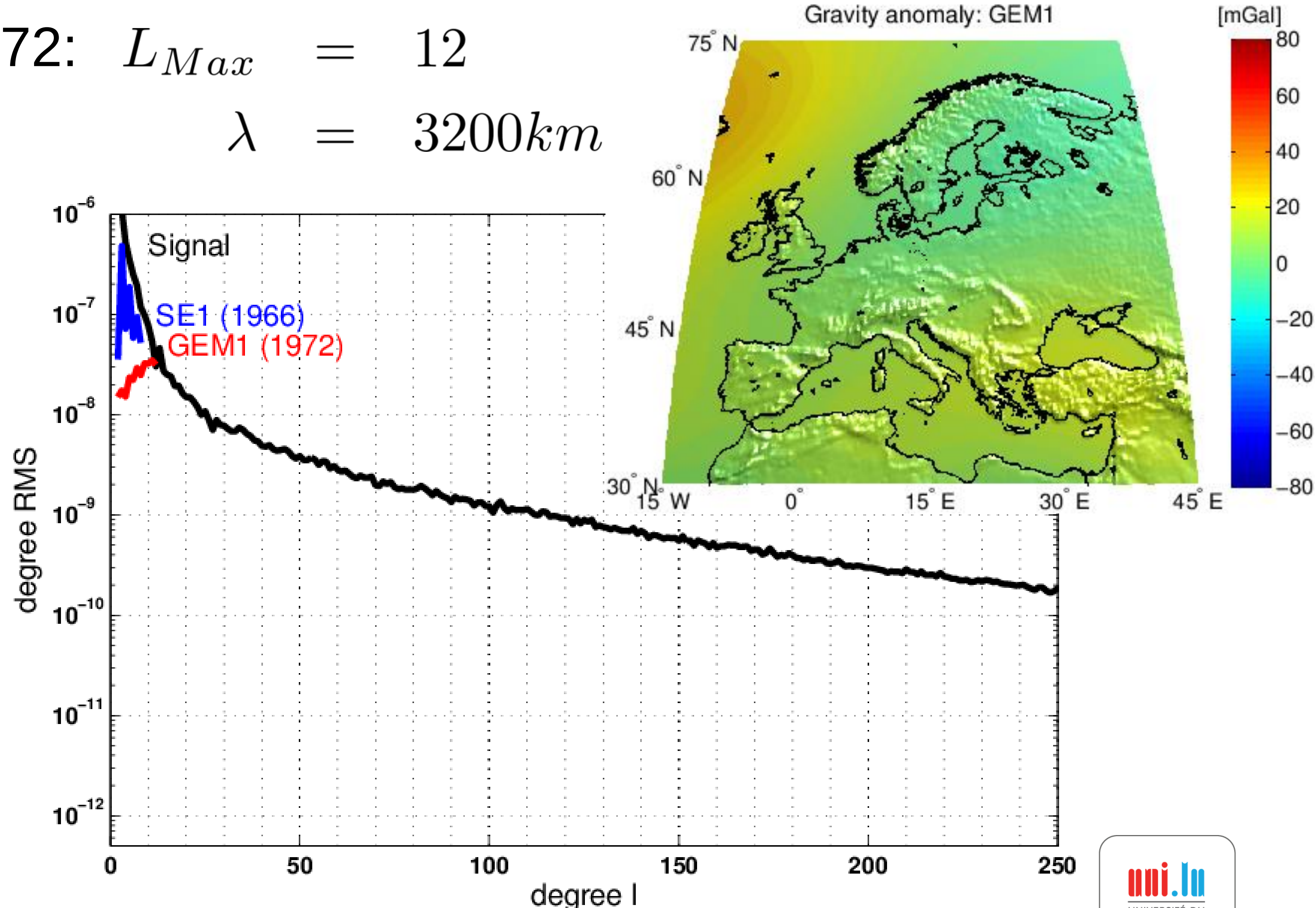
Development of the gravity field precision and resolution

$$\begin{aligned} \text{1966:} \quad L_{Max} &= 8 \\ \lambda &= 5000km \end{aligned}$$



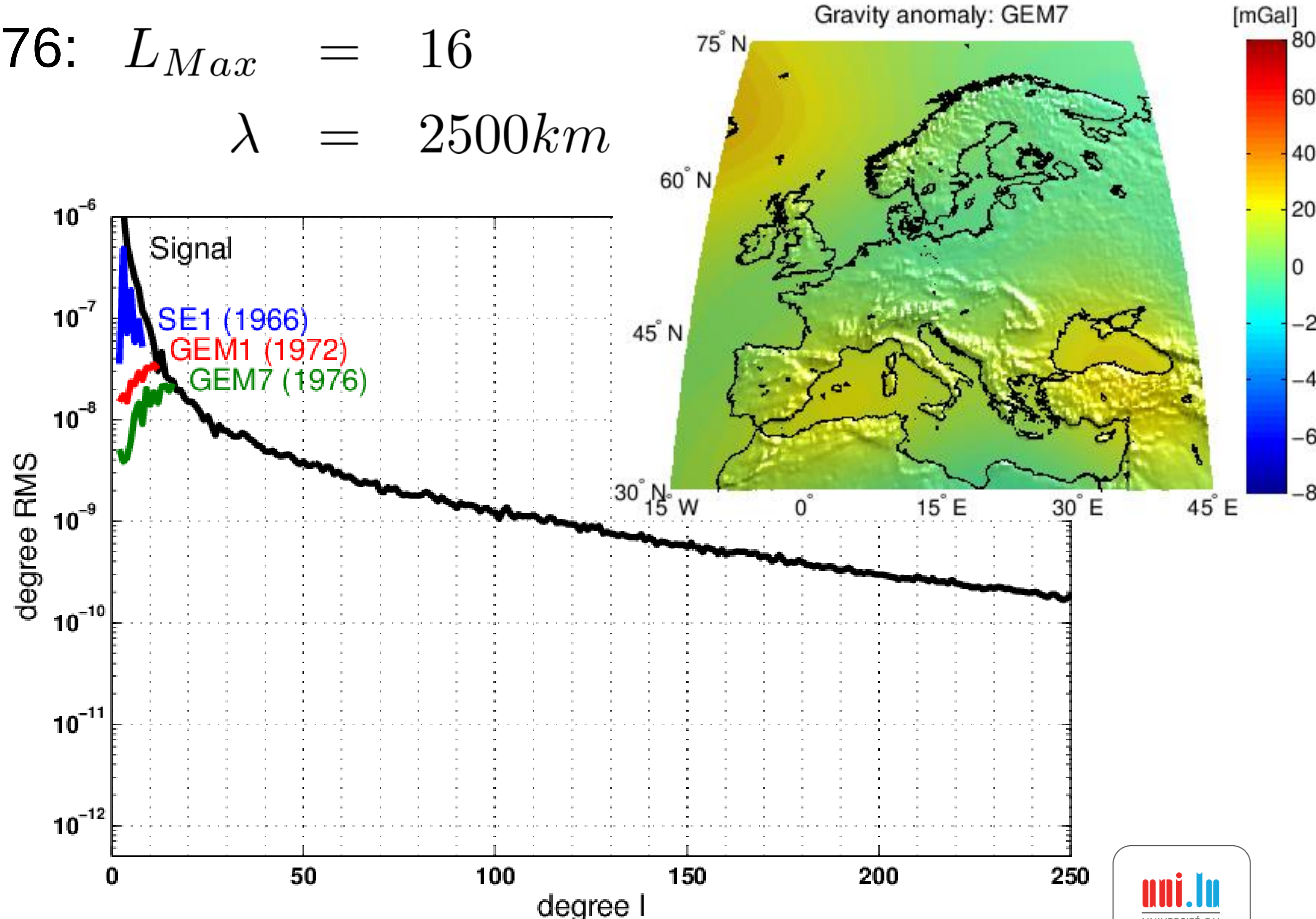
Development of the gravity field precision and resolution

1972: $$L_{Max} = 12$$
$$\lambda = 3200km$$



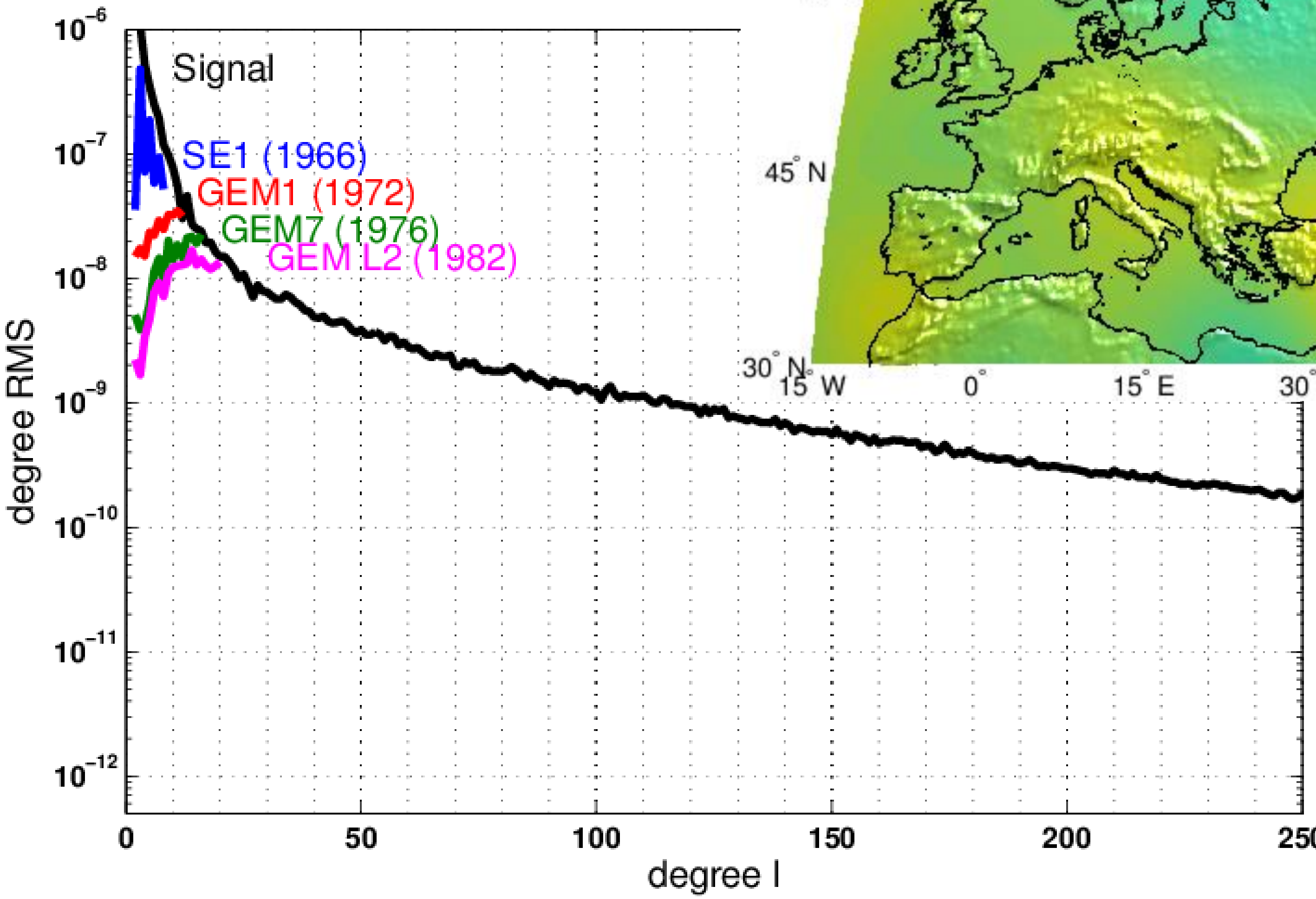
Development of the gravity field precision and resolution

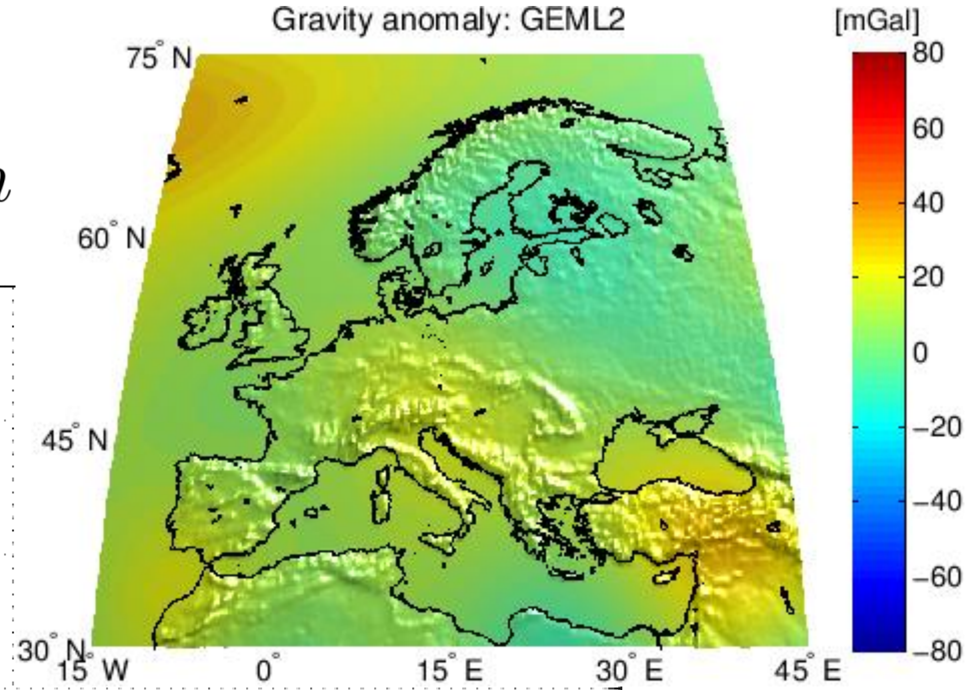
1976: $$L_{Max} = 16$$
$$\lambda = 2500km$$



Development of the gravity field precision and resolution

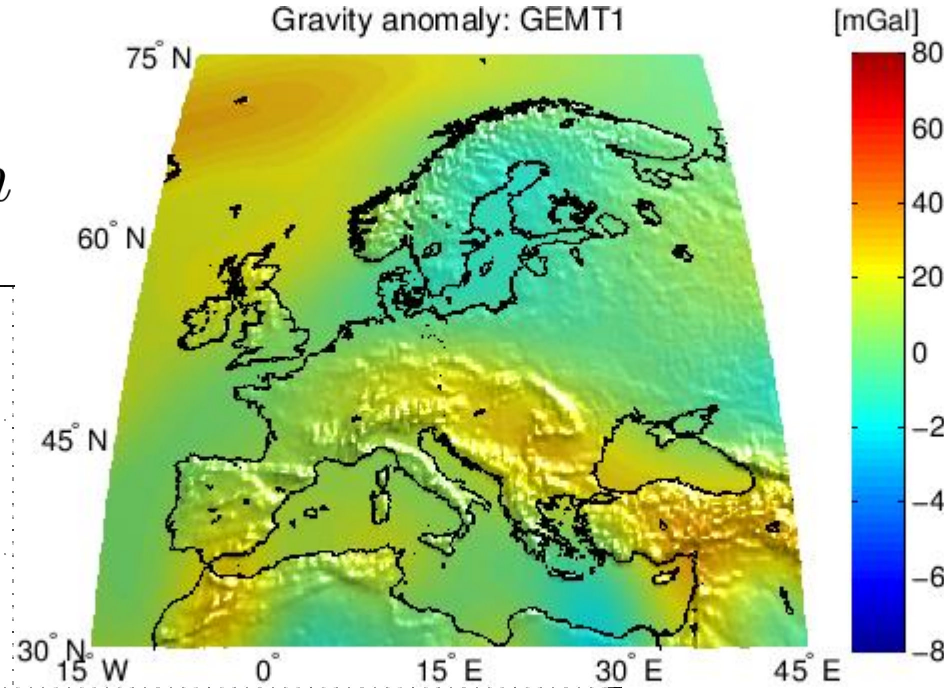
1982: $$L_{Max} = 20$$
$$\lambda = 2000km$$

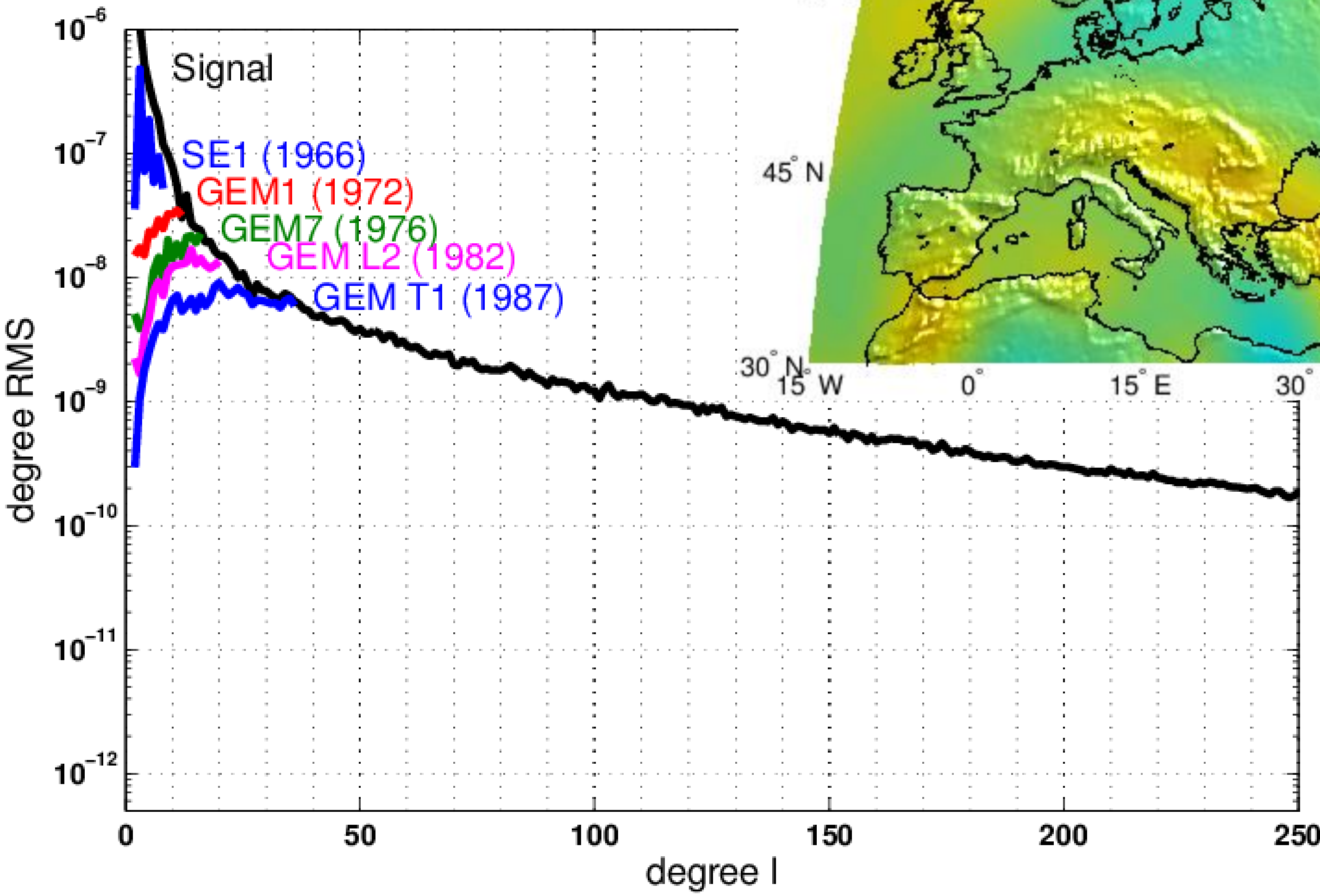




Development of the gravity field precision and resolution

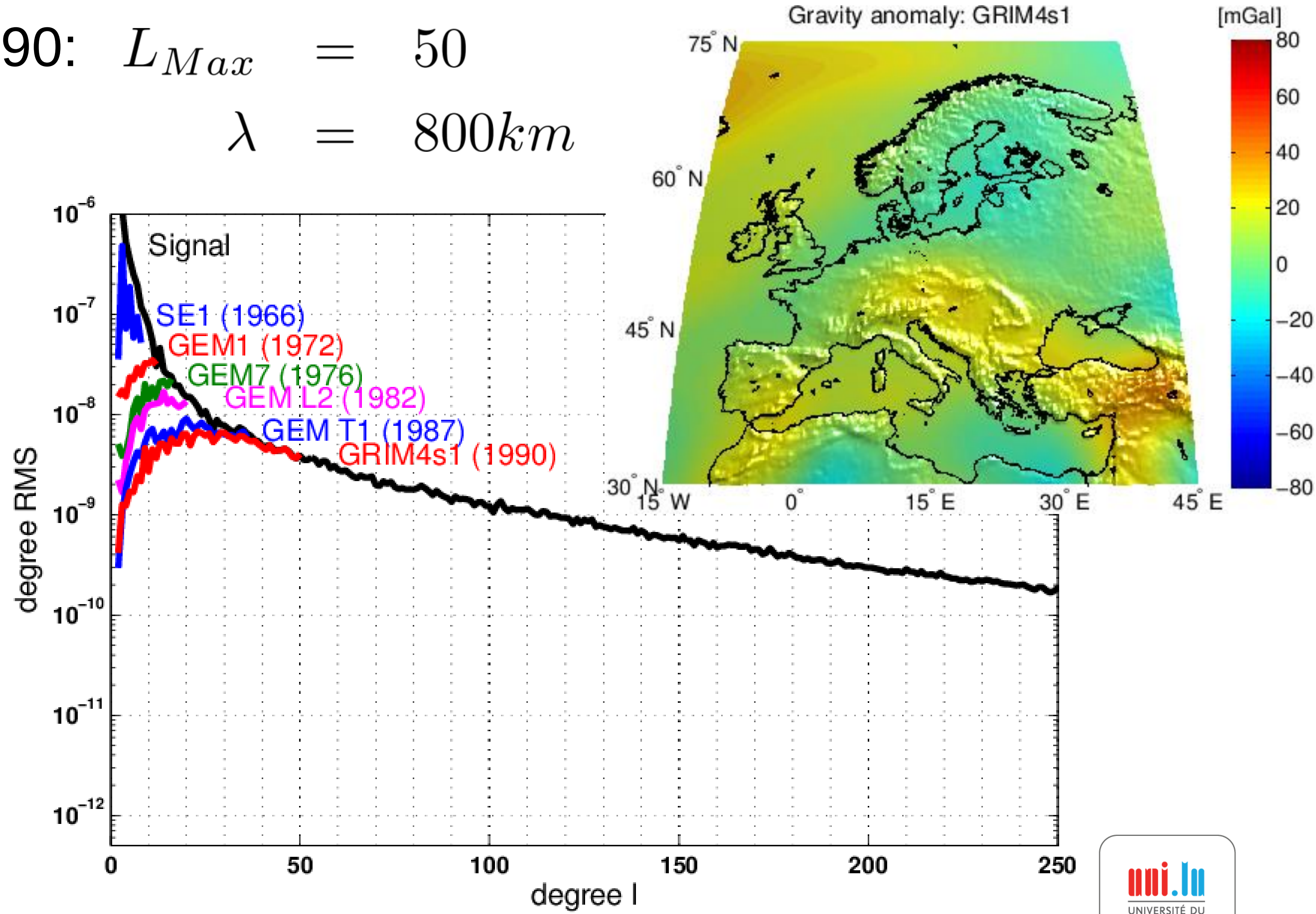
$$1987: \quad L_{Max} = 36$$
$$\lambda = 1110km$$





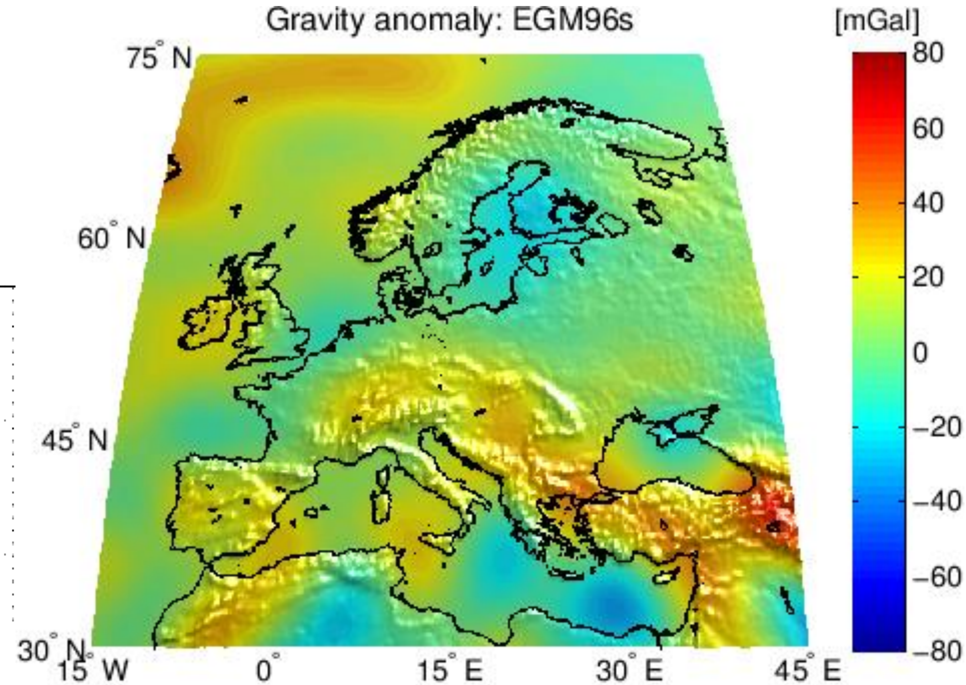
Development of the gravity field precision and resolution

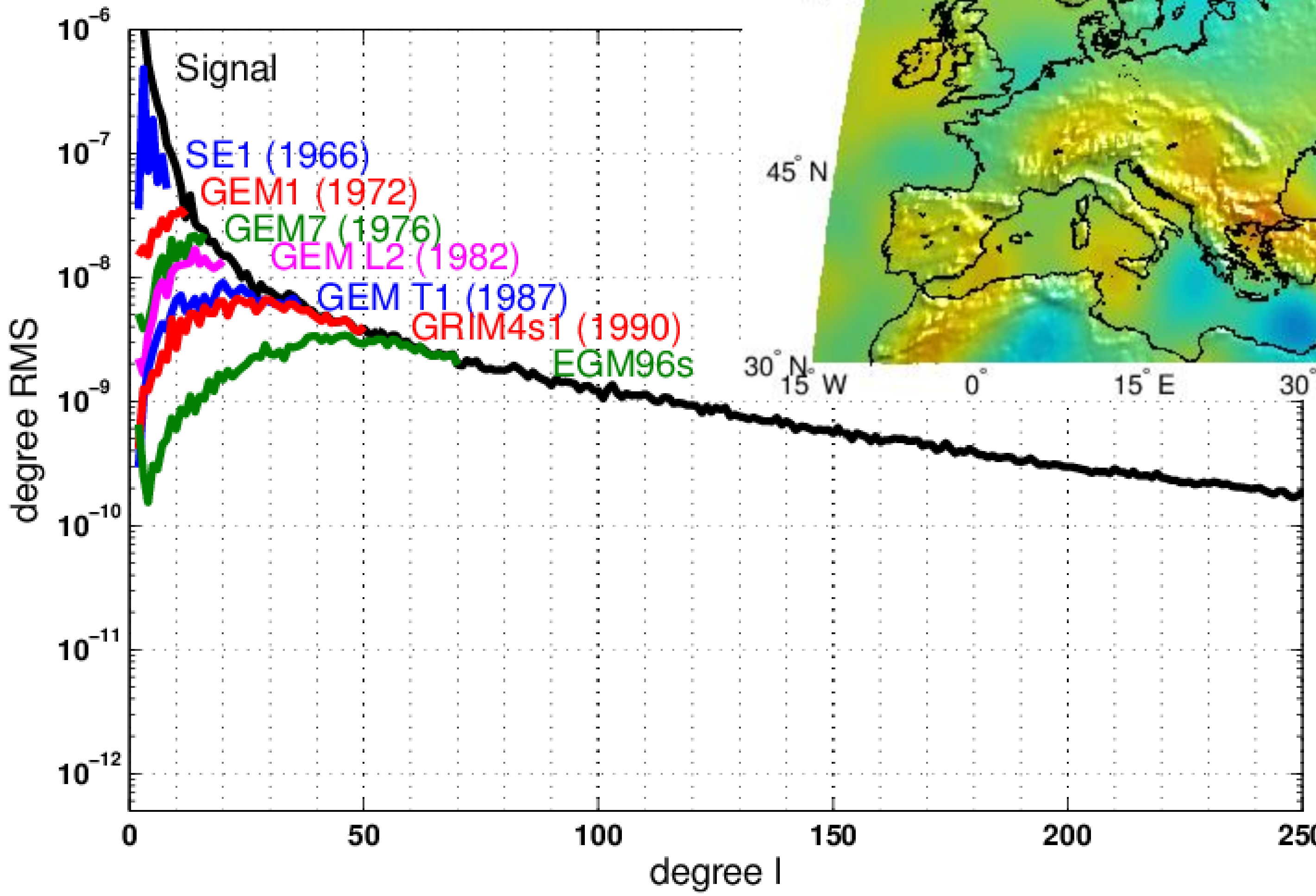
1990: $$L_{Max} = 50$$
$$\lambda = 800km$$



Development of the gravity field precision and resolution

1996: $$L_{Max} = 70$$
$$\lambda = 570km$$

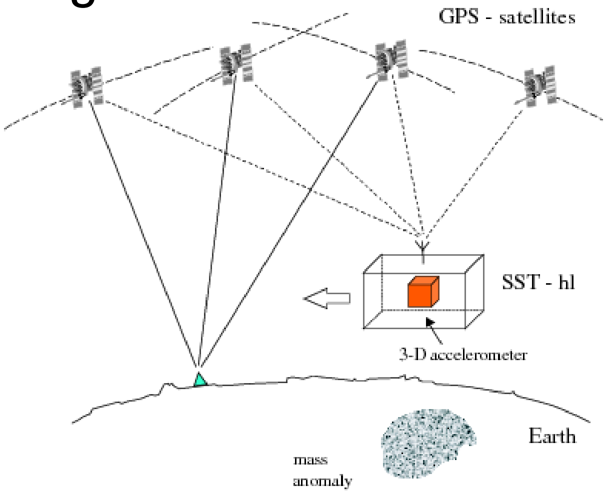




The Decade of the Geopotentials - the concepts

Satellite missions

High-low SST



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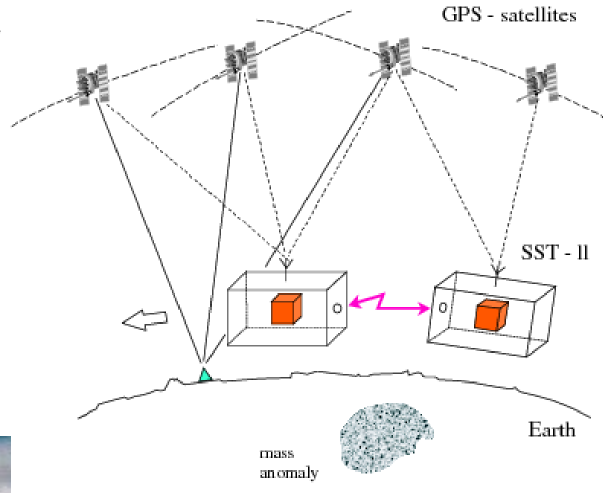
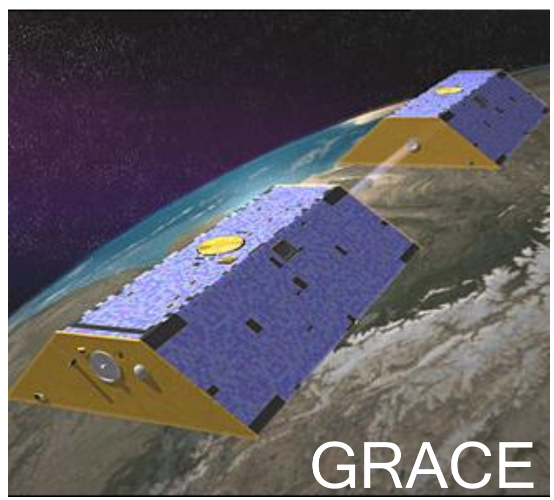
Low-low SST





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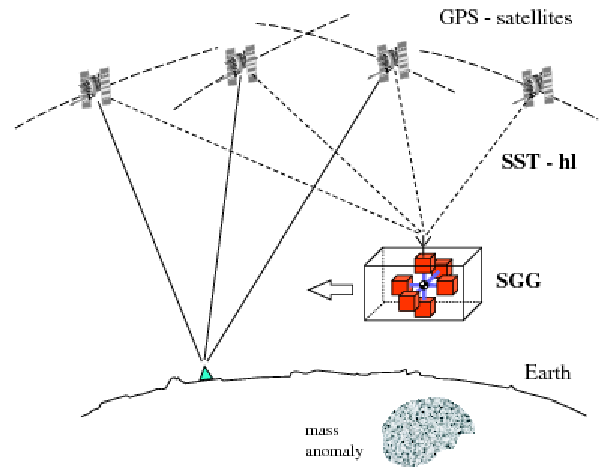
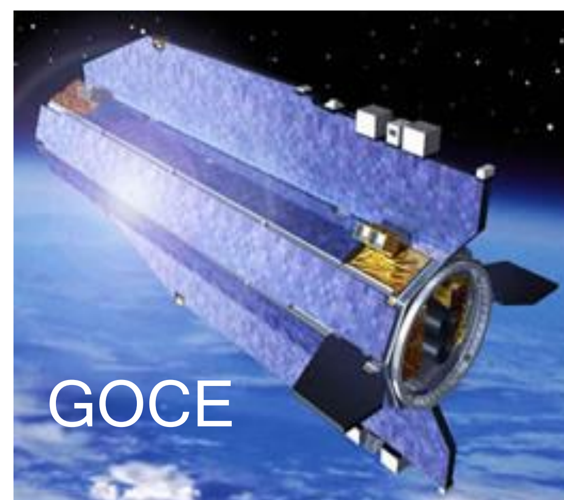
Gradiometry





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CHAMP

Satellite system CHAMP

CHAMP = CHAllenging Minisatellite Payload

- Initial orbit height: ~ 485 km
- Inclination: ~ 87.5°
- Key technologies:
  - GPS
  - Accelerometer
  - (Magnetometers)
- Observation quantity:
  - Position by GPS
  - Non-gravitational forces by accelerometers

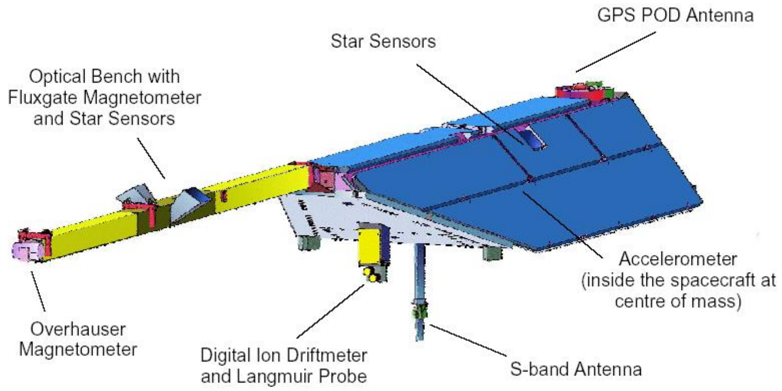
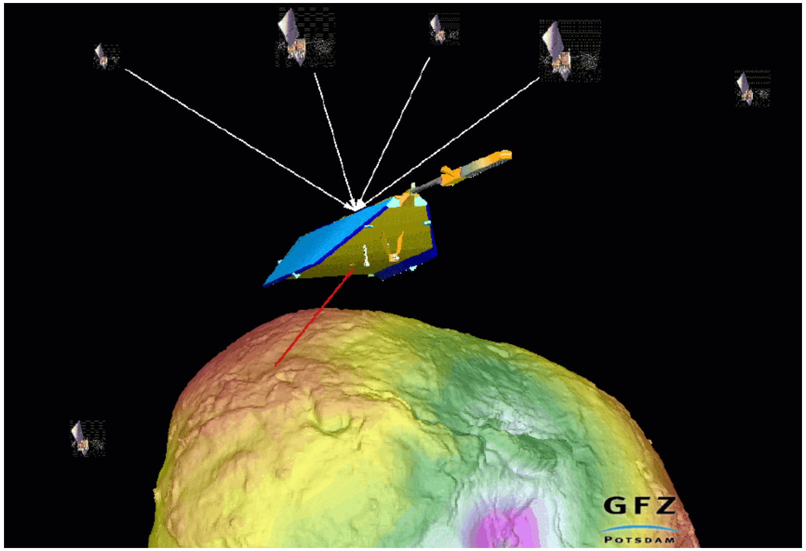


Figure 4-1: Front side view of CHAMP with location of instruments



Newton‘s equation of motion

$$F = G\frac{m_1 m_2}{r^2}$$

Newton‘s equation of motion

$$F = G\frac{m_1 m_2}{r^2} \;\rightarrow\; F = m_1 \frac{GM}{r^2}$$

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Observation: $\vec{x}$ (position)

Newton‘s equation of motion

$$F = G\frac{m_1 m_2}{r^2} \rightarrow F = m_1 \frac{GM}{r^2} \rightarrow F \approx a$$

Observation: $\vec{x}$ (position)

Differentiation: $\frac{\partial^2 \vec{x}}{\partial t^2} = \ddot{\vec{x}}$ (acceleration)

Newton‘s equation of motion

$$F = G\frac{m_1 m_2}{r^2} \;\rightarrow\; F = m_1 \frac{GM}{r^2} \;\rightarrow\; F \approx a$$

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$$\vec{F} = \vec{a} = \ddot{\vec{x}}$$

Newton's equation of motion

$$F = G\frac{m_1 m_2}{r^2} \rightarrow F = m_1 \frac{GM}{r^2} \rightarrow F \approx a$$

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Differentiation: $\frac{\partial^2 \vec{x}}{\partial t^2} = \ddot{\vec{x}}$ (acceleration)

$$\vec{F} = \vec{a} = \ddot{\vec{x}}$$

$$\vec{f}_{\text{Grav}} = \ddot{\vec{x}} - \vec{f}_{\text{3rdBody}} - \vec{f}_{\text{Tides}} - \vec{f}_{\text{Rel}} - \vec{f}_{\text{NonGrav}}$$

Newton‘s equation of motion

$$F = G\frac{m_1 m_2}{r^2} \rightarrow F = m_1\frac{GM}{r^2} \rightarrow F \approx a$$

Observation: $\vec{x}$ (position)

Differentiation: $\dfrac{\partial^2 \vec{x}}{\partial t^2} = \ddot{\vec{x}}$ (acceleration)

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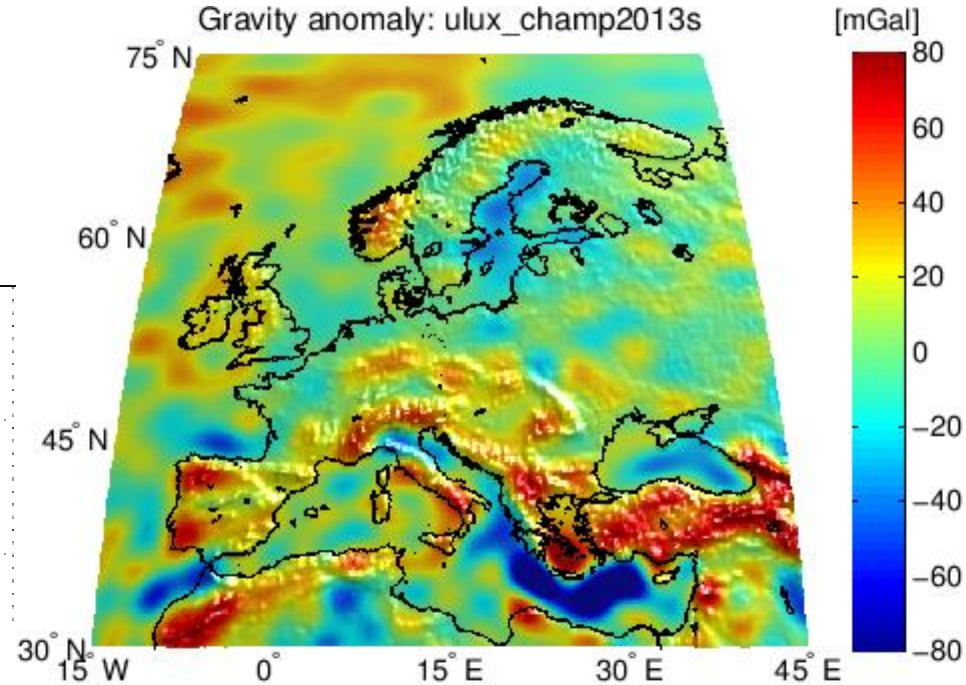
$$\vec{f}_{\mathrm{Grav}} = \ddot{\vec{x}} - \vec{f}_{\mathrm{3rdBody}} - \vec{f}_{\mathrm{Tides}} - \vec{f}_{\mathrm{Rel}} - \vec{f}_{\mathrm{NonGrav}}$$

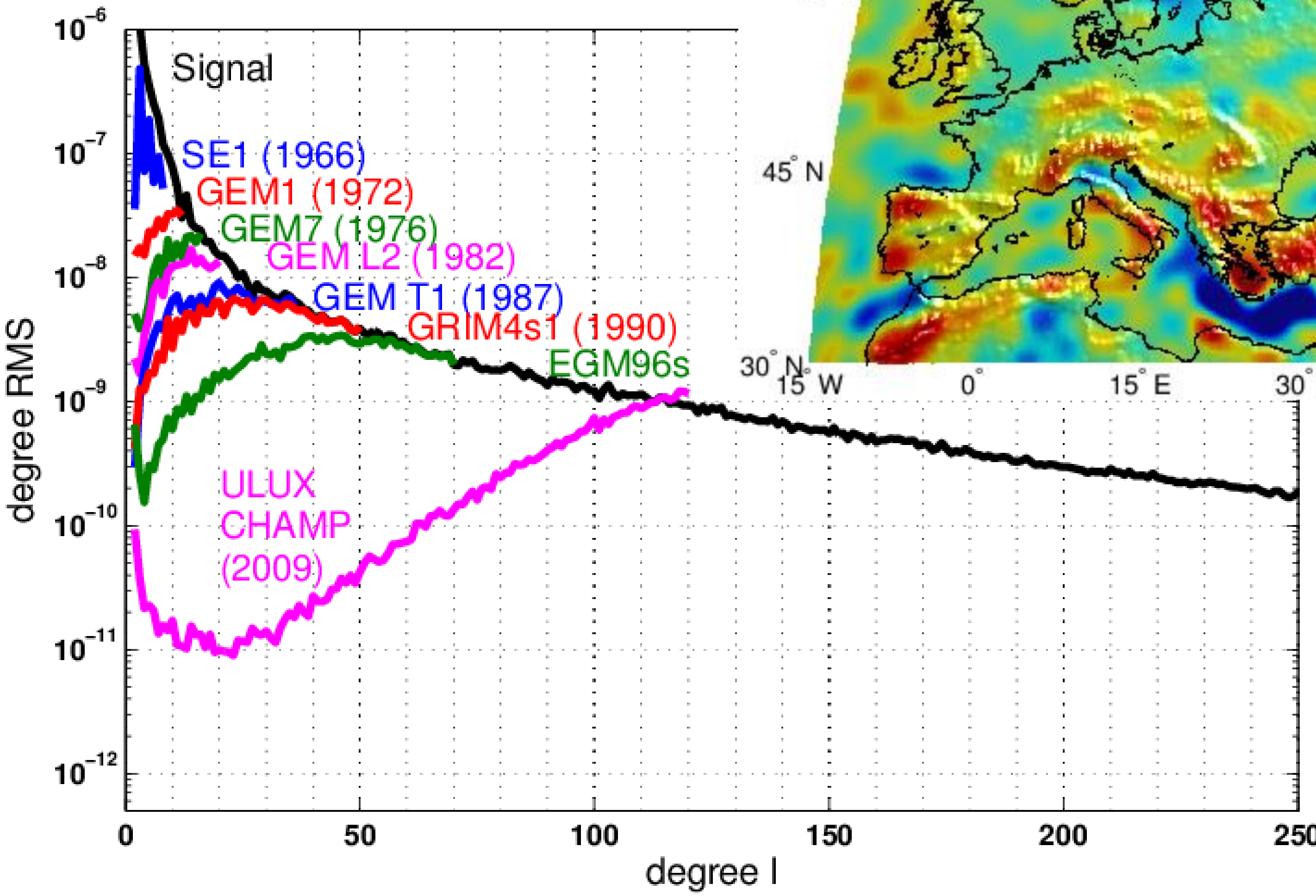
$$\nabla V = \ddot{\vec{x}} - \vec{f}_{\mathrm{3rdBody}} - \vec{f}_{\mathrm{Tides}} - \vec{f}_{\mathrm{Rel}} - \vec{f}_{\mathrm{NonGrav}}$$

GAIN due to CHAMP

2010: $L_{Max} = 120$

$\lambda = 330km$



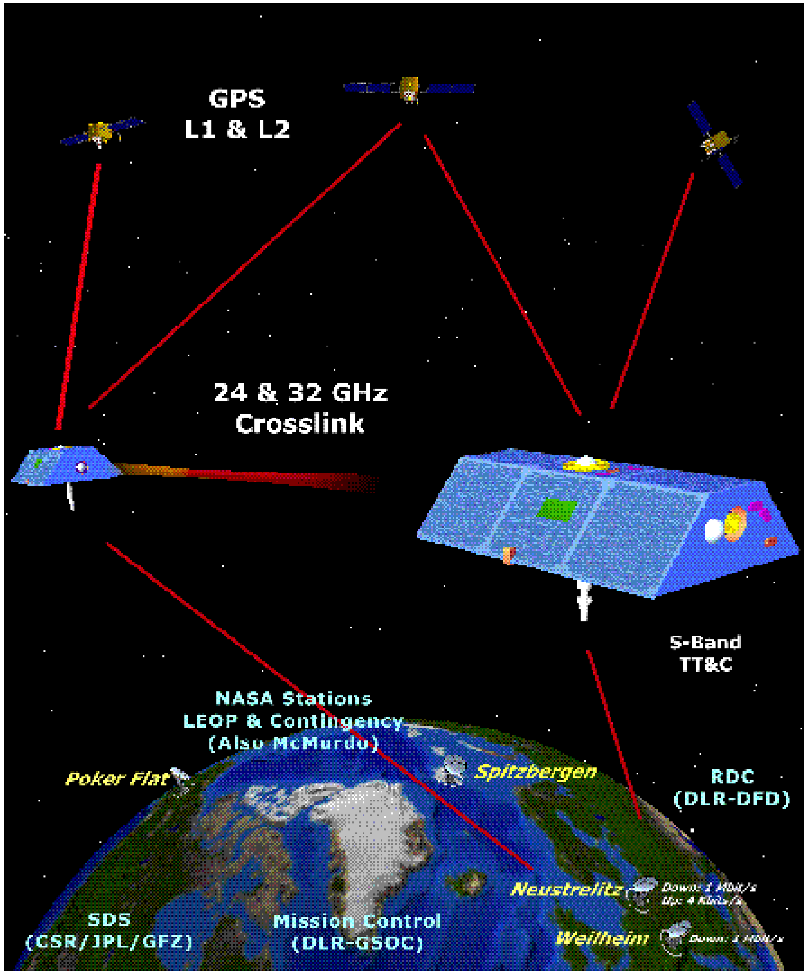


GRACE

Satellite system GRACE

GRACE = Gravity Recovery and Climate Experiment

- Initial orbit height: ~ 485 km
- Inclination: ~ 89°
- Key technologies:
  - GPS
  - accelerometer
  - K-Band ranging system
- Observation quantity:
  - distance (range)
  - change of distance (range rate)



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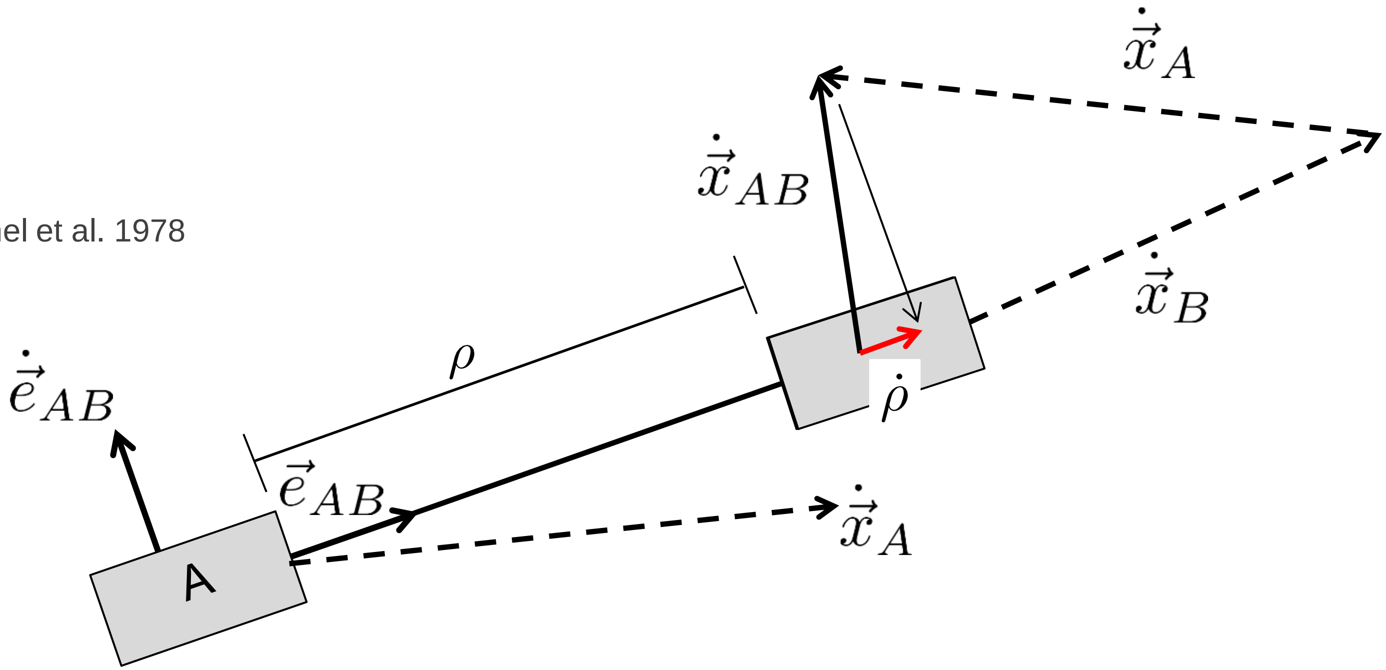
Geometry of the GRACE system



Rummel et al. 1978

Differentiation

Integration

$$\rho = \vec{x}_{AB} \cdot \vec{e}_{AB}$$

$$\dot{\rho} = \dot{\vec{x}}_{AB} \cdot \vec{e}_{AB}$$

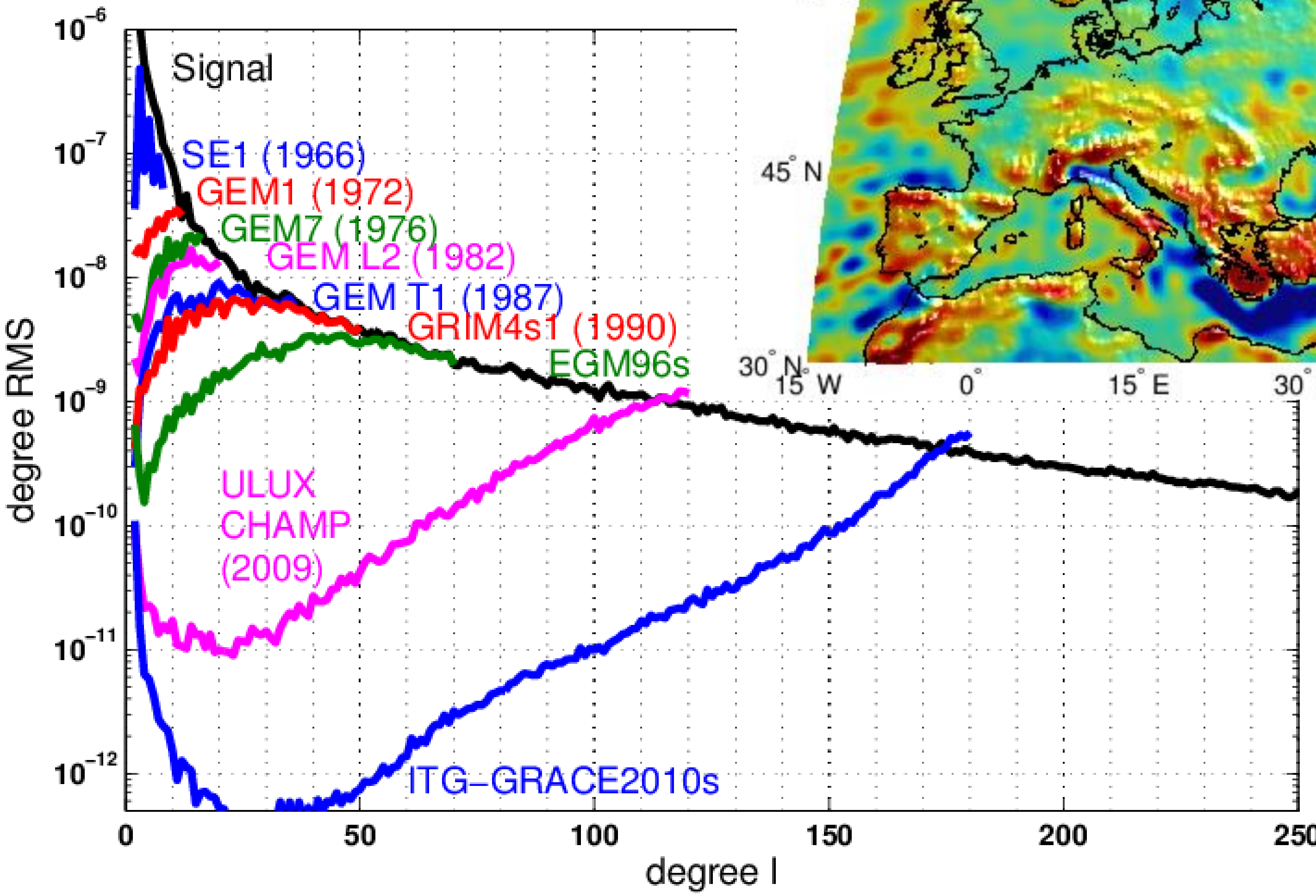
$$\ddot{\rho} = \ddot{\vec{x}}_{AB} \cdot \vec{e}_{AB} + \dot{\vec{x}}_{AB} \cdot \dot{\vec{e}}_{AB}$$

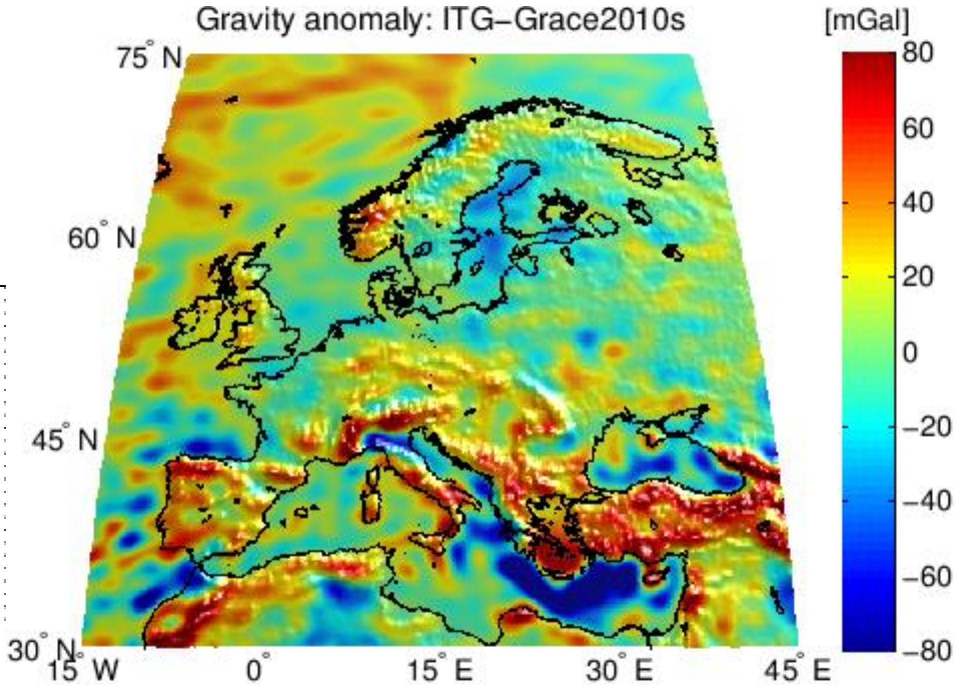
$$= \nabla V_{AB}$$

GAIN due to GRACE

2013: $L_{Max} = 180$

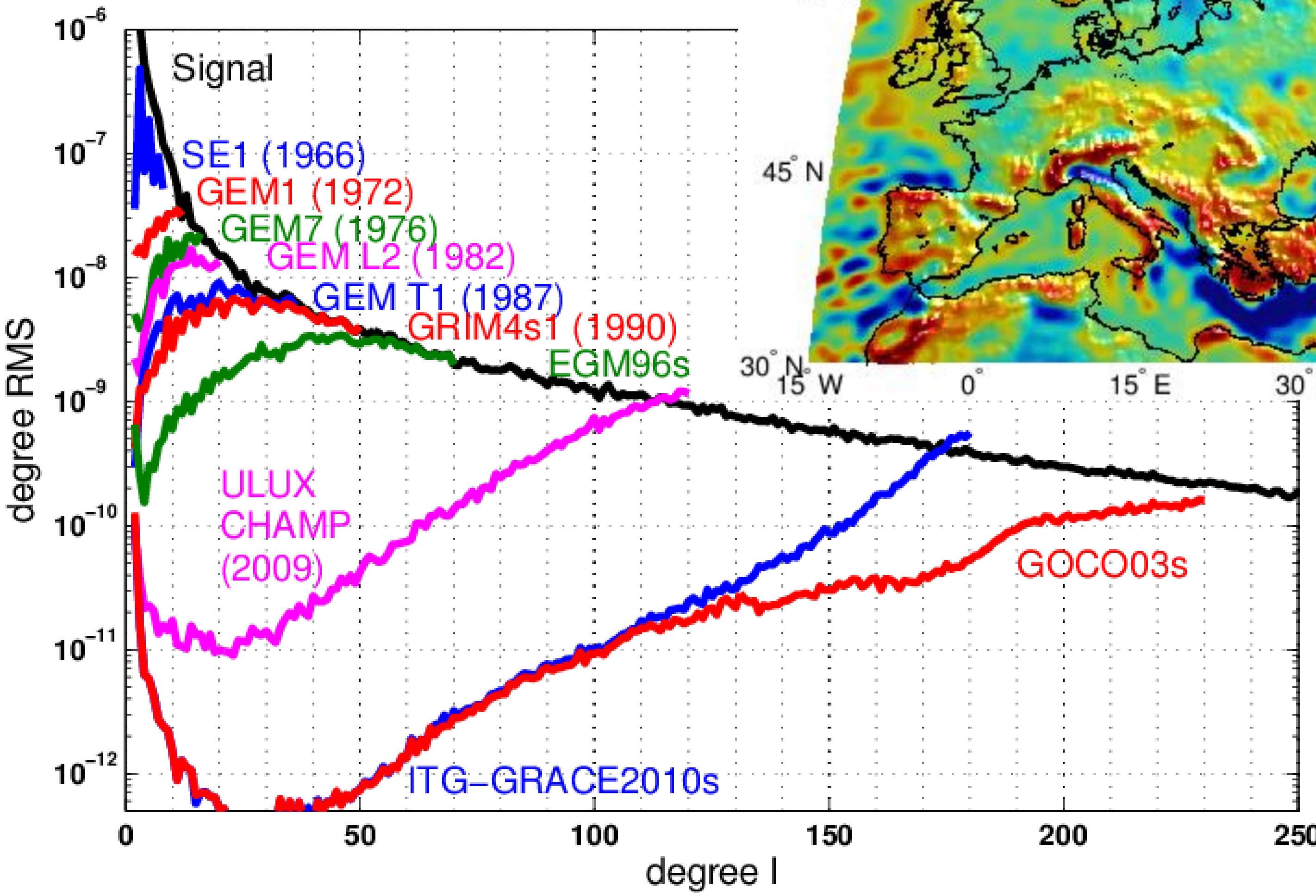
$\lambda = 220km$

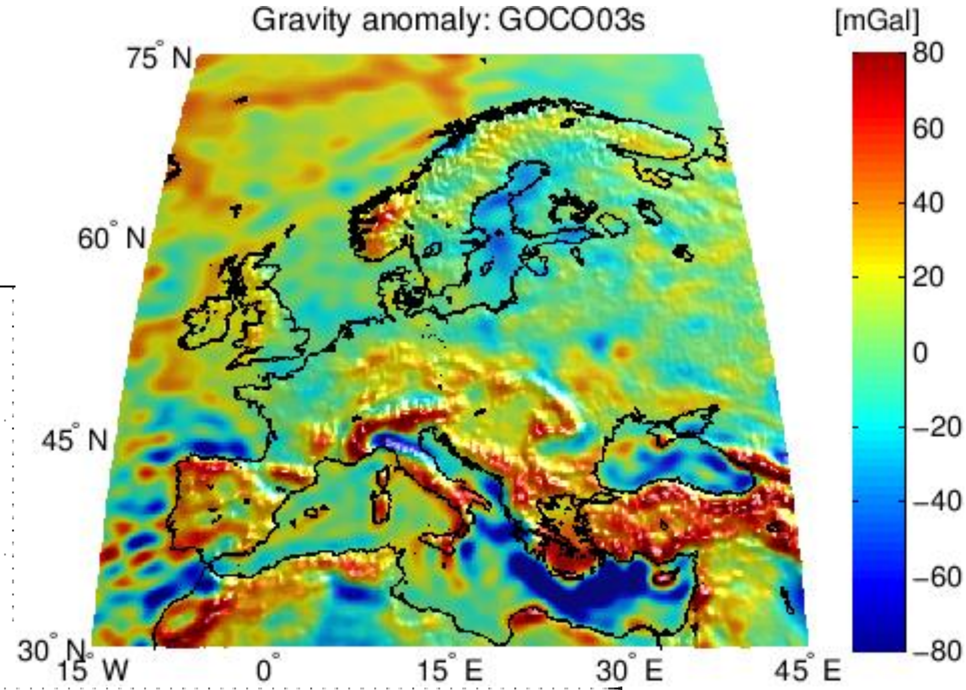




GAIN due to GRACE and GOCE

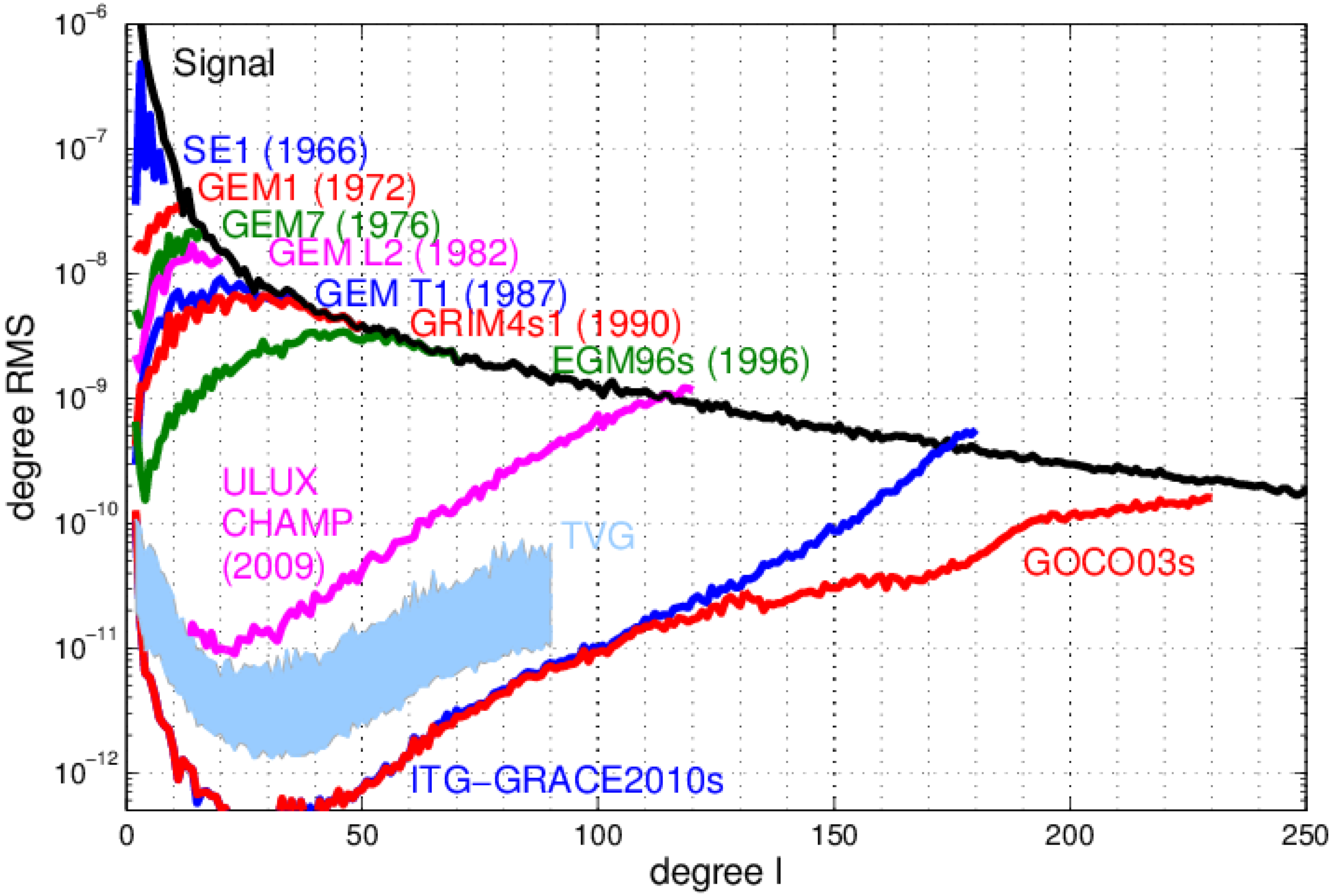
$$2013: \quad L_{Max} = 230$$
$$\lambda = 175km$$





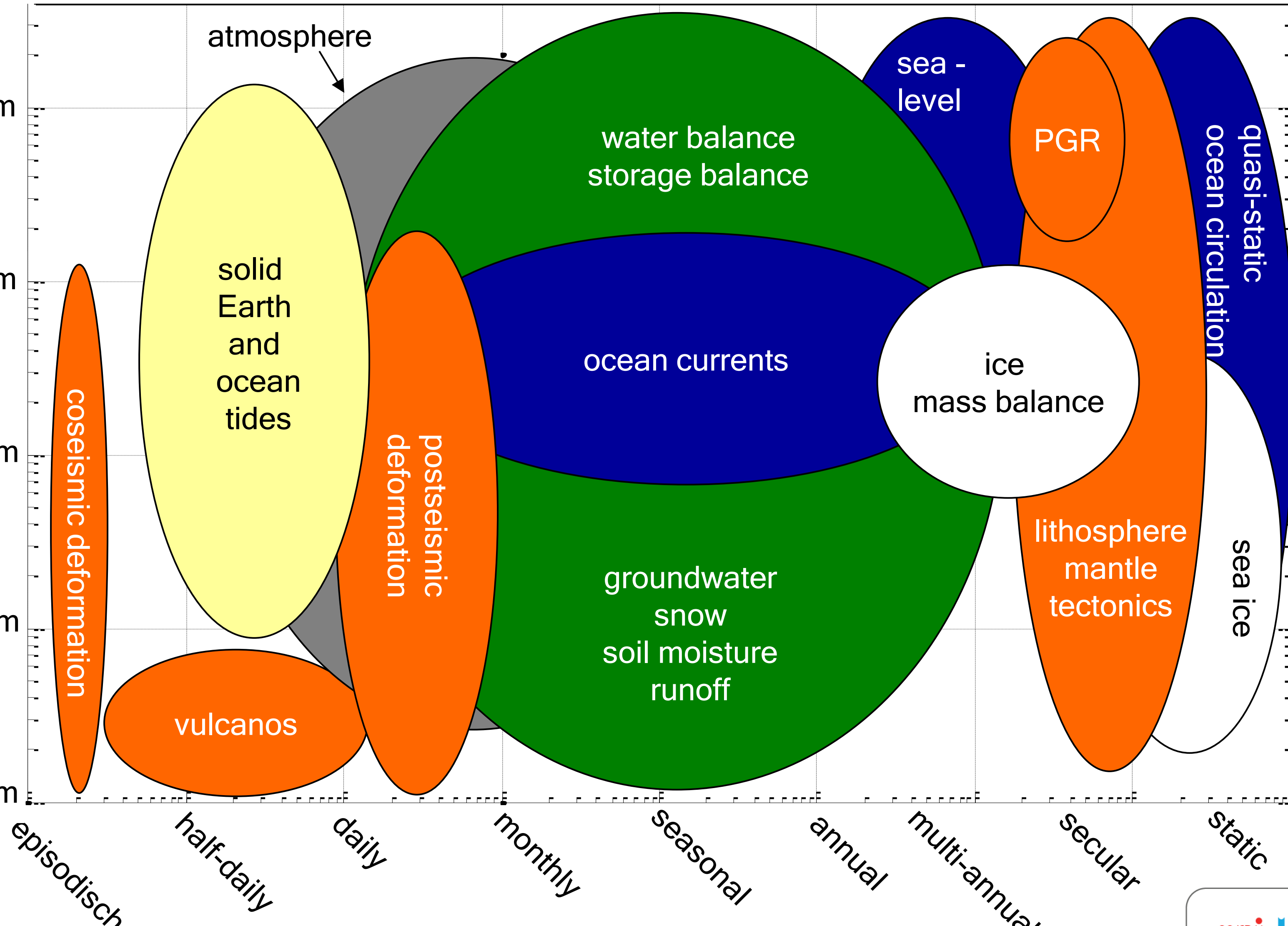
GAIN due to GRACE: time variable gravity (TVG)

2013: $$\begin{aligned} L_{Max} &= 90 \\ \lambda &= 450km \end{aligned}$$



Achievements in terms of the TVG

Spatio-temporal resolution ?



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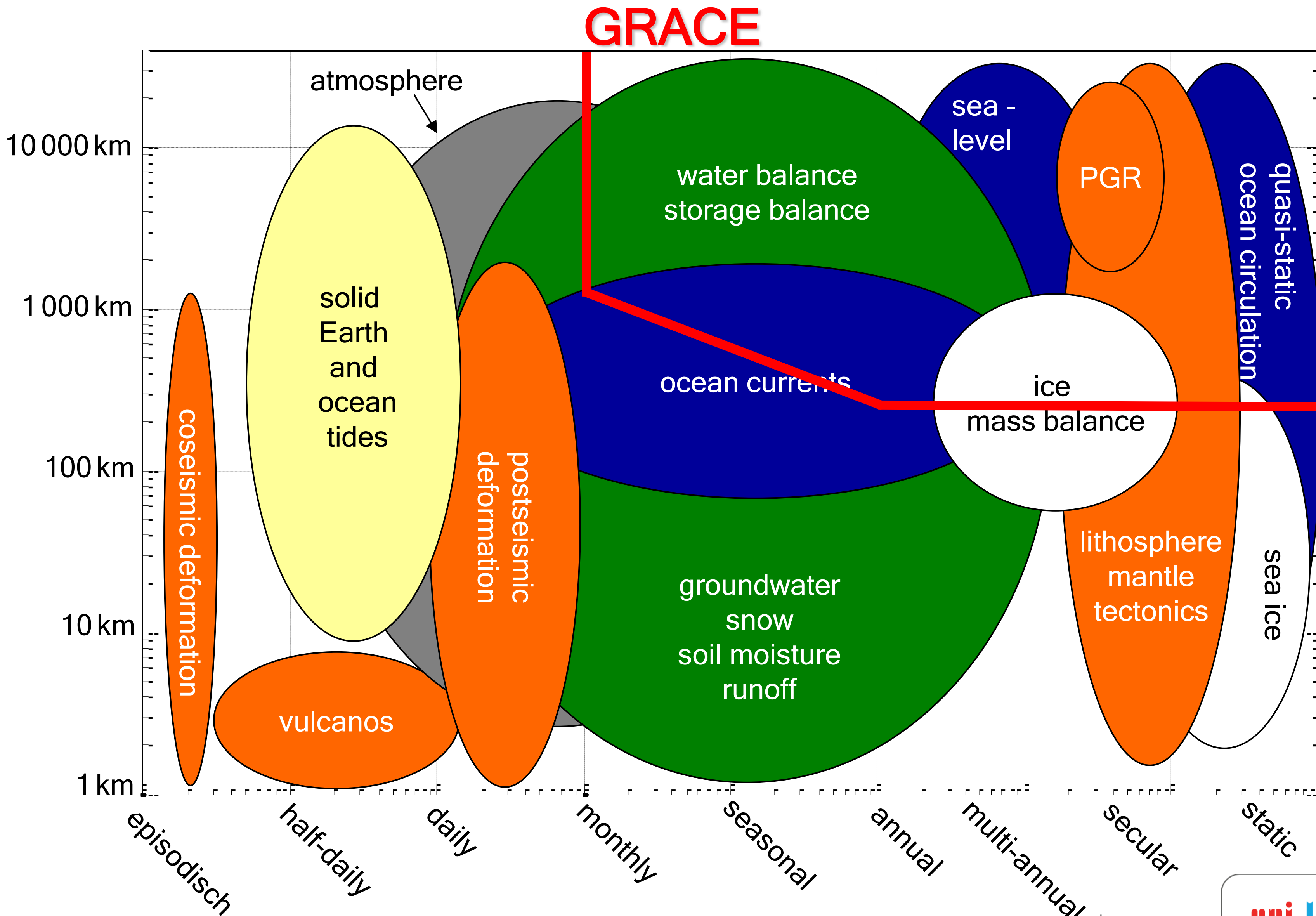
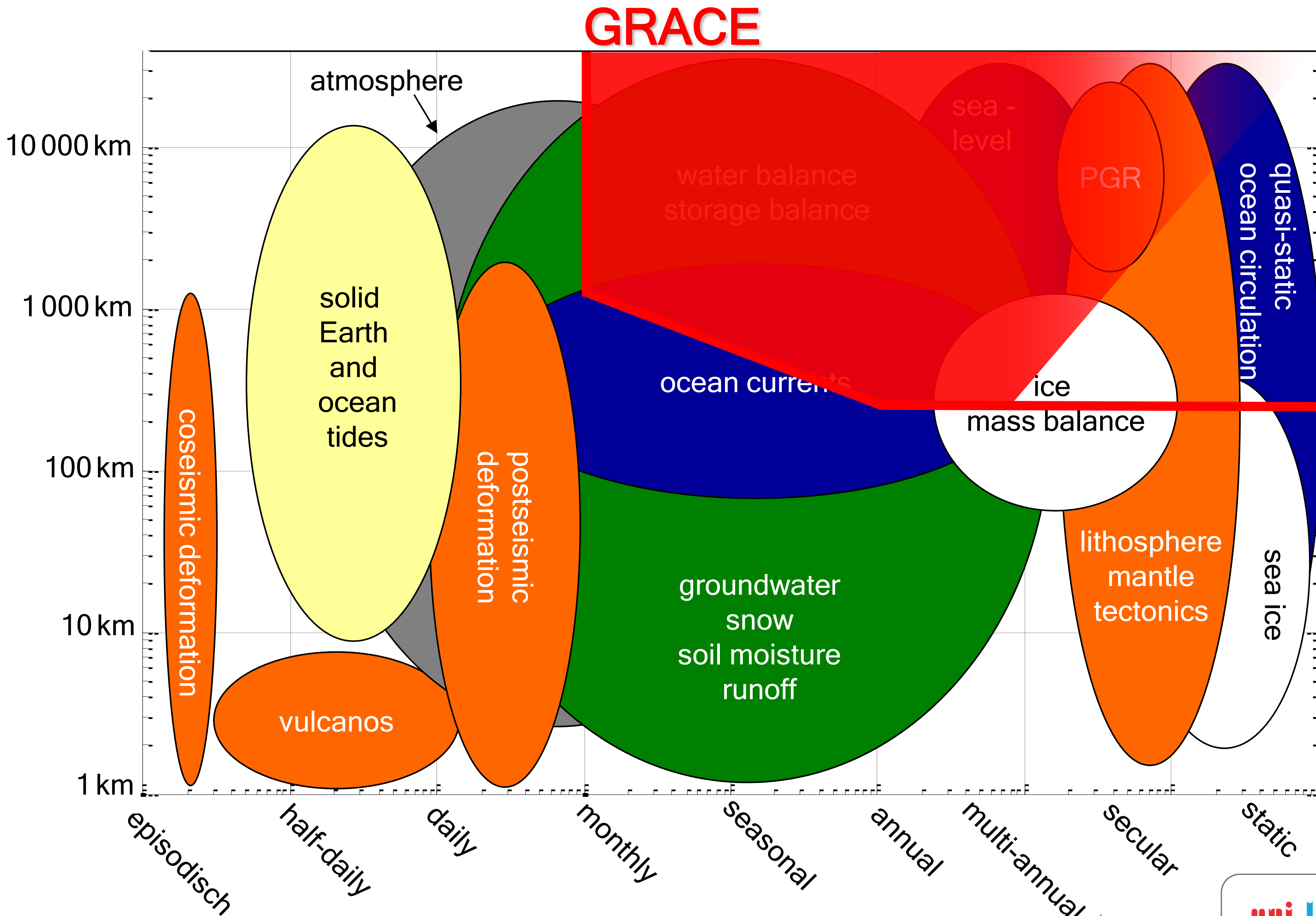
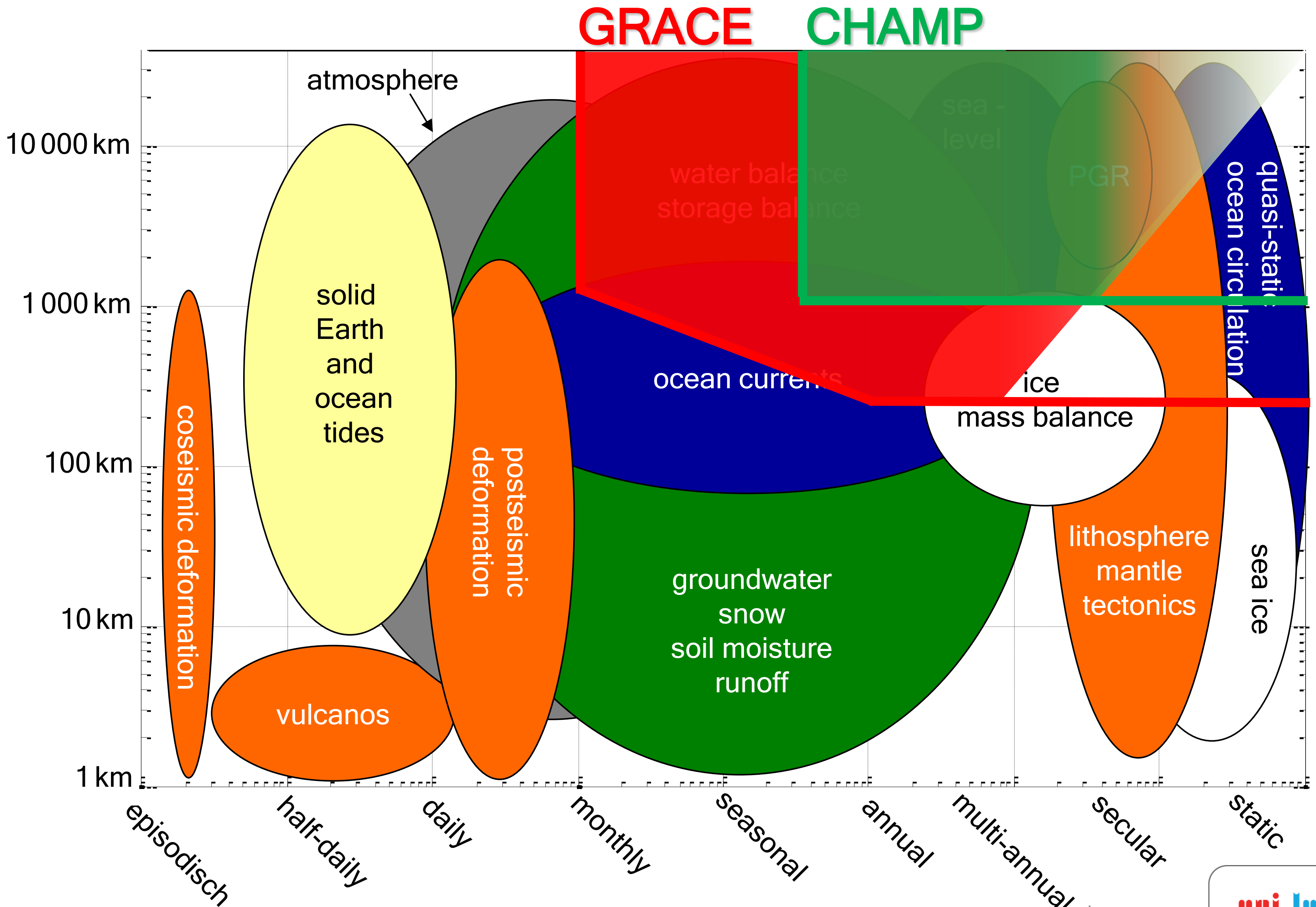
Spatio-temporal resolution ?



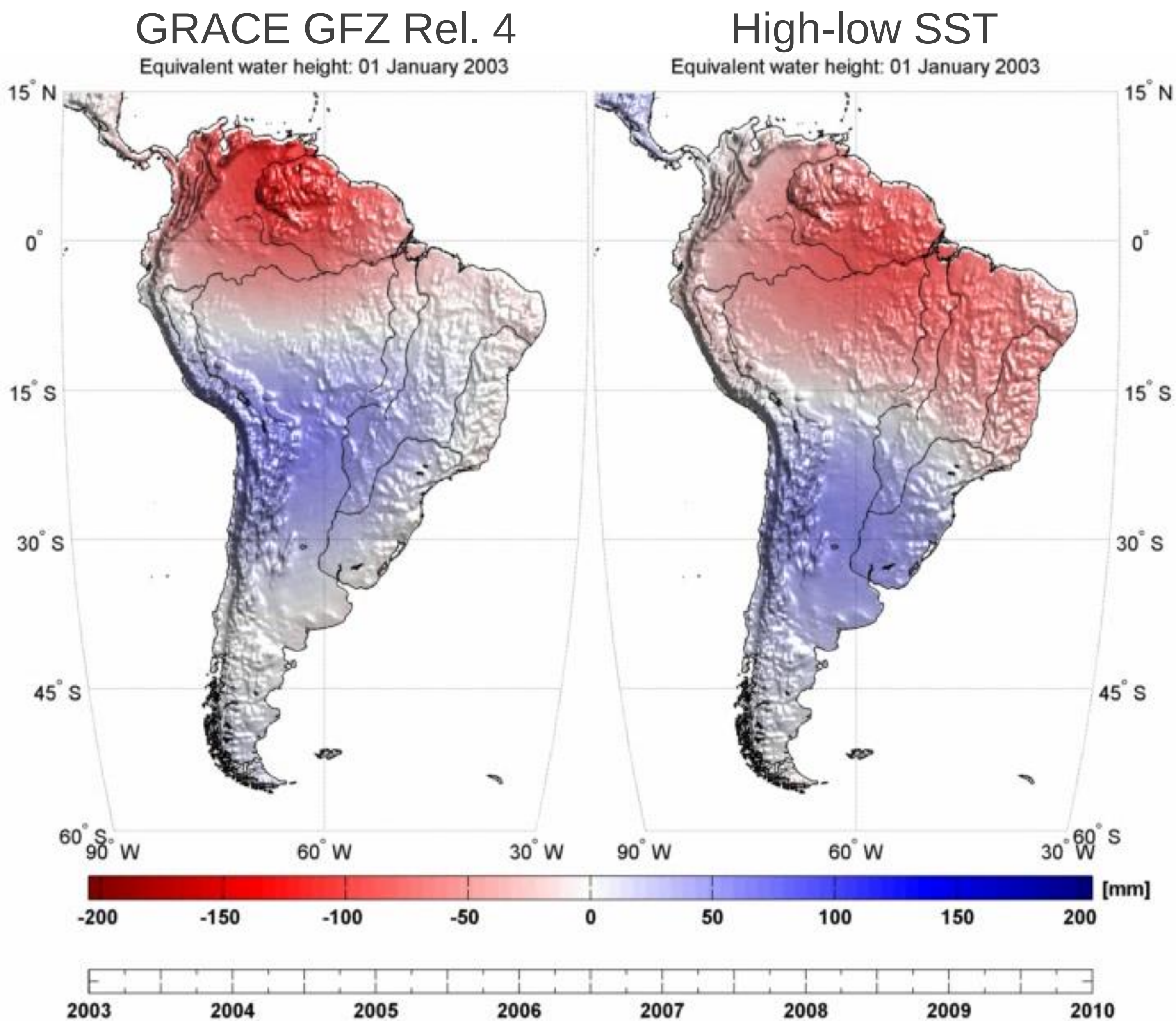
Spatio-temporal resolution ?



Spatio-temporal resolution ?

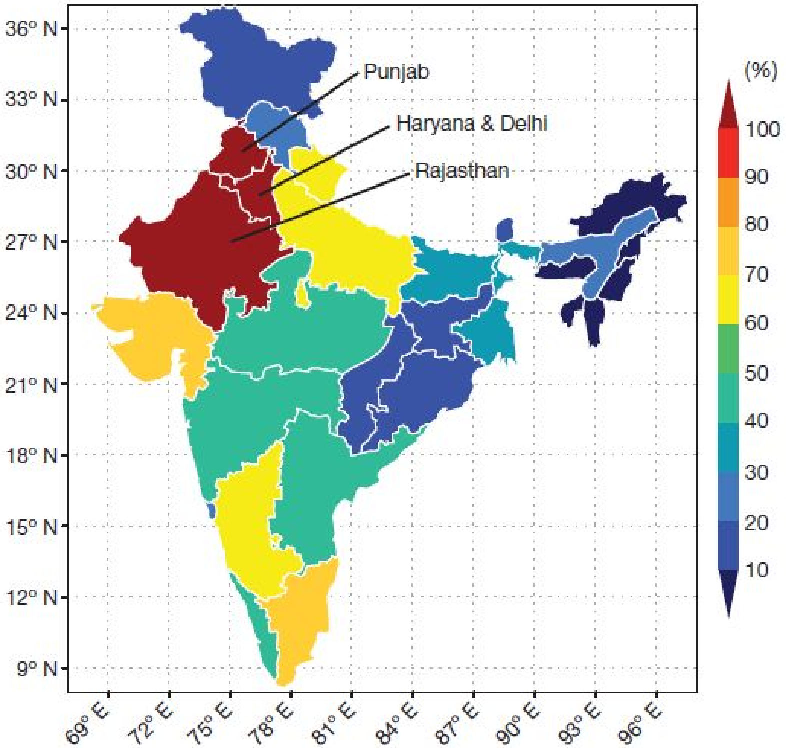


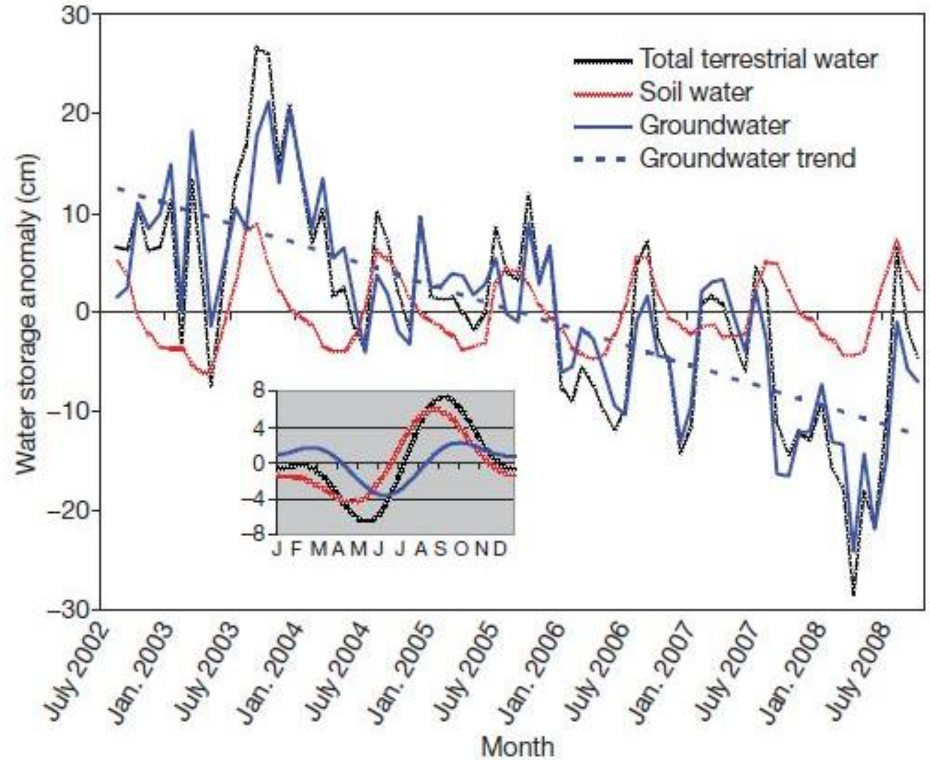
Change in water storage



Water is stressed …

- Water balance in India (Rodell et al., Nature, vol. 460, 2009)

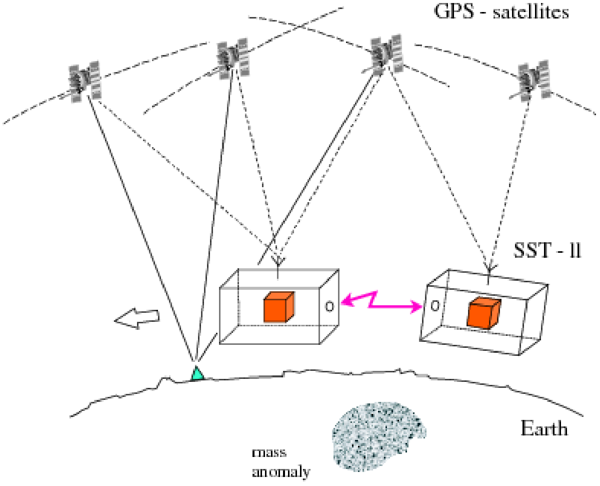


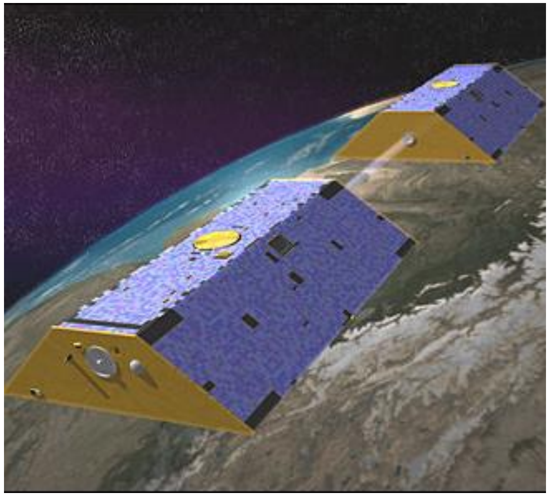


The Future

GRACE und GRACE Follow-On (GFO)

Low-low





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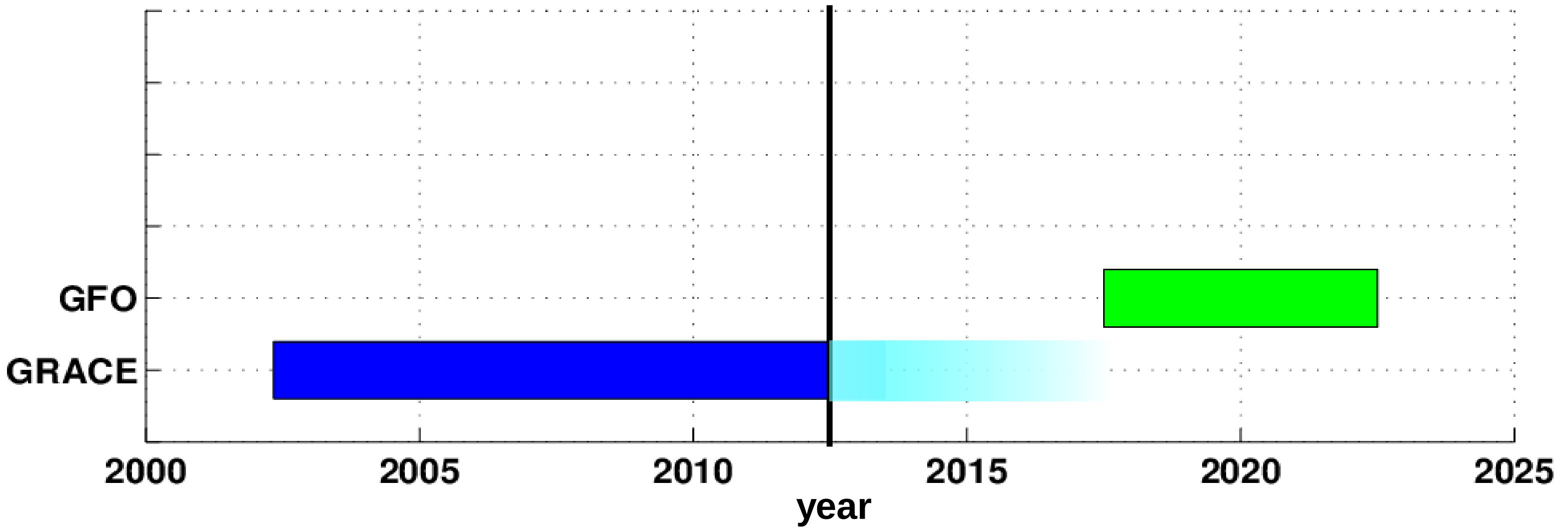
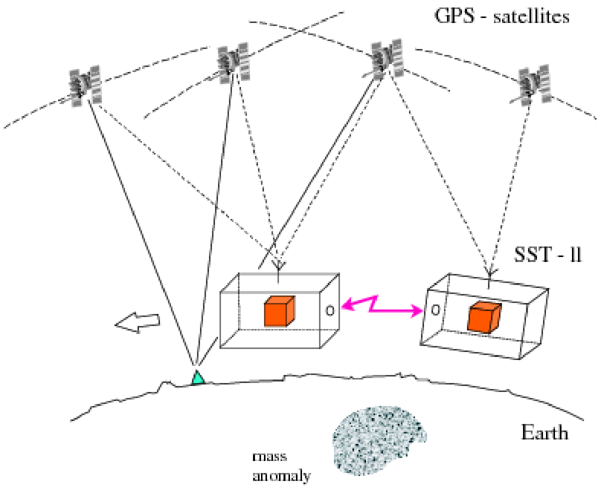
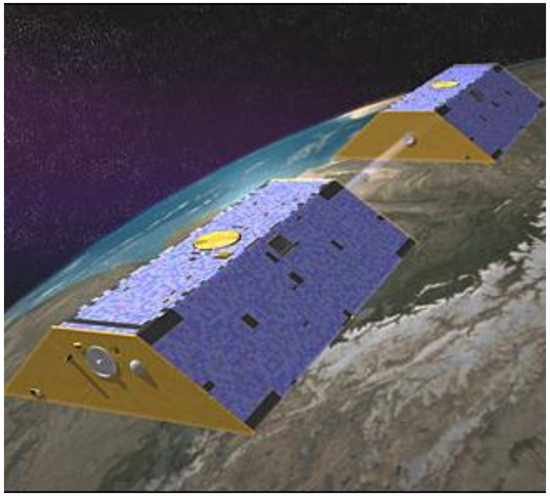
- K-Band + Laser
- GPS
- Accelerometer



GRACE und GRACE Follow-On (GFO)

Low-low





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- K-Band + Laser
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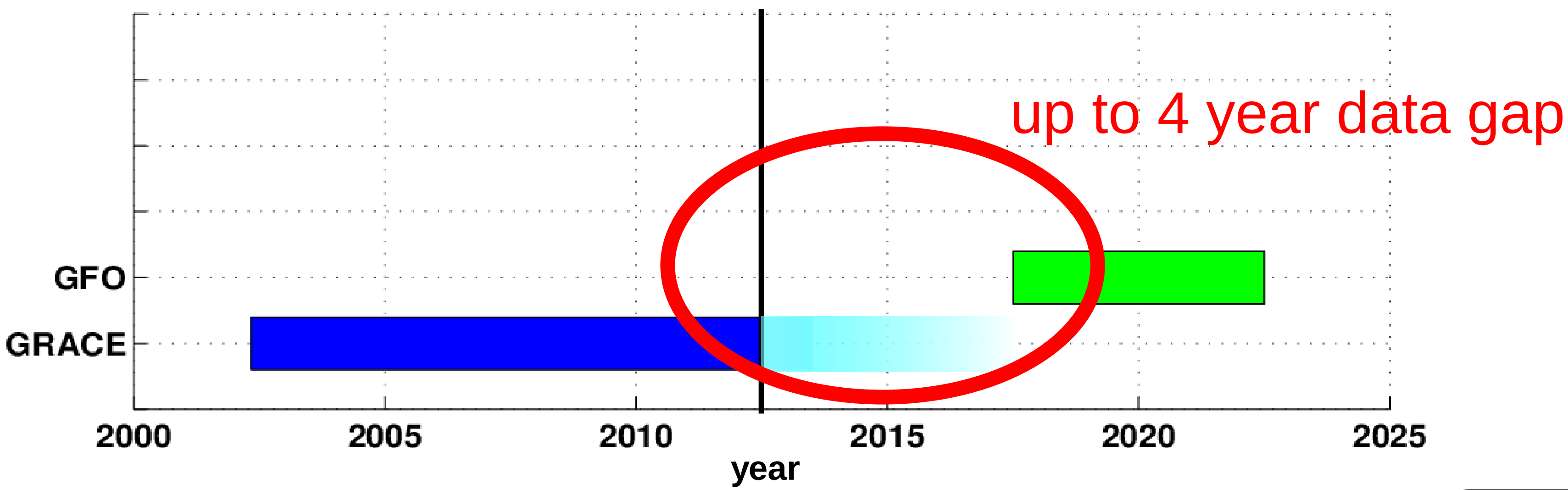
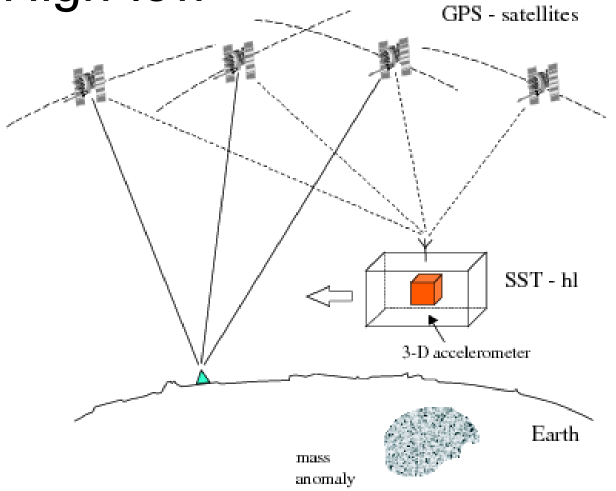
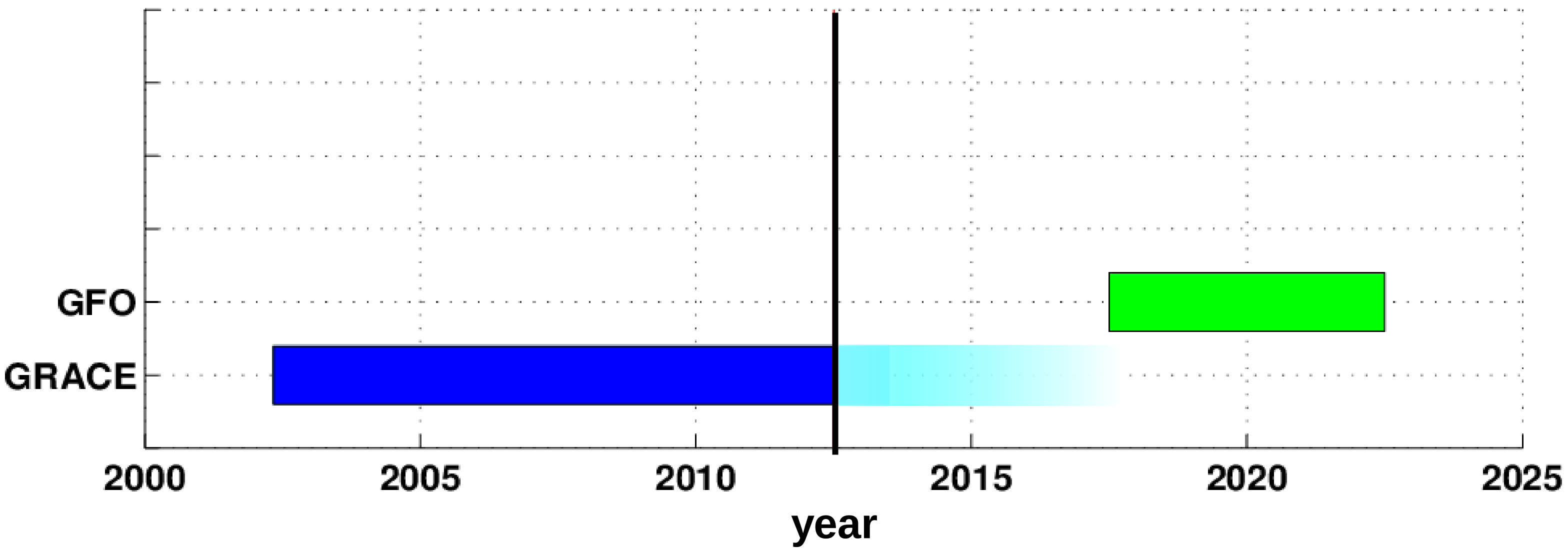


Bridging the gap …

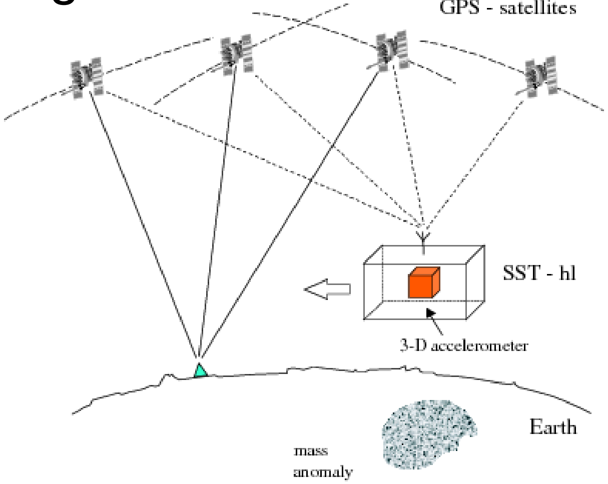
High-low

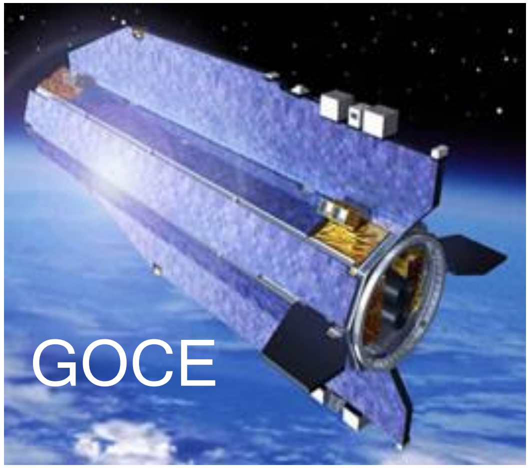




Bridging the gap …

High-low





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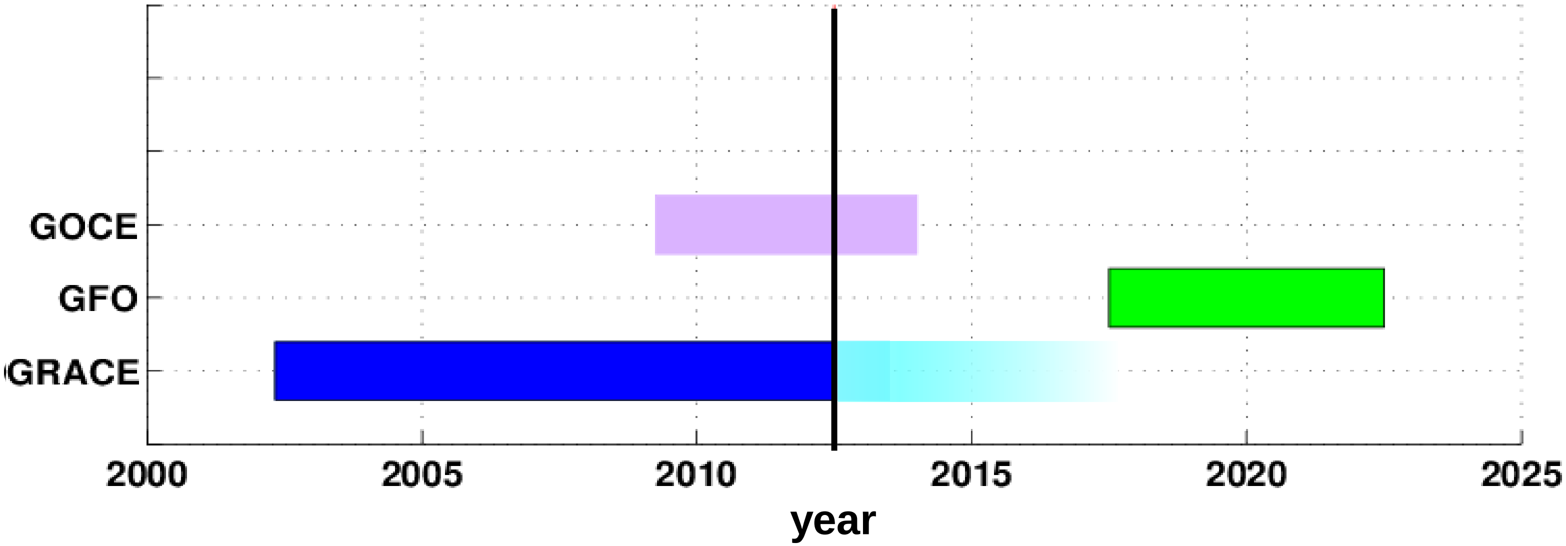
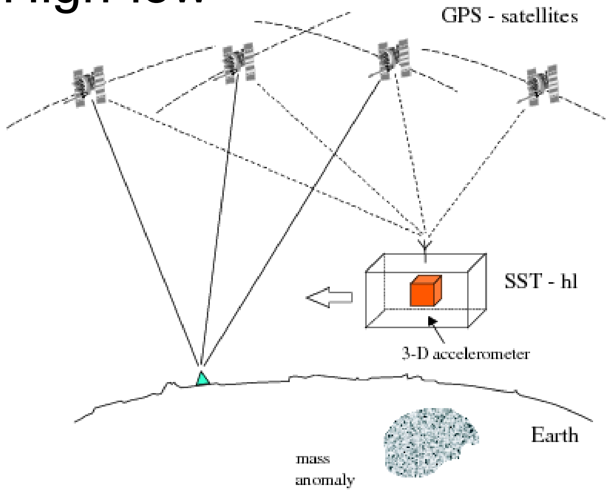
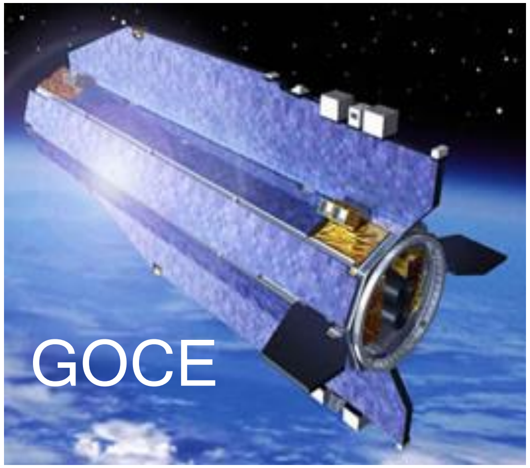


Bridging the gap …

High-low





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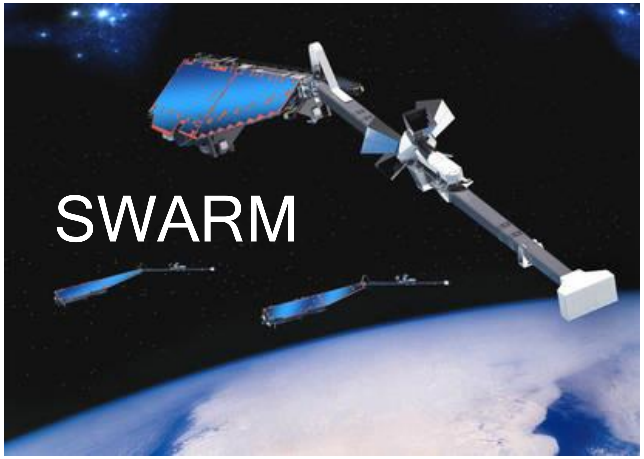

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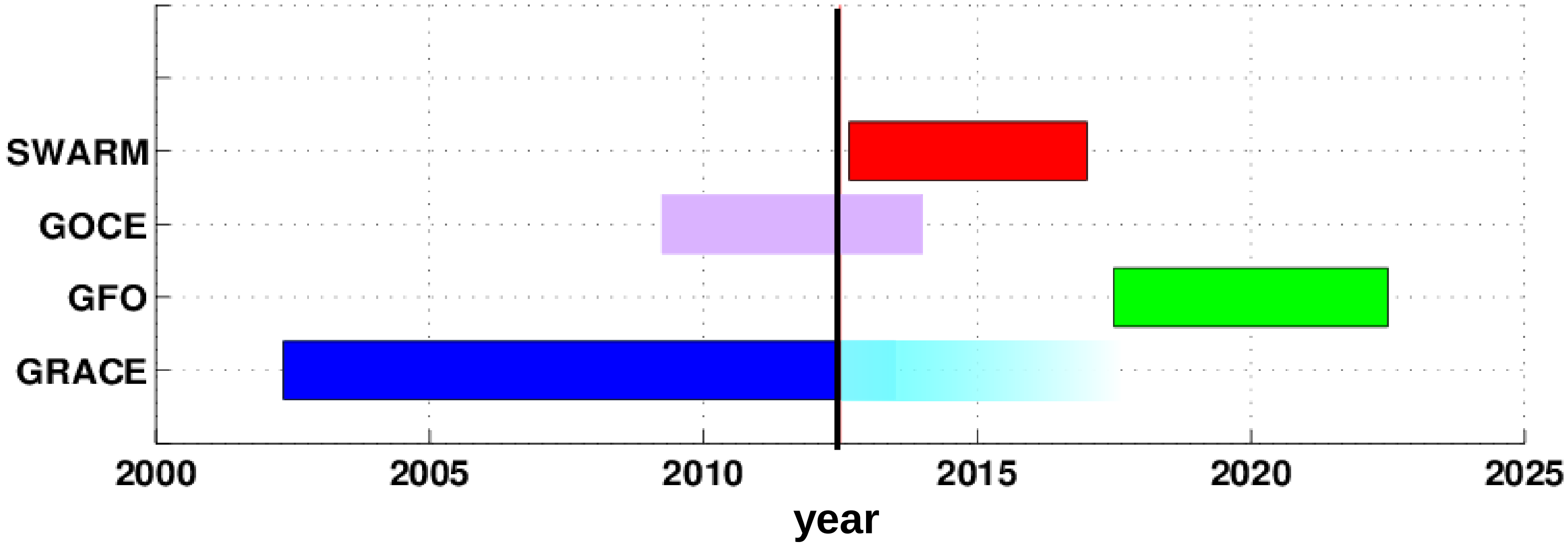

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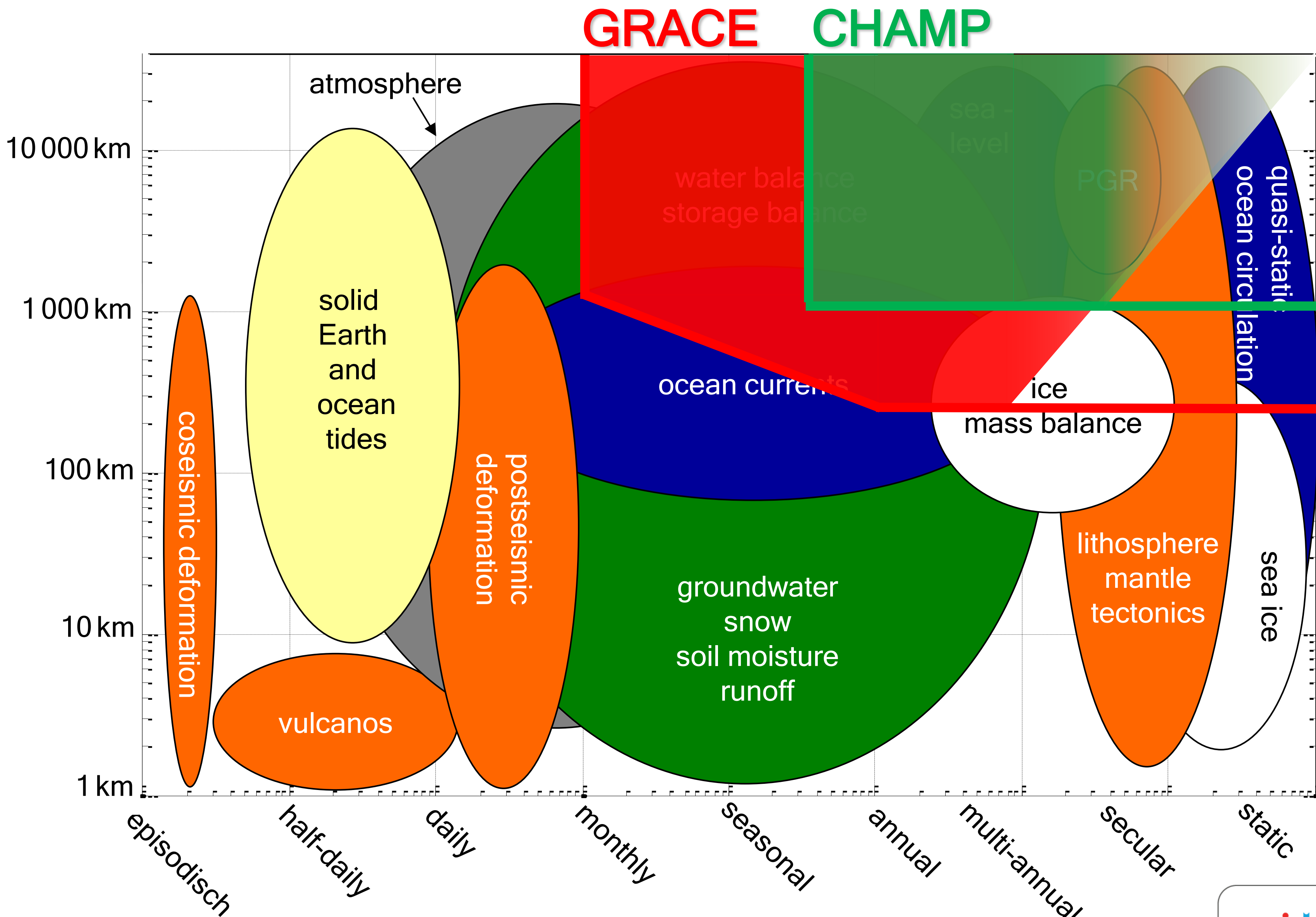
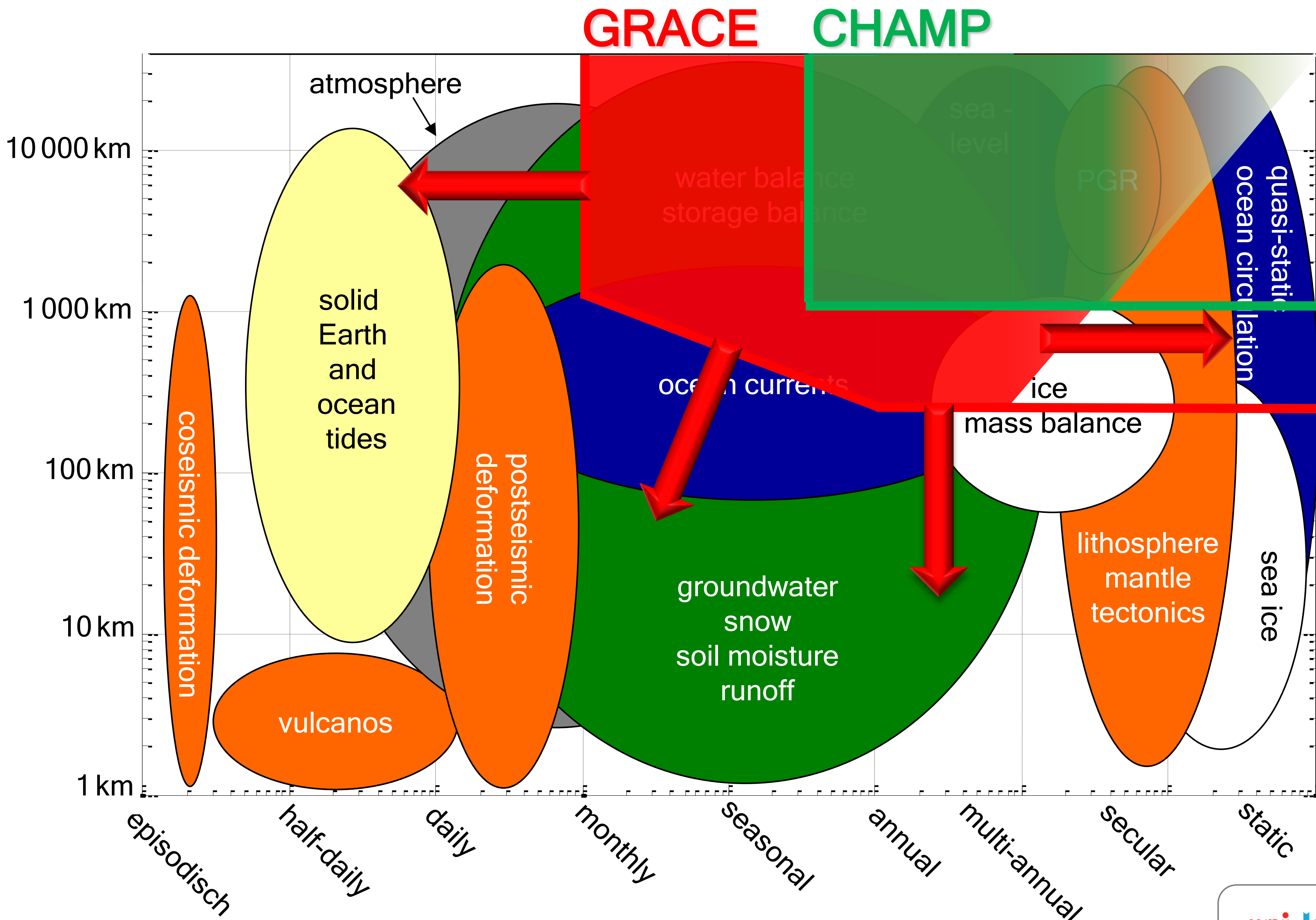
Beyond GRACE Follow-On



Beyond GRACE Follow-On



Summary

- *The decade of the potential yield an improvement in the gravity field by three orders of magnitude.*

- *Time variable gravity can be observed for the first time with a monthly sampling.*

- *Ongoing strive for higher spatial and temporal resolution (with a likely setback in the coming years).*

Thank you

Matthias Weigelt

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