# An enriched quadtree/octree implicit boundary finite element method for the simulation of incompressible hyperelastic materials

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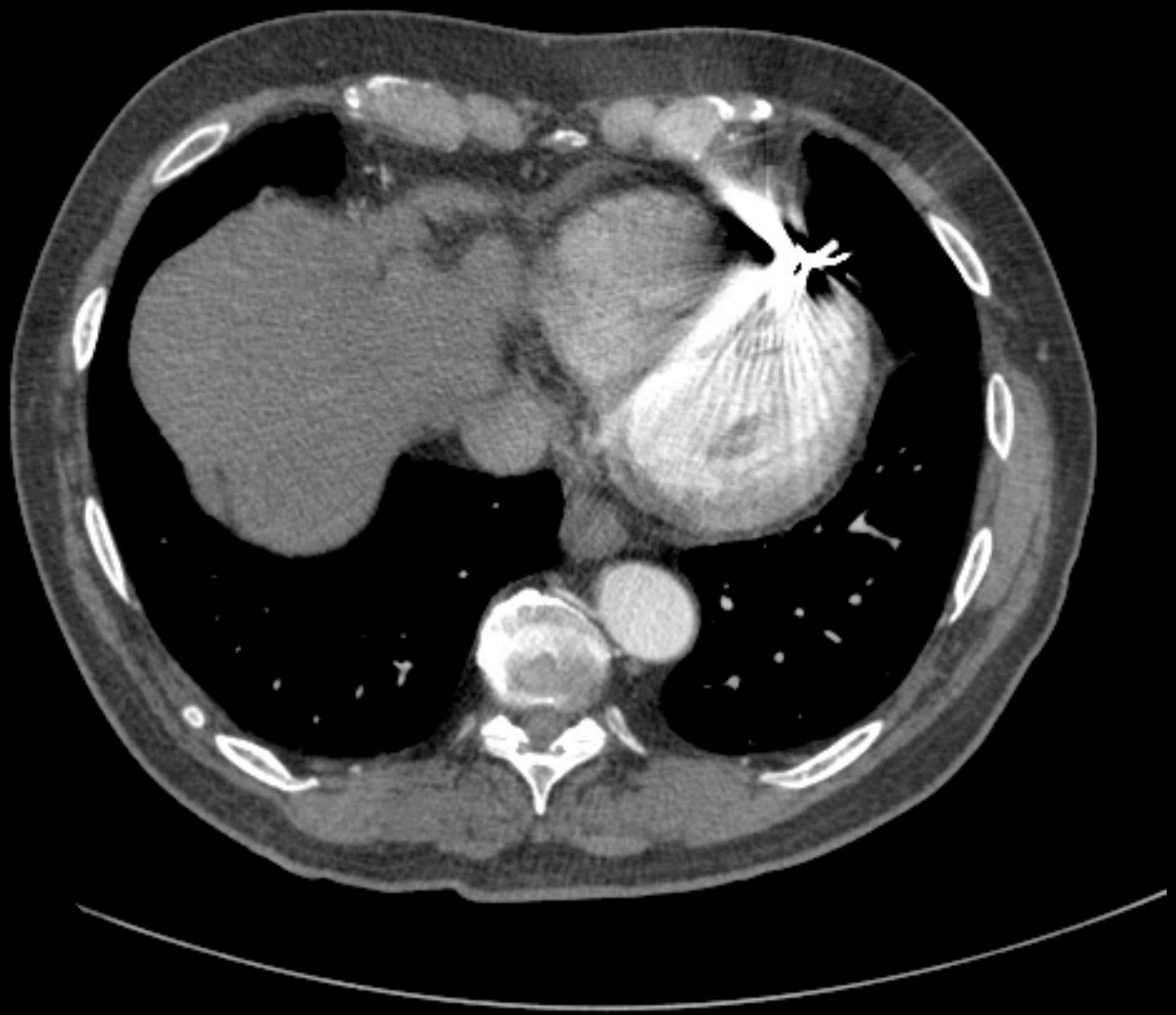
*How can we move from an image…*

*Source: COLONIX, OSIRIX*

…or perhaps a series of images…



Source: COLONIX, OSIRIX

*to a full mechanical analysis?*

Each voxel $j$ is a 32-bit floating point measurement

$z$

Planes (64)

$x$

Rows (64)

$y$

Cols (64)

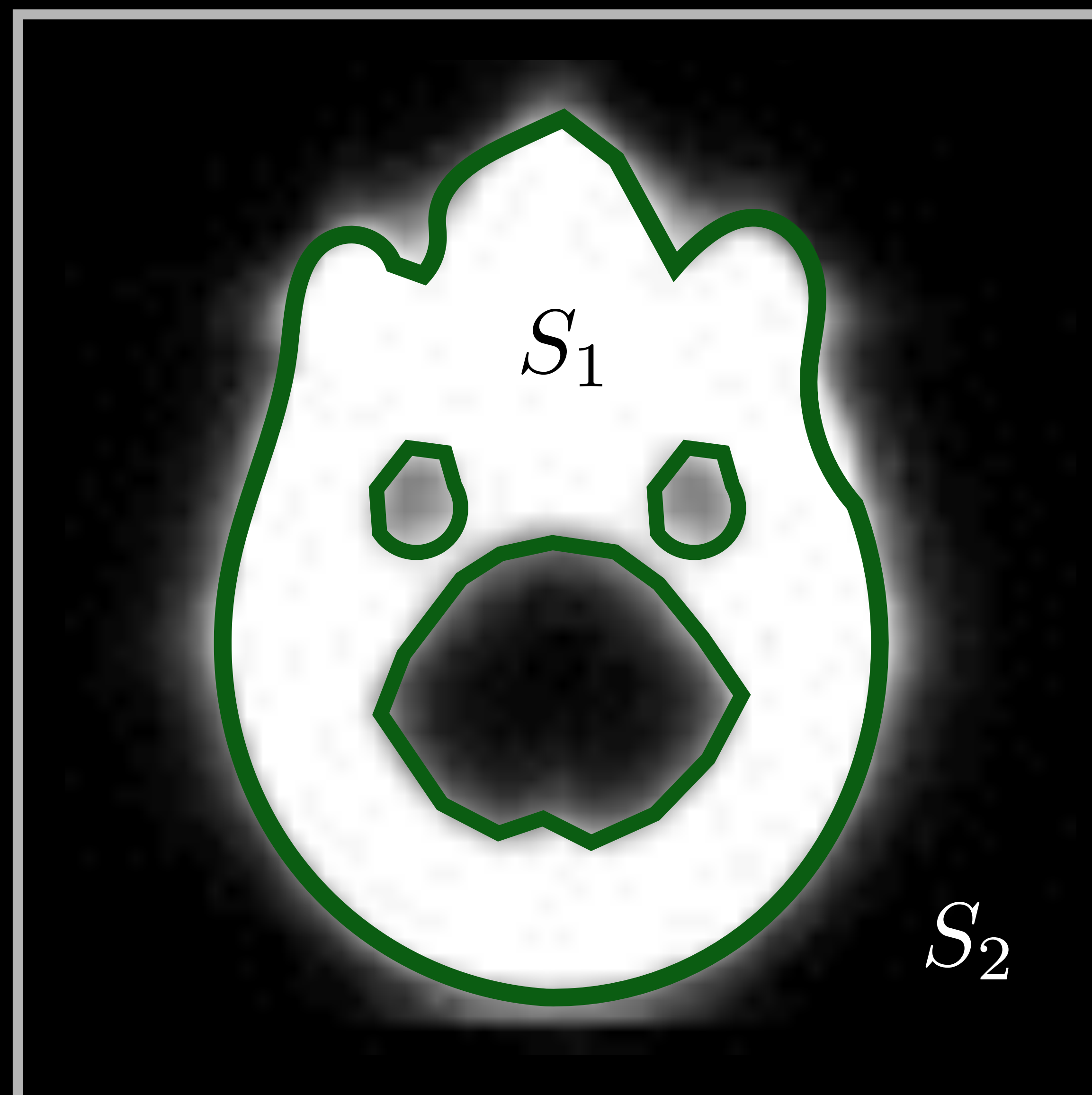
Image: CGAL Project

# Soft segmentation

$$0 < m_k(j) < 1 \quad \forall j,\, k$$

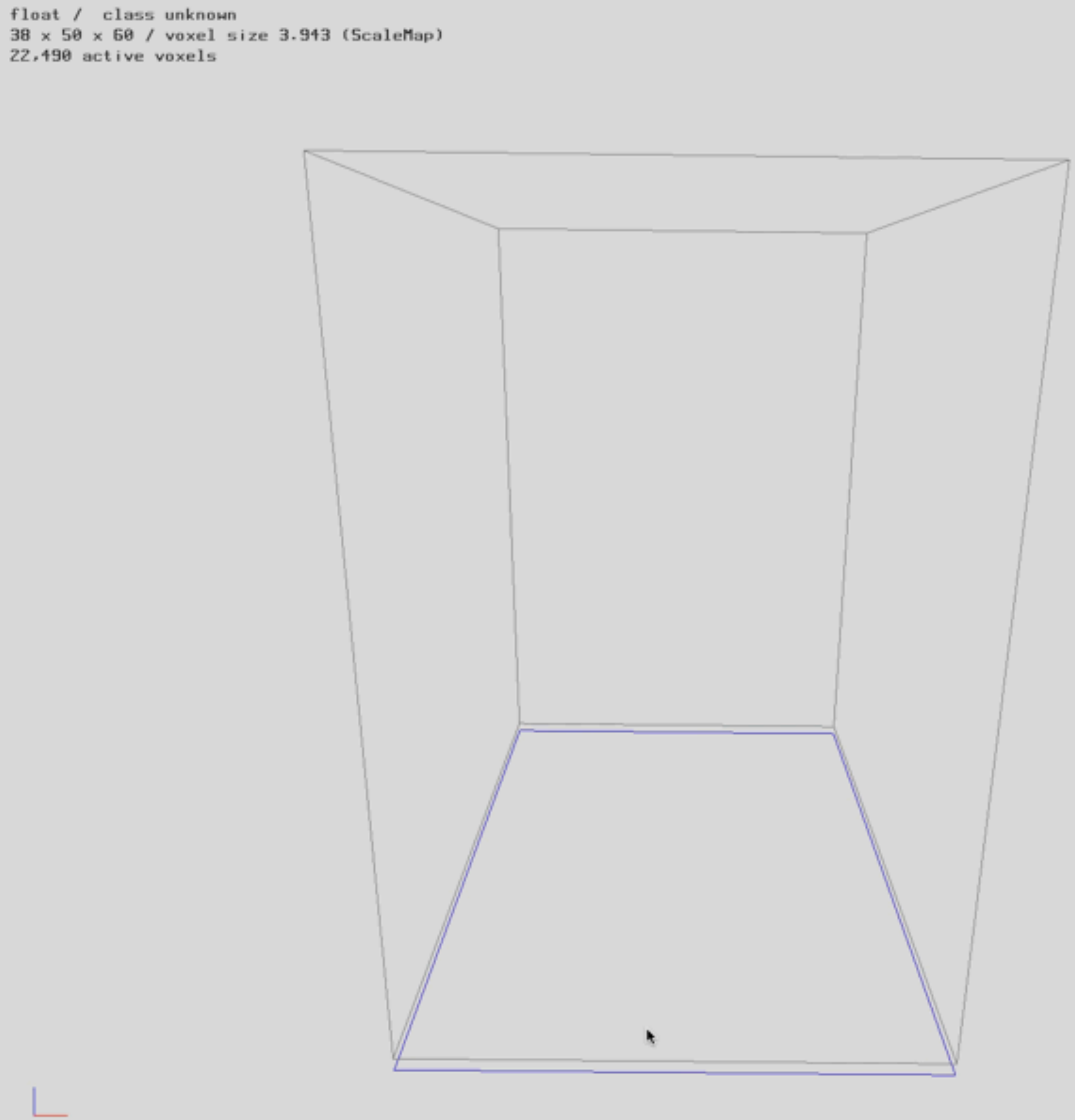
$$\sum_{k=1}^{K} m_k(j) = 1 \quad \forall j$$

# Hard segmentation

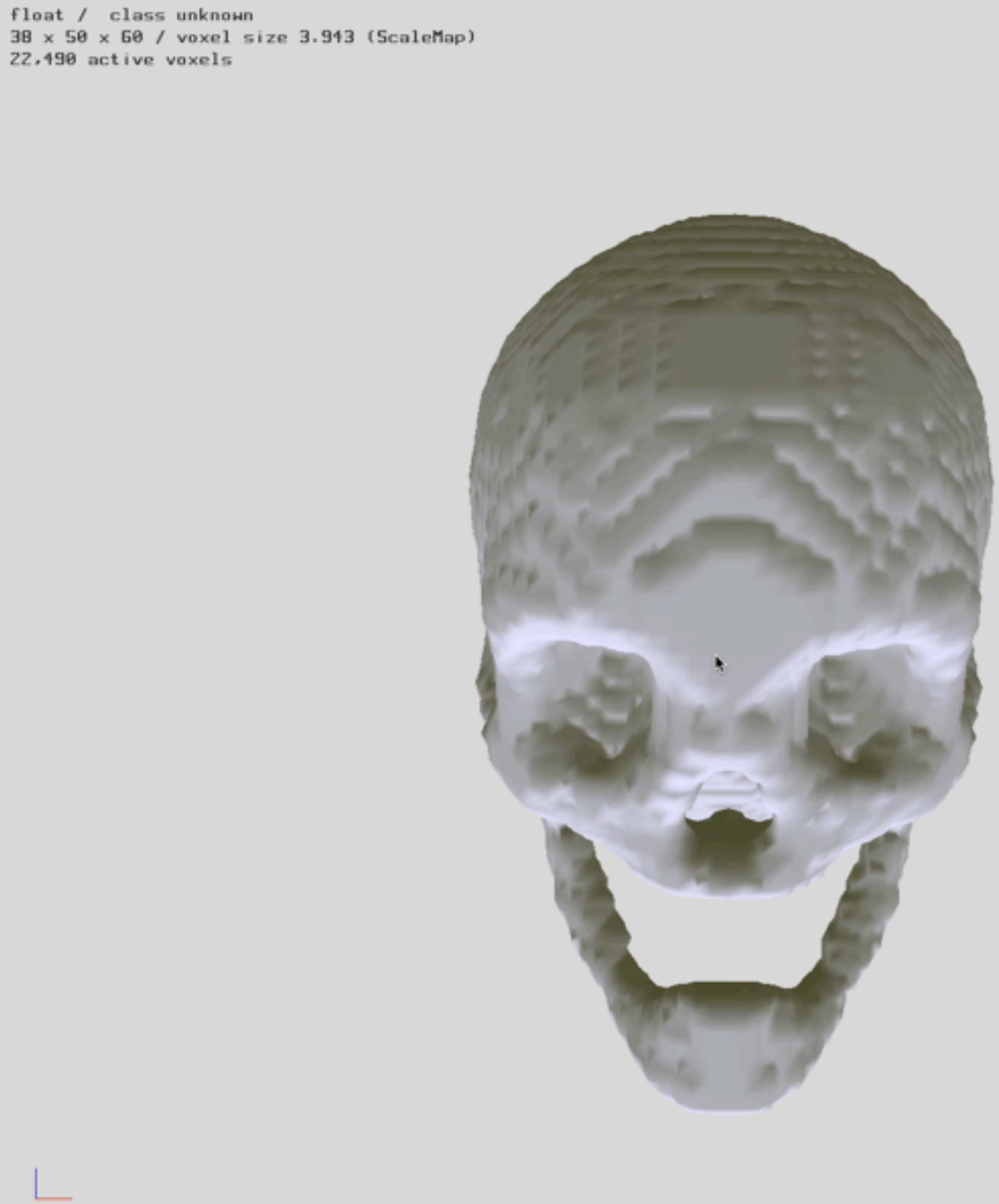



$$\Omega = \bigcup_{k=1}^{K} S_k \qquad S_k \cap S_j = \varnothing \quad \forall k \neq j$$

# Hard Segmentation at 0.2f



# Hard Segmentation at 0.2f with CGAL and OpenVDB



# The next step…

## Meshing

- Now possible with simplex or combination simplex/hex elements e.g. Simpleware, CGAL.
- For complex geometries not possible with all hex meshes.
- Automation. Ease of use in clinical environment.
- Can use existing finite element packages.

## Implicit Boundary/XFEM

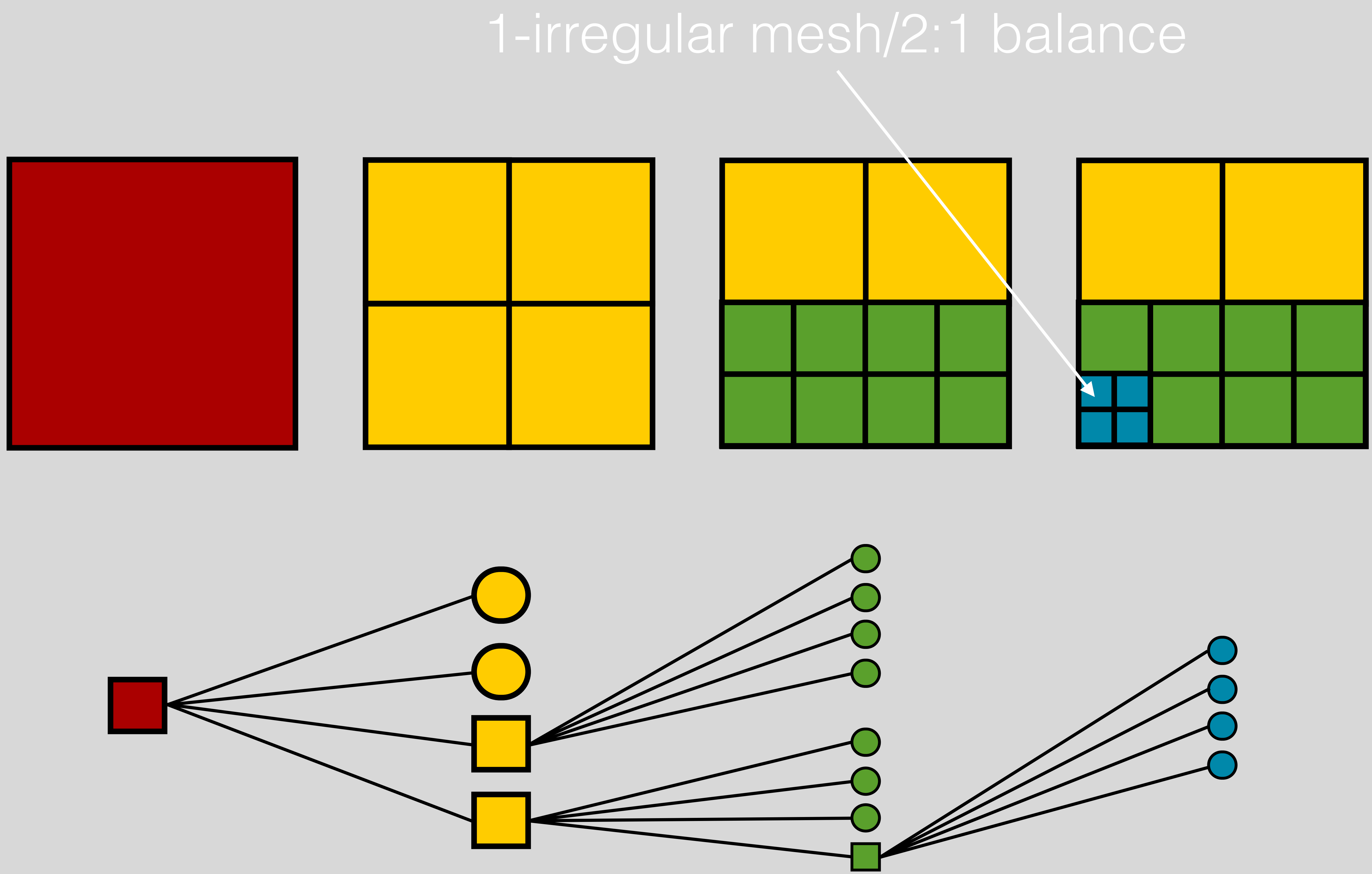
- Meshing simplified; can use regular Cartesian grid for discretisation.
- Uncoupling geometry and discretisation leads to additional flexibility.
- Potential for direct image to analysis transition.
- Largely requires development of new computational codes.

# Progress so far…

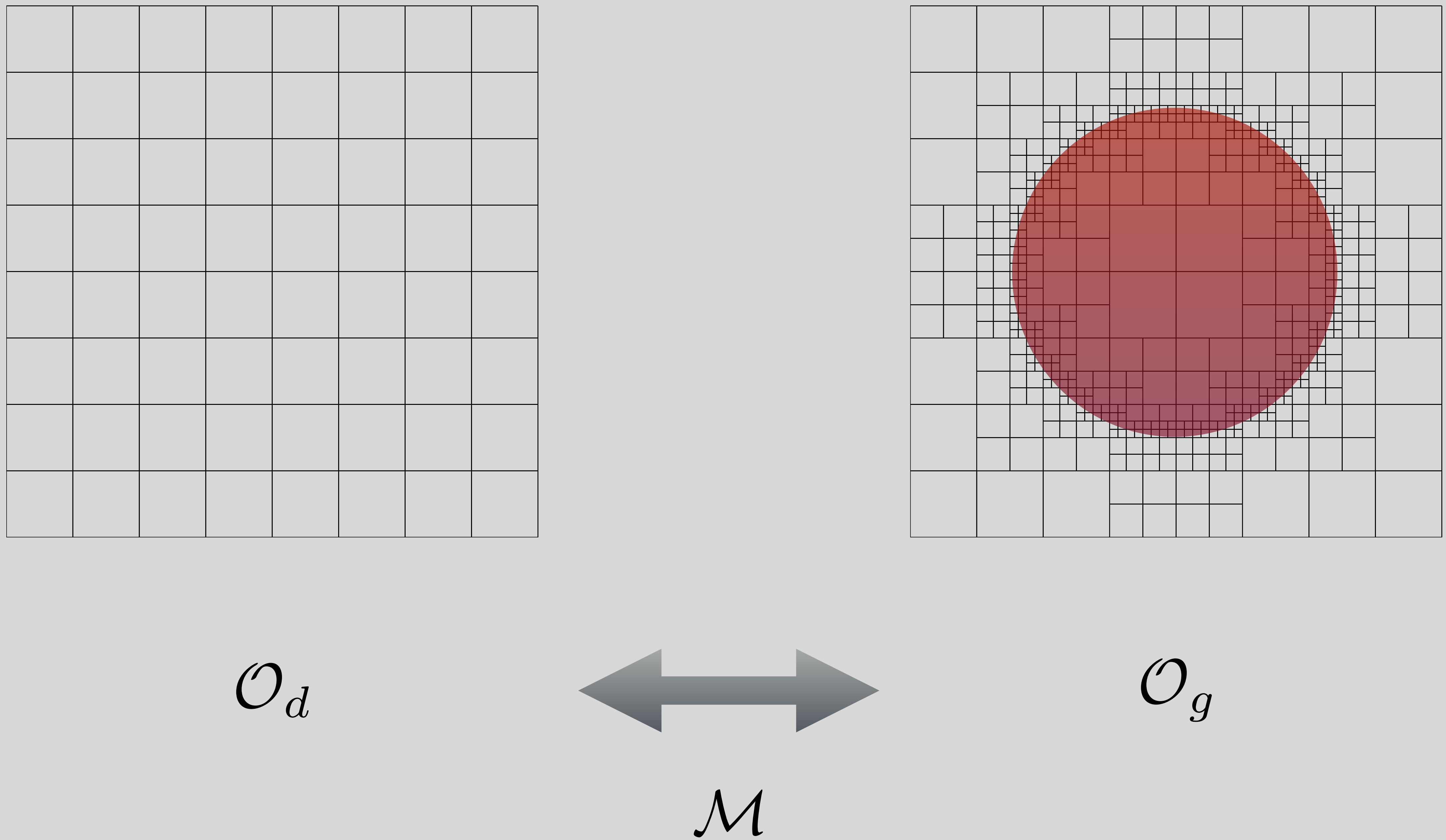
- We are developing an add-on toolkit for the deal.ii finite element package specifically designed to ease the development of cartesian grid partition of unity and implicit boundary methods.
- We will release the library under an open-source (probably GPL due to heavy use of CGAL) license once the API has stabilised.
- Lessons learned from in-house codes: OpenXFEM++ (Bordas) and UPC-Implicit (Rodenas Garcia).
- Key lesson: if someone has already coded it, don't code it yourself (CGAL, deal.ii, ITK).

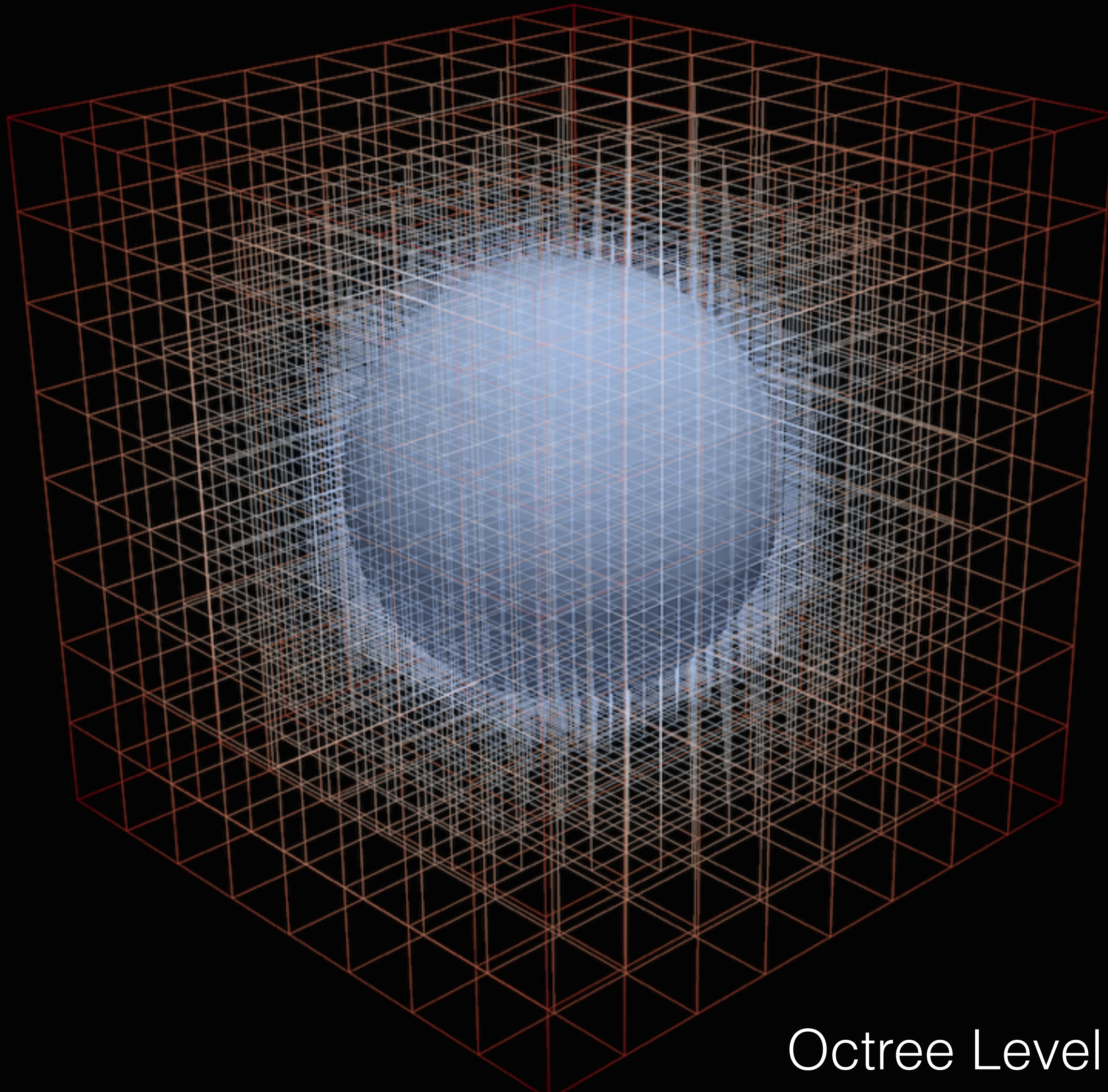
# Features

- 2D and 3D problems on the same code-path.
- parallel (MPI) assembly and solution.
    - currently each process must hold entire mesh.
        - looking at sparse octree data structures for optimal geometry representation.
    - hybrid parallel workflow (MPI and Intel TBB).
    - multigrid solvers.
- fast and robust computational geometry using CGAL.
    - automatic Delaunay tessellation of integration subdomains.
    - more efficient methods.
- completely separate representation of discretisation and geometry via nested octree data structures.
    - constructs to represent soft and hard segmentations of image data.
    - level-sets
        - level-set advection.
    - NURBS.
- independent hpe-type adaptivity on discretisation and geometry.
    - fast coarsening and refinement operations (hanging nodes).


1-irregular mesh/2:1 balance

# Nested Octree



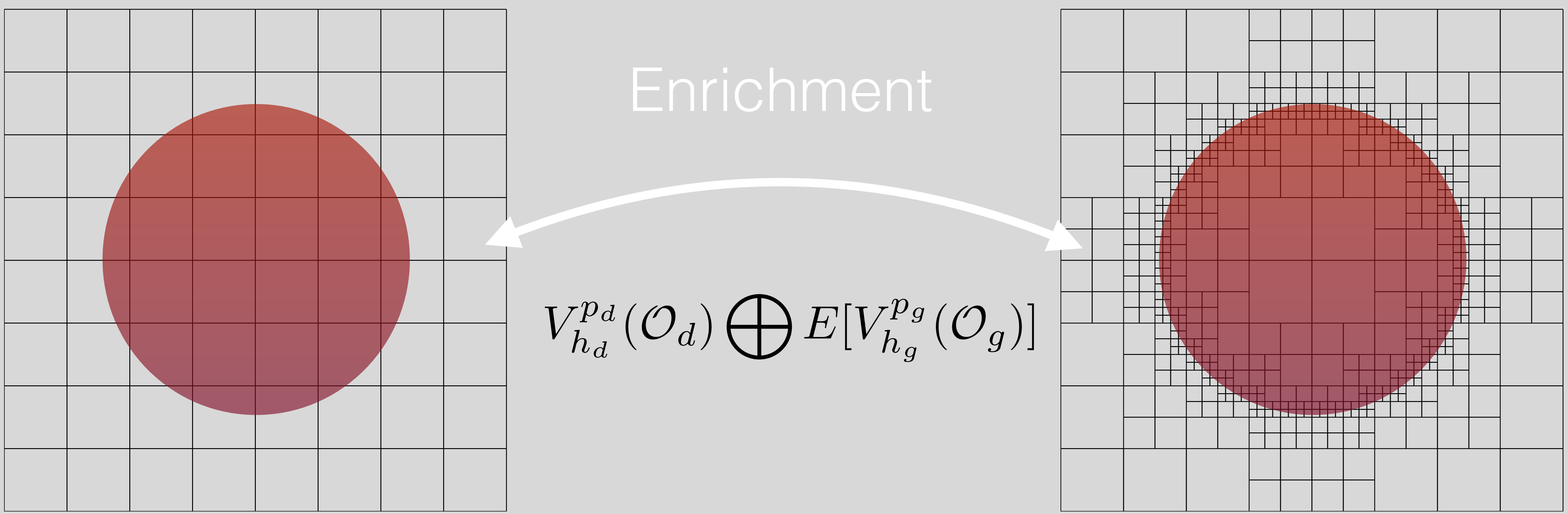


Octree Level 5/Level 3

*6a2e86c*

Surface

# How to transfer geometric information back to the discretisation?




$V_{h_d}^{p_d}(\mathcal{O}_d)$

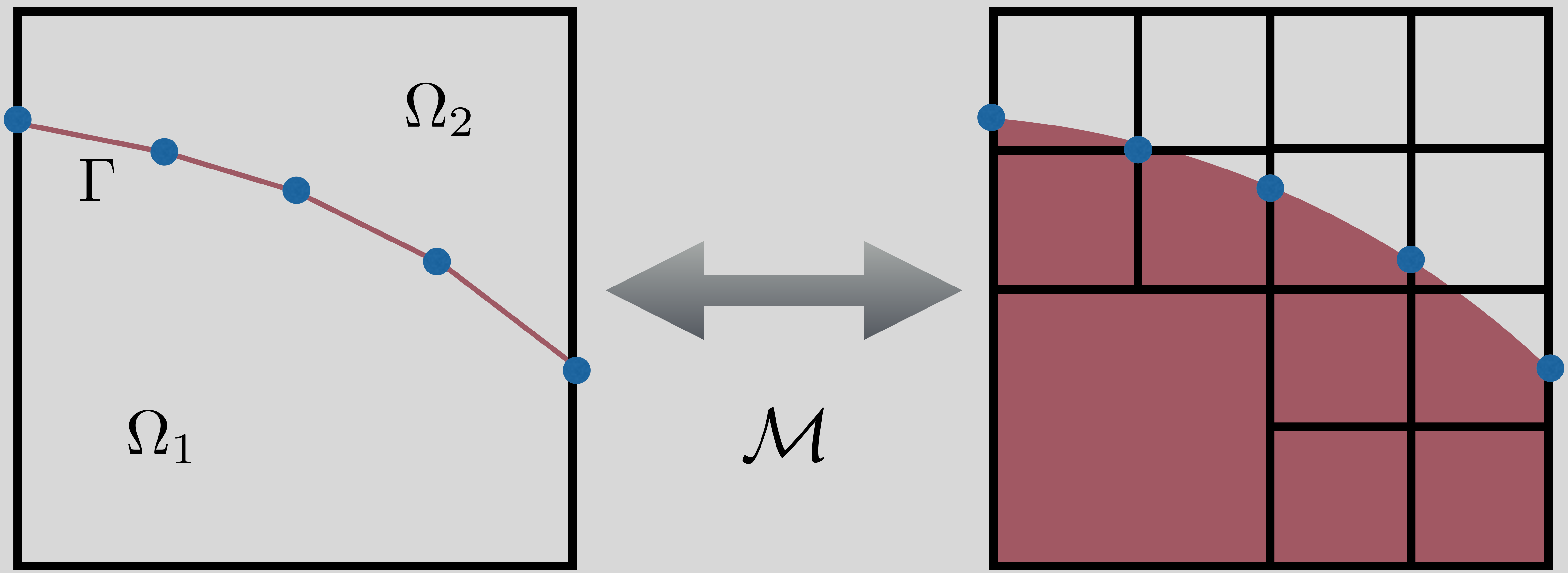
$p_d > p_g$

$h_d > h_g$

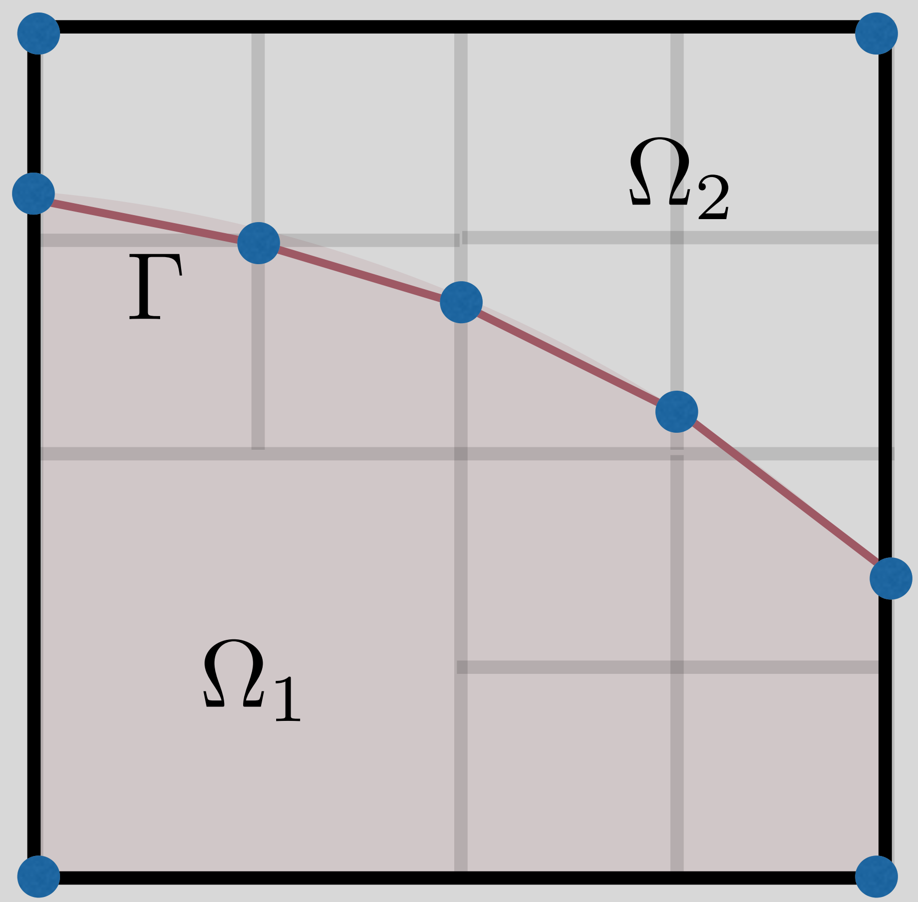
$V_{h_g}^{p_g}(\mathcal{O}_g)$

$p_g = 1$

# For each enriched cell in the discretisation…

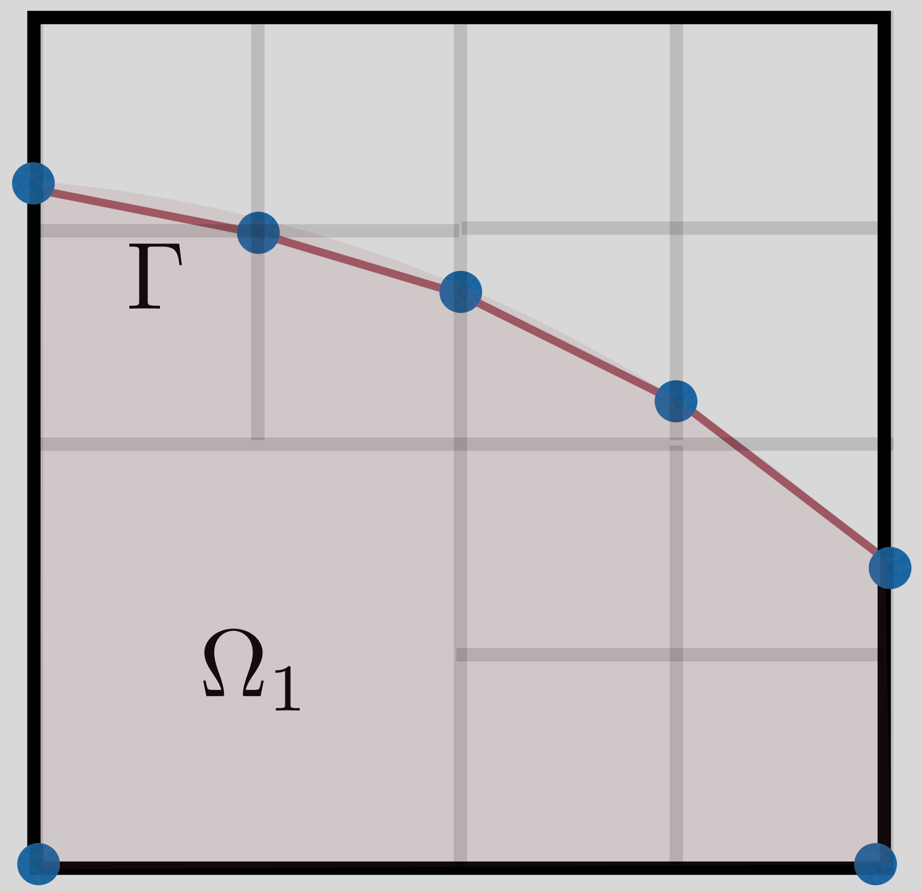


# generate local Delaunay triangulation…



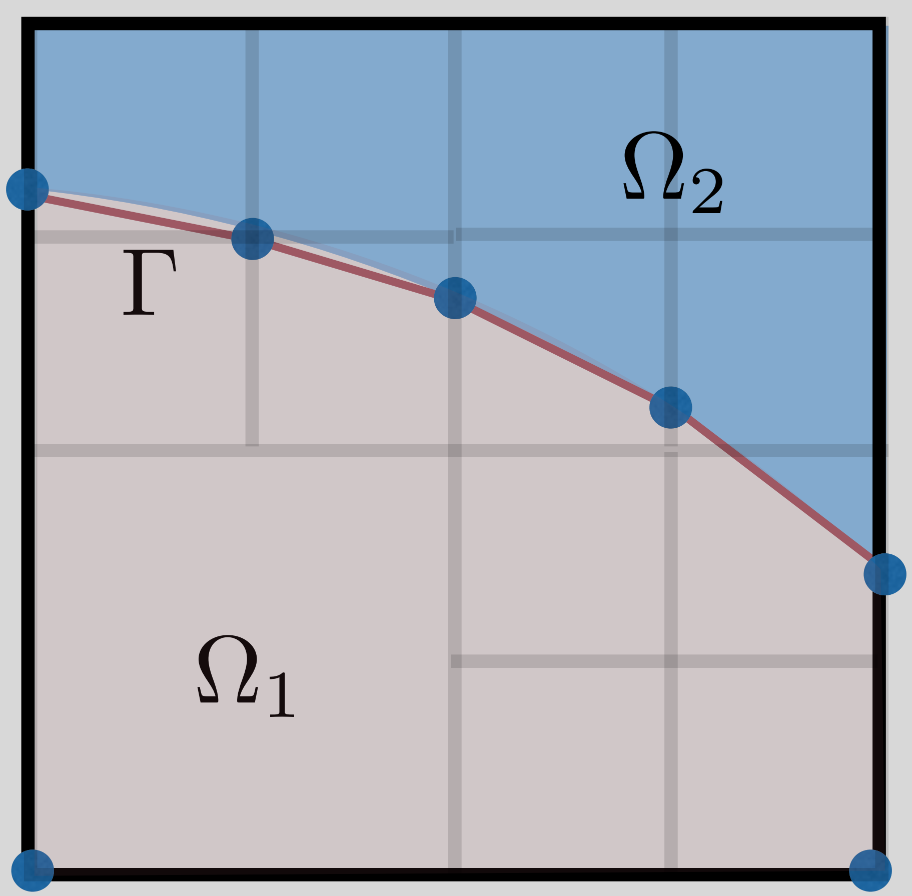
# Case 1: boundary

finite cell method, implicit boundary method…



# Case 2: inclusion

XFEM, PUM…




$$\mathbf{u}_h(\mathbf{x}) = \sum_{i=1}^{N} \mathbf{N}_i u_i + \sum_{i=1}^{N} \mathbf{N}_i \sum_{j=1}^{M} \psi_j(\mathbf{x}) a_i^j$$

# Outlook

- We are developing a cartesian grid implicit boundary/enriched finite element method toolkit within deal.ii.

- By uncoupling discretisation and geometry we hope to produce a method particularly suited to image-based analysis.

- Many challenges ahead, particularly with imposition of Dirichlet boundary conditions on hyperelastic soft-tissue model.

# Acknowledgements

Jack S. Hale would like to acknowledge the support of the Marie Curie COFUND scheme through the FNR, Luxembourg and the structural position under the chair of Prof. Stephane P. A. Bordas at the University of Luxembourg.

Stephane P. A. Bordas would like to acknowledge partial support for his time from European Research Council Starting Independent Research Grant (ERC Stg grant agreement No. 279578) entitled `Towards real time multiscale simulation of cutting in non-linear materials with applications to surgical simulation and computer guided surgery'.